Introduction

Mercury is the least explored planet in the inner solar system. Compared to the other planets, it is difficult to reach: even more energy is needed than sending a mission to the outer solar system. This is due to the fact that, along its journey towards the Sun, a spacecraft needs to decelerate, because the solar gravitational force increases with the square of the distance. On the other hand, since Mercury is difficult to observe from the Earth, due to its close proximity to the Sun, for an in-depth study of the planet and its environment it is necessary to operate a spacecraft around the planet.

The investigation of Mercury has particular importance for various aspects: being the planet the closest to the Sun, it has a privileged position for the analysis of the gravitational effects of a massive body like the Sun. Furthermore, being Mercury a now-quiescent body, it preserves evidence of the early history of the terrestrial planets, thus, its exploration can allow to understand their origin.

After the NASA's Mariner 10 mission, which, from November 1973 to March 1975, provided the first detailed data and close-up images of Mercury, the planet remained unexplored for the following decades, until the launch of the NASA Discovery class mission MESSENGER in August 2004. MESSENGER orbited Mercury from 2011 until 2015, studying the planet's chemical composition, geology, and magnetic field.

The next scheduled mission for the exploration of Mercury is the joint ESA/JAXA mission BepiColombo, executed under the ESA leadership and part of the Horizon 2000+ program. It was launched in October 2018 and the arrival at Mercury is foreseen for December 2025, after 7 years of cruise. The mission aims to study the composition, geophysics, atmosphere, magnetosphere and origin of the planet. The Mercury Orbiter Radio science Experiment (MORE) is one of the experiments on-board BepiColombo, whose scientific goals concern both the geodesy and geophysics of Mercury and some tests of fundamental physics. In particular, taking advantage from the fact that Mercury is the best-placed planet to investigate the gravitational effects of the Sun, MORE will allow an accurate test of relativistic theories of gravitation. Such test consists in processing the radio observations (range, range rate) between the MORE on-board transponder and the on-ground antennas in order to obtain an accurate estimate of some Parameterized Post-Newtonian (PPN) parameters, whose knowledge can allow to discern the validity of the theory of General Relativity from alternative theories of gravity.

The challenging scientific goals of MORE can be fulfilled only by means of the combination of very accurate tracking, which is possible thanks to state-of-the-art on-board and on-ground instrumentation, and precise orbit determination of the spacecraft and of both Mercury and the Earth. The Celestial Mechanics Group of the University of Pisa is in charge of the data analysis of the MORE experiment. The parameters estimation is performed together with the orbit determination, by means of an iterative procedure based on a classical non-linear least squares fit (LS), implemented in a dedicated software, ORBIT14, developed at the Department of Mathematics of the University of Pisa.

In this Thesis we aim to investigate a critical issue of the MORE relativity experiment: the presence of symmetries affecting the outcome of the experiment. Generally speaking, symmetries in an orbit determination problem lead to rank deficiencies in the normal matrix of the LS fit, which can result in a significant degradation of the solution. Thus, it is especially important to understand whether we are in the presence of symmetries and develop suitable methods to handle them. The investigation is undertaken by means of both theoretical considerations and numerical simulations, using the ORBIT14 software.

Until now, a detailed study of the symmetries of the BepiColombo-MORE experiment has never been performed: the existence of certain symmetries was assumed as a hypothesis and simulations were carried out adopting suitable strategies to handle these symmetries. In this Thesis we start an extensive analysis to investigate the actual presence of rank deficiencies.

When we consider the complex scenario of the MORE experiment, the spectral analysis of the normal matrix leads to results that are difficult to interpret. As a consequence, we proceed in abstract terms, by progressive approximation, studying test cases. We start analyzing a set-up in which the presence of an exact symmetry is guaranteed by theory. Then, we add to this basic scenario a list of perturbations and we study separately their influence on the symmetry. More in detail, the basic scenario consists in a 3-body problem between Mercury, the Sun and the Earth Moon Barycenter (EMB) in addition to a 2-body problem accounting for the motion of the spacecraft around Mercury. As for the perturbations, we consider the influence of the fact that some state vectors (that of the Moon and the other planets, as well as that of the on-ground antenna) are not directly computed by the ORBIT14 software, solving the related equation of motion, but, rather, they are provided by the Jet Propulsion Laboratory ephemerides or the IERS (International Earth Rotation and Reference Systems Service) orientation data.

This Thesis is structured as follows:

Chapter 1. After having recalled the fundamental principles of General Relativity, we introduce the PPN formalism. Then, we briefly review the main experimental tests that have been performed in the past in order to constrain the value of the PPN parameters. We conclude with a discussion on possible future improvements that can be achieved by exploiting the data provided by the ongoing space missions.

Chapter 2. We describe the MORE experiment, with a special focus on the relativity tests. We analyze the Mercury-centric dynamics of the spacecraft and the heliocentric dynamics of Mercury and the EMB, describing all the effects that come into play at the required level of accuracy. In the final part, we explain how to compute the range and range rate observables, taking into account the relativistic effects.

Chapter 3. We review the theoretical aspects of orbit determination and how to apply them to the MORE experiment, as well as how to interpret the results in terms of uncertainties in the determination of the desired parameters. We briefly explain the implementation of the orbit determination and parameter estimation in the ORBIT14 software. We perform a first simulation and we analyze the results, suggesting the presence of an approximate rotational symmetry.

Chapter 4. After recalling the theoretical concepts related to symmetries, we discuss the effectiveness of two strategies for curing the rank deficiency: the descoping strategy and the use of constraints to inhibit the symmetry. In particular, the second part of the Chapter contains the original work and a discussion of the obtained results.

This Thesis ends with some indications on possible issues to analyze in future work for a better understanding of the phenomena we studied.