Introduction

Space debris have been generated in the last 60 years as the by-product of the exploration and exploitation of the circumterrestrial space. Due to the dramatic growth of the number of debris in space, the active intervention by the scientific community has become necessary. Indeed, an uncontrolled increase of debris in the circumterrestrial space can provoke both environmental damages and possible risks of collision for active missions. In 2007, the United Nations General Assembly has developed a set of guidelines to limit the amount of debris generated by space launches and to minimize the possibility of later fragmentations.

In this context, the *Revolutionary Design of Spacecraft through Holistic Integration of Future Technologies* project (ReDSHIFT) was funded by the European Union. The main idea is to approach the debris mitigation issues from different perspectives, prone to suggest innovative solutions to this pressing problem, having always in mind, beyond the specific mission requirements, the minimization of the environmental impact of the spacecraft.

The first step toward this direction is to understand if the dynamical perturbations in the circumterrestrial space can drive the spacecraft toward a natural re-entry. Indeed, perturbations acting on the spacecraft can induce periodical variations in the orbital eccentricity and inclination, which can potentially be exploited. Passive solutions of this kind would be preferred if they result to be affordable for the space operators and not risky for the space environment.

In the present work, we will focus on the Low Earth Orbit (LEO) region, which constitutes the most densely populated orbital environment, and on the dynamical effects caused by three different perturbations: the high-degree zonal harmonics of the geopotential, the lunisolar gravitational attraction, and the solar radiation pressure (SRP). At specific values of inclination, these perturbations can foster the orbital decay inducing a quick increase of the eccentricity, if a well-defined resonance condition is satisfied.

Following the work of [37, 39], we make a deep analysis of the role of the resonances which act on the spacecraft dynamics in LEO. By means of a numerical computation of the Fourier transform, we characterize the evolution of the eccentricity of a dense set of orbits in terms of the main spectral components. In particular, we choose a suitable set of initial conditions for quasi-circular orbits spanning the LEO region for different values of semi-major axis a, from $a = R_\oplus + 800$ km up to $a = R_\oplus + 1600$ km, where $R_\oplus = 6378.1363$ km is the radius of the Earth, and inclination i, from $i = 0°$ up to $i = 90°$. Moreover, we selected two different values of the area-to-mass ratio of the object, in order to understand how natural perturbations, in particular SRP, can influence the orbit dynamics, depending on the A/m value.

The aim of the spectral analysis described in this thesis is to clearly link each frequency signature found in the eccentricity spectrum to the dynamical effect which generates it, in order to build a frequency cartography of the LEO region. Indeed, the detailed analysis of the principal spectral components turns out to be a powerful tool to enable a better understanding of the relative importance of each gravitational and non-gravitational perturbation in the LEO region as a function of the initial semi-major axis, eccentricity and inclination of the object. In particular, the amplitude of the spectral signature produced by a perturbation on a given orbit gives an estimate of the corresponding eccentricity variation; this quantity can be compared with the numerical results in the time domain and the analytical expressions provided by theory, in order to give a comprehensive and more robust picture of the eccentricity evolution. Together with the dynamical maps in the time domain, performed in the framework of ReDSHIFT (see, e.g., [1, 2]), our analysis can be exploited to identify the orbits where a significant growth of eccentricity, led by one or more perturbations, can assist the passive disposal of objects at their end-of-life or to optimize the design of low-cost manoeuvres, aimed at re-entering. Indeed, for some orbits, the frequency cartography is capable to disclose effects which do not show up as clearly in the temporal maps.

This thesis is organized as follows: in Chapter 1 we briefly present the issue of space debris and the recommended mitigation strategies, focusing, in particular, on the aims of the ReDSHIFT project. In Chapter 2 we describe the dynamical model adopted for the numerical propagation in the time domain. We recall the analytical developments of the natural perturbations acting on objects in LEO, and, by means of the Lagrange planetary equations, we describe the main effects on the orbital elements due to each perturbation. Furthermore, after having defined what we mean by resonant condition, we analyse different kinds of resonances, due to one or more perturbations. In particular, we focus on their effect on the eccentricity evolution, in order to compare, later in Chapter 4, our results with the analytical findings. In Chapter 3 we extensively explain the simulation scenario and the mathematical tools exploited in our study. We describe the Fourier transform and the method we used to identify the main frequency signatures, which characterize the eccentricity evolution of the orbits. Finally, in Chapter 4, we outline the numerical results of our analysis. This will be done, in turn, by describing some exemplificative cases. We provide a full description of the frequency charts, obtained with our method, comparing the results in the frequency domain with the findings in the time domain. Finally, we draw some conclusions on possible practical applications of the frequency cartography.