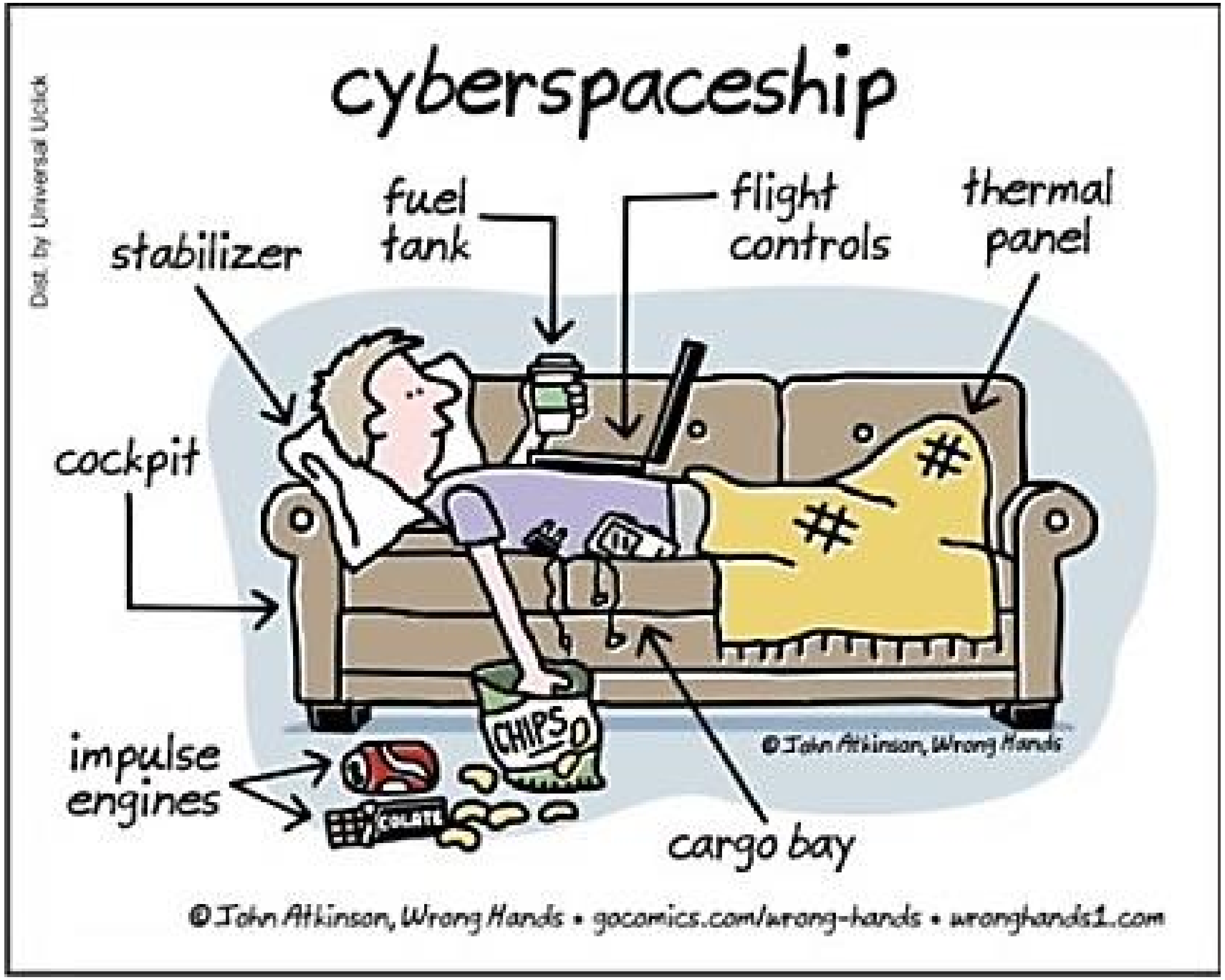

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cockpit
CHIPS
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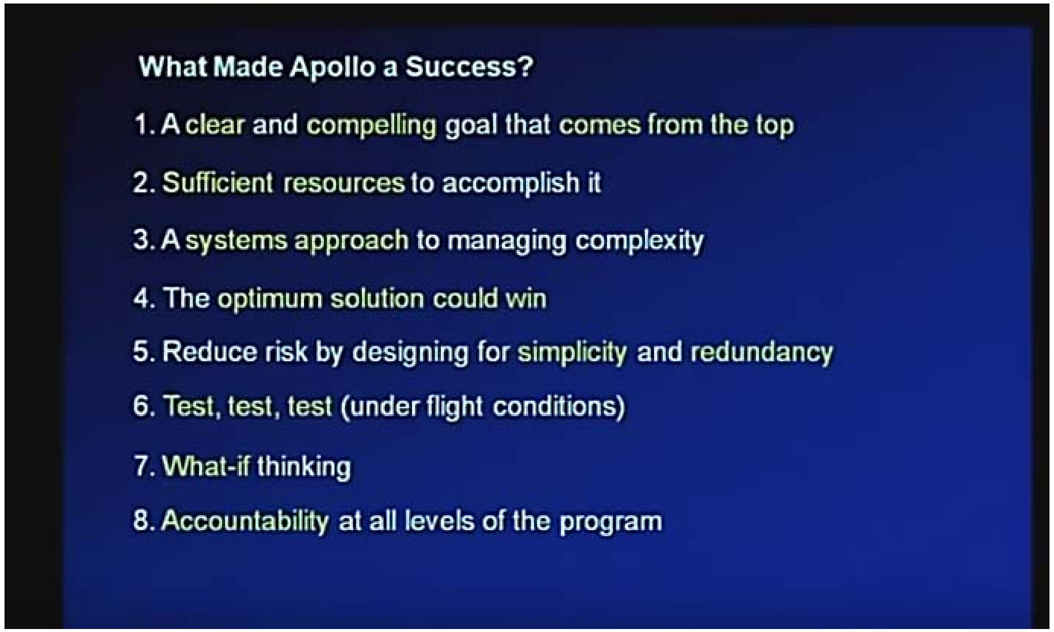
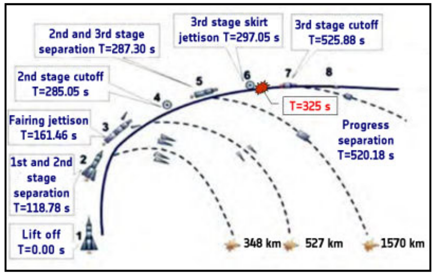
Orbit Transfer – Orbital Maneuvers

- **Two – Body Problem:** Relative forces acting on a spacecraft (orbiting body), geometry of orbits around the main body.
- **Kepler's Equation:** Relation between position at different instants and time intervals spanned by the motion.
- **Orbital Elements:** Definition of reference systems; relationship between orbital elements and position and velocity vectors in space (Ephemerides).
  - i = Inclination of the orbit
  - Ω = Right Ascension of ascending node
  - ω = Argument of periapsis
  - a = Semi major axis (or semilatus rectum)
  - e = Eccentricity of the orbit
  - f = True anomaly (or time to periapsis)
- **Lagrange Coefficients:** Determination of position and velocity at some specific time, from position and velocity at a given time (along the same orbit).
- **Time and Distance:** Measurement of time and distance computation.

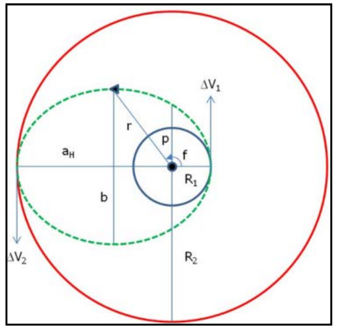
https://www.youtube.com/watch?v=B1J2RMorJXM&t=3641s



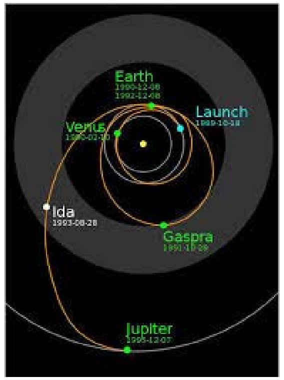
Orbit Transfer – Orbital Maneuvers



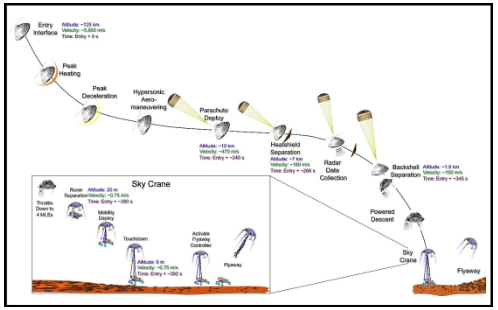
- **Launch Trajectory:** Path followed from ground to suborbital flight, orbit insertion and escape trajectory



- **Orbit Transfer:** Change of orbit (for example from parking orbit to target orbit)



- **Interplanetary Flight:** Escape trajectory, acquisition trajectory, multiple changes of orbit

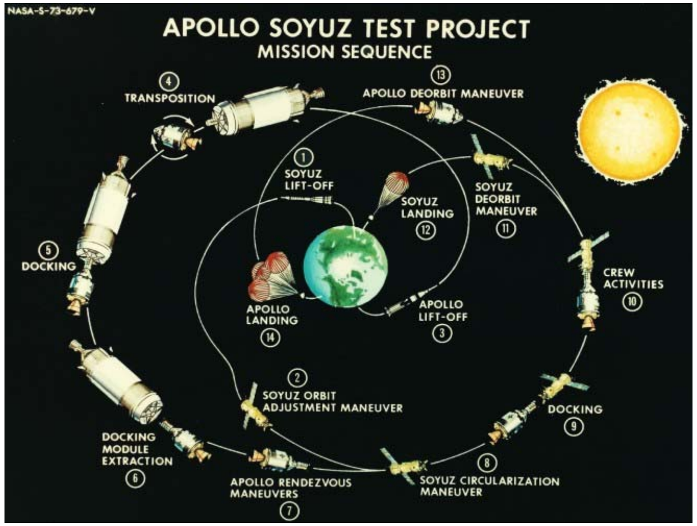


- **Descent Trajectory:** Departing parking orbit, soft landing, direct impact

  - **Proximity Operations:** Relative motion between spacecraft, docking

Orbit Transfer – Orbital Maneuvers

- **Apollo – Soyuz Rendez Vous:** Performed in 1975



What happened in 1975?

- Viene fondata la Microsoft
- Finisce la guerra nel Vietnam
- Marco Pannella viene arrestato per aver fumato hashish durante una conferenza stampa
- Margaret Thatcher diventa primo ministro
- Muoiono Francisco Franco e P.P. Pasolini
- 1 Caffè 150 lire (< 10 eurocents)
- 1 Litro di benzina 305 Lire
- Si diventa maggiorenni a 18 anni
- Wish you were here dei Pink Floyd
- Il Juventus vince il campionato …

Anni di Piombo

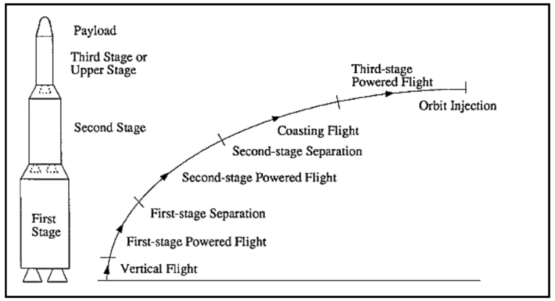
- **24 gennaio**: Empoli, a seguito di una perquisizione alla sede del Fronte Nazionale Rivoluzionario, gruppo armato di estrema destra, il terrorista Mario Tuti uccide i due carabinieri
- **18 febbraio**: Un commando delle Brigate Rosse guidato da Mara Cagol fa evadere Renato Curcio
- **13 marzo**: a Milano lo studente liceale Sergio Ramelli, militante del Fronte della Gioventù, viene aggredito sotto casa a colpi di chiave inglese da militanti di Avanguardia Operaia
- **16 aprile**: Milano, a conclusione di una manifestazione il diciassettenne Claudio Varalli, militante del gruppo di sinistra Movimento Lavoratori per il Socialismo, viene ucciso a colpi di pistola dal neofascista di Avanguardia Nazionale Antonio Braggion.
- **17 aprile**: Milano, durante la manifestazione di protesta per l'omicidio Varalli scoppiano degli scontri tra le forze dell'ordine e i dimostranti: un giovane di ventisei anni, Giannino Zibecchi, militante del Coordinamento dei Comitati Antifascisti, muore travolto da un camion dei Carabinieri.
- **6 maggio**: Roma, viene rapito da una formazione dei NAP, il magistrato Giuseppe Di Gennaro
- **25 maggio**: Milano, alle 22:30 in Via Mascagni a Milano lo studente universitario/lavoratore Alberto Brasili è ucciso a coltellate e la fidanzata Lucia Corna ferita. Saranno arrestati cinque fascisti: Antonio Bega, Pietro Croce, Giorgio Nicolosi, Enrico Caruso e Giovanni Sciabicco.
- **30 maggio**: Aversa, Giovanni Tara, membro dei NAP, muore accidentalmente a causa dell'ordigno che tentava di piazzare sul tetto del manicomio.
- **13 giugno**: Reggio Emilia, è assassinato Alceste Campanile, attivista di Lotta Continua.
- **29 ottobre**: Roma, Mario Zicchieri, membro del Fronte della Gioventù, è ucciso presso la sede MSI del quartiere Prenestino.
- **22 novembre**: Roma, negli scontri tra manifestanti e forze dell'ordine durante un corteo a favore dell'indipendenza dell'Angola, Piero Bruno, diciottenne militante di Lotta Continua viene ferito da alcuni colpi d'arma da fuoco esplosi dalle forze dell'ordine; morirà il giorno successivo in ospedale.

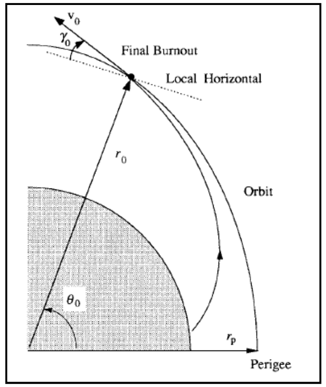
Orbit Transfer – Orbital Maneuvers

- **Launch Trajectory:** The main issues/objectives of this section are:
  - Payload weight and propellant consumption rate (exhaust velocity, velocità di scarico)
  - Selection of propulsion system (liquid, solid, combination, electric, nuclear)
  - To reach appropriate position and velocity at burn out (equivalently to achieve sufficient energy for orbit insertion)
- Propellant consumption and thrust are expressed by the specific impulse parameter I_{SP} (seconds)

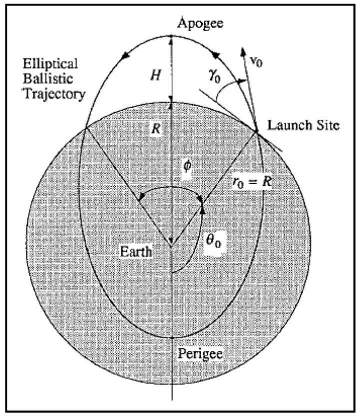
$$T = -V_e \frac{dm}{dt} + (p_e - p_{atm})A_e$$

$$V_e = g_0 I_{sp}$$





- **Flight Segments:**
  - **Vertical Ascent**: to escape denser atmosphere region
  - **Turn over Maneuver**: to achieve maximum horizontal speed component
  - **Gravity Turn Trajectory**: to optimize thrust angle to achieve maximum horizontal velocity component
  - **Orbit Injection**: to reach position and velocity sufficient for orbital motion at burn out time
  - **Suborbital flight**: to determine the apoapsis distance for a ballistic trajectory



- **Example 1:** At burnout a vehicle has the following values:

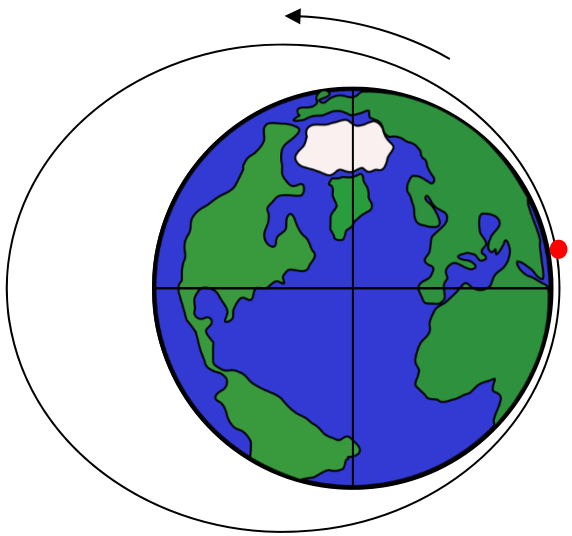
$$\begin{cases} r(t_{bo}) = 6378 + 250 = 6628\ Km \\ v(t_{bo}) = 8.5\ Km/\sec \\ \gamma(t_{bo}) = 1\deg \end{cases}$$

$$E = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = -23.923 \qquad a = -\frac{\mu}{2E} = 8330.895\ Km \qquad h = rv\cos\gamma = 56338.0\ Km^2/\sec$$

$$\begin{cases} r_p = a(1-e) = 6579.73 = (6378 + 201.73)\ Km \\ r_a = a(1+e) = 10,081.22 = (6378 + 3703.22)\ Km \end{cases}$$

$$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}} = 0.2102$$

$$f = \cos^{-1}\left[\frac{1}{e}\left(\frac{h^2}{\mu r} - 1\right)\right] = 18.19\deg$$



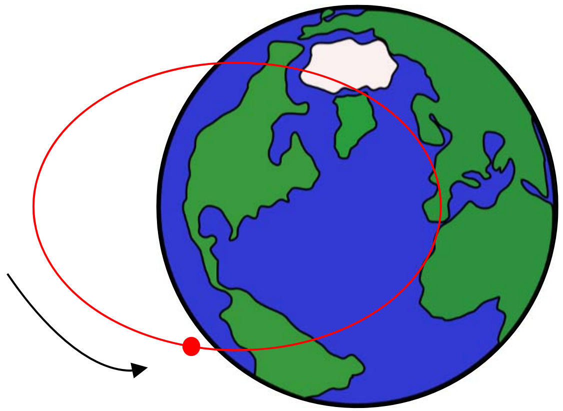
- **Example 2:** At burnout a vehicle has the following values:

$$\begin{cases} r(t_{bo}) = 6378 + 250 = 6628\,Km \\ v(t_{bo}) = 5.0\,Km/\sec \\ \gamma(t_{bo}) = 1\deg \end{cases}$$

$$E = -47.548\,Km^2/\sec^2 \qquad h = 33140.0\,Km^2/\sec \qquad e = 0.5854 \qquad a = 4191.55\,Km$$

$$\begin{cases} r_p = 1737.95\,Km \\ r_a = 6645.28 = (6378 + 267.28)\,Km \end{cases}$$

$$f = 176.47\deg$$



Orbit Transfer – Orbital Maneuvers

- **Example:** Preliminary propulsion requirements to orbit

- Recall Tsiolkovski's rocket equation (alternate form) $\Delta v = -V_e \ln \frac{m_0}{m_1}$ — $m_1 - m_0$ mass expelled by the rocket (Newton's Third Law)
- The specific Impulse is the impulse (force per unit time) per unit weight of propellant $V_e = g_0 I_{sp}$
- The thrust is the force generated by the engine and can be computed as:

$$T[N] = g_0[m\sec^{-2}]I_{SP}[\sec]\dot{m}[Kg\sec^{-1}]$$
$$= V_e[m\sec^{-1}]\dot{m}[Kg\sec^{-1}] = [Kgm] = [N]$$

Engine	Effective exhaust velocity (m/s, kg·m/(s·kg))	Specific impulse (s)
Turbofan jet engine (*actual* V is ~300 m/s)	29,000	3,000
Solid rocket	2,500	250
Bipropellant liquid rocket	4,400	450
Ion thruster	29,000	3,000
VASIMR[12][13][14]	30,000–120,000	3,000–12,000
Dual-stage 4-grid electrostatic ion thruster[15]	210,000	21,400

- **Problem:** We want to put 1 Kg of payload into circular LEO at an altitude of 250 Km

$$v_c = \sqrt{\frac{\mu_{EARTH}}{r_c}} = \sqrt{\frac{398,600}{6,628}} =$$
$$= 7.7549 = 7,754.9\, m \cdot \sec^{-1}$$

$$I_{SP} = 250\sec, m_0 = 1Kg, m_1 = ?$$

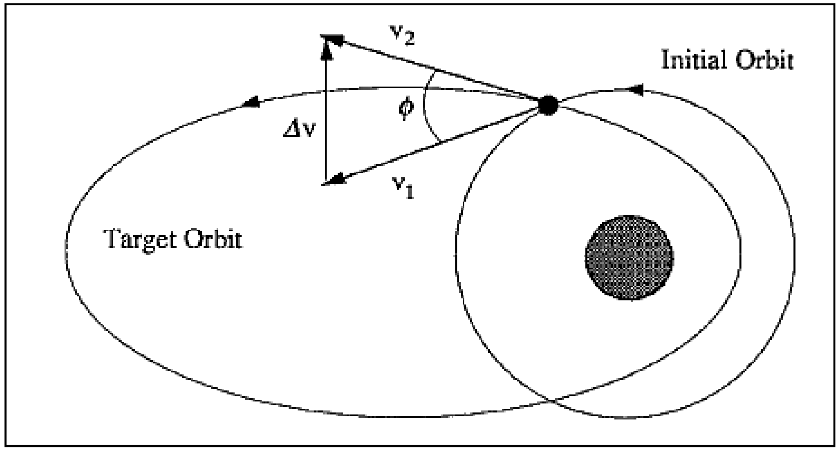
$$m_1 = m_0 e^{\left[\frac{\Delta v}{I_{SP} g_0}\right]} = 23.69\, Kg$$

- **95.78% of the total weight is due to propellant**

Orbit Transfer – Orbital Maneuvers

- **Orbit Transfer:** It is used in several segments of a mission.
  - To change from a parking orbit to the final orbit
  - As part of a more complex interplanetary flight

- **Main Assumption:** Orbit transfer is mostly based on propellant consumption, defined as Δv, that is the instantaneous change in velocity (acceleration) needed for transfer. Some Examples are:
  - Impulsive transfer (constant thrust, no position change at the impulse)
  - Continuous thrust transfer (Option for relative motion)
  - Single Impulse transfer
  - Coplanar orbit transfer
  - Minimum Fuel transfer (Hohmann)
  - Bielliptical orbit transfer
  - Out-of-plane orbit transfer
  - Planet Escape
  - Planet Capture
  - Swing by (Fly by or gravity assist)
  - …

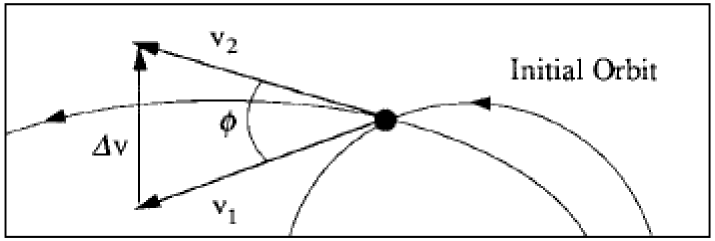


Single Impulse Maneuver

- $\Delta \boldsymbol{v}$ is typically provided by the thrust of a rocket engine, or other reaction engines (on-off thrusters). The time-rate of change of the velocity $\Delta \boldsymbol{v}$ is the magnitude of the acceleration caused by the engines, i.e., the thrust per total vehicle mass.

$$\Delta \boldsymbol{v}(t) = \int_{t0}^{t_1} \frac{\left|\boldsymbol{T}(\tau)\right|}{m(\tau)} d\tau = \int_{t0}^{t_1} \left|\boldsymbol{a}(\tau)\right| d\tau$$

$$\boldsymbol{T} = V_e \frac{dm}{dt}$$



- **The maximum Energy for a given Δv is achieved for:**

1. **v_1 and Δv collinear**
2. **Maximum v_1 (periapsis)**

- Tsiolkovsky's rocket equation

$$\Delta v = -g_0 I_{SP} \ln\left(\frac{m_0 + \Delta m}{m_0}\right); \Delta m < 0$$

$$\Delta m = m_0 \left\{1 - e^{\left(-\frac{\Delta v}{g_0 I_{SP}}\right)}\right\}$$

- Kinetic Energy Change due to Δv:

$$\boldsymbol{v}_2 = \boldsymbol{v}_1 + \Delta \boldsymbol{v}$$

$$\left|\Delta \boldsymbol{v}\right| = \Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos\phi}$$

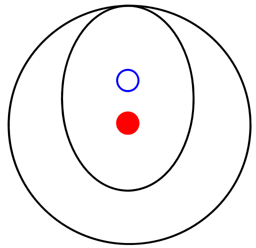
$$\Delta E = E_2 - E_1 = \frac{1}{2}(\Delta v)^2 + v_1 \Delta v \cos(180 - \phi)$$

A Δv maneuver can:

- Raise/lower the apogee/perigee
- A change in inclination
- Escape
- Reduction/Increase in period
- Change in RAAN Ω
- Begin a 2+ maneuver sequence of burns.
  - Creates a **Transfer Orbit**.

- **Problem:** Single coplanar impulse maneuver from Elliptic to Circular Earth orbit (apogee)

  - Known needed parameters: semi major axis a, eccentricity e.



At apogee, we have that

$$r_a = a(1+e)$$

From the vis-viva equation, we can calculate the velocity at apogee.

$$v_a = \sqrt{\mu\left(\frac{2}{r_a} - \frac{1}{a}\right)} = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{(1+e)}\right)} \quad \frac{d}{dt}\left(\frac{v^2}{2} - \frac{\mu}{r}\right) = 0 \Rightarrow E_{SPEC} = \mathbb{C}$$

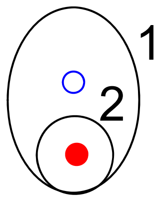
However, for a circular orbit at the same point, we calculate from vis-viva

$$v_c = \sqrt{\frac{\mu}{r_a}} = \sqrt{\frac{\mu}{a(1+e)}}$$

Therefore, the Δv required to circularize the orbit is

$$\Delta v = v_c - v_a = \sqrt{\frac{\mu}{a(1+e)}} - \sqrt{\frac{\mu}{a}\left(\frac{1-e}{(1+e)}\right)} = \frac{\mu}{a(1+e)}(1-\sqrt{1-e})$$

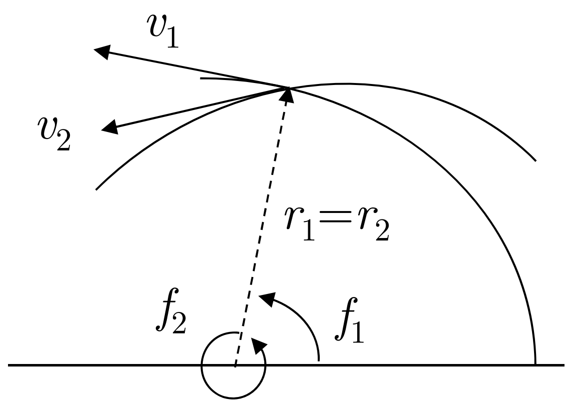
- For a change to a smaller circular orbit, the transfer should start at perigee of the elliptical orbit

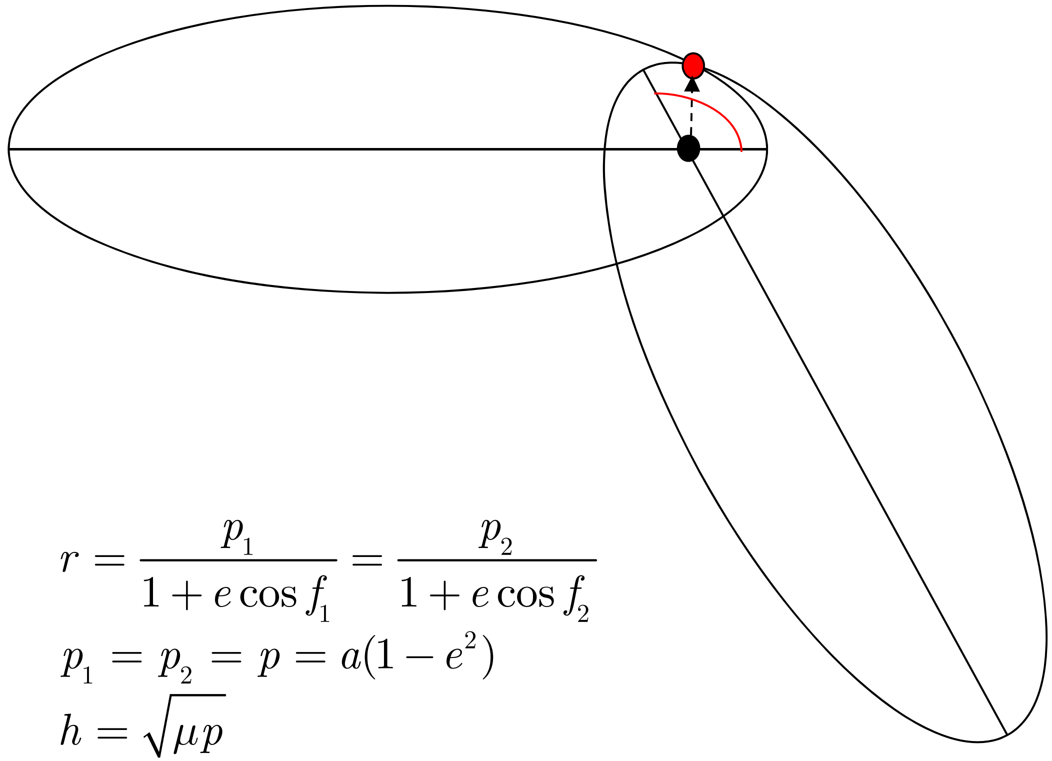


$$\begin{cases} a = 10{,}000\ Km \\ e = 0.5 \end{cases} \quad \begin{cases} r_a = 15{,}000\ Km \\ r_p = 5{,}000\ Km \end{cases} \quad \begin{cases} v_a = 3.6451\ Km\ /\ \sec \\ v_c = 5.1549\ Km\ /\ \sec \end{cases} \quad \Delta v = +1.5098\ Km\ /\ \sec$$

Orbit Transfer – Orbital Maneuvers

- **Problem:** Single impulse maneuver from Elliptic to Elliptic orbit (change of Argument of Periapsis ω)





Note: The orbits have

- The same shape (same $a,\ e$)
- The same Focus
- The same angular momentum

The spacecraft has:

- The same position and speed magnitudes
- The same flight path angle modulus

$$r = \frac{p_1}{1 + e\cos f_1} = \frac{p_2}{1 + e\cos f_2}$$

$$p_1 = p_2 = p = a(1 - e^2)$$

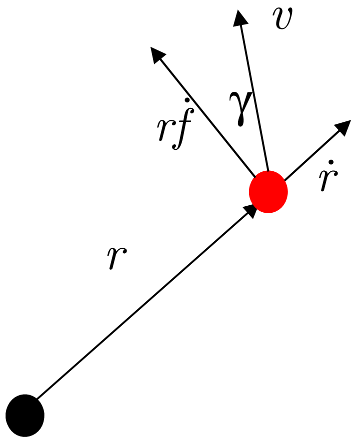
$$h = \sqrt{\mu p}$$

$$\cos f_1 = \cos f_2$$

$$f_2 = 2\pi - f_1$$

α = angle of rotation line of apsides

$$f_2 - f_1 + \alpha = 2\pi \Rightarrow f_1 = \frac{\alpha}{2}$$



- From previous computations:

$$\dot{r} = \sqrt{\frac{\mu}{p}} e \sin f; r\dot{f} = \frac{h}{r} = \sqrt{\frac{\mu}{p}} (1 + e \cos f)$$

$$v = \sqrt{\frac{\mu}{p}} \sqrt{(1 + e^2 + 2e \cos f)}$$

$$\cos\gamma = \frac{r\dot{f}}{v}; \sin\gamma = \frac{\dot{r}}{v}$$

- From above, we obtain: $\cos\gamma_1 = \cos\gamma_2$

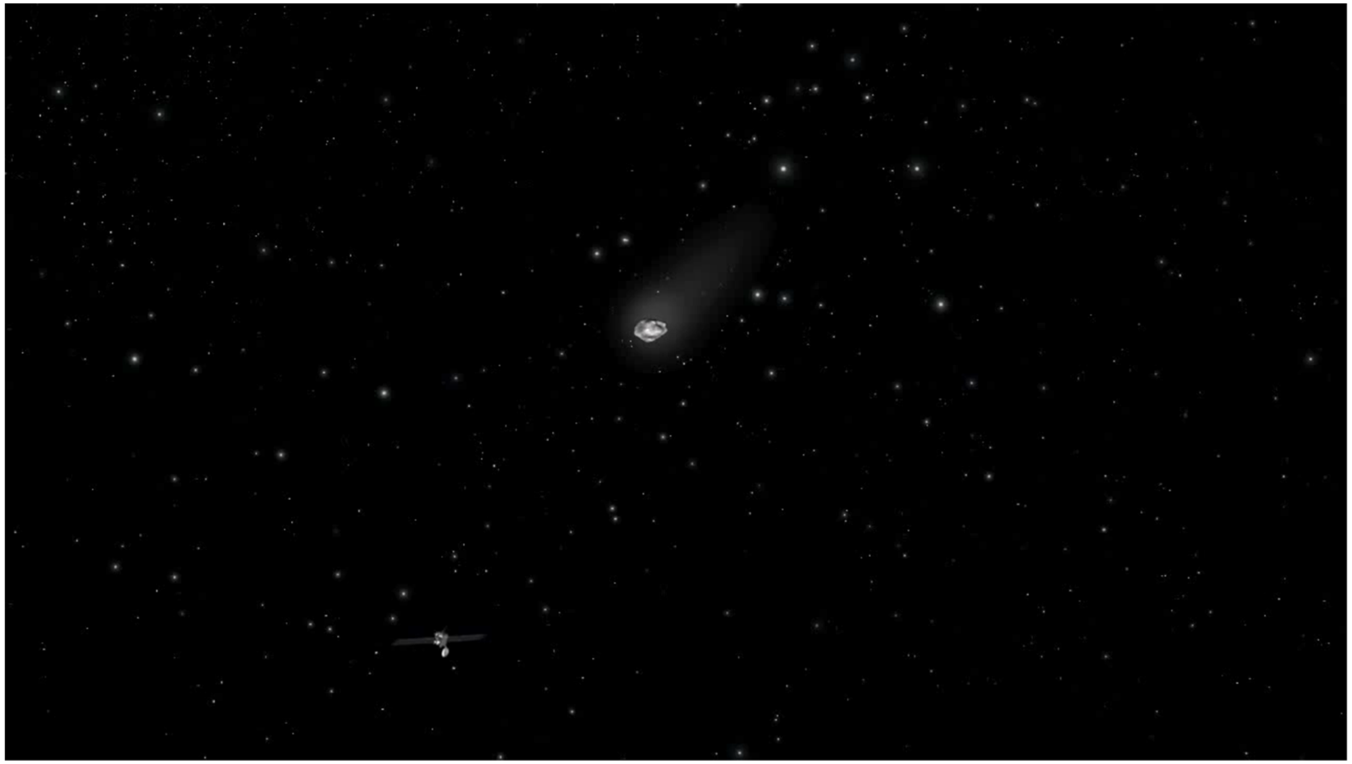
- Also: $h_1 = h_2 \Rightarrow rv_1 \cos\gamma_1 = rv_2 \cos\gamma_2 \Rightarrow v_1 = v_2 = v$

- At this point we can compute Δv, from law of cosines: $\Delta v = 2v \sin\gamma = 2\sqrt{\frac{\mu}{a(1-e^2)}} e \sin(\frac{\alpha}{2})$

$$\begin{cases} a = 10,000\ Km \\ e = 0.5 \\ f_1 = 60° \end{cases} \quad \begin{cases} h = 54,676.32 \\ \alpha = 120° \\ f_2 = 240° \end{cases} \quad \begin{cases} r_1 = r_2 = \dfrac{p}{1 + e\cos f} = 6,000 Km \\ v_1 = v_2 = 9.644\ Km / \sec \\ \gamma_1 = 19.106° \end{cases}$$

$$\Delta v = 6.3133\ Km / \sec$$

- Sequence of single impulse maneuvers: Rosetta mission, acquisition of the Comet 67P Churyumov – Gerasimenko (2014)
    - Reference system centered at the comet



Two – Impulse Maneuver, Hohmann Transfer

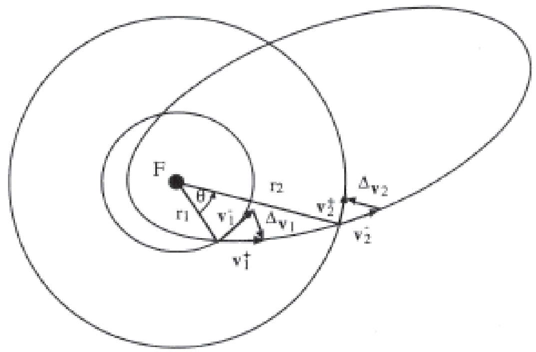
- Two (or more) impulsive maneuvers are used for complex orbit changes and also as segments for interplanetary flight mission planning. The simplest case consists of the initial orbit, the target orbit, and the transfer orbit (coplanar case for now).

Step 1: Design a transfer orbit (a,e,i, etc.).

Step 2: Calculate $\vec{v}_{tr,1}$ at the point of intersection with initial orbit.

Step 3: Calculate initial burn to maneuver into transfer orbit.

$$\Delta v_1 = \vec{v}_{tr,1} - \vec{v}_{init}$$

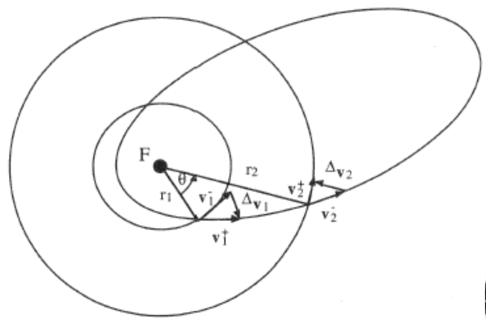


Step 4: Calculate $\vec{v}_{tr,2}$ at the point of intersection with target orbit.

Step 5: Calculate velocity of the target orbit, $\vec{v}_{fin}$, at the point of intersection with transfer orbit.

Step 6: Calculate the final burn to maneuver into target orbit.

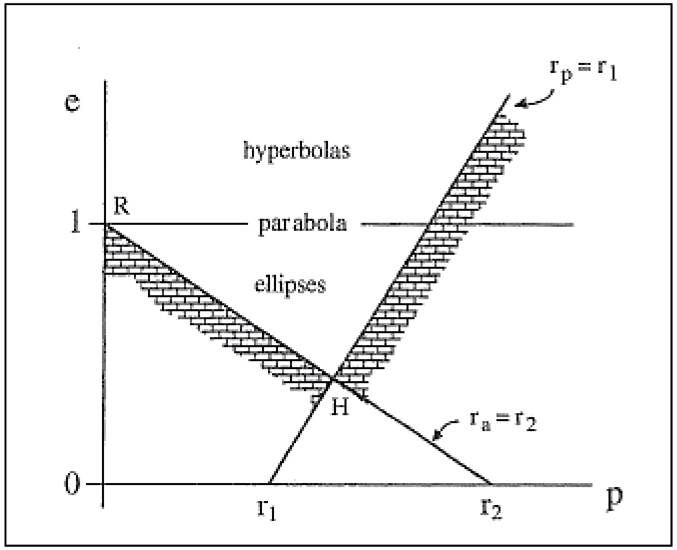
$$\Delta v_2 = \vec{v}_{fin} - \vec{v}_{tr,2}$$



- Many possible orbits satisfy the problem. There are also some constraints:

- Consider an initial (or final) circular orbit of radius r_1 and a circular final (or initial) orbit with radius $r_2 > r_1$:

$$\begin{cases} r_p = a(1-e) \leq r_1 \\ r_a = a(1+e) \geq r_2 \end{cases}$$

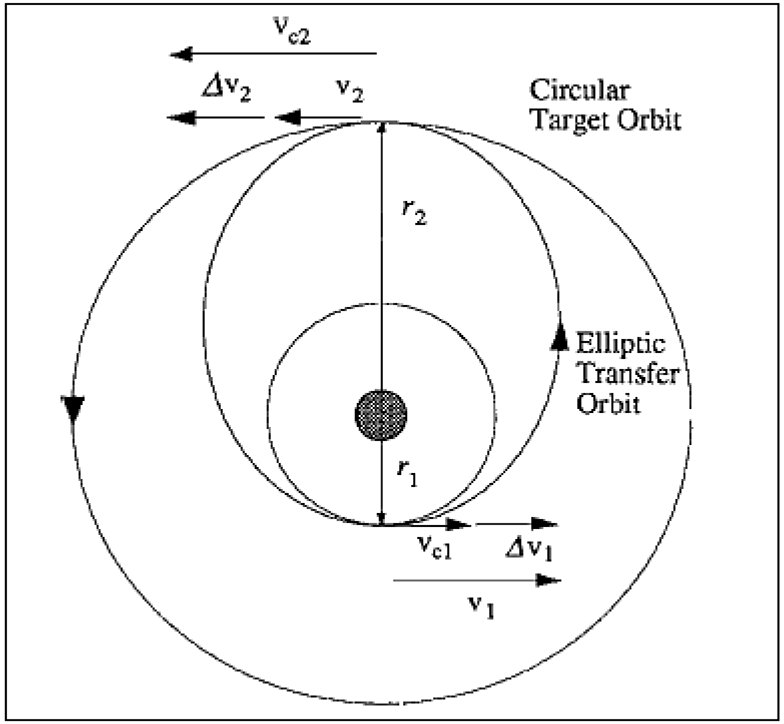


Hohmann Transfer

The Hohmann transfer is the energy-optimal two burn maneuver between any two coaxial elliptic orbits.

- Originally proposed by Hohmann (1925)
- Proved optimal for circular orbits (1952)
- Proved optimal for elliptic orbits (1986)

- **Example:** Consider an outward Hohmann transfer:



1. Speed increase from circular orbit to elliptical transfer at periapsis.
2. Speed increase at elliptical apoapsis to achieve circular target orbit

- Orbital elements of transfer orbit:

$$\begin{cases} a = \dfrac{r_p^e + r_a^e}{2} = r_1^c \\ e = \dfrac{r_p^e}{a} = \dfrac{r_2 - r_1}{r_2 + r_1} \end{cases}$$

$$v_{init} = \sqrt{\frac{\mu}{r_1}} \qquad v_{trans,p} = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a}} = \sqrt{2\mu}\sqrt{\frac{1}{r_1} - \frac{1}{r_1 + r_2}} = \sqrt{2\mu\frac{r_2}{r_1(r_1 + r_2)}}$$

- Initial impulse Δv_1:

$$\Delta v_1 = v_{trans,p} - v_{init} = \sqrt{2\mu\frac{r_2}{r_1(r_1+r_2)}} - \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{(r_1+r_2)}} - 1\right)$$

- At apoapsis:

$$v_{trans,a} = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a}} = \sqrt{2\mu\frac{r_1}{r_2(r_1+r_2)}} \qquad v_{fin} = \sqrt{\frac{\mu}{r_2}}$$

- Final impulse Δv_2:

$$\Delta v_2 = v_{fin} - v_{trans,a} = \sqrt{\frac{\mu}{r_2}} - \sqrt{2\mu\frac{r_1}{r_2(r_1+r_2)}} = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{(r_1+r_2)}}\right)$$

The transfer time is simply half the period of the orbit. Hence

$$\Delta t = \frac{\tau}{2} = \pi\sqrt{\frac{a^3}{\mu}}$$
$$= \pi\sqrt{\frac{(r_1+r_2)^3}{8\mu}}$$

Orbit Transfer – Orbital Maneuvers

- **Problem:** Transfer a satellite from parking orbit at 200 Km, to a geostationary orbit with a period of 24 hours

$$\begin{cases} r_p^e = r_1 = 6378 + 200 = 6,578\,Km \Rightarrow e = 0.7305 \\ P^2 = (4\dfrac{\pi^2}{\mu})r_2^3 \Rightarrow r_2 = 42,241\,Km \Rightarrow a = 24,409.55\,Km \end{cases}$$

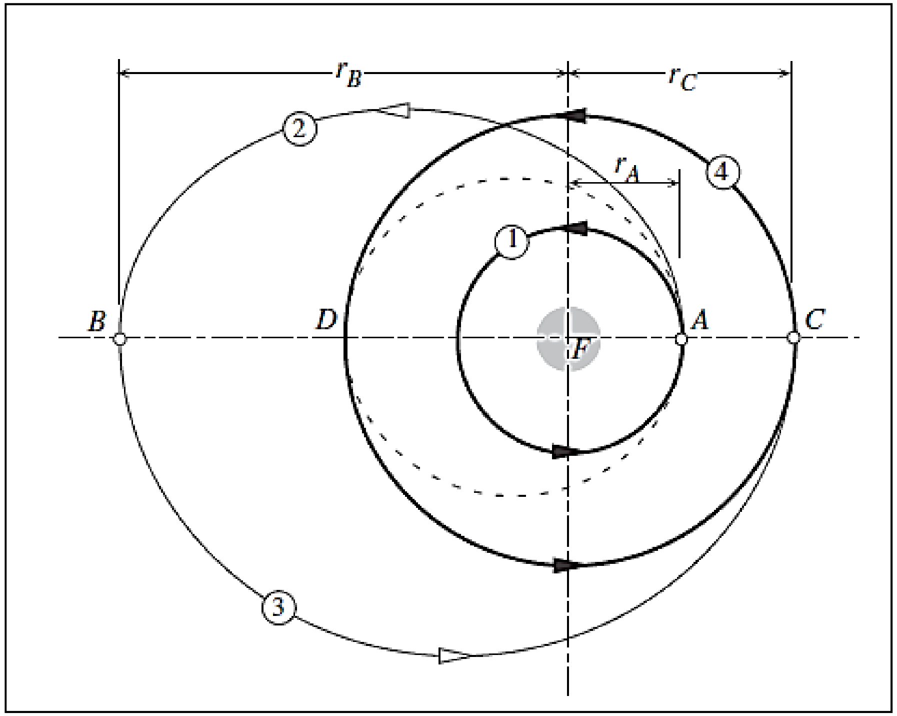
$$\begin{cases} v_1 = 7.7843 \\ v_p = 10.240 \\ v_a = 1.5947 \\ v_2 = 3.0719 \end{cases} \qquad \begin{cases} \Delta v_1 = 2.455\,Km/\sec \\ \Delta v_2 = 1.48\,Km/\sec \\ TOF = 5.27\,hours \end{cases}$$

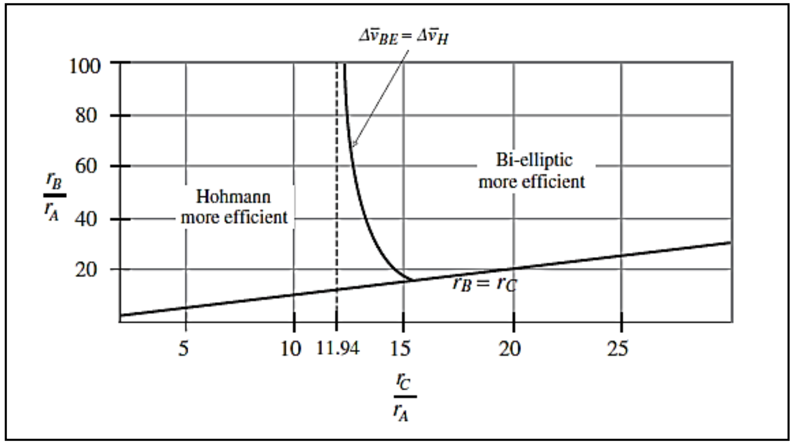
- **Properties of Hohmann Transfer:**

1. Optimal 2 – Impulse transfer in terms of fuel consumption for circular to circular and elliptic to elliptic
2. Has the longest time of flight
3. It can be suboptimal with respect to multiple impulse transfers

   - From Kinetic Energy Arguments $E_{COST} = \dfrac{\|\Delta v_1\|^2 + \|\Delta v_2\|^2}{2}$ $\qquad \Delta E_{MIN} = -\dfrac{\mu}{2a_2} + \dfrac{\mu}{2a_1}$

- **Bi-Elliptical Transfer:** There are situations in which addional fuel can be saved, at the expense of longer flight time.
  - One example is to raise the orbit of a satellite to a higher circular orbit



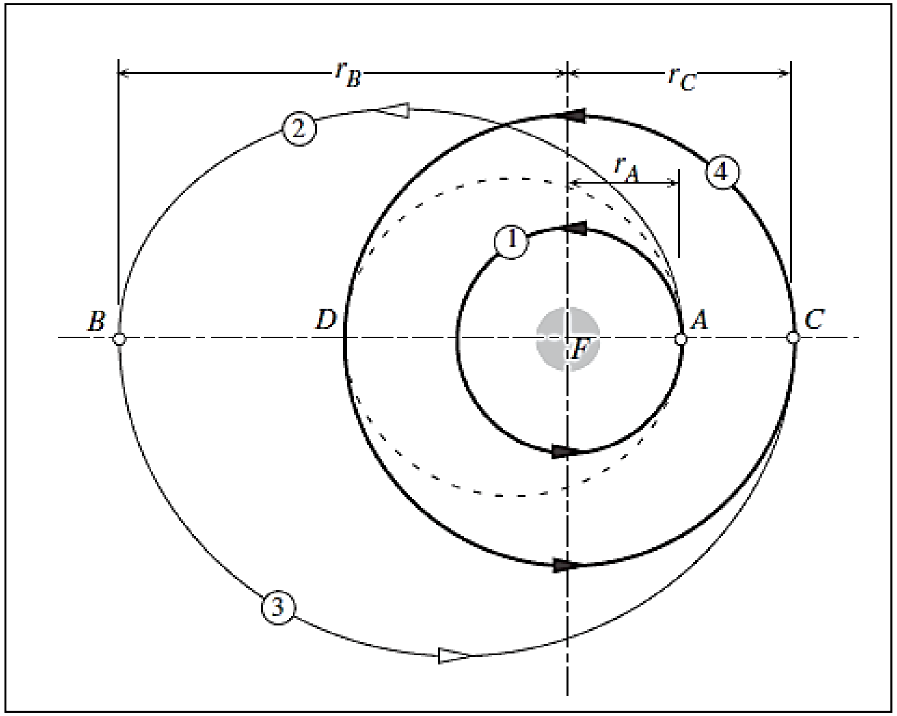


1. Original Parking Orbit (Circular)

$$v_{c1} = \sqrt{\frac{\mu}{r_A}}$$

2. First Elliptical Transfer (A – B)

$$\begin{cases} a_2 = \dfrac{r_A + r_B}{2} \\ e_2 = \dfrac{r_B - r_A}{r_A + r_B} \end{cases} \qquad \begin{cases} v_{P2} = \sqrt{2\mu \dfrac{r_B}{r_A\left(r_A + r_B\right)}} \\ v_{A2} = \sqrt{2\mu \dfrac{r_A}{r_B\left(r_A + r_B\right)}} \end{cases}$$



3. Second Elliptical Transfer (B – C)

$$\begin{cases} a_3 = \dfrac{r_B + r_C}{2} \\ e_3 = \dfrac{r_B - r_C}{r_C + r_B} \end{cases} \qquad \begin{cases} v_{P3} = \sqrt{2\mu \dfrac{r_B}{r_C\left(r_C + r_B\right)}} \\ v_{A3} = \sqrt{2\mu \dfrac{r_C}{r_B\left(r_C + r_B\right)}} \end{cases}$$

4. Final Orbit (Circular)

$$v_{c2} = \sqrt{\frac{\mu}{r_C}}$$

- Burn Sequence

$$\Delta v_1 = v_{P2} - v_{C1}\,(+)$$

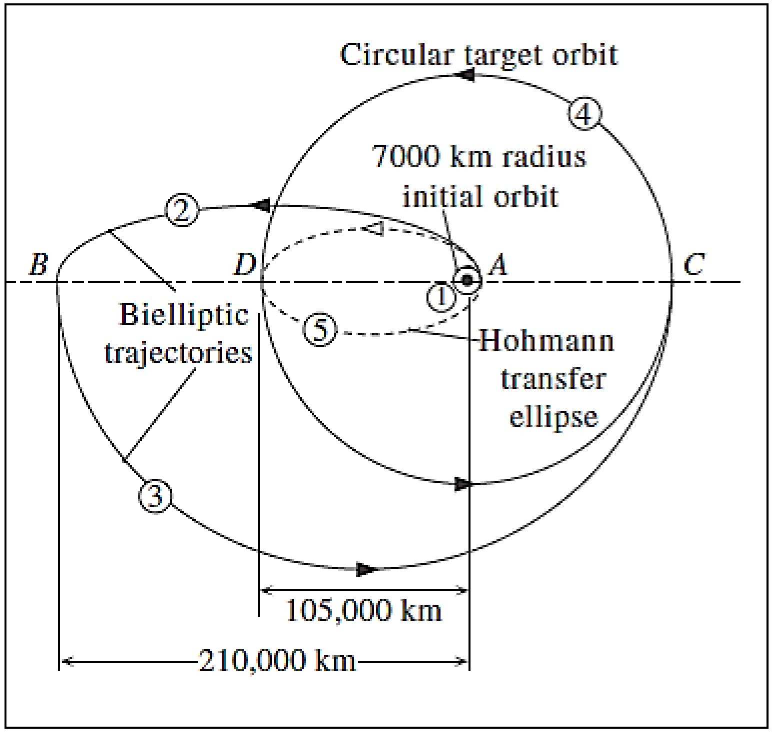
$$\Delta v_2 = v_{A3} - v_{A2}\,(+)$$

$$\Delta v_3 = v_{C2} - v_{A3}\,(-)$$

- **Problem:**

  Find the total delta-v requirement for a bi-elliptic Hohmann transfer from a geocentric circular orbit of 7000 km
  radius to one of 105,000 km radius. Let the apogee of the first ellipse be 210,000 km. Compare the delta-v,
  schedule and total flight time with that for an ordinary single Hohmann transfer ellipse.

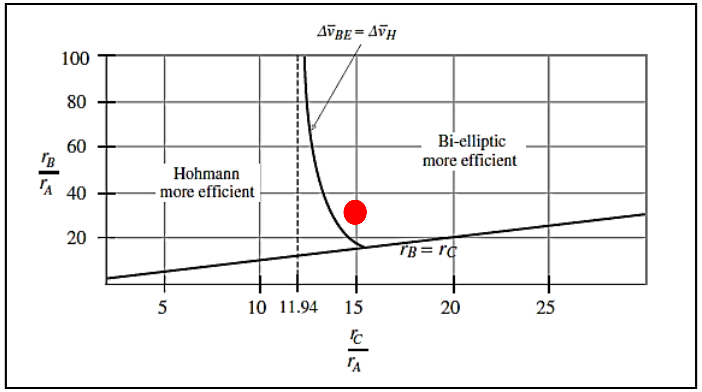
Orbit Transfer – Orbital Maneuvers



- Problem Data

$$r_{c1} = 7{,}000Km$$
$$r_{c2} = 105{,}000Km$$
$$r_{a1} = 210{,}000Km$$
$$r_{p1} = r_{c1}$$
$$r_{a2} = r_{a1}$$
$$r_{p2} = r_{c2}$$

$$\mu = 398{,}600Km^3 / \sec^2$$



$$v_{c1} = \sqrt{\frac{\mu}{r_{c1}}} = 7.54Km / \sec$$

$$\begin{cases} a_1 = \dfrac{r_{c1} + r_{a1}}{2} = 108{,}500\,Km \\ e_1 = \dfrac{r_{a1} - r_{c1}}{r_{c1} + r_{a1}} = 0.9355 \end{cases}$$

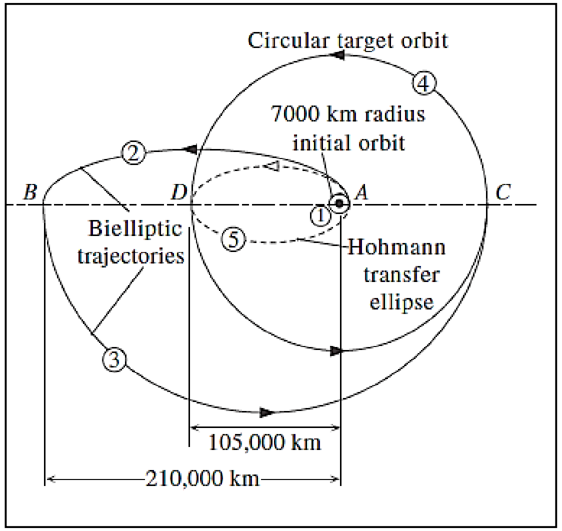
$$\begin{cases} h_1 = \sqrt{2\mu}\sqrt{\dfrac{r_{c1} r_{a1}}{r_{c1} + r_{a1}}} = 73{,}487\,Km^2 / \sec \\ E = -\dfrac{\mu}{2a} = -1.8369\,Km^4 / \sec^2 \end{cases}$$

$$\begin{cases} v_{P1} = \sqrt{2\mu \dfrac{r_B}{r_A\left(r_A + r_B\right)}} = \sqrt{2 \cdot 398{,}600 \dfrac{210{,}000}{7{,}000 \cdot 217{,}000}} = 10.4982Km / \sec \\ v_{A1} = \sqrt{2\mu \dfrac{r_A}{r_B\left(r_A + r_B\right)}} = \sqrt{2 \cdot 398{,}600 \dfrac{7{,}000}{210{,}000 \cdot 217{,}000}} = 0.3499Km / \sec \end{cases}$$

$$\begin{cases} a_2 = \dfrac{r_{c2} + r_{a1}}{2} = 157{,}500\, Km \\ e_2 = \dfrac{r_{a1} - r_{c2}}{r_{c2} + r_{a1}} = 0.333 \end{cases} \qquad \begin{cases} h_2 = \sqrt{2\mu}\sqrt{\dfrac{r_{c2} r_{a1}}{r_{c2} + r_{a1}}} = 236{,}238\, Km^2 / \sec \\ E = -\dfrac{\mu}{2a} = -1.2654\, Km^4 / \sec^2 \end{cases} \qquad \begin{cases} v_{P2} = \dfrac{h_2}{r_{c2}} = \dfrac{236{,}238}{105{,}000} = 2.24992 Km / \sec \\ v_{A2} = \dfrac{h_2}{r_{a1}} = \dfrac{236{,}238}{210{,}000} = 1.1249 Km / \sec \end{cases}$$

$$v_{c2} = \sqrt{\frac{\mu}{r_{c1}}} = 1.9484 Km / \sec \qquad \begin{array}{l} \Delta v = |10.4982 - 7.54| + |1.1249 - 0.3489| + |1.9484 - 2.24992| = 3.4327\, Km / \sec \\ TOF = 5.66\, days \end{array}$$

- Consider now a 2 impulse Hohmann

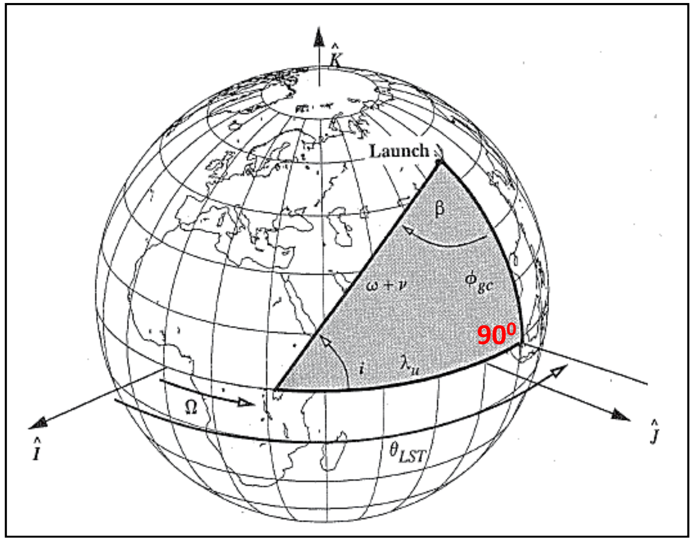


$$\begin{cases} a_H = \dfrac{r_p + r_a}{2} = 56{,}000\, Km \\ e_H = \dfrac{r_a - r_p}{r_a + r_p} = 0.875 \end{cases} \qquad \begin{cases} h = 72{,}330\, Km^2 / \sec \\ v_p = 10.333\, Km / \sec \\ v_a = 0.68886\, Km / \sec \\ TOF = 0.763\, days \end{cases}$$

$$\Delta v = |10.333 - 7.54| + |1.9484 - 0.68886| = 4.0463\, Km / \sec$$

Some Comments on Out – of – Plane Orbits

- There are many instances where orbit change requires changes in orbital plane one of these is the launch phase



The two geometric features of launch are:

- latitude of the launch site, ϕ_{gc}
- launch azimuth (direction), β.

- The orbit Inclination selection is limited

$$\cos i = \cos \phi_{gc} \sin \beta$$

- Unlike inclination, the Right Ascension of the orbital plane can be chosen by Launch Window.

Referring to the triangle our desired launch time (in Local Sidereal Time) is given by

$$\theta_{LST} = \Omega + \lambda_u$$

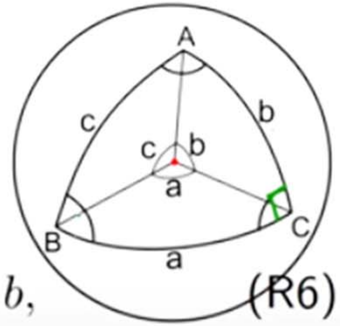
where λ_u can be found from β and i as

$$\cos \lambda_u = \frac{\cos \beta}{\sin i}$$

Note: the sum of angles is greater than 180 degrees.

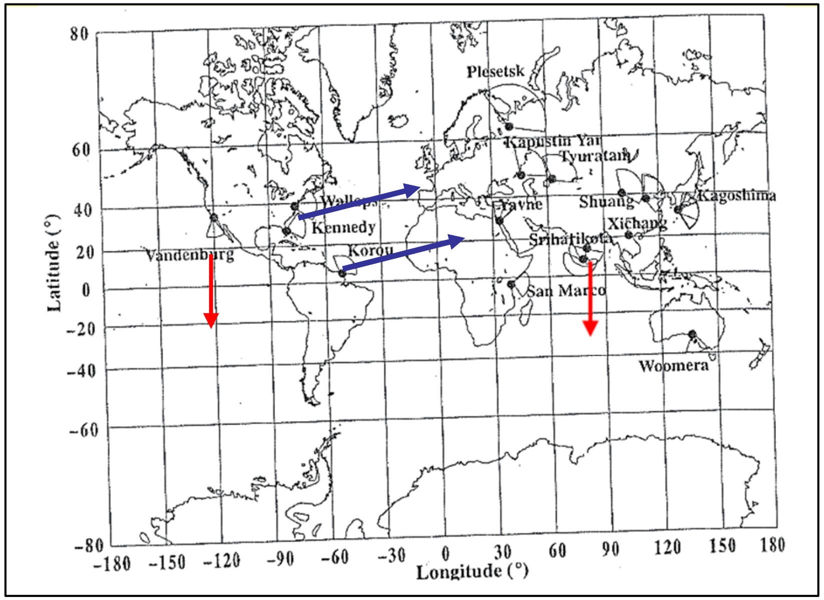
Napier's rule for spherical right triangles

- If $\beta = \pm 90^\circ$, then the minimum i is restricted by $i \geq \phi_{gc}$.
- Equation derived from Napier's rules for spherical right triangles ($C = 90^\circ$)
- $\beta = A$ (as defined on the previous figure) or $\beta = 180^\circ + A$ (using the tables)



(R1)	$\cos c = \cos a \cos b,$	(R6)	$\tan a = \cos B \tan c,$
(R2)	$\sin b = \sin B \sin c,$	(R7)	$\tan b = \cos A \tan c,$
(R3)	$\sin a = \sin A \sin c,$	(R8)	$\cos B = \sin A \cos b,$
(R4)	$\tan b = \tan B \sin a,$	(R9)	$\cos A = \sin B \cos a,$
(R5)	$\tan a = \tan A \sin b,$	(R10)	$\cos c = \cot B \cot A.$

❑ Launch sites



Site	Latitude (°)	Longitude (°)	Azimuth Min (°)	Azimuth Max (°)
Vandenberg	34.600 000	−120.600 000	147	201
Cape Kennedy	28.500 000	−80.550 000	37	112
Wallops	37.850 000	−75.466 67	30	125
Kourou	5.200 000	−52.800 000	340	100
San Marco	−2.933 333	40.200 000	50	150
Plesetsk	62.800 000	40.600 000	330	90
Kapustin Yar	48.400 000	45.800 000	350	90
Tyuratam	45.600 000	63.400 000	340	90
Sriharikota	13.700 000	80.250 000	100	290
Shuang-Ch'Eng-Tzu	40.416 667	99.833 333	350	120
Xichang	28.250 000	102.200 000	94	105
Tai-yuan	37.766 667	112.500 000	90	190
Kagoshima	31.233 333	131.083 333	20	150
Woomera	−30.950 000	136.500 000	350	15
Yavne	31.516 667	34.450 000	350	120

- Launch latitude is restricted by the site.
- Launch Azimuth is usually towards the sea or away from inhabited areas (problems with rockets), and in the eastward direction

❑ Example Vanderberg AFB

$$\phi_{gc} = 34.6\,\text{deg} \quad \text{and} \quad \beta \in [147\,\text{deg}, 201\,\text{deg}]$$

$$\cos i = \cos\phi_{gc}\sin\beta$$

Therefore

$$-.295 < \cos i < .4483$$

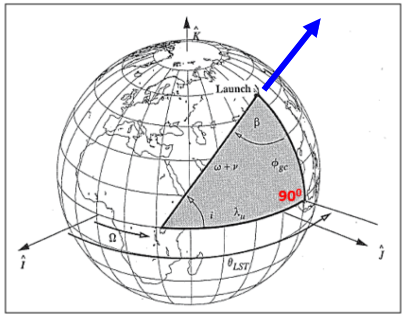
So the inclination is restricted as

$$63.36\,\text{deg} < i < 107.16\,\text{deg}$$

❑ Example

- For interplanetary flight is useful to establish an initial parking orbit on the ecliptic plane.
- Data: RAAN Ω = 0 deg, inclination 23.5 deg, Launch site: Kourou [ϕ_{GC}=5.2 deg, θ_k=-52.8 deg, β= (160 deg, 280 deg)] ->added +180
- **Find**: θ_{LST} and **verify** whether β is in the launch azimuth window.

$$\Omega = 0^0 \Rightarrow \theta_{LST} = \lambda_u \qquad \cos i = \cos\phi_{gc}\sin\beta, \qquad \cos\theta_{LST} = \frac{\cos\beta}{\sin i} \qquad \cos\lambda_u = \frac{\cos\beta}{\sin i}$$



Solving the first equation for β, we have

$$\beta = \sin^{-1}\left(\frac{\cos 23.5°}{\cos 5.2°}\right) = 67.05°, 112.95°$$

->add +180 to have prograde orbit

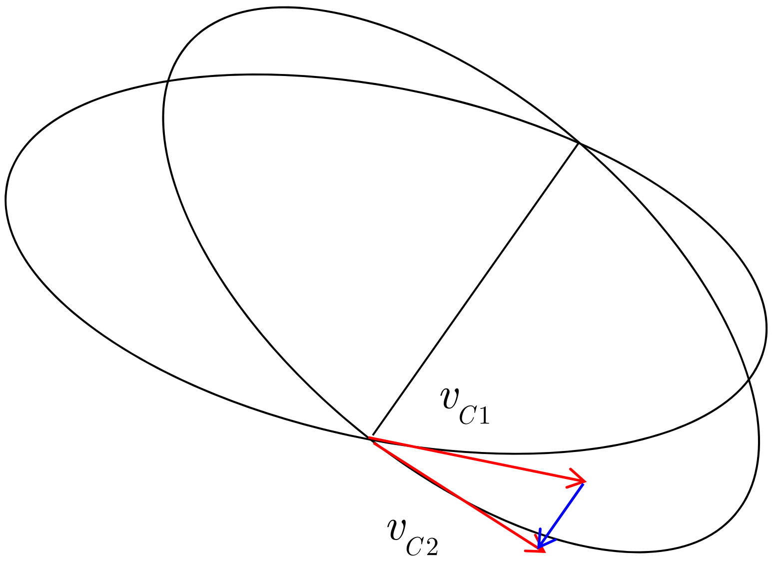
$$\beta = 67.05^0 \Rightarrow \beta' = 180^0 + \beta = 247.05^0 \in [160^0, 280^0]$$

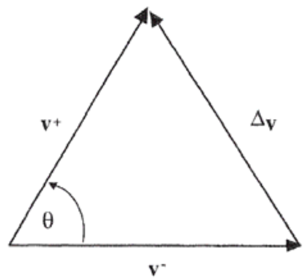
$$\beta = 112.95^0 \Rightarrow \beta' = 180^0 + \beta = 292^0 \notin [160^0, 280^0]$$

- desired launch time (in Local Sidereal Time) $\theta_{LST} = \Omega + \lambda_u = 0 + \cos^{-1}(\frac{\cos\beta}{\sin i}) = [12.074^0, 167.92^0]$

Orbit Transfer – Orbital Maneuvers

- Orbital inclination change is an orbital maneuver aimed at **changing the inclination** of an orbiting body's orbit. This maneuver is also known as an orbital plane change as the plane of the orbit is tipped. This maneuver requires a change in the orbital velocity vector at the orbital nodes.
- In general, inclination changes can require a great deal of delta-v to perform, and most mission planners try to avoid them whenever possible to conserve fuel.
- Maximum efficiency of inclination change is achieved at apoapsis, where orbital velocity is the lowest. In some cases, it may require less total delta-v to raise the satellite into a higher orbit, change the orbit plane at the higher apogee, and then lower the satellite to its original altitude.





$$\Delta v = 2v \sin\frac{\theta}{2} \qquad \theta = \Delta i$$

The direction of trust is: $90^\circ + \dfrac{\theta}{2}$

- Out of Plane maneuvers can also be used to **change both inclination and Longitude of the Ascending Node** Ω **(RAAN)**

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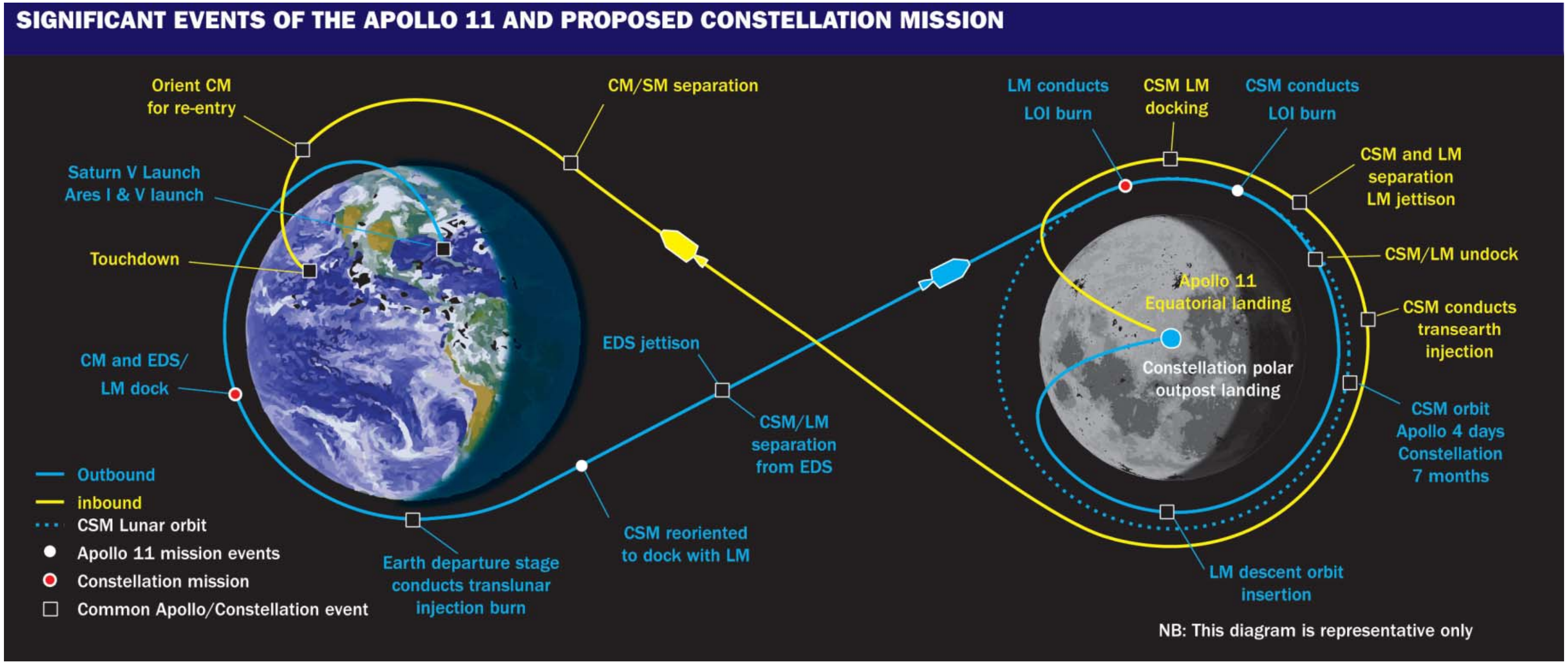
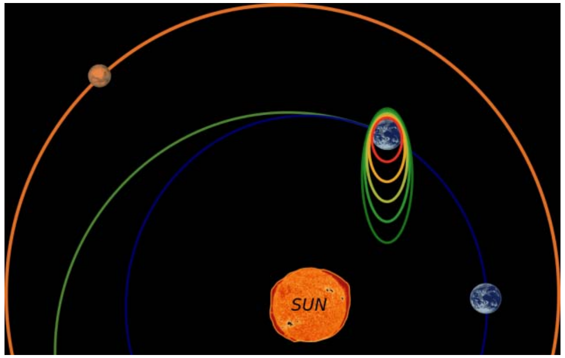
- Events during a mission



Interplanetary Flight



Timeline of Operations

Phase	Date	Event	Detail	Result	References
Geocentric phase	5 November 2013 09:08 UTC	Launch	Burn time: 15:35 min in 5 stages	Apogee: 23,550 km (14,630 mi)	[52]
	6 November 2013 19:47 UTC	Orbit raising manoeuvre	Burn time: 416 sec	Apogee: 28,825 km (17,911 mi)	[53]
	7 November 2013 20:48 UTC	Orbit raising manoeuvre	Burn time: 570.6 sec	Apogee: 40,186 km (24,970 mi)	[54][55]
	8 November 2013 20:40 UTC	Orbit raising manoeuvre	Burn time: 707 sec	Apogee: 71,636 km (44,513 mi)	[54][56]
	10 November 2013 20:36 UTC	Orbit raising manoeuvre	Incomplete burn	Apogee: 78,276 km (48,638 mi)	[57]
	11 November 2013 23:33 UTC	Orbit raising manoeuvre (supplementary)	Burn time: 303.8 sec	Apogee: 118,642 km (73,721 mi)	[54]
	15 November 2013 19:57 UTC	Orbit raising manoeuvre	Burn time: 243.5 sec	Apogee: 192,874 km (119,846 mi)	[54][58]
	30 November 2013, 19:19 UTC	Trans-Mars injection	Burn time: 1328.89 sec	Successful heliocentric insertion	[59]
Heliocentric phase	December 2013 – September 2014	*En route* to Mars – The probe travelled a distance of 780,000,000 kilometres (480,000,000 mi) in a Hohmann transfer orbit[32] around the Sun to reach Mars.[49] This phase plan included up to four trajectory corrections if needed.			[60][61][62][63][64]
	11 December 2013 01:00 UTC	1st Trajectory correction	Burn time: 40.5 sec	Success	[54][62][63][64]
	9 April 2014	2nd Trajectory correction (planned)	Not required	Rescheduled for 11 June 2014	[61][64][65][66][67]
	11 June 2014 11:00 UTC	2nd Trajectory correction	Burn time: 16 sec	Success	[65][68]
	August 2014	3rd Trajectory correction (planned)	Not required[65][69]		[61][64]
	22 September 2014	3rd Trajectory correction	Burn time: 4 sec	Success	[61][64][70]
Areocentric phase	24 September 2014	Mars orbit insertion	Burn time: 1388.67 sec	Success	[8]

- Sequence of orbit transfers called PHASING

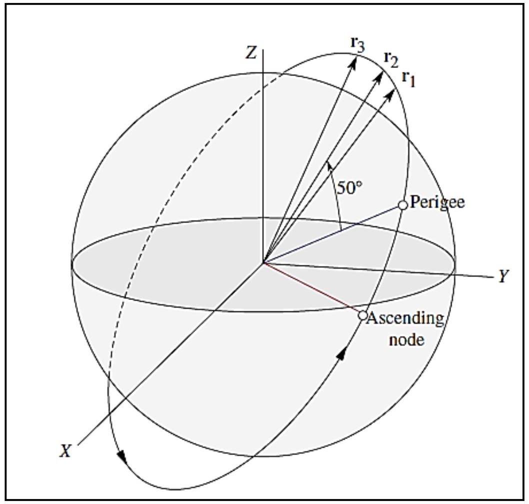
Orbit Determination

- One of the most critical issues in mission planning is the accurate position of a spacecraft and/or planet along their orbit and the orbit determination, given the measured positions at different times.
- Importance in near Earth asteroid orbit computation, and in targeting operations

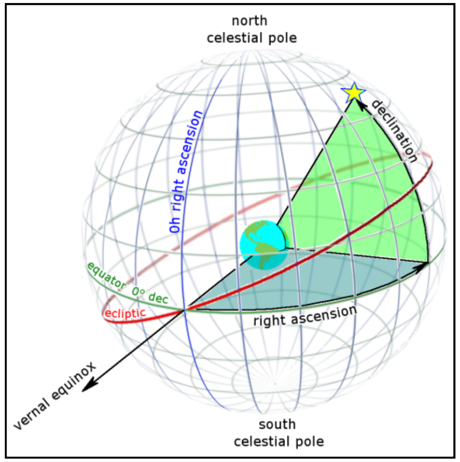
- Given two positions and the time of flight, find the orbit
- Given a position and a time interval, find the position at the final time

- **Gibbs' Method:** given the position vectors at three different times, the orbit can be found using vector calculus. The only constraint is that the three measurements be coplanar

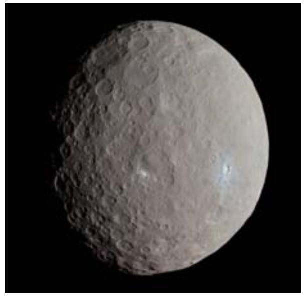
$$r_1(t_1), r_2(t_2), r_3(t_3) \rightarrow v_1(t_1), v_2(t_2)$$
$$\rightarrow (a, e, p, i, \Omega, w)$$



- Gauss Measurements for Ceres



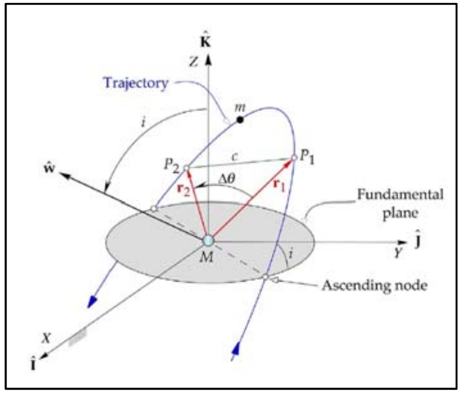
❑ **Ceres:** asteroid in the belt discovered by Piazzi (1801) with observations and orbit developed by Gauss. Visited by DAWN spacecraft

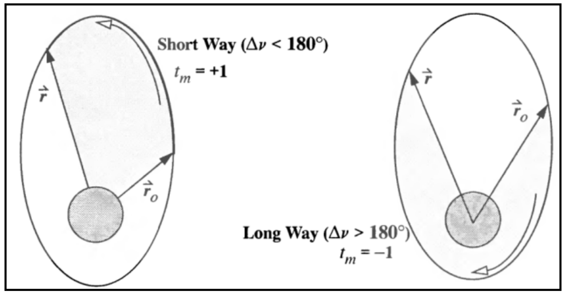


❑ **Lambert's Problem (1761):** The transfer time of a body moving between two points on a conic trajectory is a function only of the sum of the distances of the two points from the origin of the force, the linear distance between the points, and the semi major axis of the conic (Somewhat similar to Kepler's problem, but the orbit is unknown)

$$\sqrt{\mu}(t_2 - t_1) = f(a, r_1 + r_2, c)$$

- Travel between two points is not unique unless direction is specified





- The solution of Lambert's problem is very important for interplanetary flight trajectory determination, for intercepting targets in orbit (for instance dangerous asteroids), and it is obtained only numerically with methods developed even in recent years (Battin, 1978).

- The main equation governing the problem depends on the nature of the resulting orbit (elliptical, parabolic, hyperbolic), and the solution may be non unique.

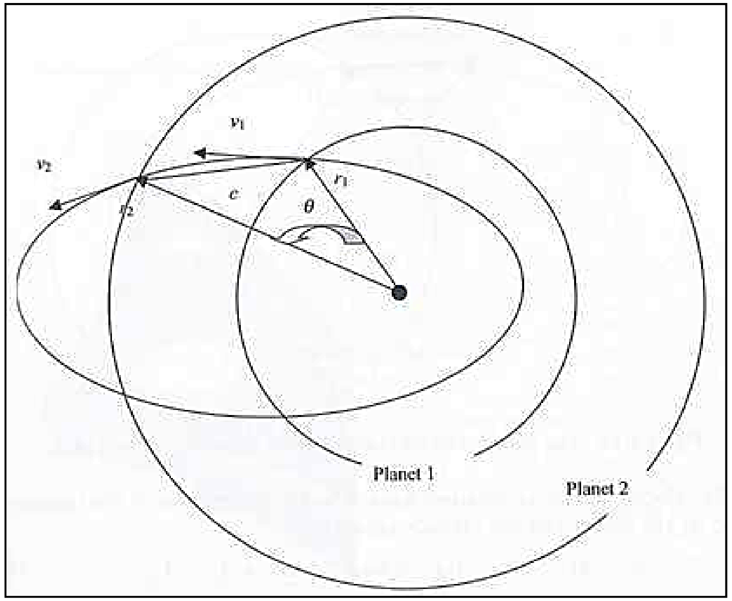
- **Lambert's equation for elliptical transfer (for proof see Mengali p. 187)**

$$(t_2 - t_1) = \frac{a^{\frac{3}{2}}}{\sqrt{\mu}}[\alpha - \beta - (\sin\alpha - \sin\beta)]$$

$$\begin{cases} \sin\dfrac{\alpha}{2} = \sqrt{\dfrac{r_1 + r_2 + c}{4a}} \\ \sin\dfrac{\beta}{2} = \sqrt{\dfrac{r_1 + r_2 - c}{4a}} \end{cases}$$

$$s = \frac{r_1 + r_2 + c}{2}$$

$$\begin{cases} \sin\dfrac{\alpha}{2} = \sqrt{\dfrac{s}{2a}} \\ \sin\dfrac{\beta}{2} = \sqrt{\dfrac{s-c}{2a}} \end{cases}$$



- Known: r_1, r_2 and Δt. Want: orbital elements
  - The presence of trigonometric functions makes the solution ambiguous.
  - Kepler's equation can be used to solve for the semi major axis a if we knew the orbit..

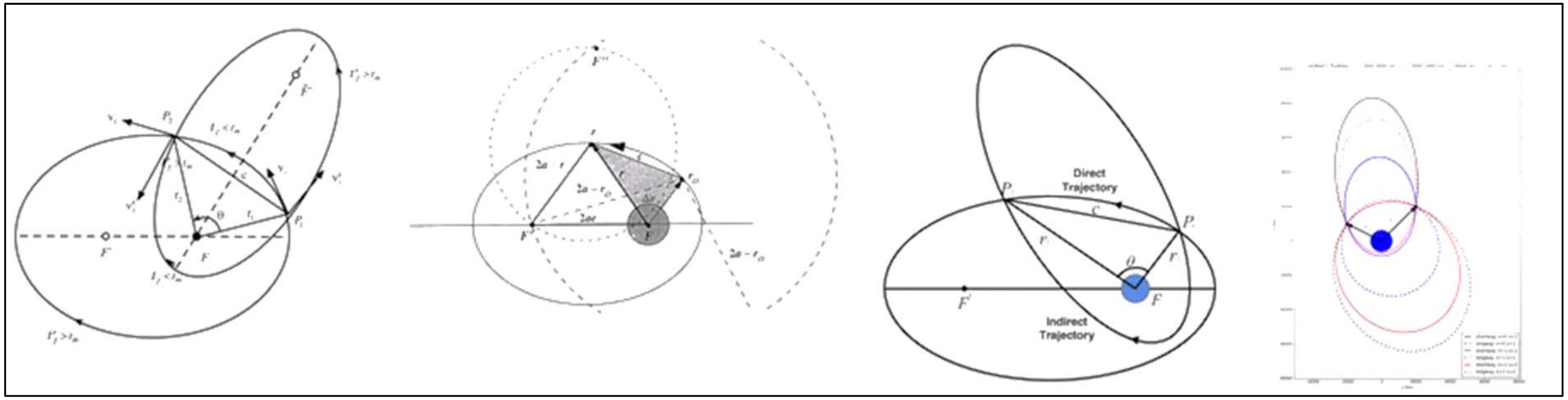
Interplanetary Flight

- Geometrical viewpoint: solving for the semimajor axis knowing time requires numerical approaches and the solution may not be unique.

$$\Delta t = (t_2 - t_1) = \frac{a^{\frac{3}{2}}}{\sqrt{\mu}}[\alpha - \beta - (\sin\alpha - \sin\beta)] \qquad \cos\Delta\theta = \frac{\boldsymbol{r}_1 \cdot \boldsymbol{r}_2}{r_1 r_2}$$

- Knowing the two positions we have the true anomaly change $\Delta\theta$ (or $360^0 - \Delta\theta$) and the orbital plane (since we know the focus of the orbit), which is not sufficient to find the orbit.

- We need to find the apparent focus or the semimajor axis



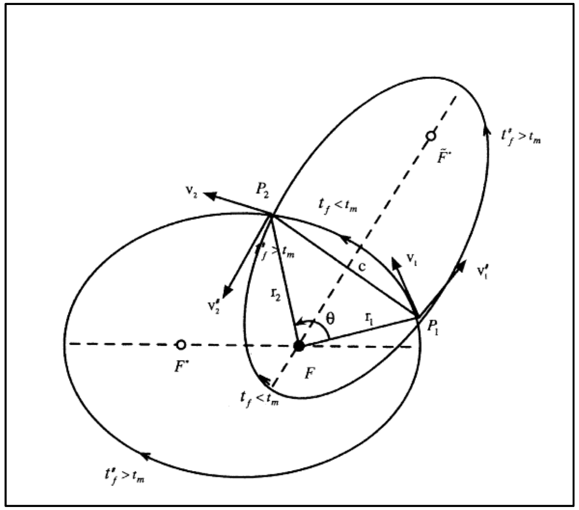
- Assume we solved for a from Δt:

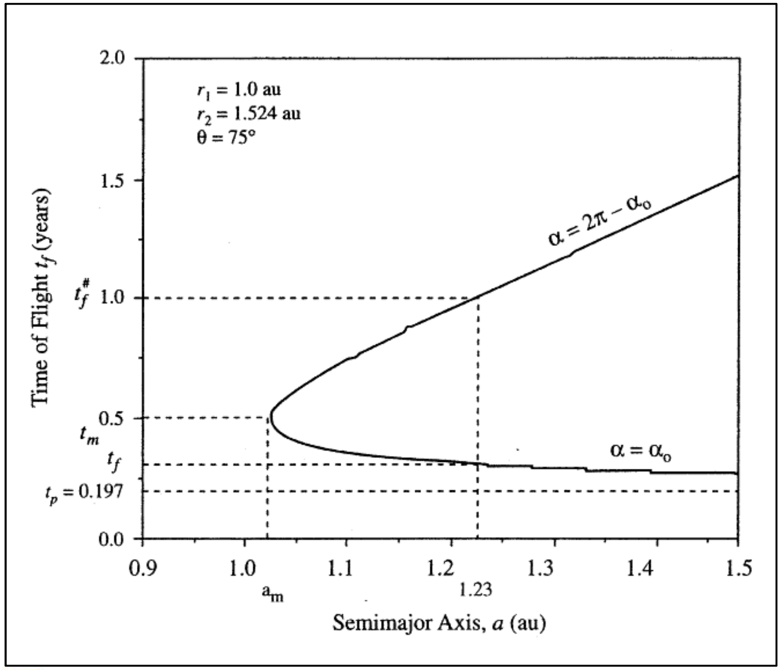
Interplanetary Flight

- Recall:

$$c = \left\| r_2 - r_1 \right\|$$

$$s = \frac{r_1 + r_2 + c}{2}$$ Half perimeter

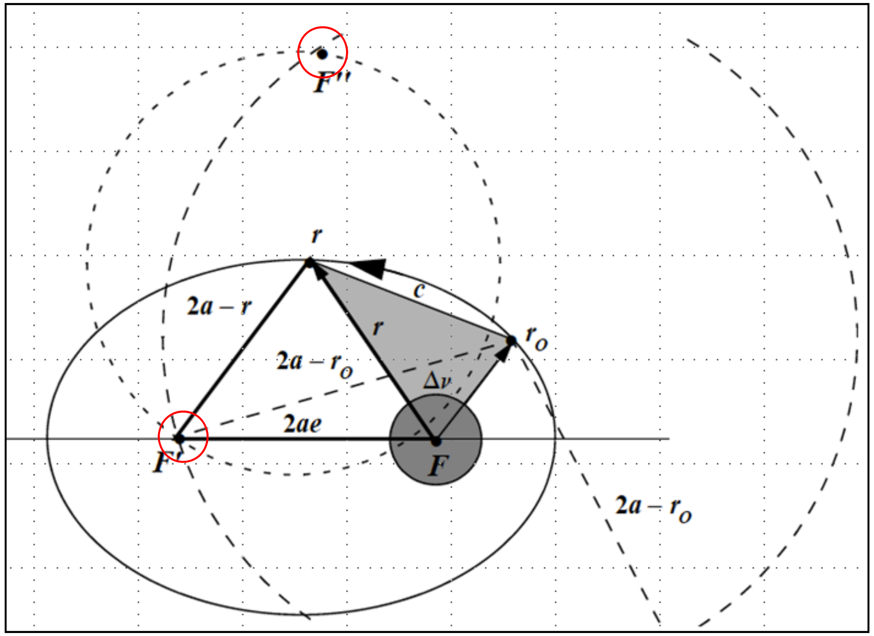




- Example of solution: In some cases there is no solution, one solution, two solutions, an extremum solution

- Use properties of ellipse: at any point the sum of the distances from the foci is always equal to $2a$.

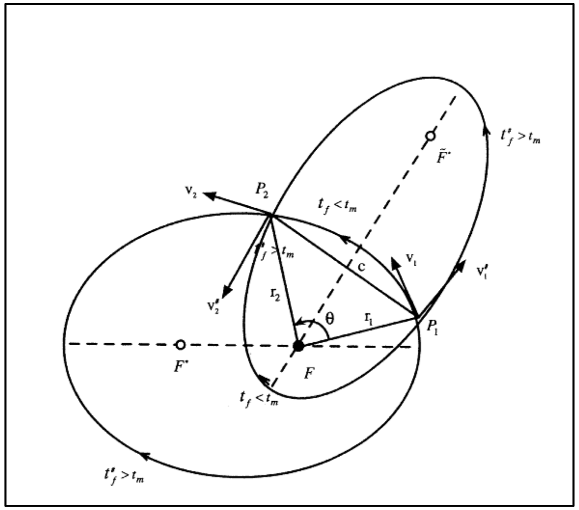
$$r_1 + r_2 = 2a$$



Circle of radius $2a - r_1$ about $\vec{r}_1$.

Circle of radius $2a - r_2$ about $\vec{r}_2$.

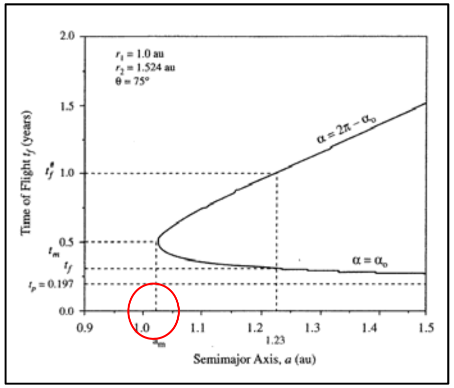
The **Intersection** of these circles gives the two possible locations of the hidden focus, F' or F''



- The **Short Way** Corresponds to the small Δt solution.
- The **Long Way** Corresponds to the large Δt solution.

In addition, the direction of travel along each ellipse can be reversed to obtain the **Retrograde Path**

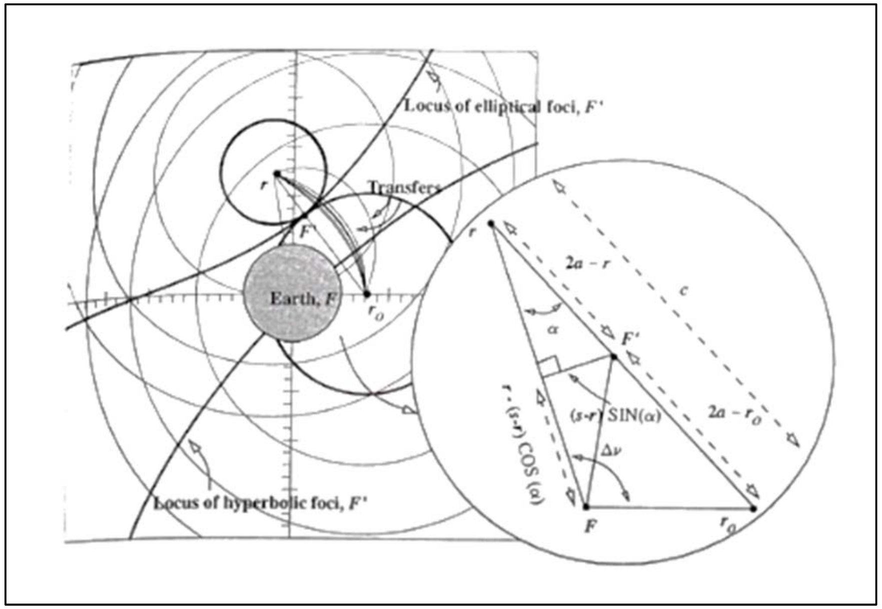
- Minimum energy transfer:
  - Corresponds to the smallest semimajor axis



Note the focal locations vary continuously as we change a.

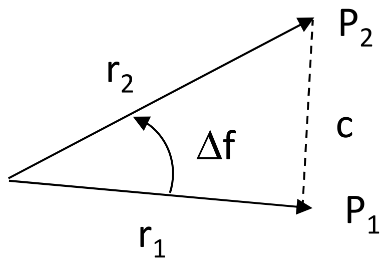
- As we vary a, the set of foci points F' and F'' form a *hyperbola*.
- As a increases, the two possible foci get farther apart.
- At a_{min}, The foci F' and F'' coincide.

  This is the minimum energy transfer ellipse



- $a_{\min} = \frac{r_1+r_2+c}{4}$ where $c = \|\vec{r}_2 - \vec{r}_1\|$.
- The minimum energy transfer yields the smallest a for which it is possible to have the two points on the same orbit.
- Hidden focus for the minimum energy transfer lies on the line between $\vec{r}_1$ and $\vec{r}_2$.
- This is NOT the Hohman transfer.

- **Trace**: Solution of Lambert's problem (elliptical case)



- From Kepler's Equation:

$$\sqrt{\frac{\mu}{a^3}}(t_2 - t_1) = (E_2 - e\sin E_2) - (E_1 - e\sin E_1)$$

- From elliptical orbit property:

$$r = a(1 - e\cos E) \Rightarrow (r_1 + r_2) = a[2 - e(\cos E_1 + \cos E_2)]$$

- Let:

$$E_P \triangleq \frac{E_2 + E_1}{2}; E_M \triangleq \frac{E_2 - E_1}{2} \Rightarrow note : E_M > 0 \;\Rightarrow (r_1 + r_2) = 2a(1 - e\cos E_P \cos E_M)$$

- From geometry of the ellipse, with the x axis along the major axis, we can compute the chord c:

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = \ldots = 4a^2 \sin^2 E_M (1 - e^2 \cos E_P)$$

- Let: $\cos\xi = e\cos E_P \; (e < 1, \xi > 0)$

- Define the angles:

$$\begin{cases} \alpha \triangleq \xi + E_M \\ \beta \triangleq \xi - E_M \end{cases}$$

- We obtain:

$$\begin{cases} r_1 + r_2 + c = 4a\sin^2\dfrac{\alpha}{2} \\ r_1 + r_2 - c = 4a\sin^2\dfrac{\beta}{2} \end{cases}$$

- Note that $\alpha > 0$ but $\beta < \alpha$ can be negative

- So Kepler's equation can be rewritten as:

$$\sqrt{\mu}(t_2 - t_1) = \sqrt{a^3}\left[(E_2 - E_1) - e(\sin E_2 - \sin E_1)\right] = 2\sqrt{a^3}\left[E_M - \sin E_M e\cos E_P\right]$$

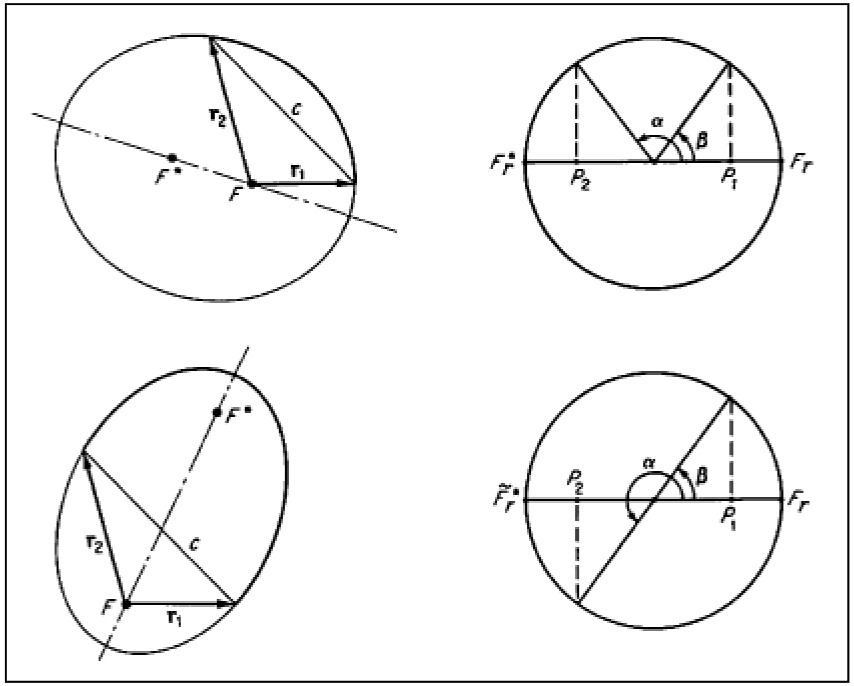
- using:

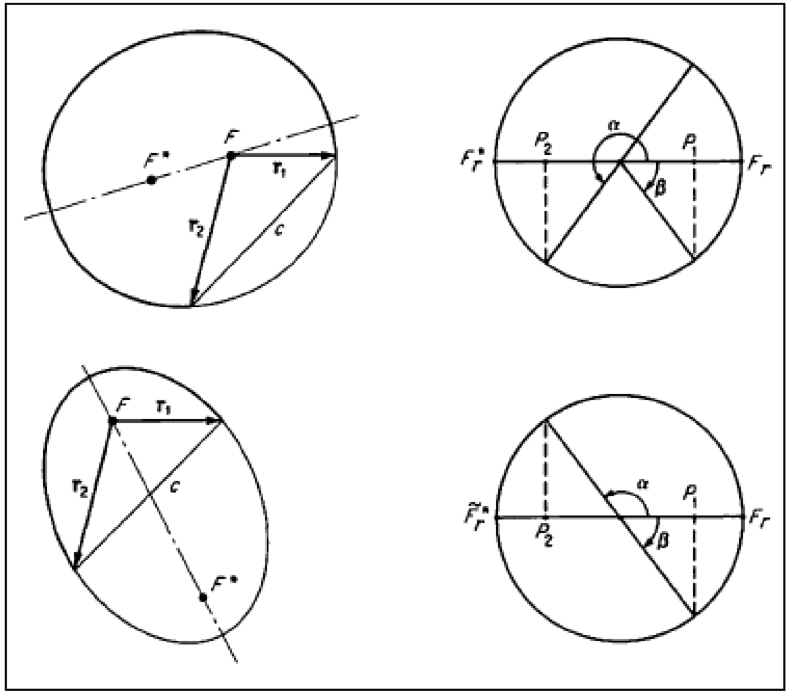
$$\begin{cases} E_2 - E_1 = 2E_M \\ \alpha - \beta = 2E_M \end{cases}$$

$$(t_2 - t_1) = \frac{a^{\frac{3}{2}}}{\sqrt{\mu}}[\alpha - \beta - (\sin\alpha - \sin\beta)]$$

$$\begin{cases} \sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{r_1 + r_2 + c}{4a}} \\ \sin\dfrac{\beta}{2} = \pm\sqrt{\dfrac{r_1 + r_2 - c}{4a}} \end{cases}$$

- The uncertainty in the values of α and β is due to the possible ellipses with Focus F and semi major axis a.
- Example of possible ellipses for given pairs of α and β angles:





- Since the positions are known, we can use the Lagrange coefficients to determine the velocities.

$$\begin{aligned} r_2(t_2) &= F r_1(t_1) + G v1(t_1) \\ v_2(t_2) &= \dot{F} r_1(t_1) + \dot{G} v_1(t_1) \end{aligned} \qquad \mathbf{v}_1 = \frac{1}{g}(\mathbf{r}_2 - f\,\mathbf{r}_1) \qquad \mathbf{v}_2 = \dot{f}\,\mathbf{r}_1 + \frac{\dot{g}}{g}(\mathbf{r}_2 - f\,\mathbf{r}_1) = \frac{\dot{g}}{g}\mathbf{r}_2 - \frac{f\dot{g} - \dot{f}g}{g}\mathbf{r}_1$$

$$f\dot{g} - \dot{f}g = 1 \qquad \mathbf{v}_2 = \frac{1}{g}(\dot{g}\,\mathbf{r}_2 - \mathbf{r}_1)$$

$$\boxed{\begin{aligned} f &= 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\theta) \\ g &= \frac{r r_0}{h}\sin\Delta\theta \\ \dot{f} &= \frac{\mu}{h}\,\frac{1-\cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1-\cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right] \\ \dot{g} &= 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\theta) \end{aligned}}$$

- Orbital elements can then be found from (r_1, v_1) or (r_2, v_2), see Curtis Ch. 5, section 3.

Interplanetary Flight

There are several ways to solve Lambert's Equation

- Newton Iteration
  - More Complicated than Kepler's Equation
- Series Expansion
  - Probably the easiest...
- Bisection
  - Relatively Slow, but easy to understand
  - Only works for *monotone* functions.

Define $g(a) = \sqrt{\frac{a^3}{\mu}}\,(\alpha - \beta - (\sin\alpha - \sin\beta))$.

Root-Finding Problem:

Find a :

such that $g(a) = \Delta t$

Bisection Algorithm:

1. Choose $a_{\min} = \frac{s}{2} = \frac{r_0 + r + c}{4}$
2. Choose $a_{\max} >> a_{\min}$
3. Set $a = \frac{a_{\max} + a_{\min}}{2}$
4. If $g(a) > \Delta t$, set $a_{\min} = a$
5. If $g(a) < \Delta t$, set $a_{\max} = a$
6. Goto 3

This is guaranteed to converge to the unique solution (if it exists).

- We assume *Elliptic* solutions.

ALGORITHM 5.2

To solve Lambert's problem use the MATLAB implementation that appears in Appendix D.25. Given $\mathbf{r}_1$, $\mathbf{r}_2$, and Δt, the steps are as follows:

1. Calculate r_1 and r_2 using Eq. (5.24).
2. Choose either a prograde or a retrograde trajectory and calculate $\Delta\theta$ using Eq. (5.26).
3. Calculate A in Eq. (5.35).
4. By iteration, using Eqs. (5.40), (5.43), and (5.45), solve Eq. (5.39) for z. The sign of z tells us whether the orbit is a hyperbola ($z < 0$), parabola ($z = 0$), or ellipse ($z = 0$).
5. Calculate y using Eq. (5.38).
6. Calculate the Lagrange f, g, and $\dot{g}$ functions using Eqs. (5.46a), (5.46b), (5.46c), and (5.46d).

- Similar expressions can be found for parabolic and hyperbolic transfer orbits between two points P_1 and P_2.

  - **Lambert's equation for hyperbolic transfer**

$$(t_2 - t_1) = \frac{a^{\frac{3}{2}}}{\sqrt{\mu}}[\sinh\alpha - \sinh\beta - (\alpha - \beta)]$$

$$\begin{cases} \sinh\dfrac{\alpha}{2} = \sqrt{\dfrac{r_1 + r_2 + c}{4a}} = \sqrt{-\dfrac{s}{2a}} \\ \sinh\dfrac{\beta}{2} = \sqrt{\dfrac{r_1 + r_2 - c}{4a}} = \sqrt{-\dfrac{s-c}{2a}} \end{cases}$$

- **Example:**

  The position of an earth satellite is first determined to be $\boldsymbol{r_1}$ = $5{,}000\mathbf{i}+ 10{,}000\mathbf{j}+2{,}100\mathbf{k}$ (Km). After one hour the position vector is $\boldsymbol{r_2}$ = $-14{,}600\mathbf{i}+2{,}500\mathbf{j}+7{,}000\mathbf{k}$. Determine the orbital elements and find the perigee altitude and the time since perigee passage of the first sighting (t_1).

  1. Select a prograde trajectory. The angle between the two positions is given by

$$\Delta\theta = \cos^{-1}\frac{r_1 \times r_2}{r_1 \cdot r_2} = 100.29^0$$

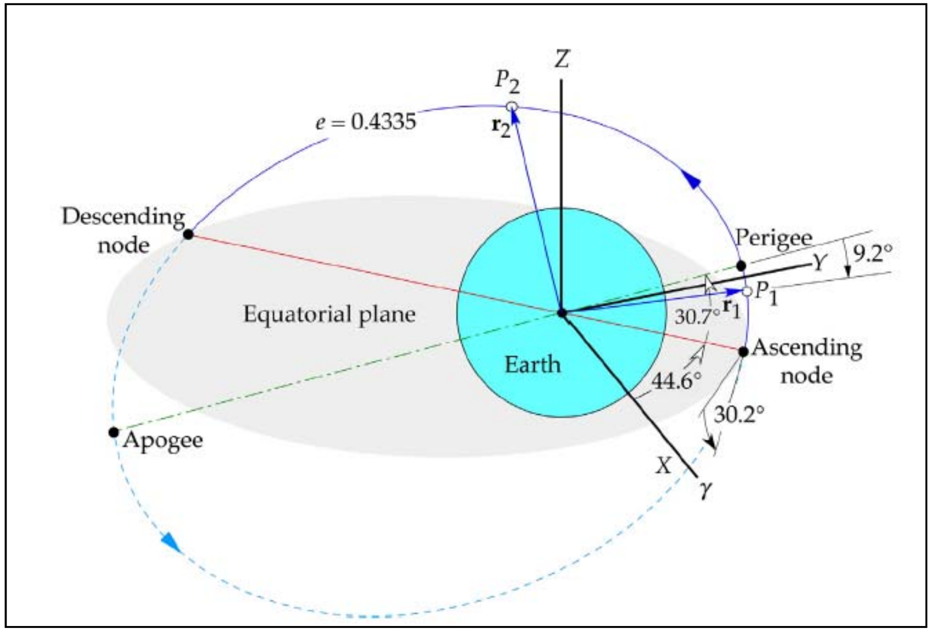
2. Compute iteratively initial and final velocities using a Lambert's problem solver

$$\mathbf{v}_1 = -5.9925\hat{\mathbf{I}} + 1.9254\hat{\mathbf{J}} + 3.2456\hat{\mathbf{K}}\ (\text{km})$$

$$\mathbf{v}_2 = -3.3125\hat{\mathbf{I}} - 4.1966\hat{\mathbf{J}} - 0.38529\hat{\mathbf{K}}\ (\text{km})$$

3. Compute orbital elements from position and velocity data

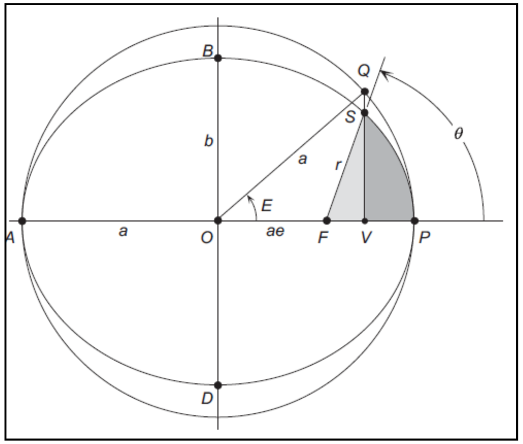
$$h = 80{,}470\ \text{km}^2/\text{s}$$
$$a = 20{,}000\ \text{km}$$
$$e = 0.4335$$
$$\Omega = 44.60°$$
$$i = 30.19°$$
$$\omega = 30.71°$$
$$\theta_1 = 350.8°$$



$$r_p = \frac{h^2}{\mu}\frac{1}{1+e\cos(0)} = \frac{80{,}470^2}{398{,}600}\frac{1}{1+0.4335} = 11{,}330\ \text{k}$$

Therefore, the perigee altitude is $11330 - 6378 = \boxed{4952\ \text{km}}$.

4. Compute the Eccentric Anomaly E_1



$$\tan\frac{E}{2} = \sqrt{\frac{1-e^2}{1+e}}\tan\frac{\theta}{2} \Rightarrow E_1 = -0.1007\, rad$$

5. Compute the Mean Anomaly M_1 from Kepler's Equation

$$M_1 = E_1 - e\sin E_1 = -0.05715\, rad$$

5. Compute the time of periapsis passage

$$t_p = \frac{\sqrt{\frac{p^3}{\mu}}}{(1+e\cos\theta)^2}\theta_p = -256.1\sec$$

- The minus sign means that there are 256.1 seconds until perigee encounter after the first sighting

- **Example:** A meteoroid is sighted at an altitude of 267,000 km. After 13.5 h and a change in true anomaly of 5°, the altitude is observed to be 140,000 km. Calculate the perigee altitude and the time to perigee after the second sighting.

P_1: $r_1 = 6378 + 267{,}000 = 273{,}378\,\text{km}$

P_2: $r_2 = 6378 + 140{,}000 = 146{,}378\,\text{km}$

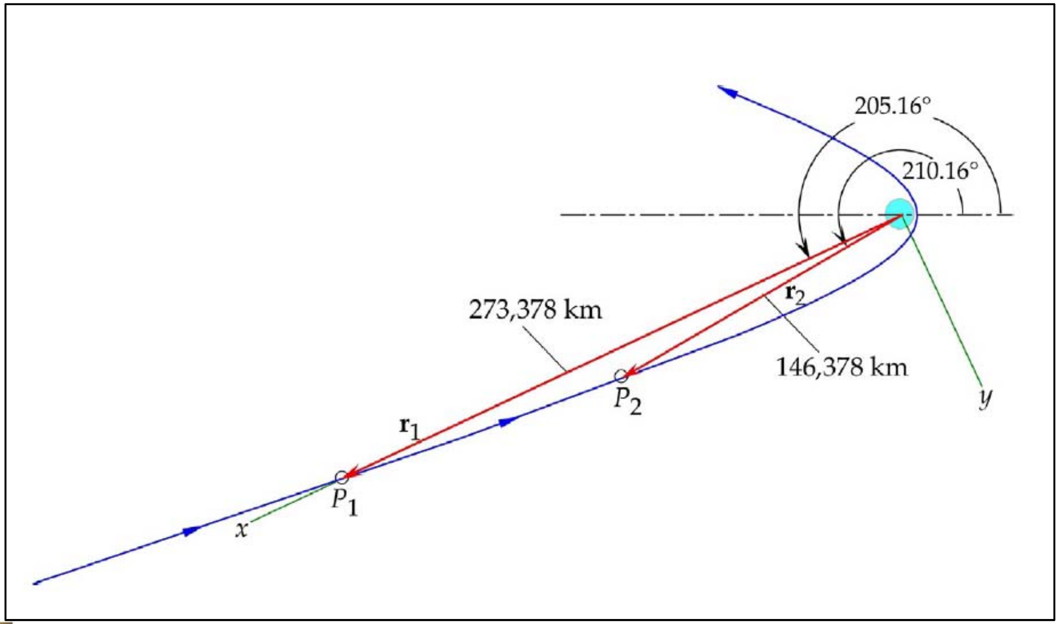
$\Delta t = 13.5 \times 3600 = 48{,}600\,\text{s}$

$\Delta\theta = 5°$

$\mathbf{r}_1 = r_1\hat{\mathbf{i}} = 273{,}378\hat{\mathbf{i}}\,(\text{km})$

$\mathbf{v}_1 = \frac{1}{g}(\mathbf{r}_2 - f\mathbf{r}_1) = -2.4356\hat{\mathbf{i}} + 0.26741\hat{\mathbf{j}}\,(\text{km/s})$

$h = 73{,}105\,\text{km}^2/\text{s}$ $\quad e = 1.0506$ $\quad \theta_1 = 205.06°$



$$r_p = \frac{h^2}{\mu}\frac{1}{1 + e\cos(0)} = 6538.2\,\text{km}$$

$$\theta_2 = \theta_1 + 5° = 210.16°$$

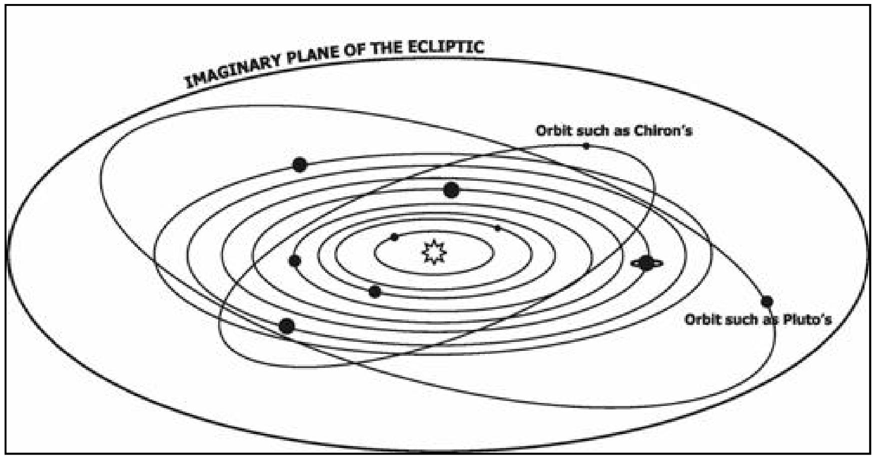
$$t_2 = \frac{M_{h_2} h^3}{\mu^2 (e^2 - 1)^{3/2}} = -38{,}396\,\text{s}$$

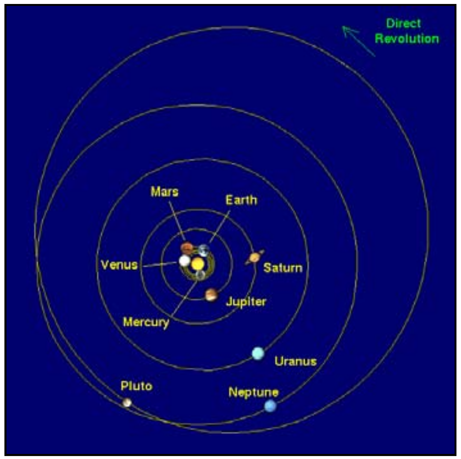
Interplanetary Flight

- **Preliminary interplanetary Mission planning:** The design of an interplanetary mission is a complex – multi year project, and the mission itself may last many decades. Elements such as orbit perturbations, and multiple body gravitational attraction must be taken into account, among other parameters outside of the scope of this course.

- **Assumptions:**
  - Coplanar trajectories (except for special cases)
  - Earth (home planet) Departure – hyperbolic escape trajectory
  - Target Planet Arrival – hyperbolic capture trajectory
  - Heliocentric Transfer8S9
  - Planetary Assist
  - Landing

- Coplanar trajectories

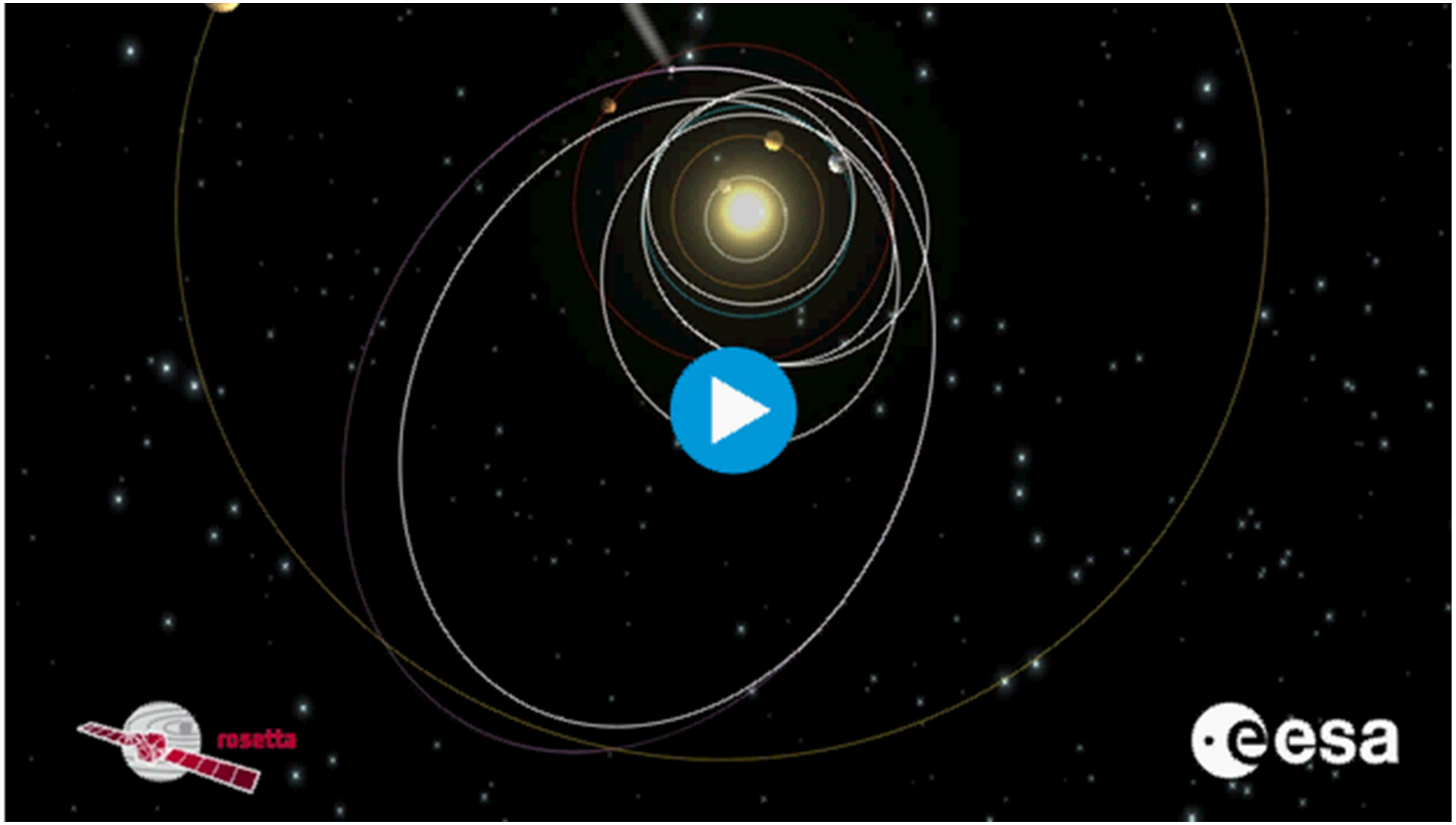




Solar System

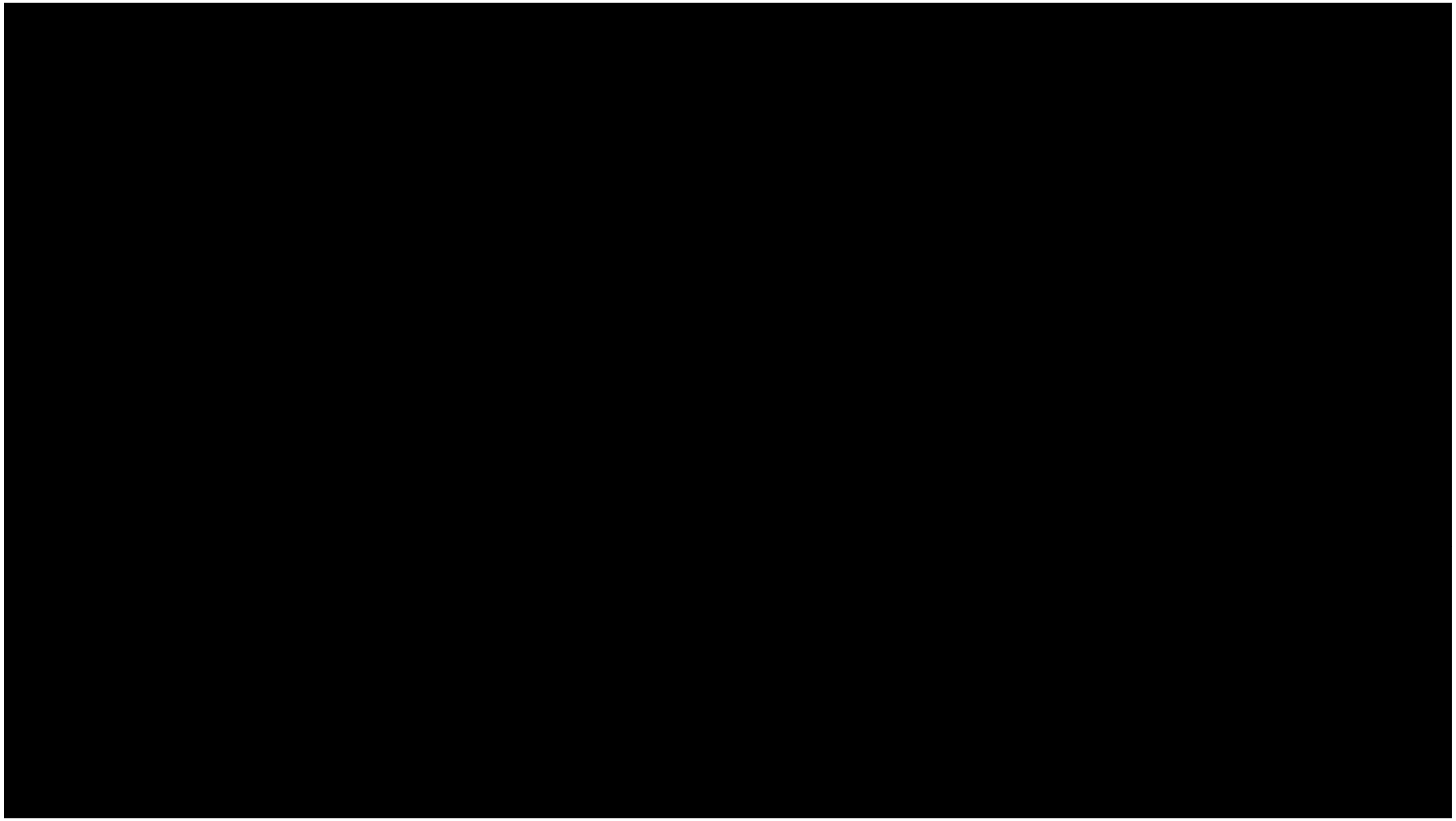
❑ Interplanetary preliminary Mission Analysis

Comet 67P / Churyumov – Gerasimenko encounter



Interplanetary Flight

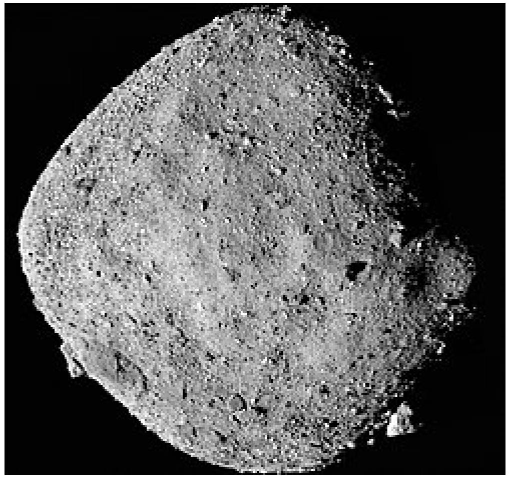
https://www.youtube.com/watch?v=ppdEzii5aDs

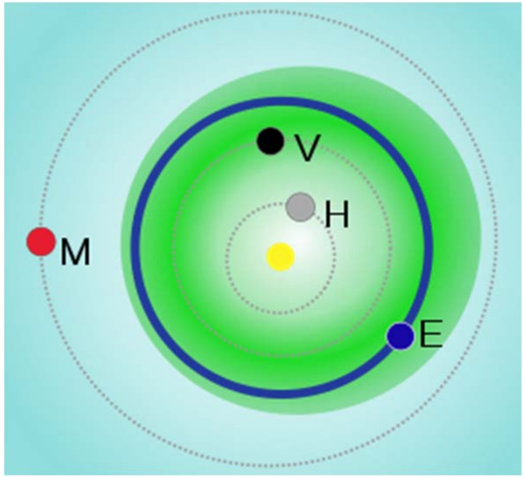


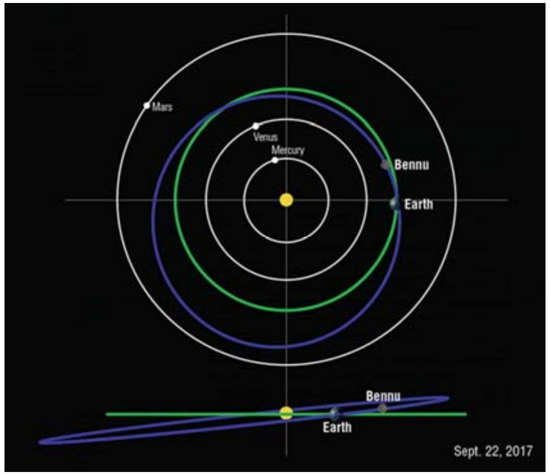
Interplanetary Flight

https://www.youtube.com/watch?v=NYGHbl_esgw

https://www.nasa.gov/press-release/nasa-s-osiris-rex-spacecraft-successfully-touches-asteroid



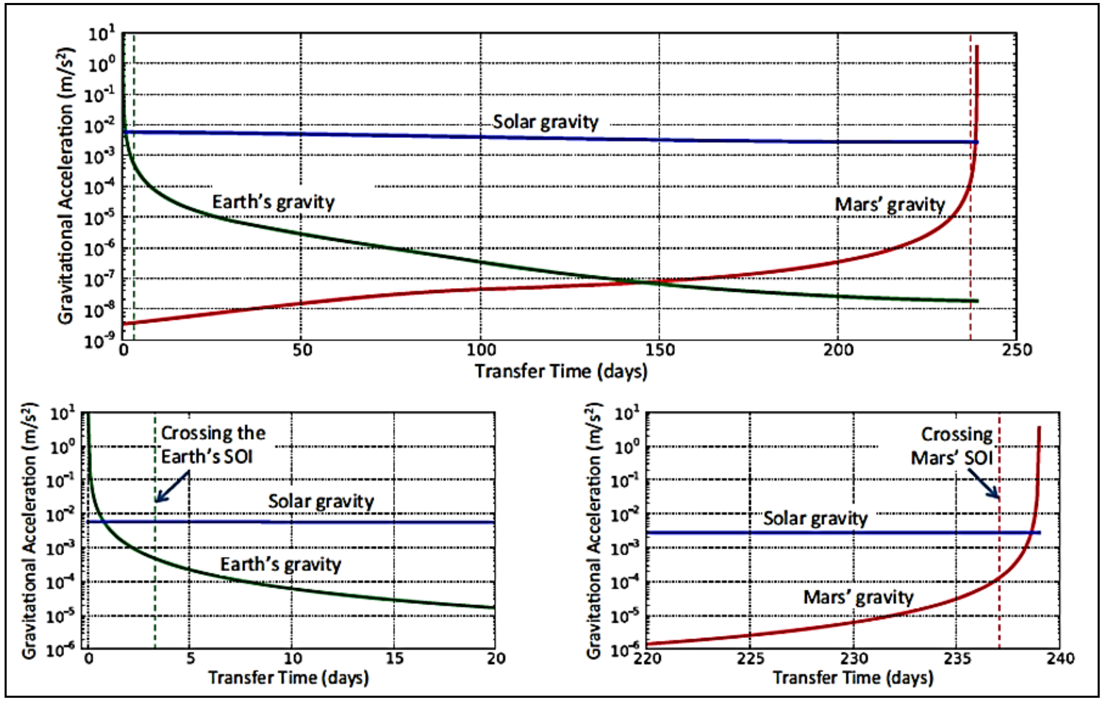




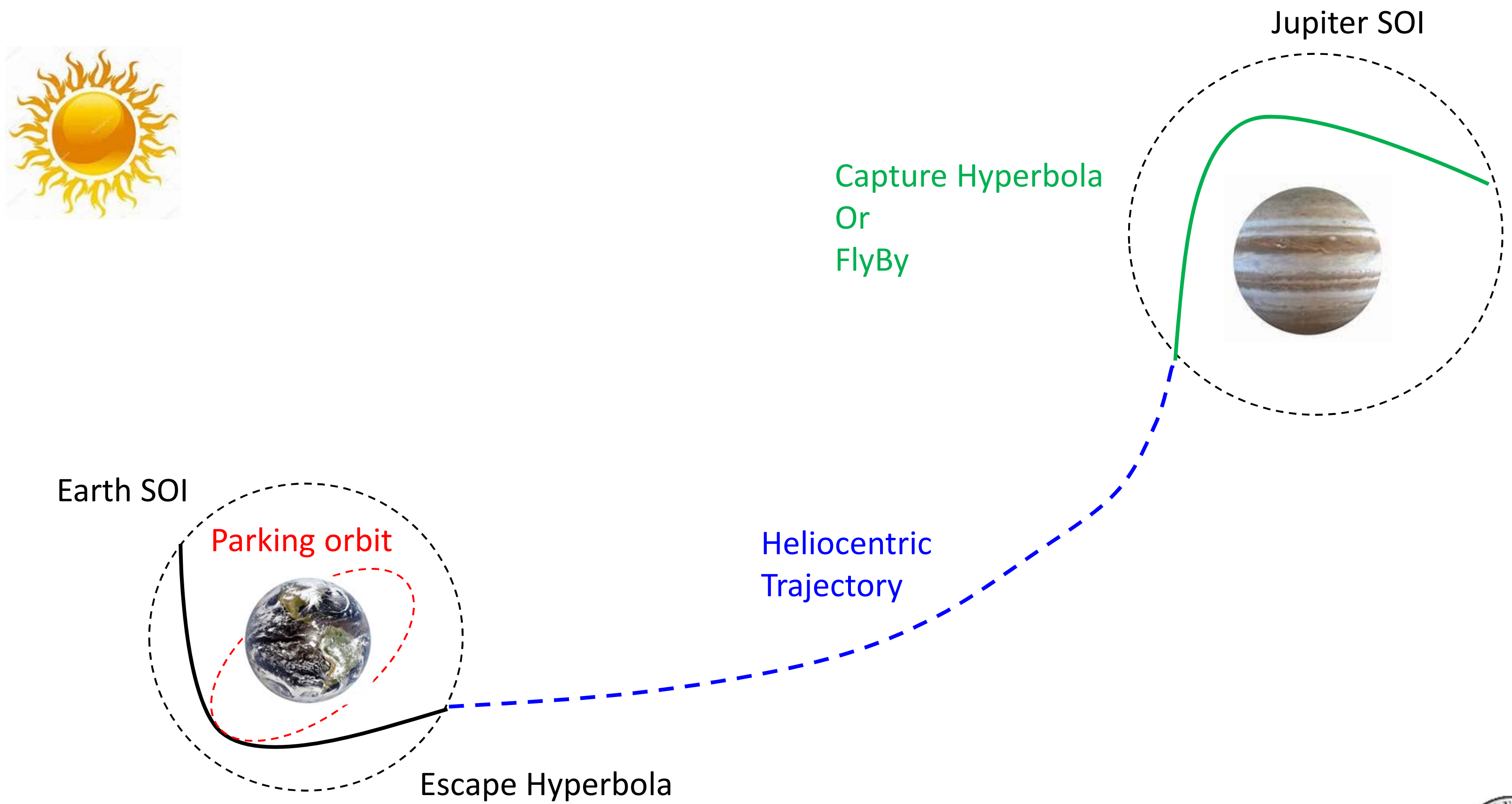
- OSIRIS-REx launched from Cape Canaveral, Florida, on an Atlas V 411 rocket on Sept. 8, 2016. In September 2017, OSIRIS-REx used Earth's gravitational field to assist it on its way to Bennu. On Dec, 3, 2018, OSIRIS-REx will use an array of small rocket thrusters to match the velocity of Bennu and rendezvous with the asteroid. n March 2021, the window for departure from the asteroid will open, and OSIRIS-REx will begin its return journey to Earth, arriving two and a half years later in September 2023. The sample return capsule will separate from the spacecraft and enter the Earth's atmosphere.

- **Patched Conics Method:** The trajectory is composed of Keplerian orbits (or orbit sections), with the main body given by the planets of interest and the Sun.

- **Sphere of Influence:** A sphere of influence (SOI) is the oblate-spheroid-shaped region around a celestial body where the primary gravitational influence on an orbiting object is that body. This is usually used to describe the areas in the Solar System where planets dominate the orbits of surrounding objects (such as moons), despite the presence of the much more massive (but distant) Sun. In a more general sense, the patched conic approximation is only valid within the SOI.

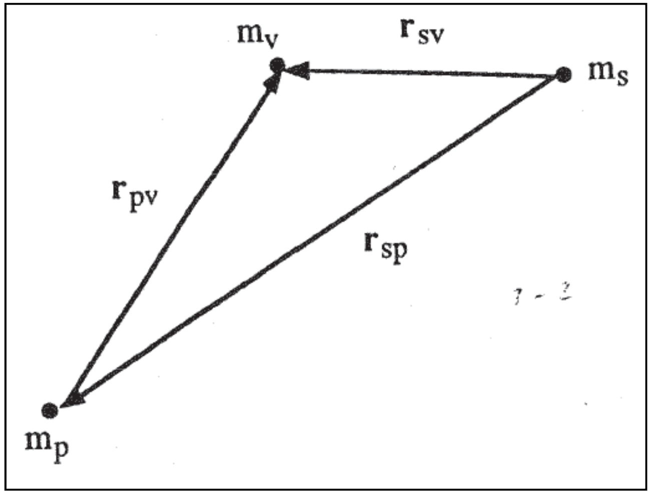


- **Patched Conics Method:** The trajectory is composed of Keplerian orbits (or orbit sections), with the main body given by the planets of interest and the Sun.



Interplanetary Flight

- The computation of the SOI is obtained by defining a 3 – body problem and treating the effect of the Sun as a perturbation, which may be neglected in proximity of the planet. For example, at its surface the Earth's gravitational force is over 1600 times greater than the Sun's.



1. Motion of a Spacecraft w.r.t. the Sun

$$\ddot{\boldsymbol{r}}_{sv} = -Gm_s \frac{\boldsymbol{r}_{sv}}{\left\|\boldsymbol{r}_{sv}\right\|^3} - Gm_p \left[\frac{\boldsymbol{r}_{pv}}{\left\|\boldsymbol{r}_{pv}\right\|^3} + \frac{\boldsymbol{r}_{sp}}{\left\|\boldsymbol{r}_{sp}\right\|^3}\right]$$

2. Motion of a Spacecraft w.r.t. the Planet

$$\ddot{\boldsymbol{r}}_{pv} = -Gm_p \frac{\boldsymbol{r}_{pv}}{\left\|\boldsymbol{r}_{pv}\right\|^3} - Gm_s \left[\frac{\boldsymbol{r}_{sv}}{\left\|\boldsymbol{r}_{sv}\right\|^3} - \frac{\boldsymbol{r}_{sp}}{\left\|\boldsymbol{r}_{sp}\right\|^3}\right]$$

- Secondary Body influence modeled as a disturbance

$$\ddot{\boldsymbol{r}}_s = \ddot{\boldsymbol{r}}_{central,s} + \ddot{\boldsymbol{r}}_{dist.,s}$$

$$\ddot{\boldsymbol{r}}_p = \ddot{\boldsymbol{r}}_{central,p} + \ddot{\boldsymbol{r}}_{dist.,p}$$

Definition 1.

An object is in the **Sphere of Influence**(SOI) of body 1 if

$$\frac{\|\ddot{\vec{r}}_{dist,1}\|}{\|\ddot{\vec{r}}_{central,1}\|} < \frac{\|\ddot{\vec{r}}_{dist,2}\|}{\|\ddot{\vec{r}}_{central,2}\|}$$

for any other body 2.

Sun Perspective: Lets group the forces as central and disturbing. Consider motion of a spacecraft relative to the sun:

$$\ddot{\vec{r}}_{sv} + \underbrace{Gm_s \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}}_{\text{Effect of sun on object}} = -Gm_p \left[\underbrace{\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}}_{\text{Effect of planet on object}} + \underbrace{\frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}}_{\text{Effect of planet on sun}} \right]$$

where p denotes planet, v denotes vehicles and s denotes sun.

The Central "Force" is

$$\ddot{\vec{r}}_{central,s} = -Gm_s \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}$$

The Disturbing "Force" is

$$\ddot{\vec{r}}_{dist,s} = \underbrace{-Gm_p \left[\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]}_{\text{Acceleration of object due to planet}}$$

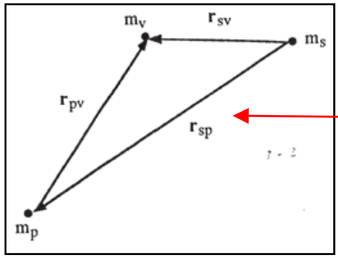
- For the sun-moon system, e.g., the vectors

$$\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} >> \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \cong 0$$

so

$$\frac{\ddot{\vec{r}}_{dist,s}}{\ddot{\vec{r}}_{central,s}} \cong \frac{m_p}{m_s} \frac{\|\vec{r}_{sv}\|^2}{\|\vec{r}_{pv}\|^2}$$

- So if $\|\vec{r}_{pv}\|$ is small and $\|\vec{r}_{sv}\|$ is big, the disturbing force dominates.



Acceleration of Sun due to Earth is negligible

Planet Perspective: The relative motion of the spacecraft with respect to the planet is

$$\ddot{\vec{r}}_{pv} + \underbrace{Gm_p \frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}}_{\text{Effect of planet on object}} = -Gm_s \left[\underbrace{\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}}_{\text{Effect of sun on object}} - \underbrace{\frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}}_{\text{Effect of sun on planet}} \right]$$

The Central "Force" for the planet is

$$\ddot{\vec{r}}_{central,p} = -Gm_p \frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}$$

The Disturbing "Force" for the planet is

$$\ddot{\vec{r}}_{dist,p} = \underbrace{-Gm_s \left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} - \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]}_{\text{Acceleration of object due to sun}}$$

- When the vehicle is near the planet, $\vec{r}_{sp} \cong \vec{r}_{sv}$ and hence

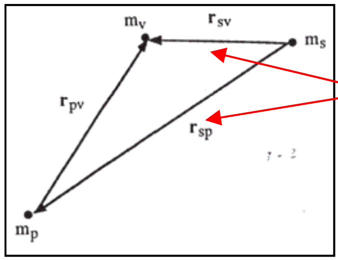
$$\frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \cong \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}$$

so $\ddot{\vec{r}}_{dist,p} \cong 0$ and

$$\frac{\ddot{\vec{r}}_{dist,p}}{\ddot{\vec{r}}_{central,p}} \cong \frac{m_s}{m_p} \cdot 0 \cong 0$$

and hence the relative size of the disturbance is small.

- Sphere of influence is based on the relative distance.



Acceleration of Earth and Moon due to Sun are about same

Interplanetary Flight

- The ratio between disturbing force to central force defines the size of the SOI

- For planets, an approximation for determining the SOI of a planet of mass m_p at distance d_p from the sun is:

$$\boxed{R_{SOI} \cong \left(\frac{m_p}{m_s}\right)^{2/5} d_p}$$

$$\frac{\|\ddot{\vec{r}}_{dist,p}\|}{\|\ddot{\vec{r}}_{central,p}\|} < \frac{\|\ddot{\vec{r}}_{dist,s}\|}{\|\ddot{\vec{r}}_{central,s}\|}$$

$$\frac{m_p}{m_s}\frac{\|\vec{r}_{sv}\|^2}{\|\vec{r}_{pv}\|^2} > \frac{m_s\left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} - \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}\right]}{m_p\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}} \cong \frac{m_s\left[\vec{r}_{sv} - \vec{r}_{sp}\right]}{m_p\frac{\vec{r}_{pv}\|\vec{r}_{sv}\|^3}{\|\vec{r}_{pv}\|^3}}$$

$$\frac{m_p^2}{m_s^2}\frac{\|\vec{r}_{sv}\|^5}{\|\vec{r}_{pv}\|^5} > \frac{\left[\vec{r}_{sv} - \vec{r}_{sp}\right]}{\vec{r}_{pv}} \cong 1$$

$$\frac{m_p^2}{m_s^2}\|\vec{r}_{sv}\|^5 > \|\vec{r}_{pv}\|^5$$

$$\|\vec{r}_{pv}\| < \left(\frac{m_p}{m_s}\right)^{2/5}\|\vec{r}_{sv}\|$$

Interplanetary Flight

Celestial Body	Equatorial Radius (*km*)	SOI Radius (*km*)	SOI Radius (*body radii*)
Mercury	2487	1.13×10^5	45
Venus	6187	6.17×10^5	100
Earth	6378	9.24×10^5	145
Mars	3380	5.74×10^5	170
Jupiter	71370	4.83×10^7	677
Neptune	22320	8.67×10^7	3886
Moon	1738	6.61×10^4	38

- The SOI of the Moon with respect to the Earth is inside of the SOI of the Earth with respect to the Sun.
- An interplanetary mission is a sequence of two – body problem conic sections as a first approximation.

Problem: Suppose we want to plan a lunar-lander mission. Determine the spheres of influence to consider for a patched-conic approach.

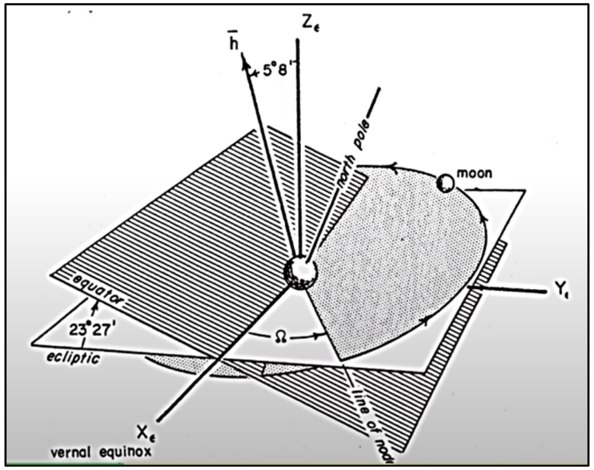
- The SOI of the earth is of radius 924,000km.
- The SOI of the moon is of radius 66,100km.

Solution: The moon orbits at a distance of 385,000km. The spacecraft will transition to the lunar sphere at distance

$$r = 385,000 - 66,100 = 318,900km$$

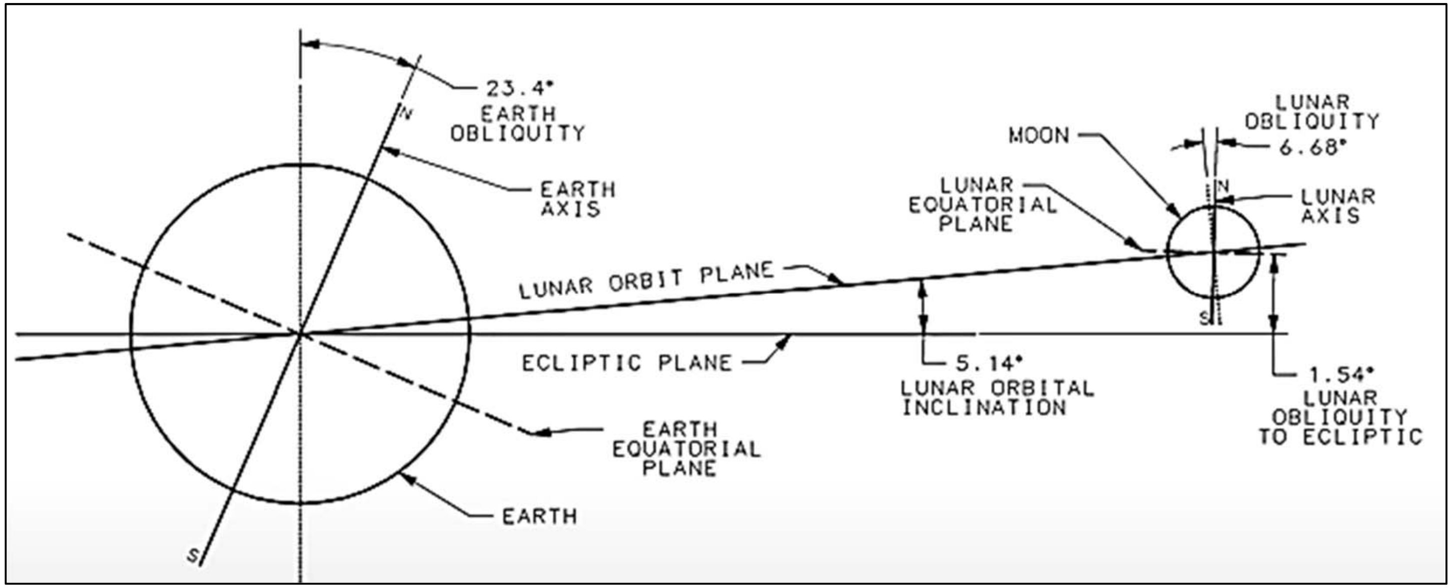
We will probably also need a plane change. A reasonable mission design is

1. Depart earth on a Hohmann transfer to radius $317,900$ km.
2. Perform inclination change near apogee.
3. Enter sphere of influence of the moon.
4. Establish parking orbit.

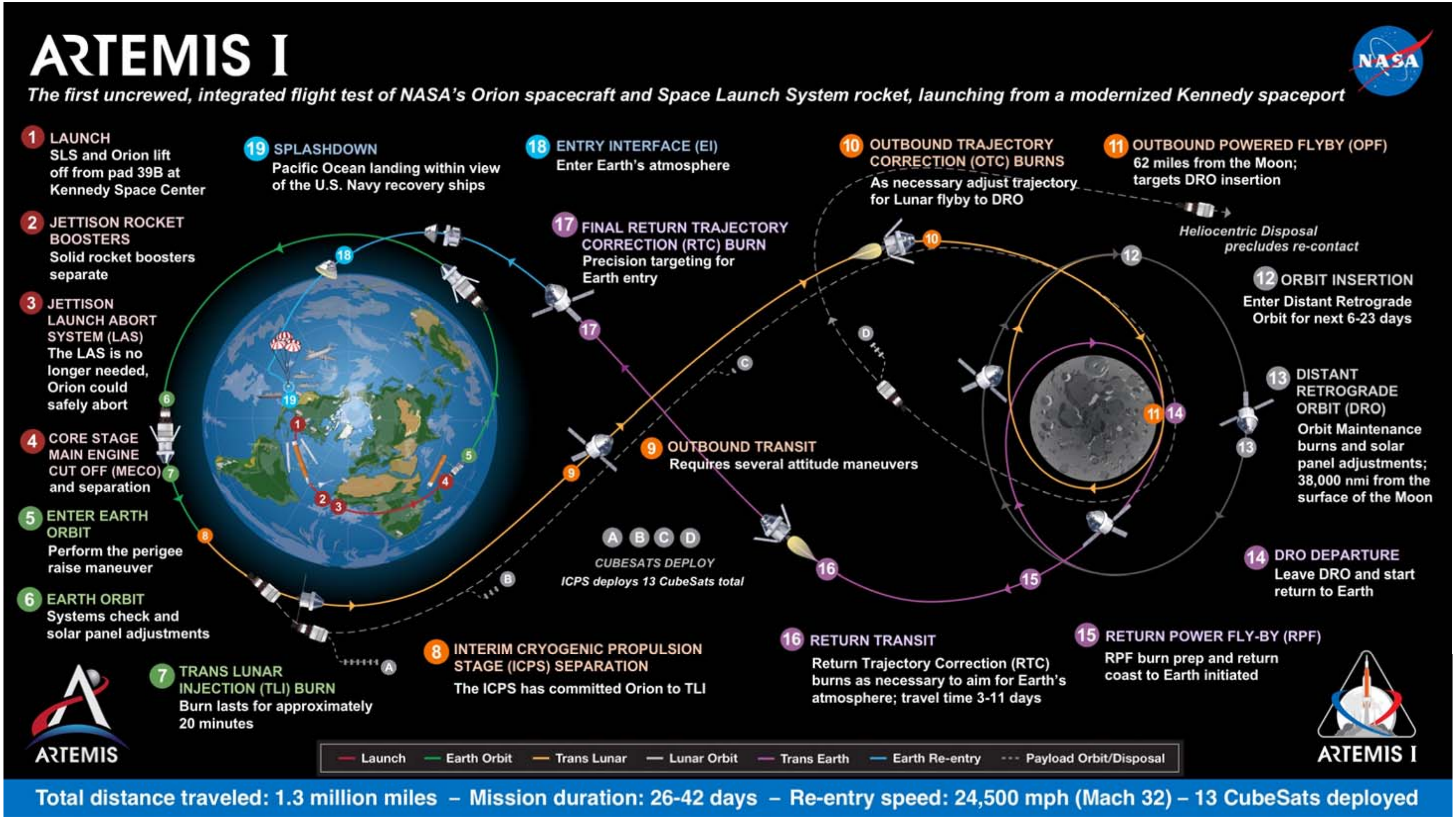


Why a Plane Change is needed.

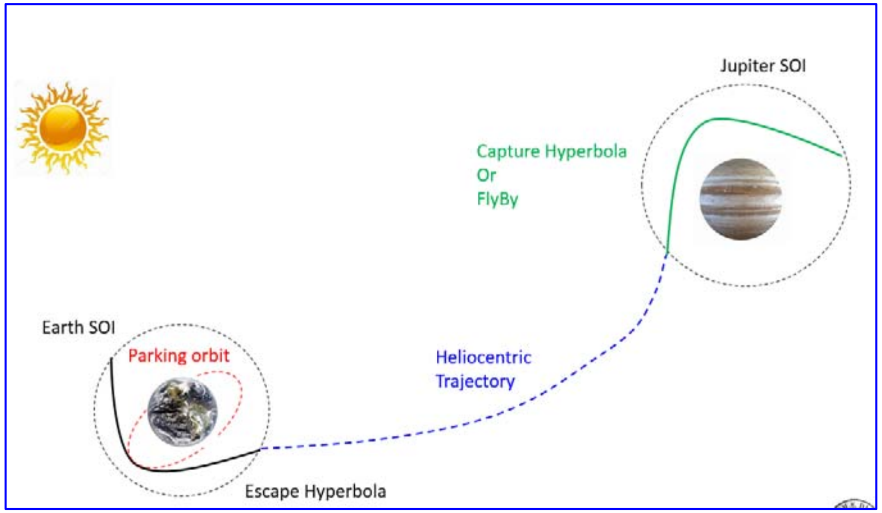
- Note that the lunar orbit is inclined at about $4.99°–5.30°$ to the ecliptic plane.
- The inclination of the lunar orbit is almost fixed with respect to the ecliptic.
- Not fixed relative to the equatorial plane (Saros cycle - Solar and $J2$).
- Inclination to equator varies $= 21.3° \pm 5.8°$ every 18 years.



Interplanetary Flight



Interplanetary Flight

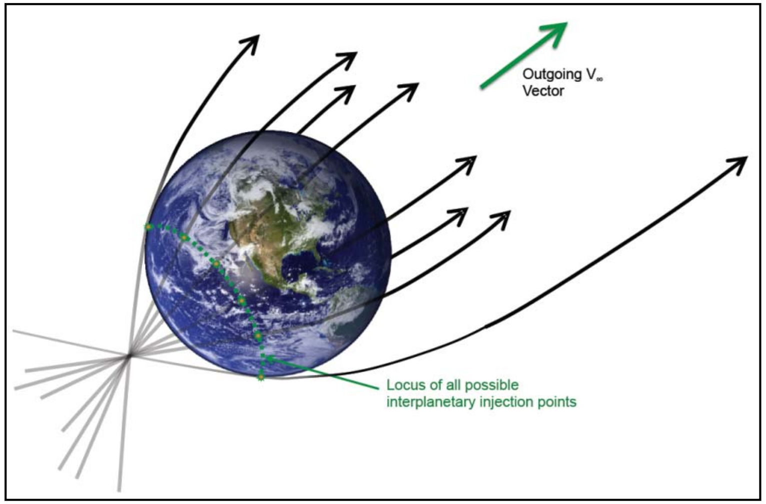


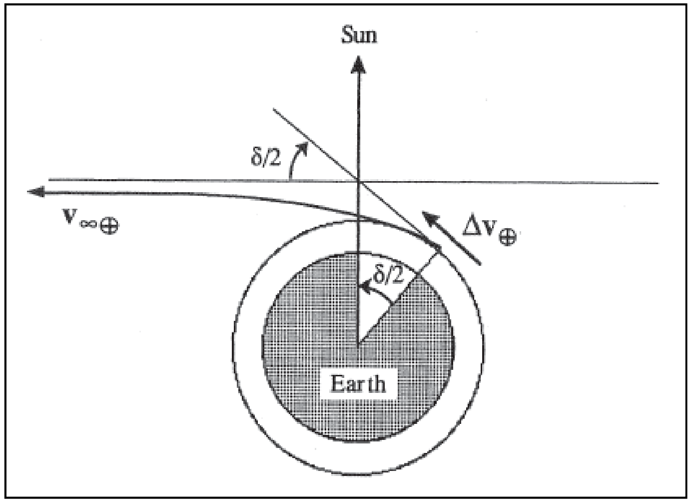
- Launch into a parking orbit
  - Possibly change inclination and RAAN depending on launch site and Heliocentric trajectories
- Leave planet with hyperbolic escape (P SOI)
  - Reach appropriate v_∞ according to Heliocentric section
  - Adjust timing to reach heliocentric section
  - Possibly plane change
- Heliocentric trajectory (Sun SOI)
  - Selected apriori as a function of TOF. DV, etc.
  - Use Lambert's problem and/or other transfers

- Capture hyperbolic trajectory (P SOI)
  - Possibly change plane as function of target parking orbit
- Select properties of hyperbolic capture based on target parking orbit
- Enter parking orbit

- Heliocentric trajectory
  - May include several segments achieved with intermediate firing
  - Each section may require solution of Lambert's problems
  - Possibly plane changes
- Intermediate gravity assists
  - Change specific energy wrt the Sun
  - Increase/decrease heliocentric speed and direction

- **Escape Trajectory:** A spacecraft needs to leave the departing planet (either at launch, or from a parking orbit) along a hyperbolic trajectory. We can assume that plane changes have been performed already

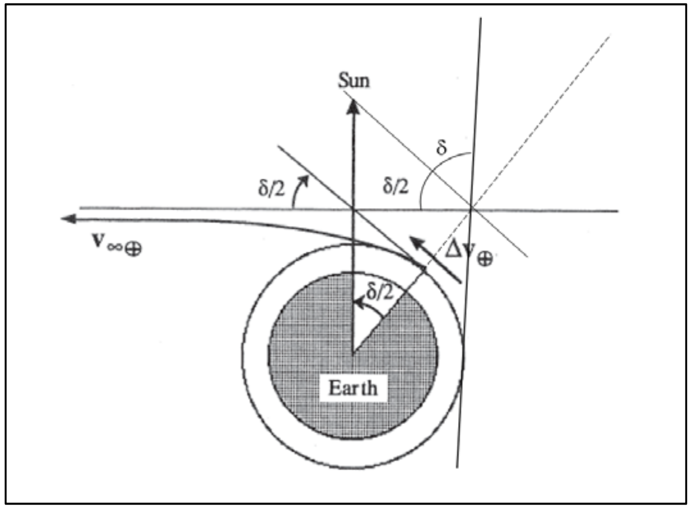


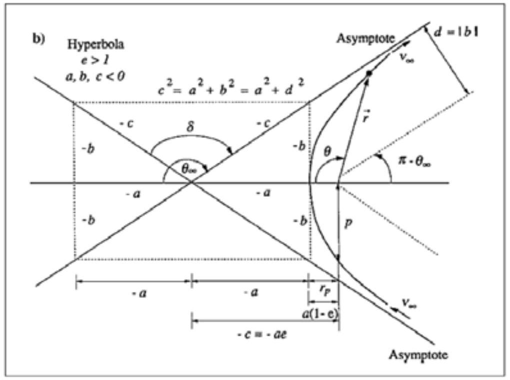


- For an elliptical parking orbit the escape is usually at the periapsis (highest speed) into the periapsis of the escape hyperbola
- At the end of the departing planet SOI, the spacecraft should have the correct speed of the Heliocentric transfer orbit
- The transfer time, and the departing time must be found according to Lambert's theorem, so that the spacecraft will reach the SOI of the target planet at the same time as the planet itself

Interplanetary Flight

- **Departing angle δ/2:**





- From property of hyperbola

$$\delta = 2\sin^{-1}\frac{1}{e}$$

- From energy compute semimajor axis

$$E = \frac{1}{2}v_\infty^2 = -\frac{\mu}{2a} > 0$$

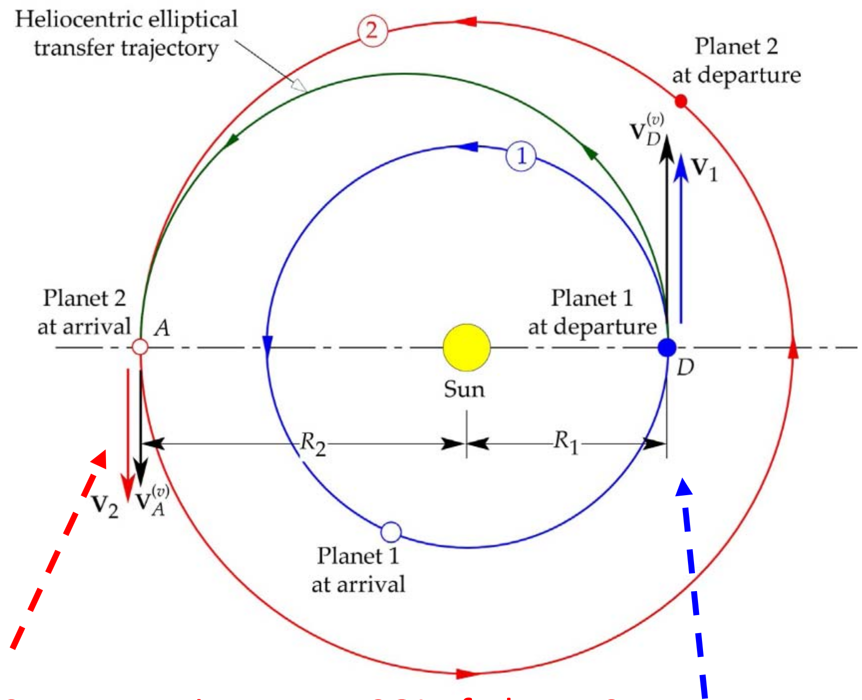
- At periapsis (assuming circular parking orbit)

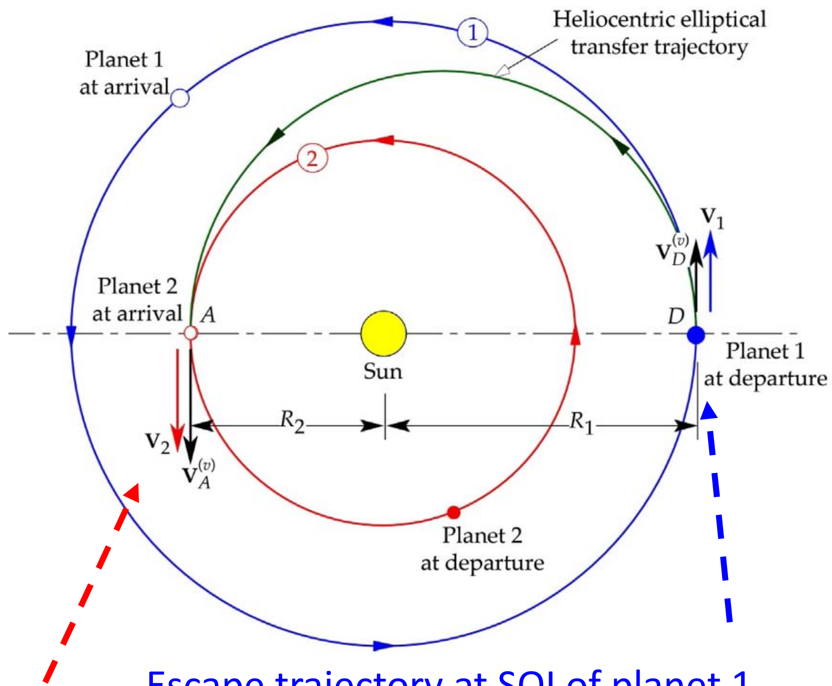
$$r_P = r_C = a(1-e) \Rightarrow \boldsymbol{e}$$

- From Earth – Sun vector the figure departing is before noon (if on the light side)

Interplanetary Flight

- Recall interplanetary Hohmann transfer trajectory





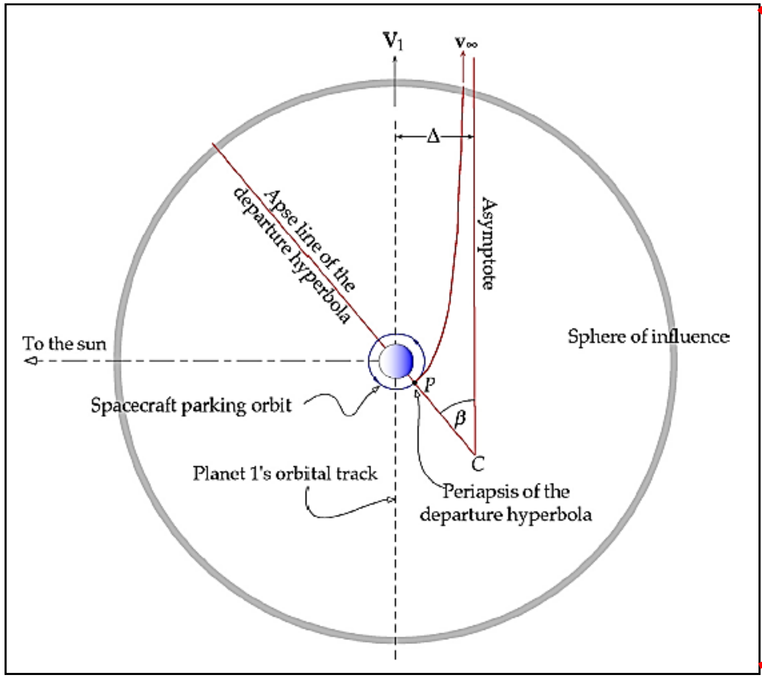
Capture trajectory at SOI of planet 2

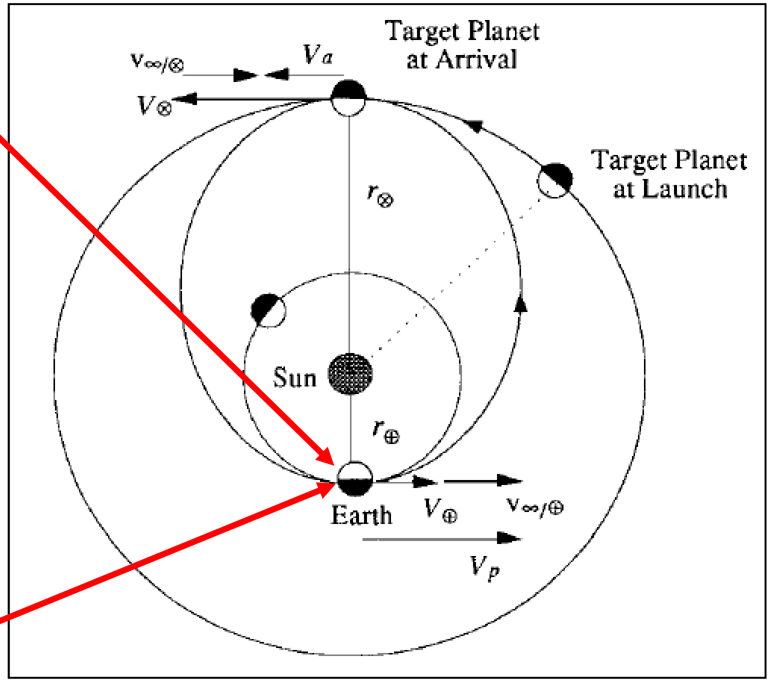
Escape trajectory at SOI of planet 1

Escape trajectory at SOI of planet 1

Capture trajectory at SOI of planet 2

- **Example:** Escape trajectory to enter a Hohmann transfer to Mars from Earth circular parking orbit





- Properties of Hohmann transfer orbit

$$a_{HOH} = \frac{r_{EARTH} + r_{MARS}}{2}$$

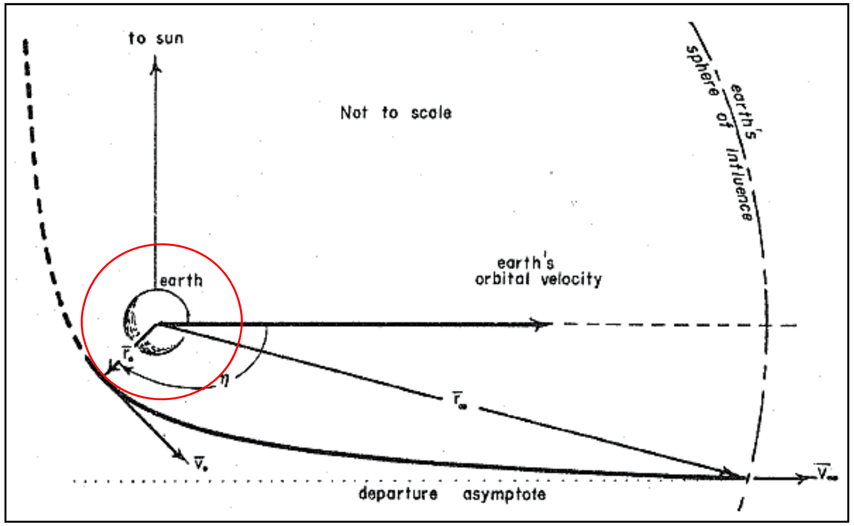
$$\begin{cases} v_P^{HOH} = \sqrt{\dfrac{\mu_{SUN}}{a_{HOH}} \cdot \dfrac{r_{MARS}}{r_{EARTH}}} \\ v_A^{HOH} = \sqrt{\dfrac{\mu_{SUN}}{a_{HOH}} \cdot \dfrac{r_{EARTH}}{r_{MARS}}} \end{cases}$$

- Excess hyperbolic speed

$$v_\infty^{EARTH} = v_P^{HOH} - v^{EARTH}$$

$$= v_P^{HOH} - \sqrt{\frac{\mu_{SUN}}{r_{EARTH}}}$$

- Properties of Escape Hyperbola (light side for inner planets, dark side for outer planets)



$$a_{HYP} = -\frac{\mu_{EARTH}}{\left(v_{\infty}^{EARTH}\right)^2} \qquad e_{HYP} \geq 1 - \frac{r_{PARK}^{EARTH}}{a_{HYP}}$$

- **Note:** the minimum energy required is when the periapsis of the hyperbola coincides with the parking orbit radius.

$$r_P^{HYP} = r_{PARK}^{EARTH} = -a_{HYP}(e_{HYP} - 1)$$

- Parking Orbit Speed:

$$v_{PARK}^{EARTH} = \sqrt{\frac{\mu_{EARTH}}{r_{PARK}^{EARTH}}}$$

- Speed at Escape Hyperbola Perigee (from specific Energy):

$$v_P^{HYP} = \sqrt{\frac{2\mu_{EARTH}}{r_P^{HYP}} - \left(v_{\infty}^{EARTH}\right)^2}$$

$$\boxed{\Delta v_1 = v_P^{HYP} - v_{PARK}^{EARTH}}$$

- Speed Increase (acceleration)

Interplanetary Flight

- **Example:** A spacecraft is launched on a mission to Mars starting from a 300-km circular parking orbit. Calculate:
  - the delta-v required,
  - the location of perigee of the departure hyperbola,
  - the amount of propellant required as a percentage of the spacecraft mass before the delta-v burn, assuming a specific impulse of 300 s.

- Problem data:

$$\mu_{\text{sun}} = 1.327(10^{11})\,\text{km}^3/\text{s}^2 \qquad R_{\text{earth}} = 149.6(10^6)\,\text{km}$$

$$\mu_{\text{earth}} = 398,600\,\text{km}^3/\text{s}^2 \qquad R_{\text{Mars}} = 227.9(10^6)\,\text{km}$$

- the hyperbolic excess speed is: $v_\infty = \sqrt{\dfrac{\mu_{\text{sun}}}{R_{\text{earth}}}}\left(\sqrt{\dfrac{2R_{\text{Mars}}}{R_{\text{earth}}+R_{\text{Mars}}}}-1\right) = 2.943\,\text{km/s}$

- The speed of the spacecraft in its 300-km circular parking orbit is: $v_c = \sqrt{\dfrac{\mu_{\text{earth}}}{r_{\text{earth}}+300}} = \sqrt{\dfrac{398,600}{6678}} = 7.726\,\text{km/s}$

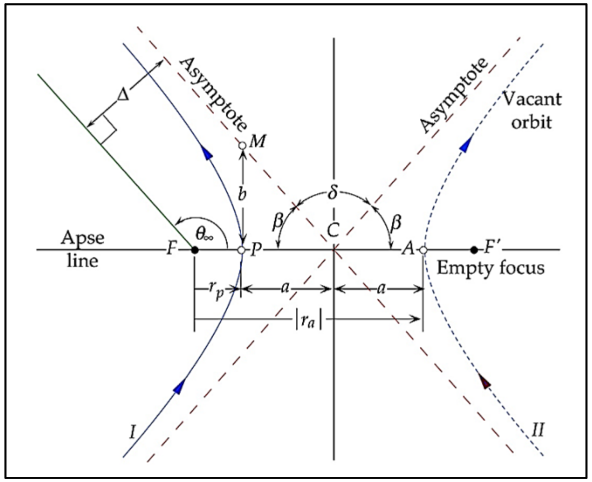
- The speed at the escape hyperbola perigee is: $v_p = \dfrac{h}{r_p} = \sqrt{{v_\infty}^2 + \dfrac{2\mu_1}{r_p}}$

- the delta-v required to put the vehicle onto the hyperbolic departure trajectory is:

$$\boxed{\Delta v = 3.590\,\text{km/s}}$$

Interplanetary Flight

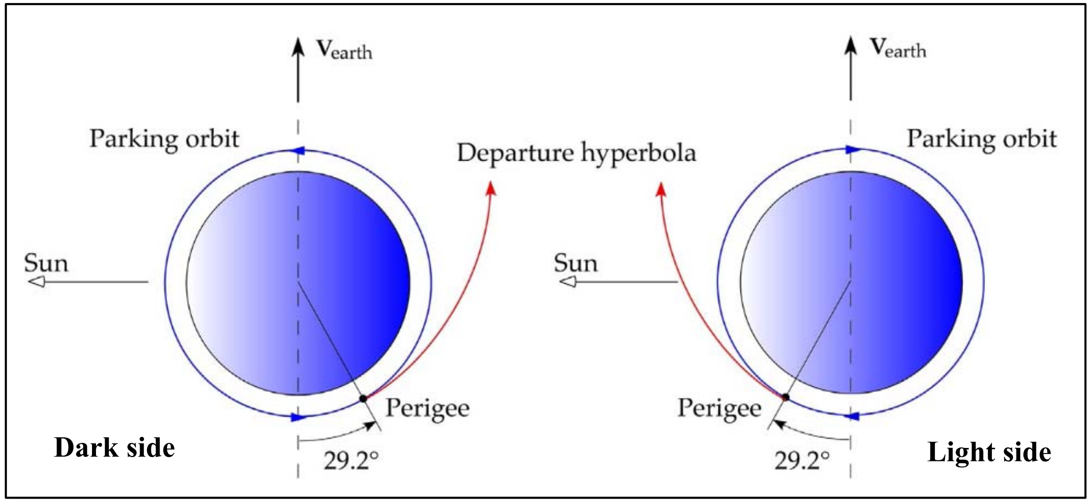
- The location of the perigee is found from the hyperbola geometry:



$$\beta = \cos^{-1}(1/e) \qquad \beta = \cos^{-1}\left(\frac{1}{1+\frac{r_p v_\infty{}^2}{\mu_{\text{earth}}}}\right)$$

$$\boxed{\beta = 29.16^\circ}$$

- Since the parking orbit is **usually** prograde, the departure is from the dark side.

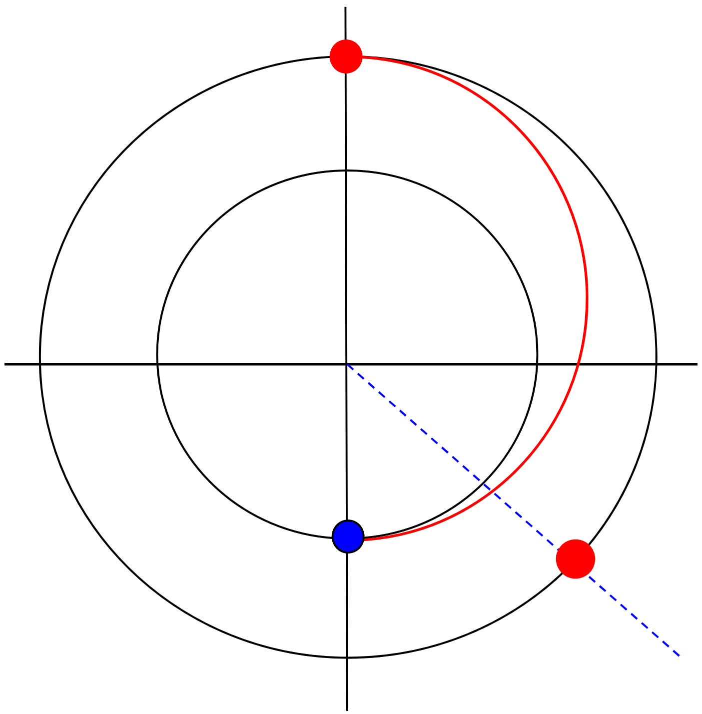


$$\frac{\Delta m}{m} = 1 - \exp\left(-\frac{\Delta v}{I_{\text{sp}} g_0}\right) \qquad \boxed{\frac{\Delta m}{m} = 0.705}$$

Before the delta-v maneuver is over, 70% of the spacecraft mass must be propellant.

- **Note on Timing:** Knowledge of time of flight is necessary in order to start the interplanetary maneuver so that the spacecraft reaches the SOI of Mars when Mars is there. This requires the use of Kepler's equation, Lambert's theorem and the **Ephemeris** of Earth and Mars (from JPL spice site).

- Time constraints may require "phasing" of the parking orbit, the necessity of alternate Heliocentric trajectories, or even indirect paths through sphere of influence of different bodies prior the arrival to the target planet (Fly-By).



- The Hohmann transfer has the following properties:
- Semi major axis

$$a_{HOH} = \frac{r_{EARTH} + r_{MARS}}{2} \approx 189 \times 10^6\ Km$$

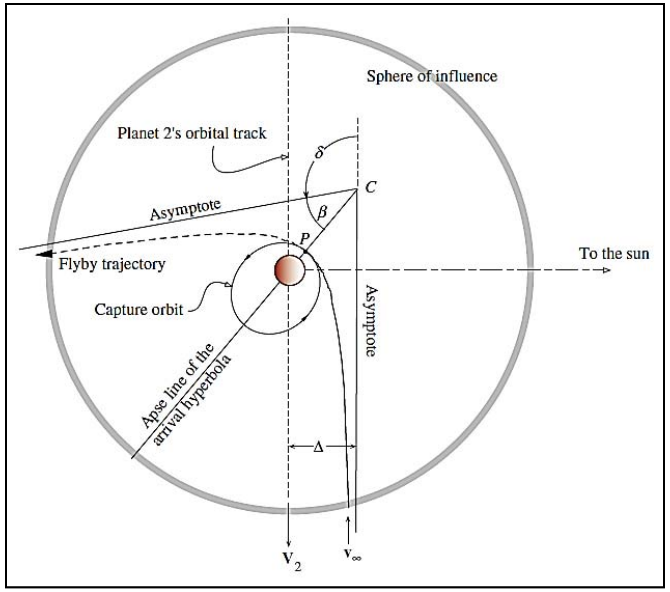
- Half Period

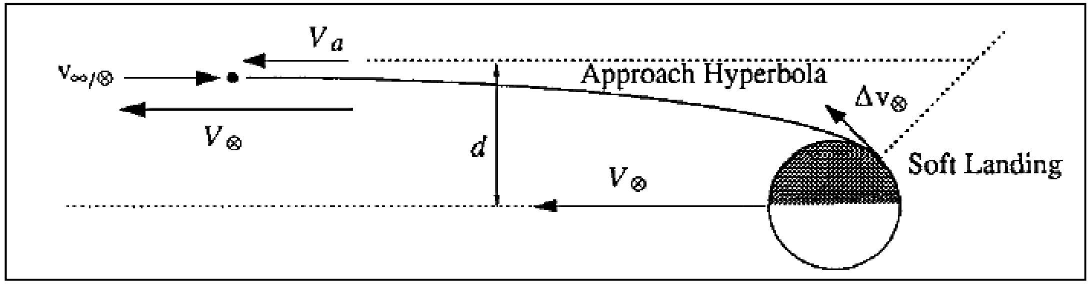
$$\frac{P}{2} = \sqrt{\frac{\pi^2 a_{HOH}^3}{\mu_{SUN}}} \approx 259.34\, d$$

- Mars orbital period = 1.881 Y = 686.565 d

- Location of Mars at Departure (135.9° from rendez vous)

- Capture Trajectory: Assuming that the Hohmann apohelion coincides with the entrance in Mars SOI, at this point the spacecraft should enter a hyperbolic orbit around the target planet.





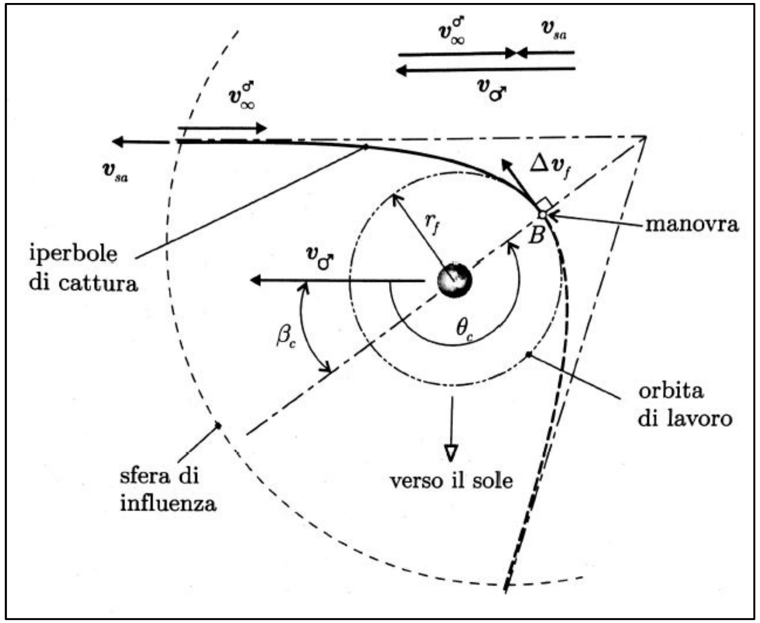
- Several options exist based on the mission:
  - Planetary Fly – By (see more later)
  - Soft (hard) landing
  - Planetary Capture

- **Planetary Capture:** this case implies a transfer from hyperbolic capture to a parking orbit. We assume a transfer change at the periapsis of the hyperbola, into a circular orbit around Mars

$$v_A^{HOH} = \sqrt{\frac{\mu_{SUN}}{a_{HOH}} \cdot \frac{r_{EARTH}}{r_{MARS}}}; v_{SUN}^{MARS} = \sqrt{\frac{\mu_{SUN}}{r_{MARS}}}$$

$$\boxed{v_\infty^{MARS} = v^{MARS} - v_A^{HOH}}$$

- This case shows the arrival **AHEAD** of the SOI of Mars (one possibility, depending on the planet location with respect to Earth). The other is a **BEHIND** arrival (see later)



$$v_\infty^{MARS} = v^{MARS} - \sqrt{\frac{2\mu_{SUN}}{r_{MARS} + r_{EARTH}}\left(\frac{r_{EARTH}}{r_{MARS}}\right)}$$

- The minimum energy transfer to a parking orbit is achieved at the periapsis of the hyperbola

$$r_P^{HYP} = r_{PARK}^{MARS} = -a_{HYP}(e_{HYP} - 1)$$

$$a_{HYP} = -\frac{\mu_{MARS}}{\left(v_\infty^{MARS}\right)^2}$$

- Parking Orbit Speed:

$$v_{PARK}^{MARS} = \sqrt{\frac{\mu_{MARS}}{r_{PARK}^{MARS}}}$$

- Speed at Capture Hyperbola Periapsis (from specific Energy):

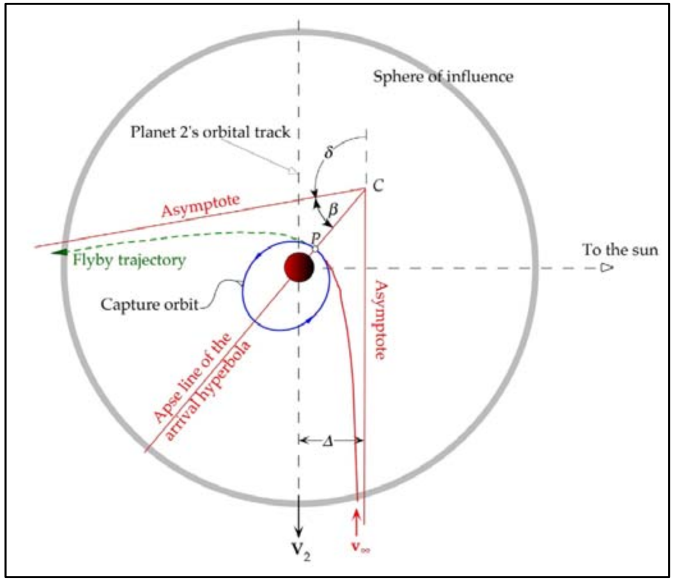
$$v_P^{HYP} = \sqrt{\frac{2\mu_{MARS}}{r_P^{HYP}} - \left(v_\infty^{MARS}\right)^2}$$

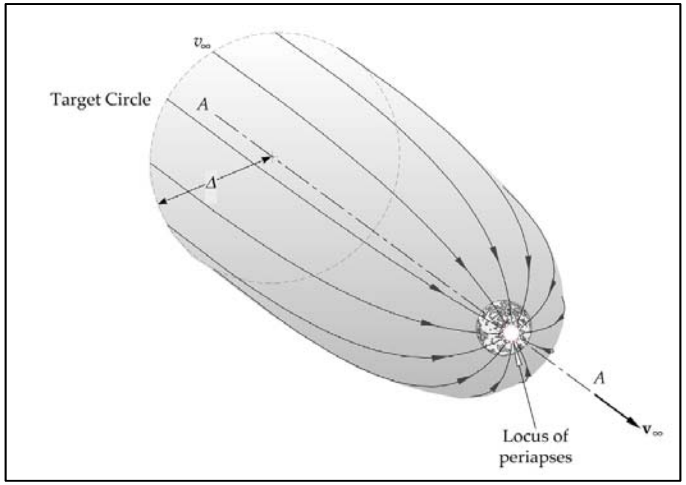
$$\boxed{\Delta v_2 = v_P^{HYP} - v_{PARK}^{MARS}}$$

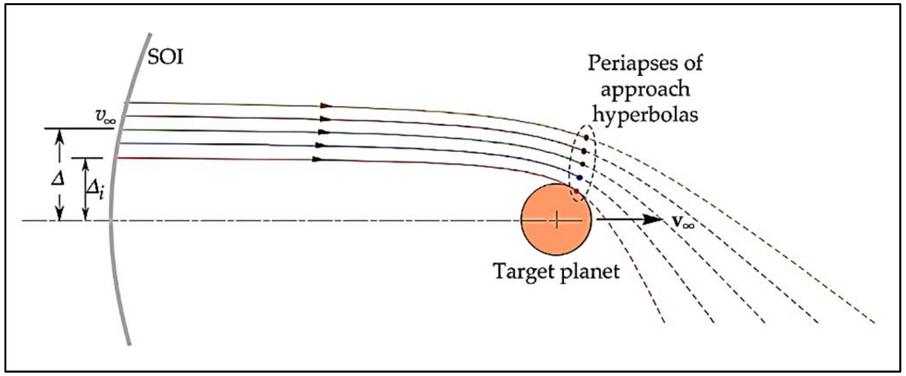
- Speed Decrease (deceleration)

Interplanetary Flight

- Some considerations about aiming radius and angle of hyperbolic arrival







- Knowing the hyperbolic excess speed and periapsis

$$\left[v_\infty^{MARS};r_P^{HYP}\right]$$

- We can find eccentricity and angular momentum of the hyperbola

$$e=1+\frac{r_p {v_\infty}^2}{\mu_2} \qquad h=r_p\sqrt{{v_\infty}^2+\frac{2\mu_2}{r_p}}$$

$$\boxed{\delta=2\sin^{-1}\left(\frac{1}{1+\frac{r_p {v_\infty}^2}{\mu_2}}\right)} \qquad \boxed{\Delta=\frac{h^2}{\mu_2}\frac{1}{\sqrt{e^2-1}}}$$

- **Example:** After a Hohmann transfer from earth to Mars, calculate:
  - the minimum delta-v required to place a spacecraft in orbit with a period of 7 h
  - the periapsis radius
  - the aiming radius
  - the angle between periapsis and Mars' velocity vector.

- Problem data:

$$\mu_{\text{sun}} = 1.327\left(10^{11}\right)\text{km}^3/\text{s}^2 \qquad R_{\text{earth}} = 149.6\left(10^6\right)\text{km}$$

$$\mu_{\text{Mars}} = 42,830\text{km}^3/\text{s}^2 \qquad R_{\text{Mars}} = 227.9\left(10^6\right)\text{km}$$

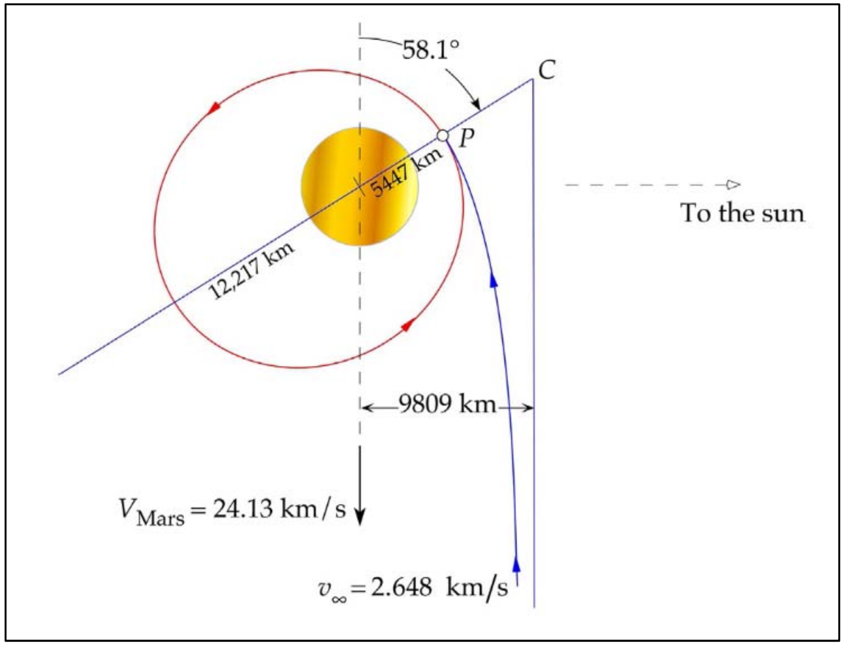
$$r_{\text{Mars}} = 3396\text{km}$$

- Hyperbolic excess speed: $v_\infty = \sqrt{\frac{\mu_{\text{sun}}}{R_{\text{Mars}}}}\left(1 - \sqrt{\frac{2R_{\text{earth}}}{R_{\text{earth}} + R_{\text{Mars}}}}\right) = 2.648\,\text{km/s}$

- the semimajor axis a of the capture orbit from Kepler's third law: $a = \left(\frac{25,200\sqrt{42,830}}{2\pi}\right)^{2/3} = 8832\text{km}$

- the eccentricity of the Mars parking orbit is:

$$a = \frac{r_p}{1-e} \qquad a = \frac{2\mu_{\text{Mars}}}{{v_\infty}^2}\frac{1}{1+e} \qquad e = \frac{2\mu_{\text{Mars}}}{a{v_\infty}^2} - 1 = \frac{2 \cdot 42,830}{8832 \cdot 2.648^2} - 1 = 0.3833$$



- Δv necessary to achieve Mars parking orbit:

$$\Delta v = v_\infty \sqrt{\frac{1-e}{2}} = 2.648\sqrt{\frac{1-03833}{2}} = \boxed{1.470\text{km/s}}$$

- The periapsis radius is:

$$r_p = \frac{2\mu_{\text{Mars}}}{v_\infty{}^2}\frac{1-e}{1+e} = \frac{2\cdot 42,830}{2.648^2}\frac{1-0.3833}{1+0.3833} = \boxed{5447\,\text{km}}$$

- The aiming radius is:

$$\Delta = r_p\sqrt{\frac{2}{1-e}} = 5447\sqrt{\frac{2}{1-0.3833}} = \boxed{9809\,\text{km}}$$

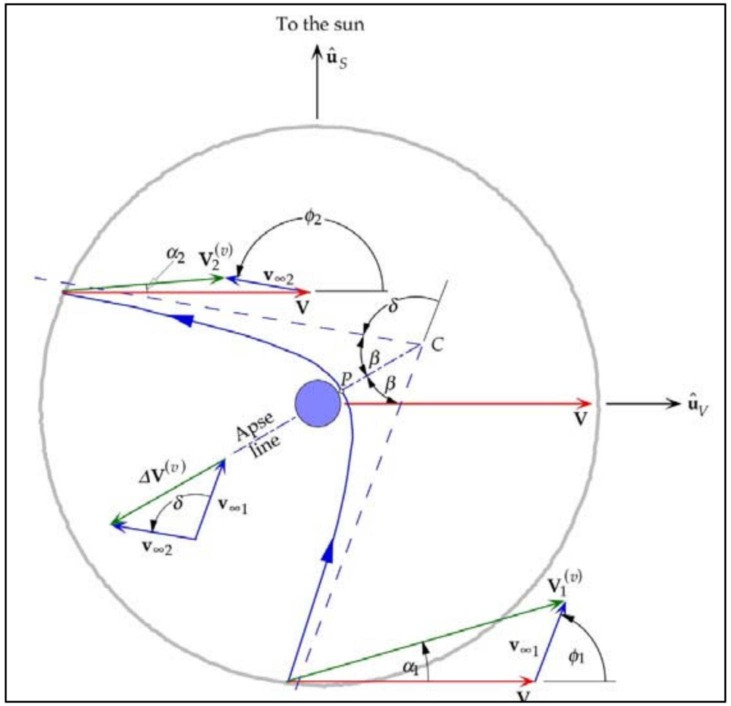
- The angle to periapsis is:

$$\beta = \cos^{-1}\left(\frac{1}{1+\dfrac{r_p v_\infty{}^2}{\mu_{\text{Mars}}}}\right) = \cos^{-1}\left(\frac{1}{1+\dfrac{5447\cdot 2.648^2}{42,830}}\right) = \boxed{58.09°}$$

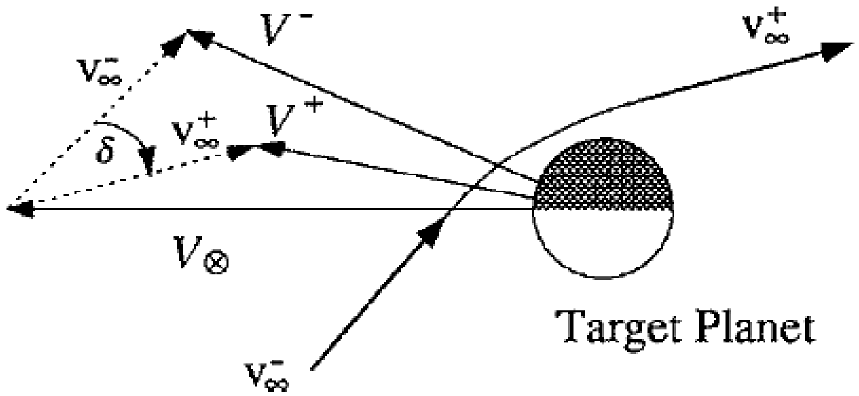
Note: the approach could also be made from the dark side of the planet instead of the sunlit side. The approach hyperbola and capture ellipse would be the mirror image of that shown.

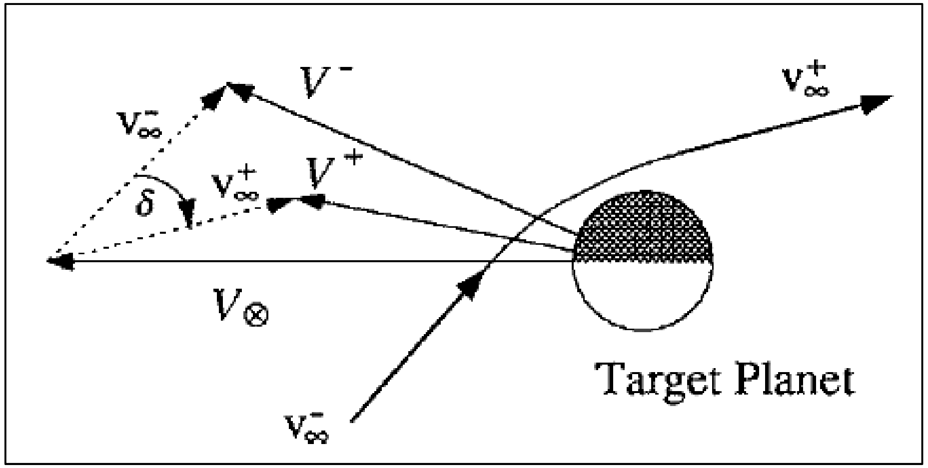
- **Planet Fly – By (Also called Gravity Assist):** This is a common maneuver to achieve:
  - Change of trajectory
  - Increase of Decrease of spacecraft energy (by changing the speed with respect to the Sun).

  - A spacecraft that enters a planet's sphere of influence and does not impact the planet or go into orbit around it will continue in its hyperbolic trajectory through periapsis P and exit the sphere of influence. After its planetary fly-by may gain or lose its energy depending on whether it passes behind or ahead of the planet. The gain or loss of energy is caused by the rotation of the spacecraft's velocity vector with respect to the planet.



- Fly-By Ahead of the planet

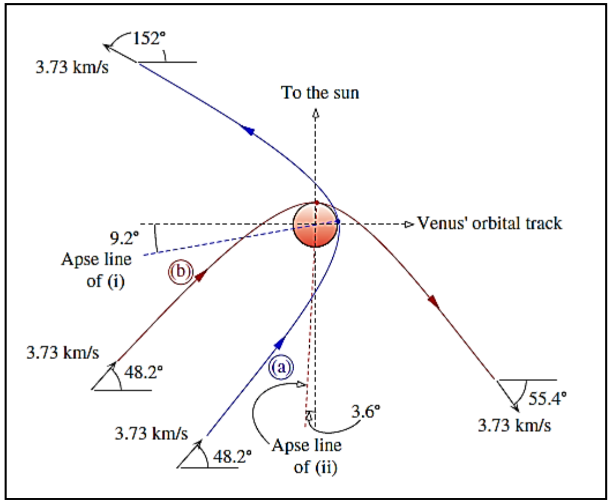




- $V_\otimes$: Planet inertial velocity
- V^- : Spacecraft inbound inertial velocity
- V^+ : Spacecraft outbound inertial velocity
- v_∞^- : Spacecraft inbound velocity relative to the Planet
- v_∞^+ : Spacecraft outbound velocity relative to the Planet

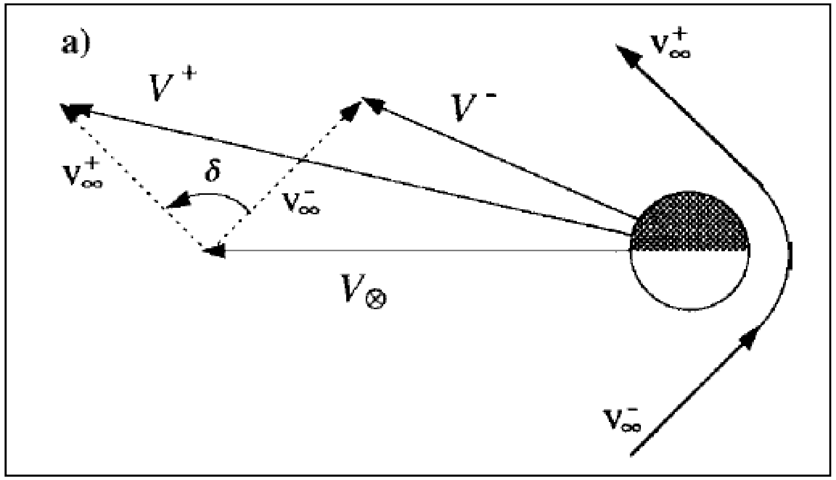
- The Fly-By ahead of a planet reduces the inertial speed of the spacecraft, essentially slowing it down, in addition of changing the direction of the velocity vector of an amount δ depending on the particular hyperbola selected. <u>This is the case of a Fly-By to an outer planet, which has a Heliocentric velocity higher than the Hohmann velocity of a spacecraft</u>.

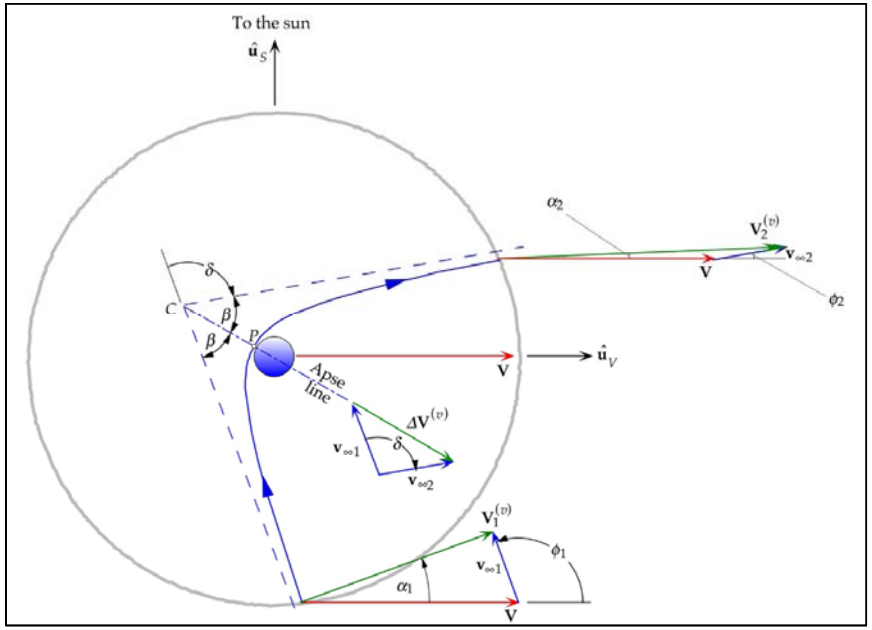
- **Note:** Fly-By ahead can be performed from the dark side or the sunlit side depending on the arrival direction and the request for a prograde (west-east) or a retrograde (east-west) parking orbit

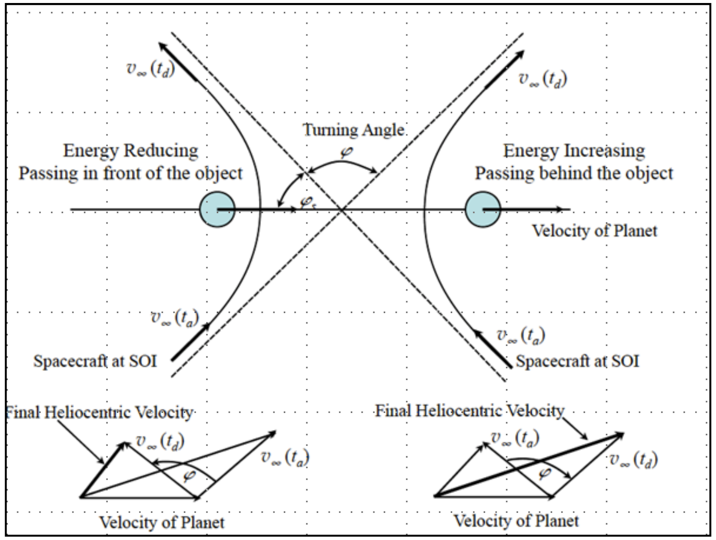


Interplanetary Flight

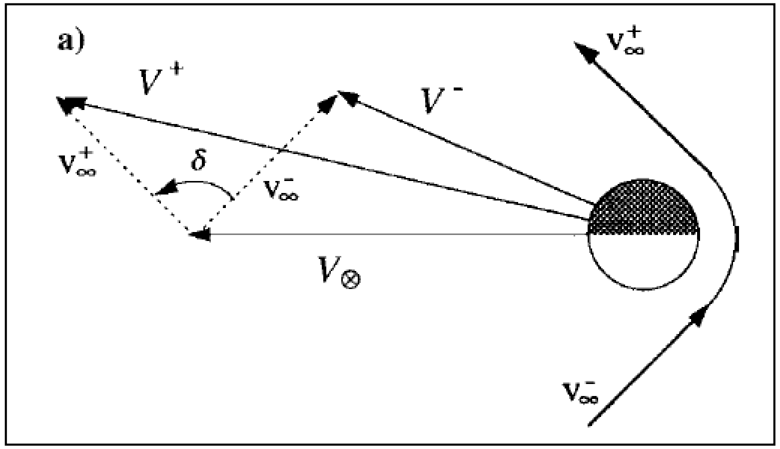
- Fly-By Behind the planet





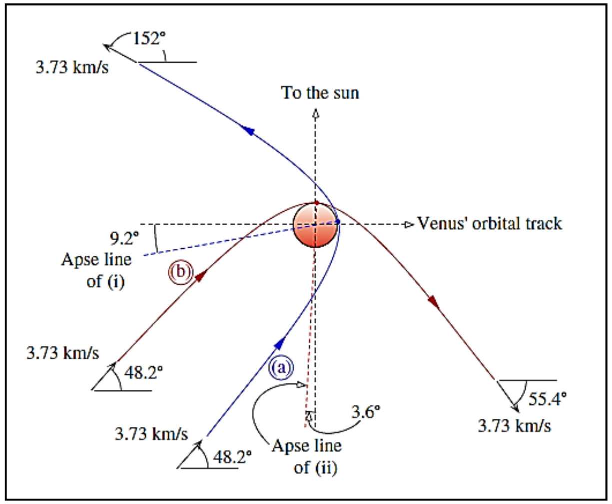


- The Fly-By behind a planet increases the inertial speed of the spacecraft, essentially speeding it up, in addition of changing the direction of the velocity vector of an amount $\boldsymbol{\delta}$ depending on the particular hyperbola selected. <u>This is the case of a Fly-By to an inner planet , which has a Heliocentric velocity lower than the Hohmann velocity of the spacecraft</u>.



$V_\otimes$	**Planet inertial velocity**
V^-	**Spacecraft inbound inertial velocity**
V^+	**Spacecraft outbound inertial velocity**
v_∞^-	**Spacecraft inbound velocity relative to the Planet**
v_∞^+	**Spacecraft outbound velocity relative to the Planet**

- **Note:** Fly-By behind can be performed from the dark side or the sunlit side depending on the arrival direction and the request for a prograde (west-east) or a retrograde (east-west) parking orbit



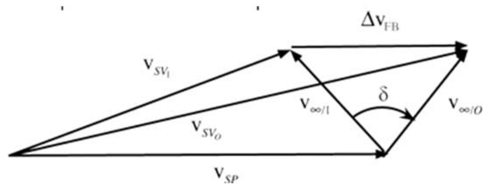
- **Note on velocity changes:**

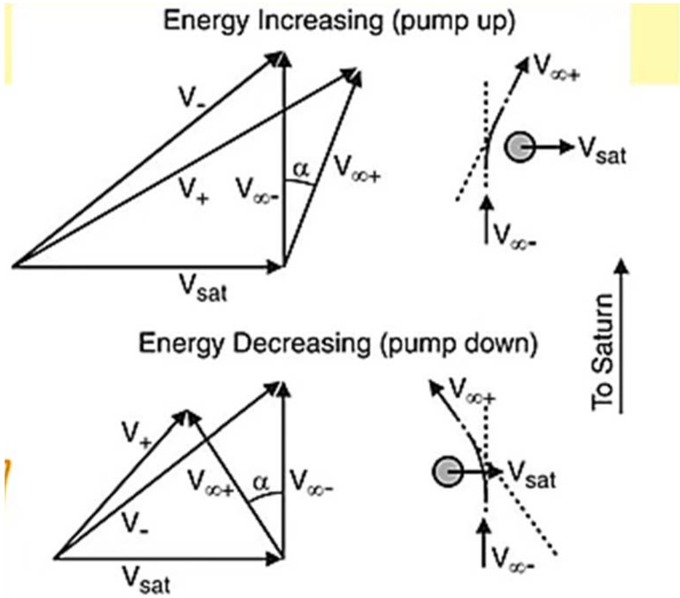
$$\vec{v}_f - \vec{v}_{planet} = R_1(\delta)\,(\vec{v}_i - \vec{v}_{planet}) \qquad \vec{v}_f = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} (\vec{v}_i - \vec{v}_{planet}) + \vec{v}_{planet}$$

Example: If $\delta = 180°$ and $\vec{v}_i = \begin{bmatrix} -20 \\ 0 \end{bmatrix} km/s$ and $\vec{v}_p = \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s$, then

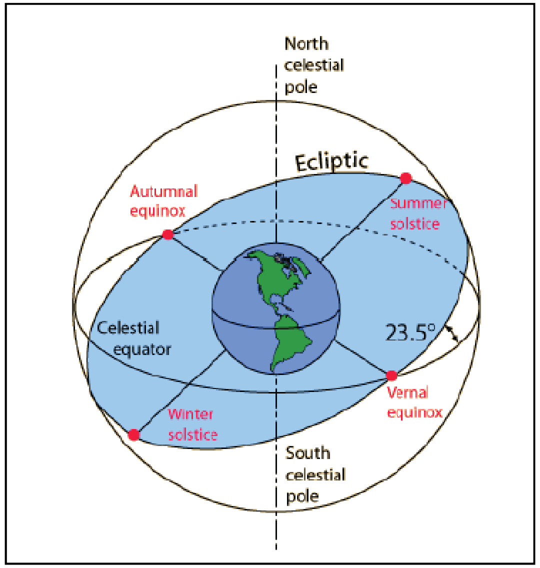
$$v_f = R(180°)\begin{bmatrix} -40 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 40 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 60 \\ 0 \end{bmatrix} km/s$$

Thus a probe can potentially *triple* its velocity!





- **Transition to Ecliptic Plane:** Most planets orbit around the Sun on a plane (Ecliptic), Pluto is a notable exception. Most launch sites do not put a vehicle in orbit directly on the Ecliptic plane, thus a non planar orbit change may be required. The transition implies satisfaction of two objectives:
  - Out of Plane transfer (change inclination)
  - Computation of Launch Window



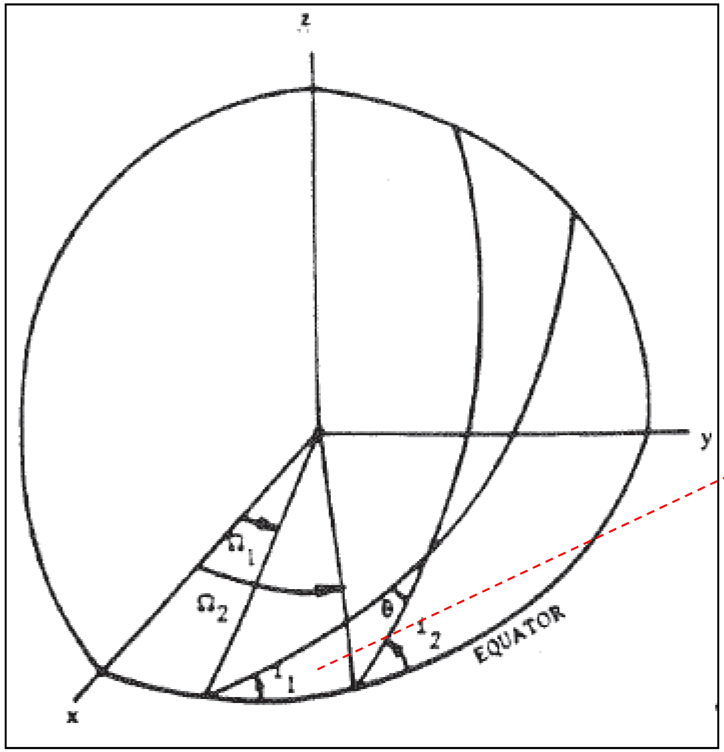
- Transition must occur when the orbital plane and ecliptic planes intersect.
- Any orbit about the Earth passes through the ecliptic twice per orbit.

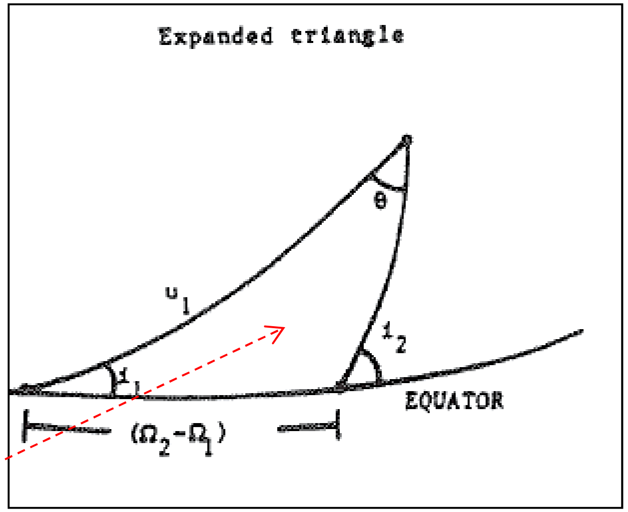
- But not at the ascending node (w. r. t. the equatorial plane).
- But not at the correct time.

  - Orbital Parameters of interest are:
  - i = Inclination (typical target i = 23.5 deg.)
  - Ω = Right Ascension of the Ascending Node (RAAN)

- We know that simultaneous change of i and Ω is preferable from propellant consumption standpoint

Interplanetary Flight





Our desired orbit has

- $i_2 = \epsilon = 23.5°$ - Inclination to the ecliptic
- $\Omega_2 = 0°$ - by definition: Ω is measured from FPOA (intersection of equatorial and ecliptic planes).

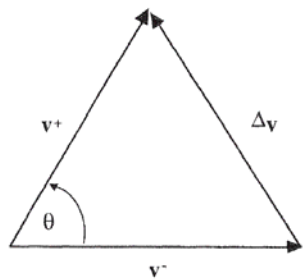
If our initial orbit has inclination i_1 and RAAN Ω_1, then the angle change is

$$\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1)$$

- The position in the orbit is then given by:

$$\cos(\omega + \upsilon) = \frac{\cos i_1 \cos\theta - \cos i_2}{\sin i_1 \sin\theta}$$

- The Δv is found as



$$\Delta v = 2v \sin\frac{\theta}{2}$$

Interplanetary Flight

- **Problem:** Design an Earth-Venus rendez-vous. Final Venus orbit should be prograde of altitude 500km.

  - **Launch Data:** Launch from Cape Kennedy into a circular orbit with altitude 200 Km.

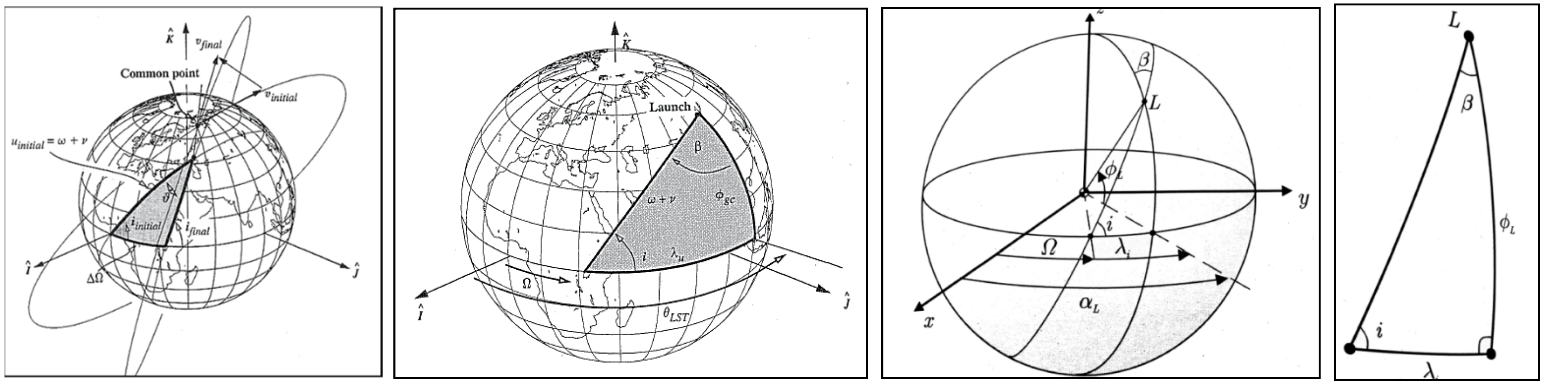
    Lat = 28.5^0, Long = -80.55^0, A_{zmin} = 320, Az_{max} = 112^0

    $$\cos i = \cos(Lat)\cos(Az = \beta) \quad 45^0 < i < 109.22^0$$

  - **Initial orbit:** i = 55^0, Ω_1 (RAAN) = 0^0, r = 6578 Km, e = 0.

    $$v = \sqrt{\frac{\mu_E}{r}} = 7.7843\, Km / \sec; \mathrm{P} = 1.4741\, h$$

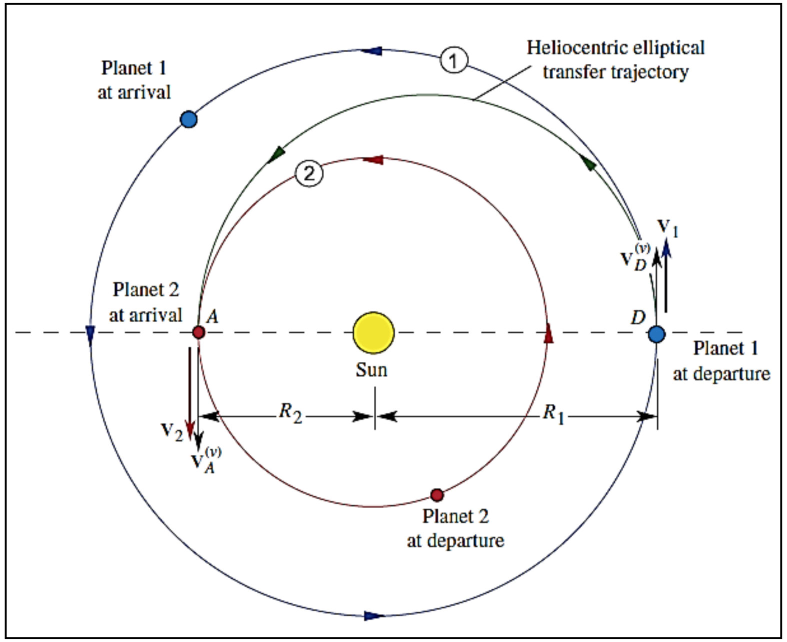
  - **Final orbit on the Ecliptic Plane:** i = 23.5^0, Ω_2 (RAAN) = 45^0, r = 6578 Km, e = 0.



- **From spherical trigonometry:** $\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1)$ $\theta = 40.8^0$

$$\cos(\omega + \nu) = \frac{\cos i_1 \cos\theta - \cos i_2}{\sin i_1 \sin\theta} \qquad \nu = 154.44^0 \qquad \boxed{\Delta v = 2v\sin\frac{\theta}{2} = 5.4368\, Km / \sec}$$

- **Select a Hohmann Transfer to Venus:** most of the travel is around Sun



- Heliocentric velocities of transfer orbit (perihelion and aphelion), from energy

$$r_e = 1AU \qquad a = 0.8615AU \qquad \prod = 293.09\,d$$
$$r_v = 0.723AU \qquad e = 0.1608$$

$$v_1^+ = v_p = \sqrt{2\mu_{sun}\frac{r_e}{r_v(r_e + r_v)}} = 37.73km/s$$

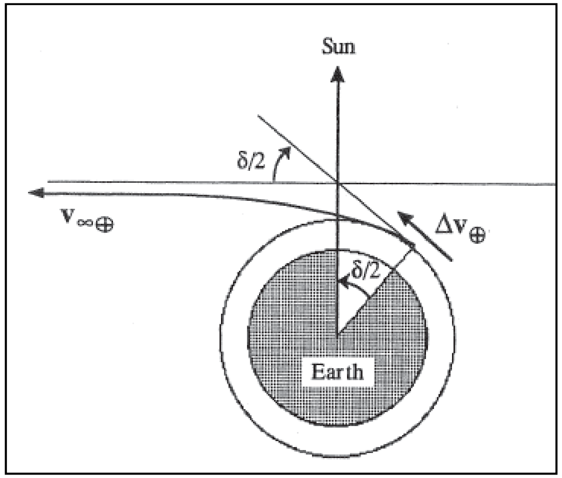
$$v_2^+ = v_a = \sqrt{2\mu_{sun}\frac{r_v}{r_e(r_e + r_v)}} = 27.29km/s$$

- Earth velocity around the Sun is given by: $29.78\ Km/sec$

- The escape velocity from Earth is therefore:

$$v_{\infty,e} = v_a - v_e^- = 27.29 - 29.78 = -2.49 km/s$$

- **Hyperbolic Escape from Earth:** The energy at the end of Earth's SOI is then



$$E_+ = \frac{1}{2} v_{\infty,e}^2 = 2.067$$

- Recall the velocity in the parking orbit:

$$v_{park} = \sqrt{\frac{\mu_E}{r}} = 7.7843\, Km / \sec$$

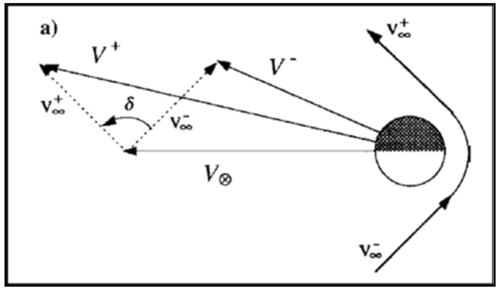
- Since the energy is constant, we can find the velocity at the burn location (perigee):

$$E = \frac{1}{2} v_{b,e}^2 - \frac{\mu_e}{r_{park}} = +2.067$$

$$v_{b,e} = \sqrt{2E + 2\frac{\mu_e}{r_{park}}} = \sqrt{v_{\infty,e}^2 + 2\frac{\mu_e}{r_{park}}} = 11.195 km/s$$

$$\boxed{\Delta v_e = v_{b,e} - v_{park} = 3.4106 km/s}$$

- Assuming counterclockwise orbit of the spacecraft, and knowing that the same holds for Earth's motion around Sun, the Hyperbolic path should be on the light side of the Earth



- **Burn timing:** The burn into hyperbolic escape should occur before noon where the angle δ is

$$\delta = 2\sin^{-1}\frac{1}{e}$$

- The orbital parameters of the Hyperbola can be found from energy considerations

$$E = \frac{1}{2}v_{\infty,e}^2 = 2.067 = -\frac{\mu}{2a} \qquad a = -\frac{\mu}{v_{\infty,e}^2} = -\frac{\mu}{2E} = -96,420km$$

$$r_{p,e} = r_c = a(1-e) = 6578km \qquad e = 1 - \frac{r_{p,e}}{a} = 1.0682$$

$$\delta = 2.423rad = 138.83^\circ$$

Thus the spacecraft should depart at $\delta/2 = 69.4^\circ$ before noon

- **Venus Arrival:** The arrival is programmed for 145.54 days after departure (half period of Hohmann ellipse.

- The spacecraft excess velocity with respect to Venus is given by:

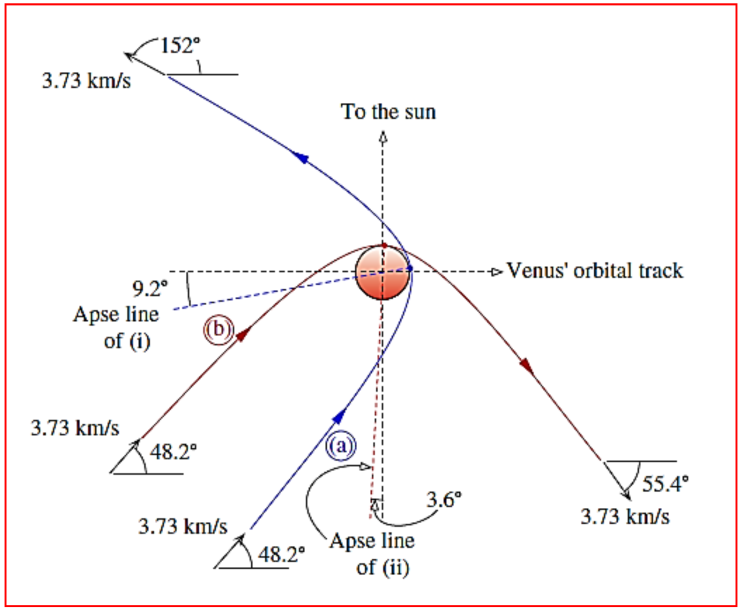
$$v_{\infty,v} = v_p - v_v = v_1^- - v_1^+ = 37.81km/s - 35.09km/s = 2.71km/s$$

Where Venus velocity with respect to the Sun is: $v_1^+ = v_v = \sqrt{\frac{\mu_s}{r_v}}$

And the Perihelion velocity of the Hohmann transfer is: $v_p = 37.81\,Km\,/\,\sec$

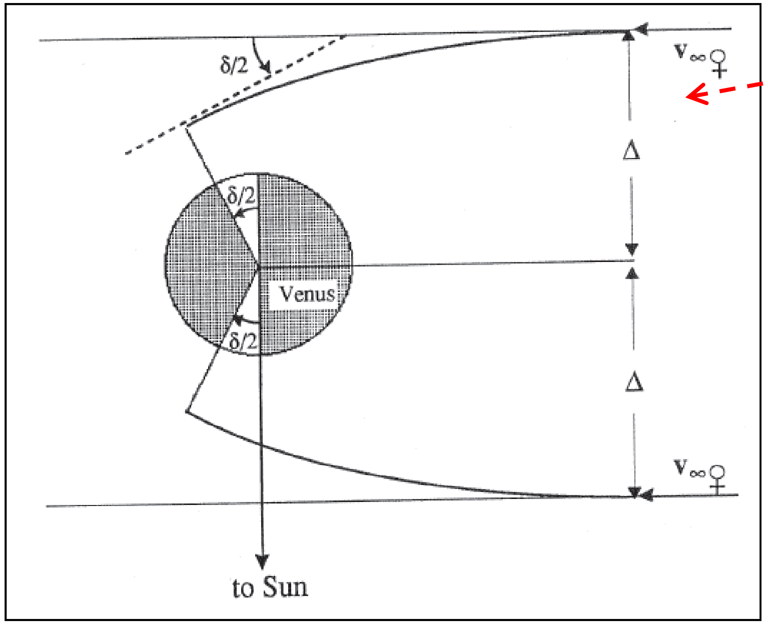
Because $v_{\infty,v} > 0$, the spacecraft will approach Venus from behind.

- Spacecraft is catching up to planet (not vice-versa)
- The back door



- Venus Data: $R_v = 6187km, \quad \mu_v = 324859, \quad a_{venus} = 1.08 \cdot 10^8$

- We want a prograde (counterclockwise) circular orbit around Venus with radius $r_c = 6187 + 500 = 6687\ Km$, therefore the approach to Venus is from the <u>dark side</u> of the planet



- For orbital insertion, we want to perform a retrograde burn at periapsis of the incoming hyperbola.
- To achieve the desired circular orbit of radius, we need the periapsis of our incoming hyperbola to occur at

$$r_{p,v} = a(1-e) = 6687km$$

- The arrival Hyperbola parameters can be found from energy

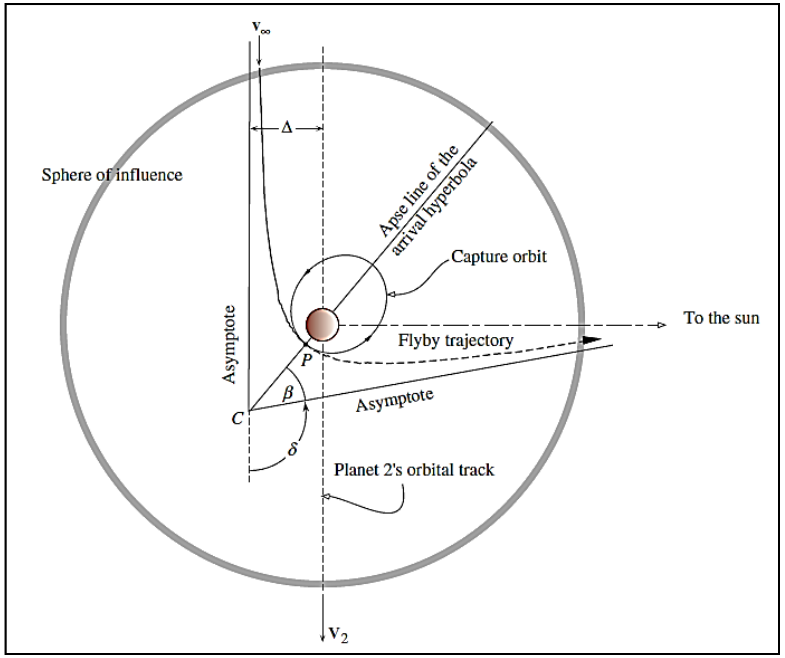
$$E = \frac{1}{2}v_{\infty,v}^2 = 3.67$$

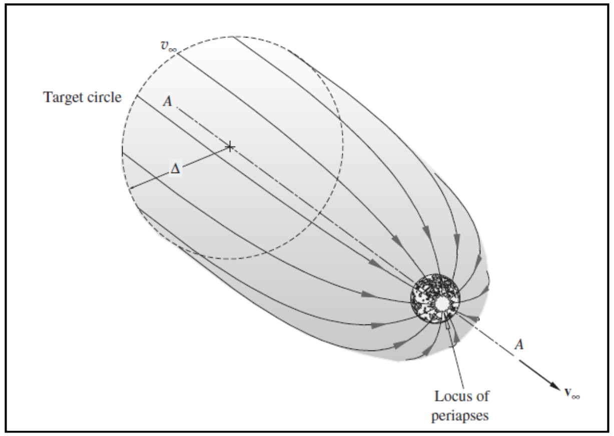
$$a = -\frac{\mu_v}{v_{inf,v}^2} = -44,232km$$

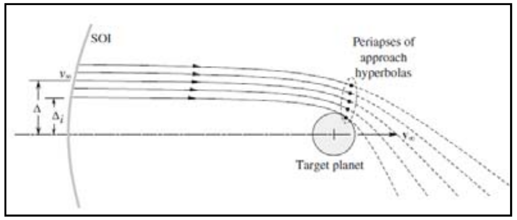
$$e = 1 - \frac{r_p}{a} = 1.15$$

Interplanetary Flight

- Target Radius control (distance Δ and angle δ) to achieve correct capture Hyperbola can be done using property of angular momentum







A smaller target radius Δ implies smaller eccentricity (closer to parabola), since energy is fixed by a.

- From the definition: $\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} \Rightarrow \left|\boldsymbol{h}\right| = \Delta \cdot v_{\infty,v}$ $\quad p = \dfrac{h^2}{\mu_v} = \dfrac{\Delta^2 v_{\infty,v}^2}{\mu_v}$

$a = -44,232km$ and $e = 1.15$, we get $p = 14,265km.$

$$\Delta = \sqrt{\frac{p\mu_v}{v_{\infty,v}^2}} = \sqrt{\frac{a(1-e^2)\mu_v}{v_{\infty,v}^2}} = 25,120km \qquad \frac{\delta}{2} = \sin^{-1}\frac{1}{e = 1.15} = 60.41^o$$

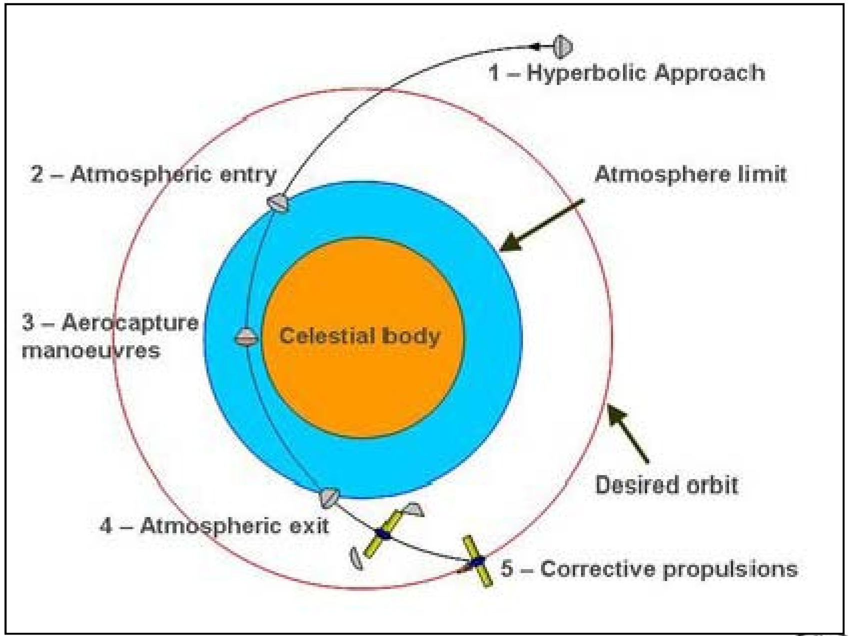
- **Venus parking orbit insertion:** from energy applied to capture Hyperbola

$$v = \sqrt{\frac{2\mu_v}{r_{p,v}} - \frac{\mu_v}{a}} = 10.223 km/s$$

$$v_c = \sqrt{\mu_v r_{p,v}} = 6.97 km/s$$

$$\boxed{\Delta v = 6.97 - 10.223 = -3.253 km/s}$$

- **Alternate Procedure:** Aero braking Assist



- **Total Propellant Effort:**
  - Inclination change: 5.4368 Km/sec
  - Enter Hohmann transfer: 3.4106 Km/sec
  - Enter Venus parking orbit: -3.2530 Km/sec

$$\Delta v = 12.1004 km / \sec$$

Interplanetary Flight

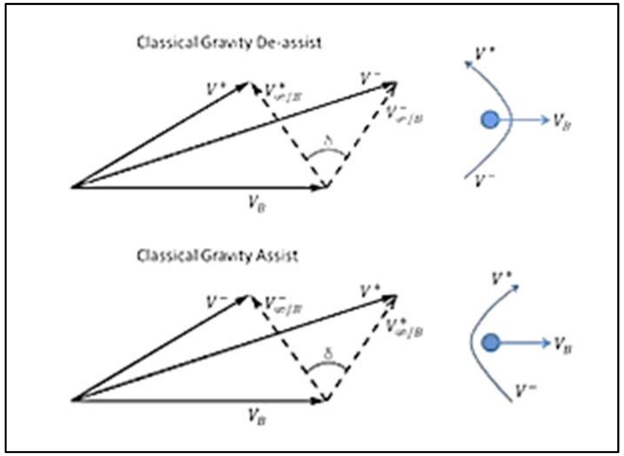
- **Problem:** Hohmann transfer Earth – Jupiter. Search for the best gravity assist.

- **Hint:** $$\vec{v}_i = v_a = \sqrt{2\mu_{sun}\frac{r_e}{r_j(r_j + r_e)}} = 7.414km/s$$

$$\vec{v}_{planet} = v_j = \sqrt{\frac{\mu_s}{d_j}} = 13.0573km/s$$

Since this is an outer planet, we approach from the front door. In the Jupiter $R - T - N$ frame we have

$$\vec{v}_i = \begin{bmatrix} 7.414 \\ 0 \end{bmatrix}, \qquad \vec{v}_{planet} = \begin{bmatrix} 13.0573 \\ 0 \end{bmatrix}$$



The velocity of the spacecraft relative to Jupiter is $\vec{v}_\infty = \vec{v}_i - \vec{v}_p = \begin{bmatrix} -5.6429 \\ 0 \end{bmatrix}$

Jupiter Data: Radius $r_j = 11.209ER$; Distance $d_j = 5.2028AU$; $\mu_j = 317.938\mu_e$.

The velocity of the spacecraft relative to jupiter is

$$\vec{v}_\infty = \vec{v}_i - \vec{v}_p = \begin{bmatrix} -5.6429 \\ 0 \end{bmatrix} km/s$$

Thus we can calculate the energy of the hyperbolic approach as

$$a = -\frac{\mu_s}{\|\vec{v}_i - \vec{v}_p\|^2} = -3.98E6km$$

The closest we can approach jupiter is its radius. If we use this for periapse, we get

$$e = 1 - \frac{r_j}{a} = 1.018$$

The eccentricity yields the maximum turning angle as

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right) = 158.44°$$

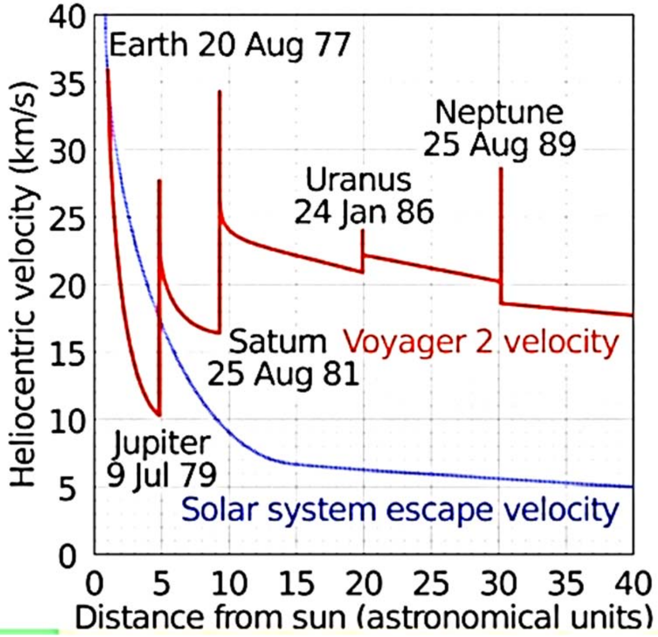
Applying this rotation (light-side approach), we get

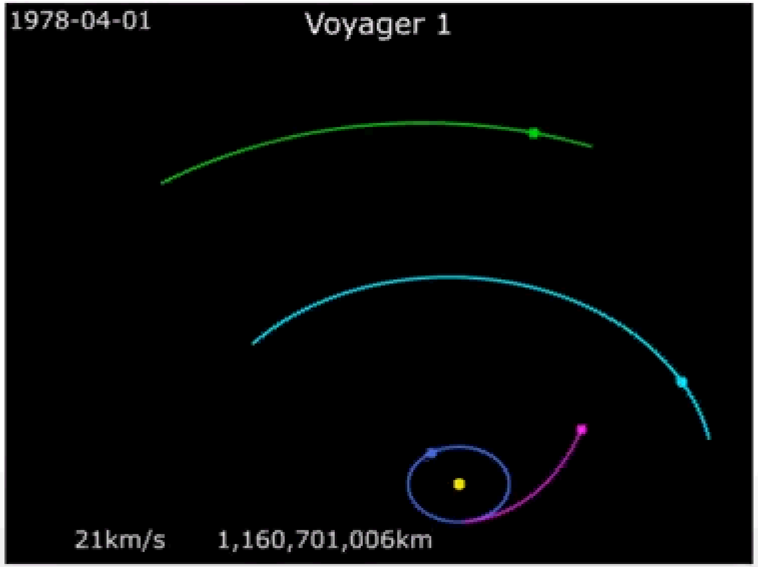
$$\vec{v}_f = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} (\vec{v}_i - v_{planet}) + \vec{v}_{planet} = \begin{bmatrix} 18.305 \\ 2.076 \end{bmatrix}$$

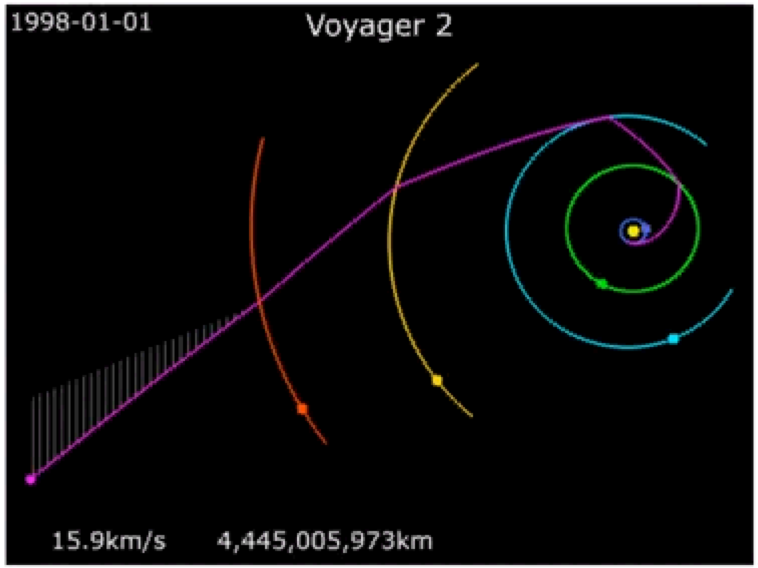
The magnitude of the Δv from this flyby is 11.01km/s. A factor of 2.5.

Note that if we could have reversed our direction of flight (clockwise approach), we could achieve a $\Delta v = 20.05 km/s$.

Interplanetary Flight







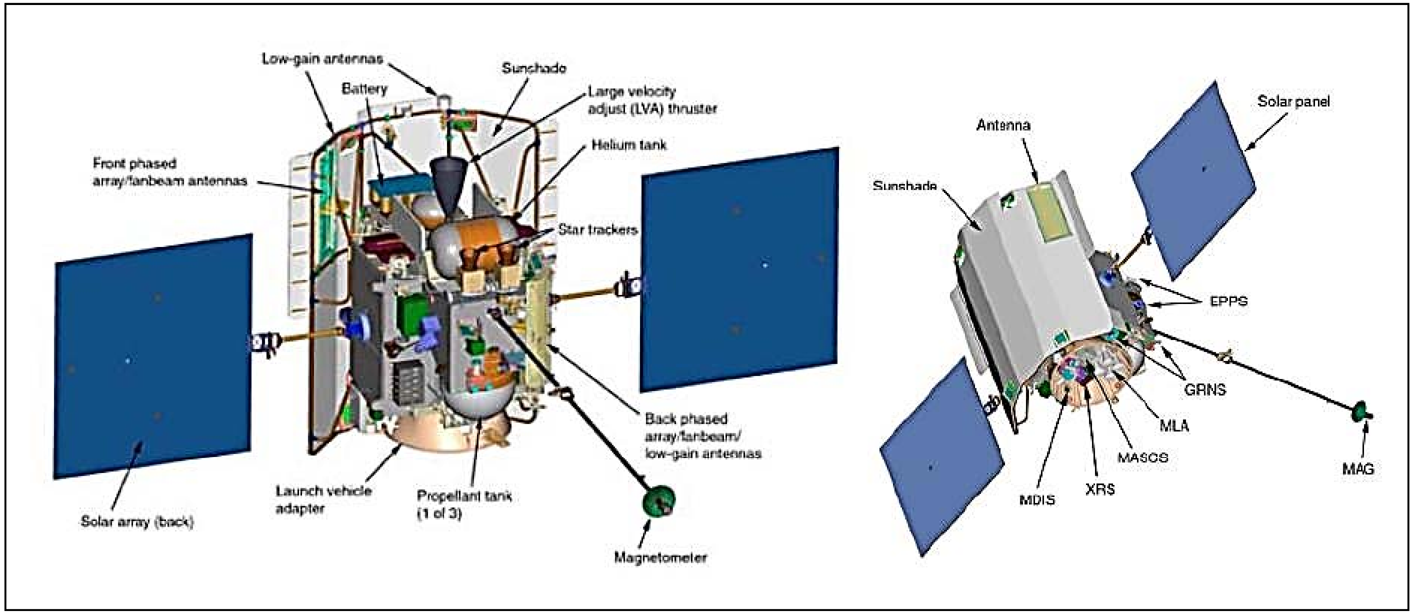
Links: Additional videos by Alfonso Gonzalez - Astrodynamics & SE Podcast (using Python)

[Gravity Assist]

[Lambert's Problem]

- **Example:** Messenger Spacecraft interplanetary Mission   Messenger

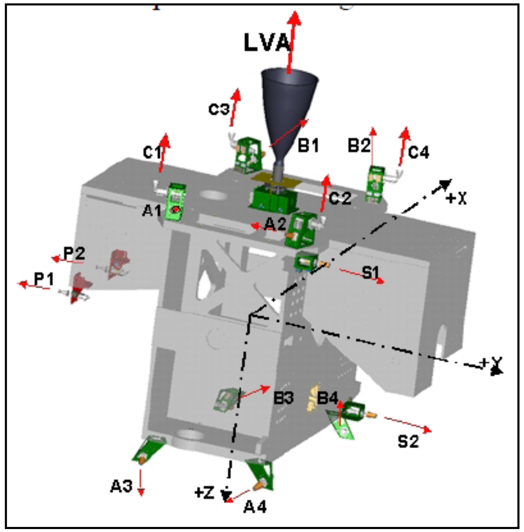
- First mission after Mariner X in 1975
- Planet mapping < 100 m resolution, elemental and chemical composition of surface
- Radar reflecting materials properties
- Planet's magnetic field, libration and gravitational field
- Launch: August 3, 2004
- End of mission: April 2015, planet impact



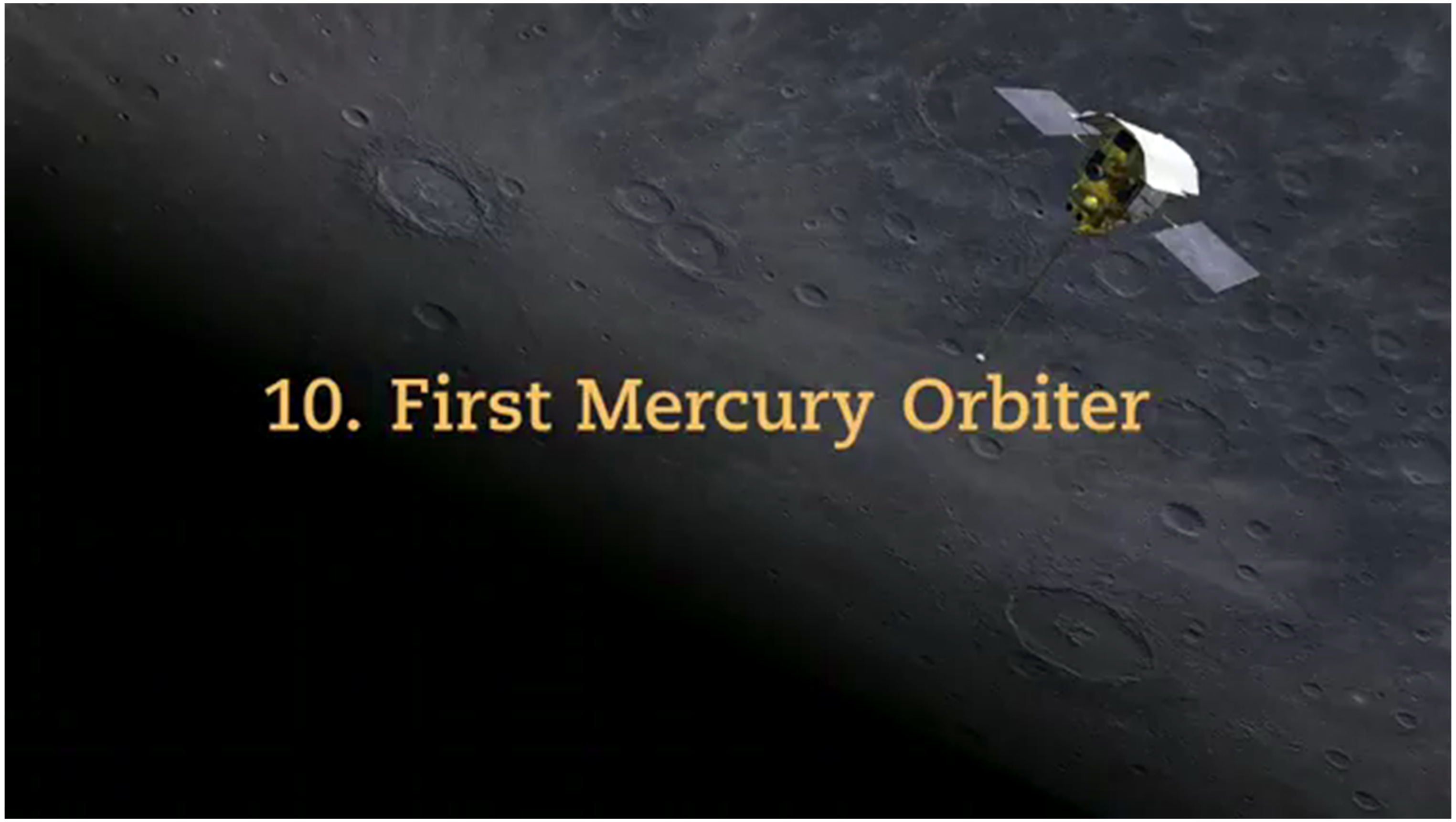
Interplanetary Flight

- MESSENGER is well protected against the heat, but it must always know its orientation relative to Mercury, Earth, and the Sun and be "smart" enough to keep its sunshade pointed at the Sun. Attitude determination – knowing in which direction MESSENGER is facing – is performed using star-tracking cameras, digital Sun sensors, and an Inertial Measurement Unit (IMU, which contains gyroscopes and accelerometers). Attitude control for the 3-axis stabilized craft is accomplished using four internal reaction wheels and, when necessary, MESSENGER's small thrusters.

- The IMU accurately determines the spacecraft's rotation rate, and MESSENGER tracks its own orientation by checking the location of stars and the Sun. Star-tracking cameras on MESSENGER's top deck store a complete map of the heavens; 10 times a second, one of the cameras takes a wide-angle picture of space, compares the locations of stars to its onboard map, and then calculates the spacecraft's orientation. The Guidance and Control software also automatically rotates the spacecraft and solar panels to the desired Sun-relative orientation, ensuring that the panels produce sufficient power while maintaining safe temperatures.



- Five Sun sensors back up the star trackers, continuously measuring MESSENGER's angle to the Sun. If the flight software detects that the Sun is "moving" out of a designated safe zone it can initiate an automatic turn to ensure that the shade faces the Sun. Ground controllers can then analyze the situation while the spacecraft turns its antennas to Earth and awaits instructions – an operating condition known as "safe" mode.



- Critical Issues: High level of radiations due to proximity to Sun. High maneuver energy required for Hohmann transfers (classical and bielliptic, due to Sun's gravitational pull)

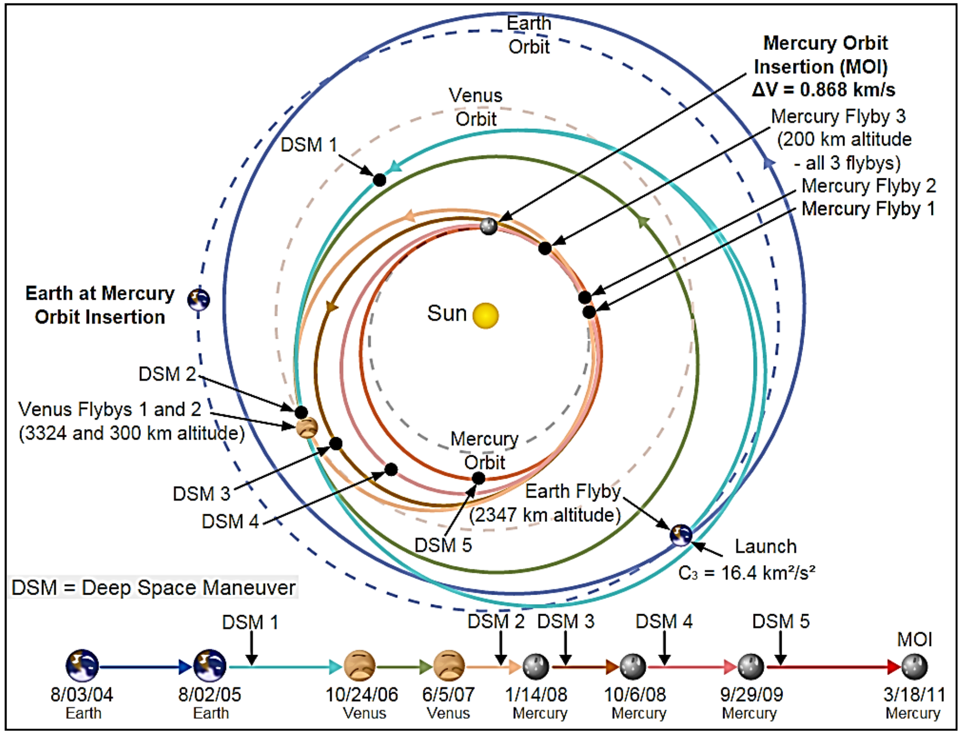
Table 3 Current ΔV allocation

ΔV budget category	ΔV, m/s
DSMs	1008
Launch vehicle, navigation errors (99%)	121
MOI	867
OCMs	85
Contingency	169
Total	2250

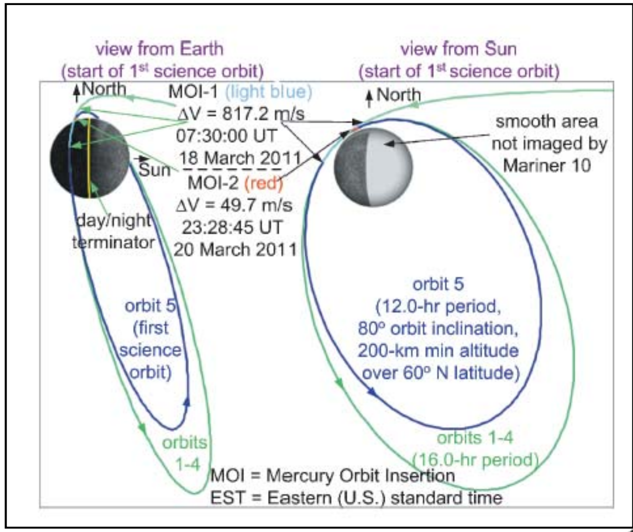
[Animation vimeo.com]

[Another Messenger Animation]

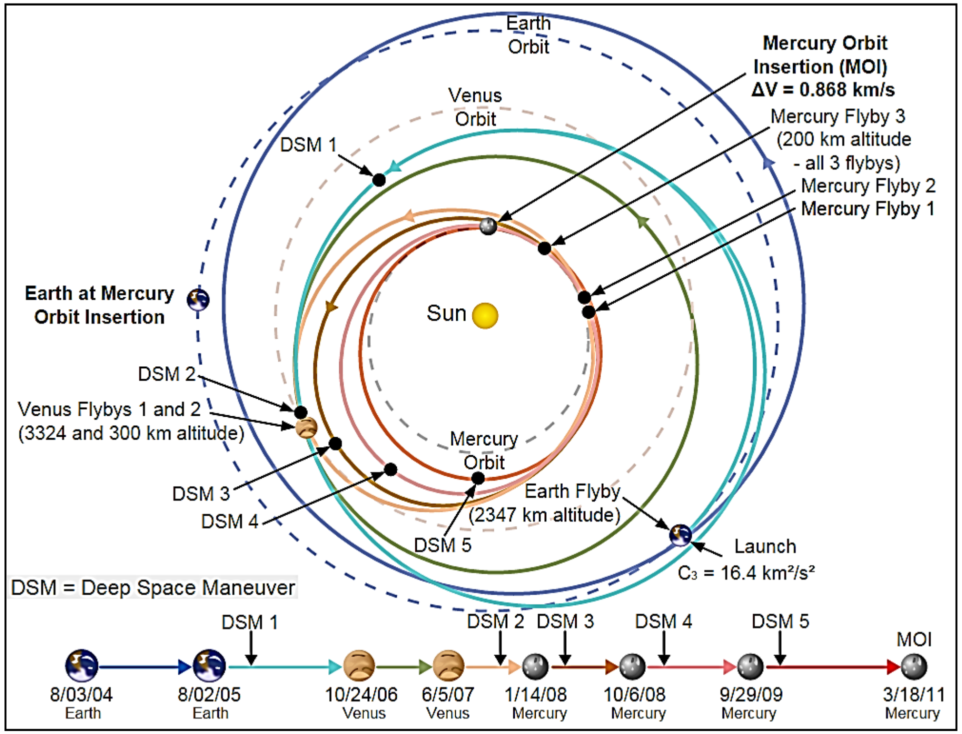
Interplanetary Flight



- Snapshot of Earth-Moon from 183million Km



Interplanetary Flight



- Launch to achieve hyperbolic escape with energy C_3 = 16.4

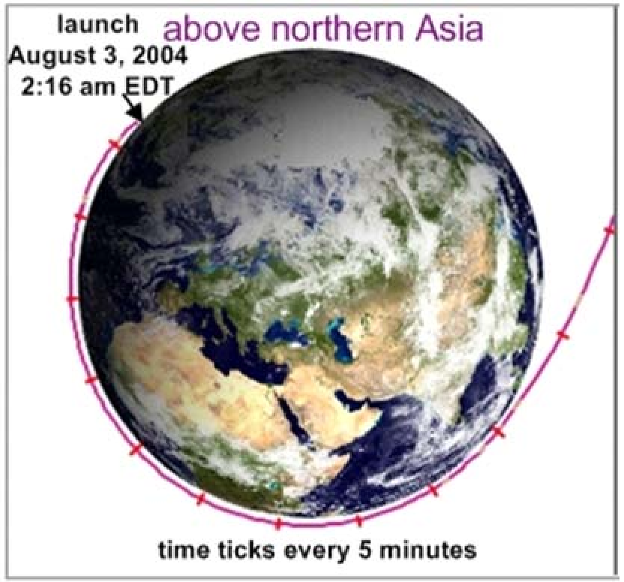


Table 2 MESSENGER launch options for 2004

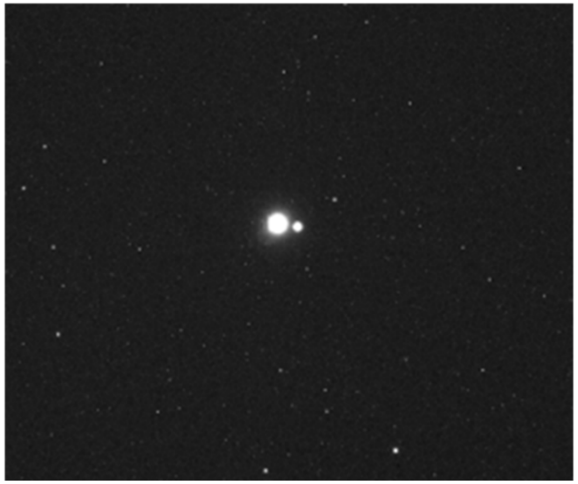
	Month		
Month	March	May	Aug. (final)
Launch dates	10–29	11–22	30 July–13 Aug.[a] 3 Aug.
Launch period, days	20	12	15
Launch energy, km^2/s^2	$\leq$15.700	$\leq$17.472	$\leq$16.887 (16.388)
Earth flybys	0	0	1
Venus flybys	2	3	2
Mercury flybys	2	2	3
Deterministic ΔV, m/s	$\leq$2026	$\leq$2074	$\leq$1991 (1966)
Total ΔV, m/s	2300	2276	2277 (2251)[b]
Orbit insertion date	6 April 2009	2 July 2009	18 March 2011

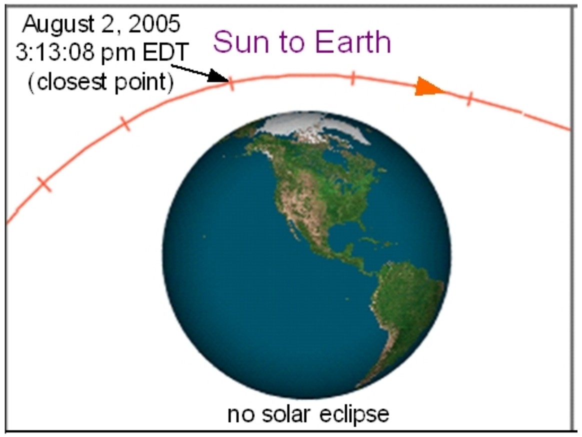
[a]Final launch period started on 2 August because of delays in availability of launch facility.
[b]Lower total ΔV reflects reduced propellant load required to meet spacecraft launch weight limit.

MESSENGER Launch and Earth Escape

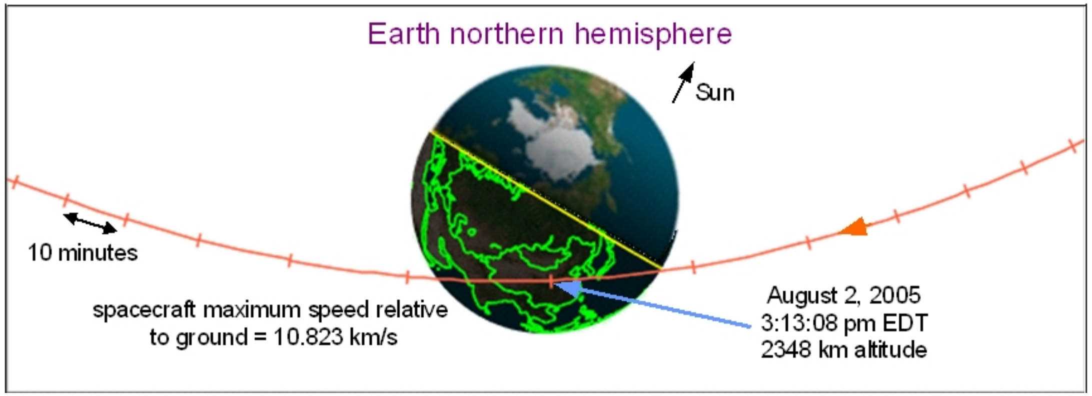
Interplanetary Flight

- Snapshot of Earth-Moon from 183million Km



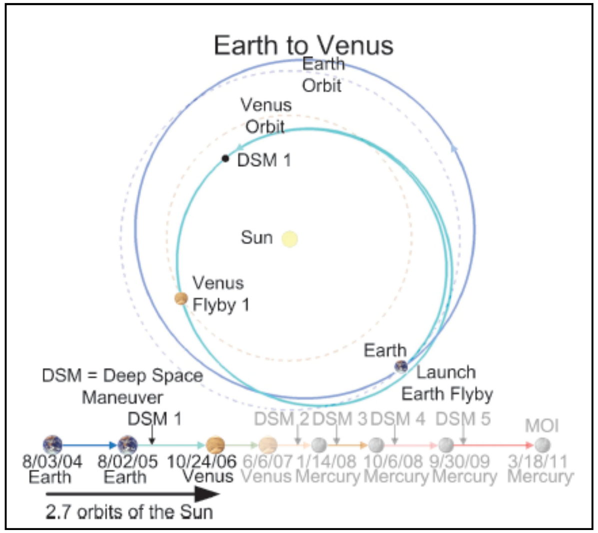


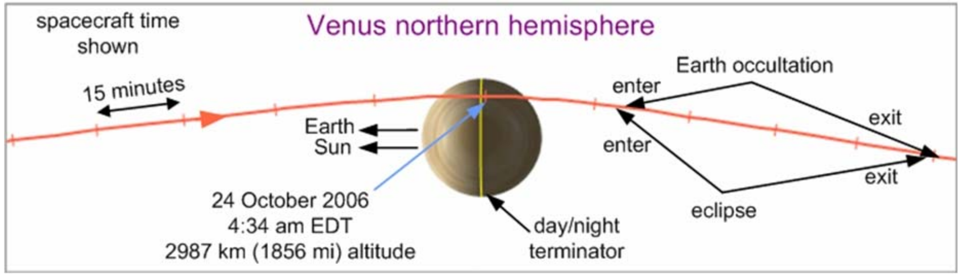
- Earth Fly By after 364 days from launch

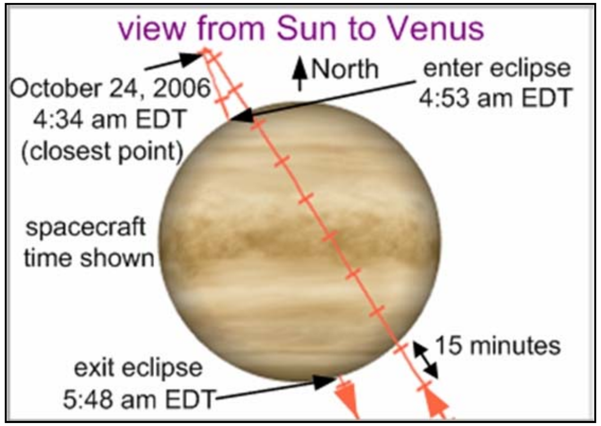


Interplanetary Flight

- First Venus Fly By







Highlights (January 2007 update) for Venus Flyby 1

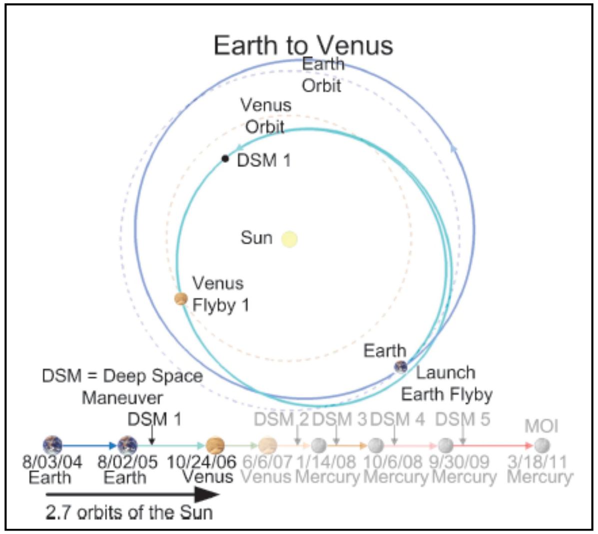
Minimum altitude (approximate since the surface is not perfectly round) above Venus:
2,987.3 kilometers (1,856.2 miles) on October 24, 2006 8:34 am UTC

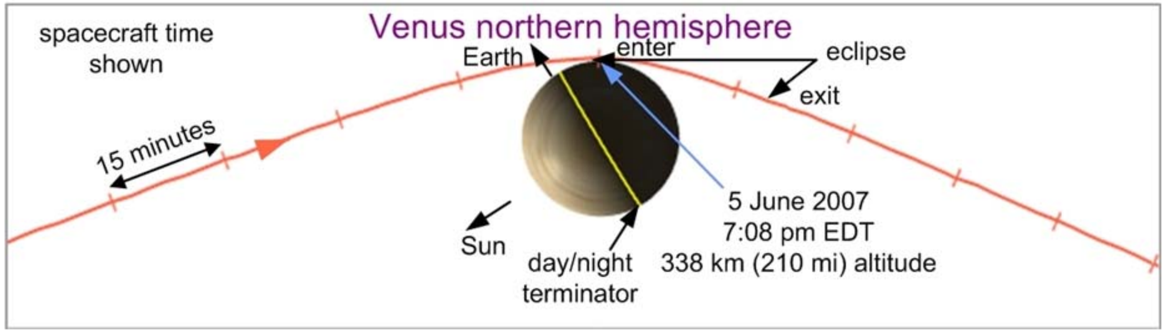
Maximum spacecraft speed relative to center of Venus:
12.395 kilometers per second (7.702 miles per second or 27,726 miles per hour)

Distance from Earth and Sun (1 AU = average Earth-Sun distance):
Earth-spacecraft: 1.7163 AU (256.75 million kilometers, 159.54 million miles)
Sun-spacecraft: 0.7222 AU (108.05 million kilometers, 67.136 million miles)

Interplanetary Flight

- Second Venus Fly By





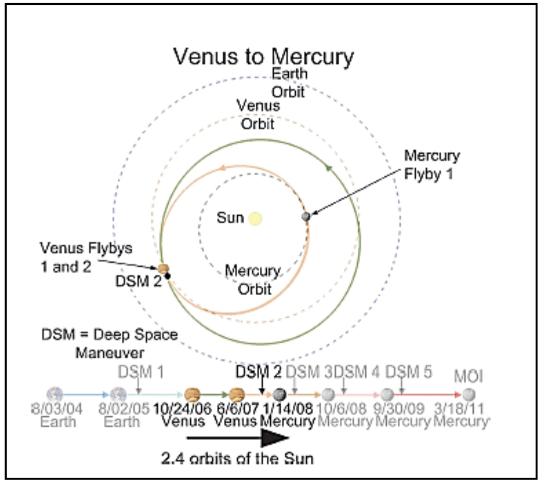
Minimum altitude above Venus:
338.2 kilometers (210.1 miles) on June 5, 2007 11:08 pm UTC

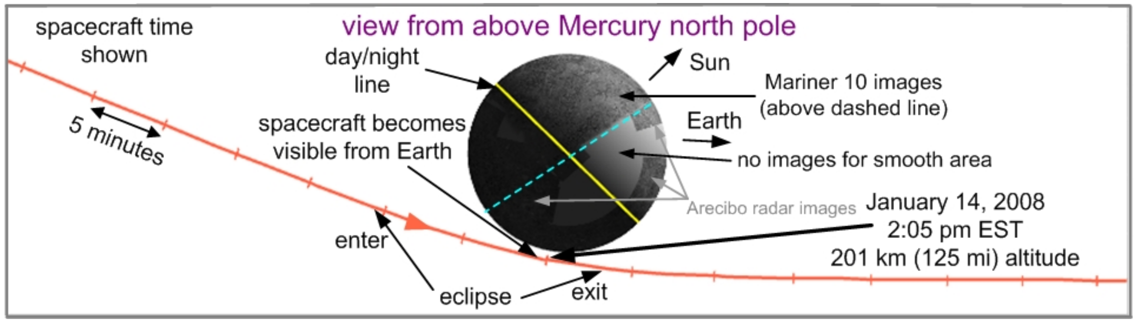
Maximum spacecraft speed relative to center of Venus:
13.531 kilometers per second (8.408 miles per second or 30,268 miles per hour)

Distance from Earth and Sun (1 AU = average Earth-Sun distance):
Earth-spacecraft: 0.7345 AU (109.87 million kilometers, 68.27 million miles)
Sun-spacecraft: 0.7222 AU (108.04 million kilometers, 67.14 million miles)

MESSENGER Second Venus Flyby

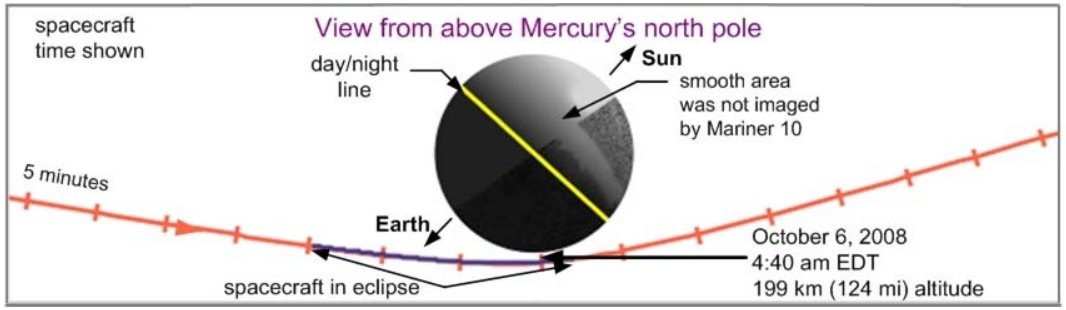
- Venus to Mercury

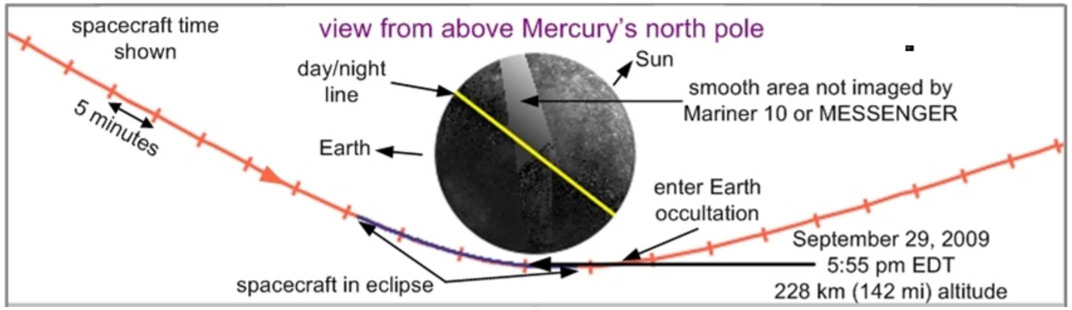


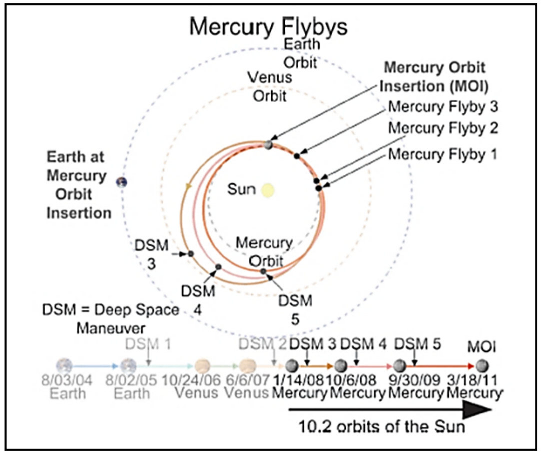


Interplanetary Flight

- Venus to Mercury Fly By's



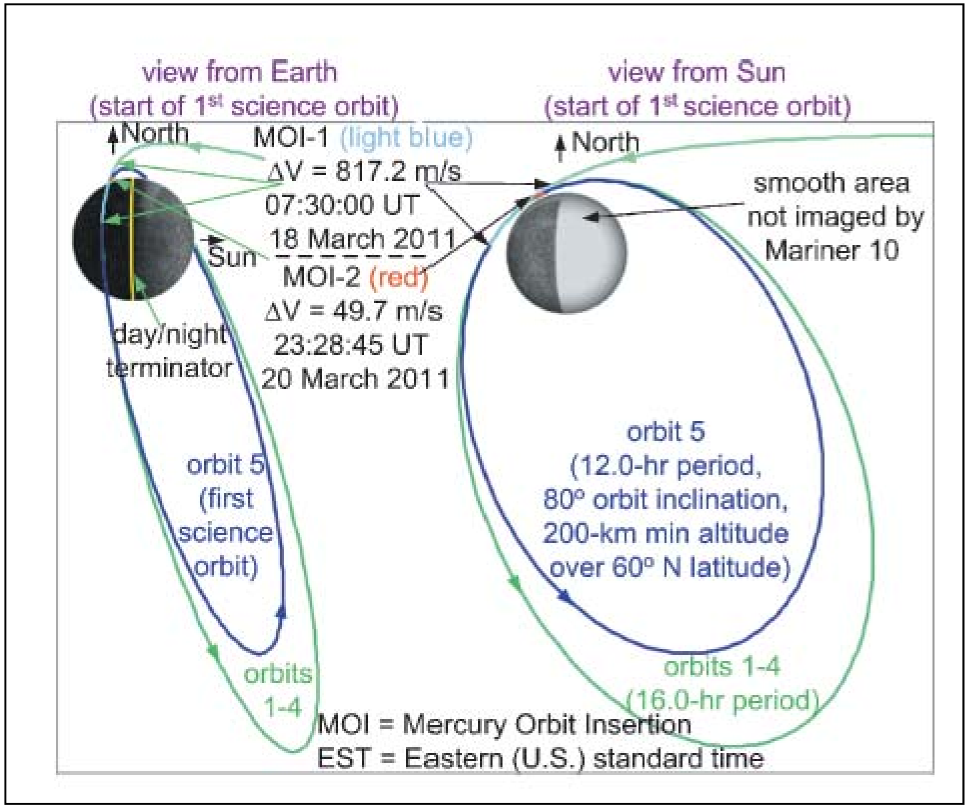




MESSENGER
Second Mercury
Flyby

MESSENGER
Third Mercury
Flyby

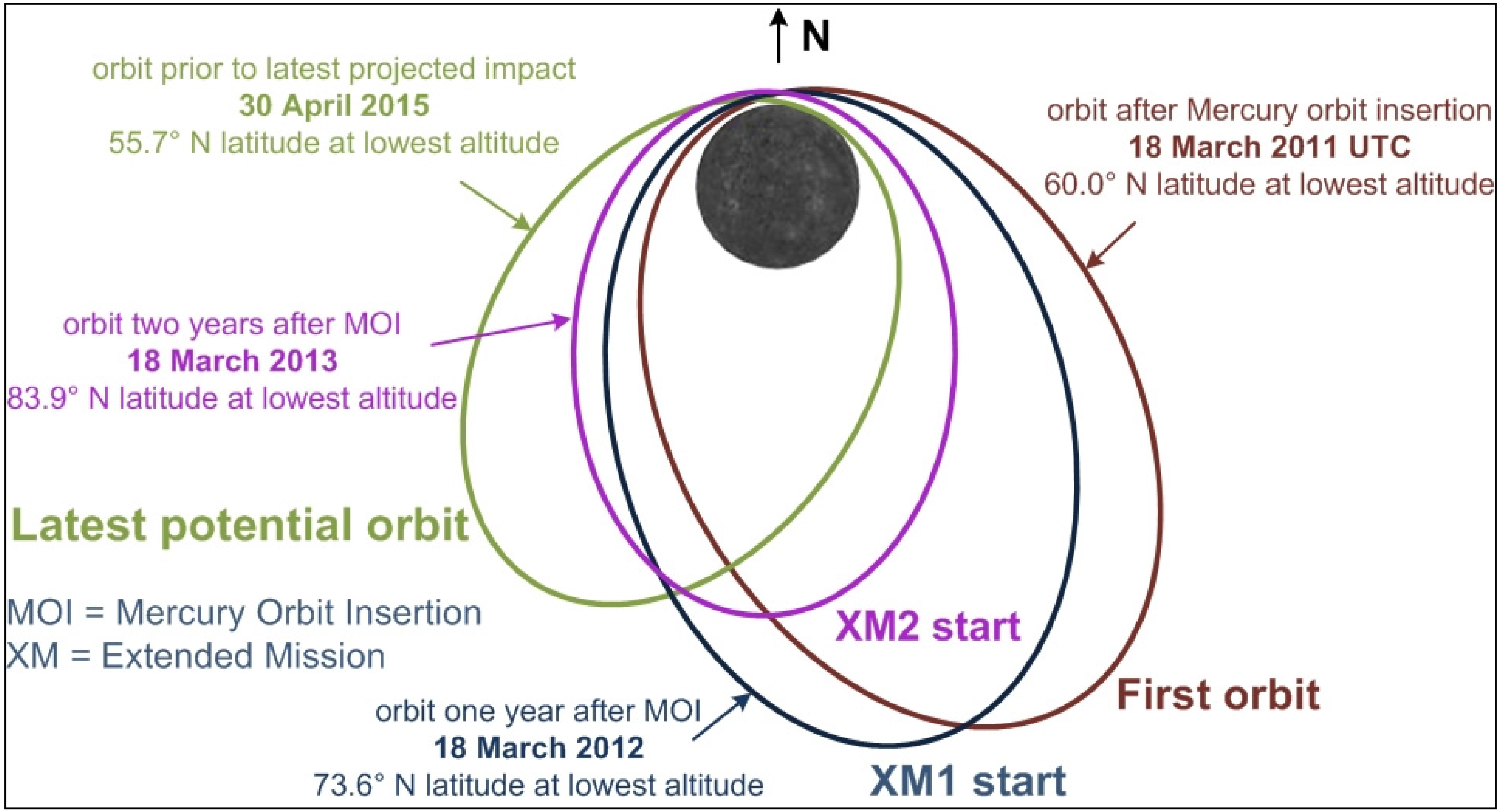
- Mercury Mission



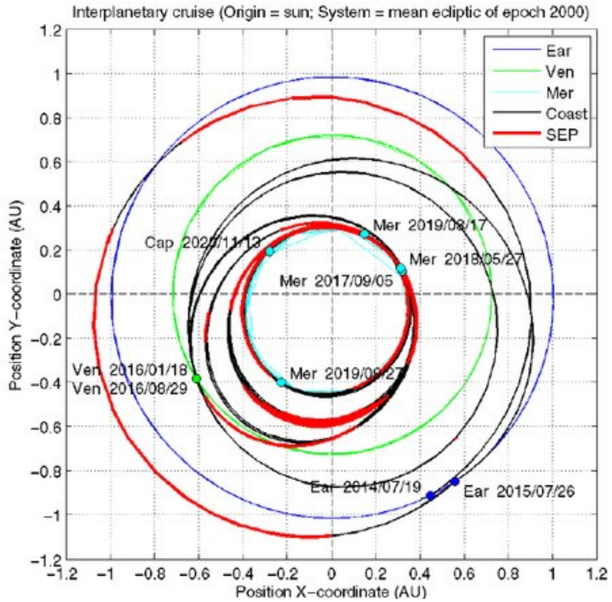
Maneuver segment	Date	Time, hr	ΔV,[a] m/s	Earth–spacecraft range, AU	Sun–spacecraft range, AU	Sun–spacecraft $-\Delta V$, deg	Sun–Earth–spacecraft, deg
Requirement→						(78–102)	(>3)
OCM 1 start	15 June 2011	1129:20	4.22	1.319	0.311	96.1	3.3
OCM 2 start	16 June 2011	1806:18	26.35	1.314	0.313	99.6	4.8
OCM 3 start	09 Sept. 2011	1207:57	3.92	1.097	0.308	86.0	16.1
OCM 4 start	10 Sept. 2011	1842:20	24.18	1.129	0.309	93.8	15.3
OCM 5 start	05 Dec. 2011	1244:25	3.59	0.683	0.308	82.1	3.2
OCM 6 start	06 Dec. 2011	1916:13	22.22	0.693	0.309	89.9	6.0

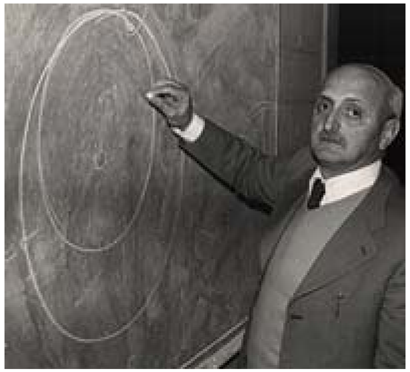
[a]Orbit phase total $\Delta V = 84.48$.

- End of Mission



- ESA Bepicolombo Mission



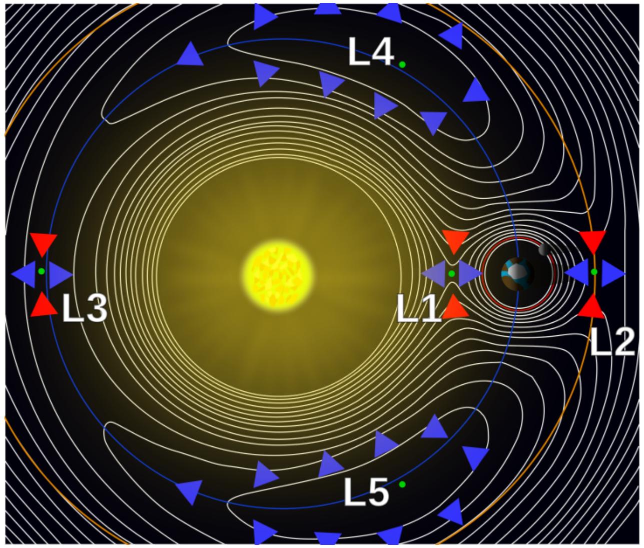


https://www.youtube.com/watch?v=BK3F4fmqtbA&t=109s

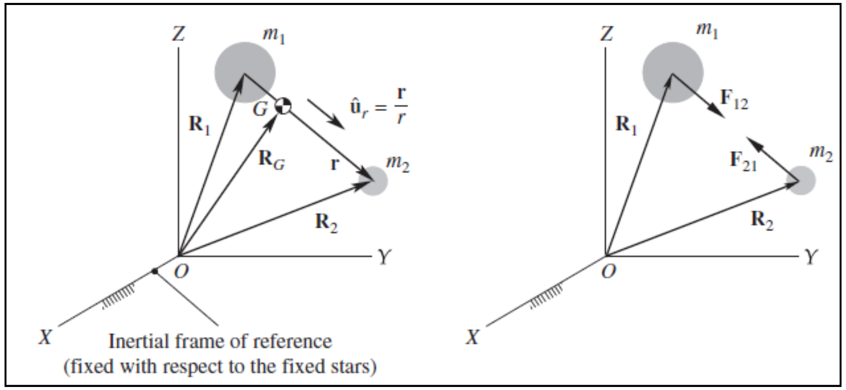
https://sci.esa.int/web/bepicolombo/home

[Maimeri] [Dawn]

Restricted three Body Problem



- **Consideration:** Two – Body problem gives an approximate analysis of motion, especially for (but not limited to) the trajectory of a spacecraft with respect to planets.
  - Euler 1771 Earth-Moon-Sun
  - Orbit of spacecraft near target planets
  - …
- **Recall the 2 – Body problem:**



- Inertial dynamics of each mass w.r.t. the other

$$m_1\ddot{\mathbf{R}}_1 = \frac{Gm_1m_2}{r^2}\hat{\mathbf{u}}_r$$

$$m_2\ddot{\mathbf{R}}_2 = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{u}}_r$$

- Relative position of mass 2 w.r.t. mass 1

$$\mathbf{r} = \mathbf{R}_2 - \mathbf{R}_1$$

- Inertial Motion of Center of Mass

$$\mathbf{R}_G = \frac{m_1\mathbf{R}_1 + m_2\mathbf{R}_2}{m_1 + m_2}$$

$$\mathbf{v}_G = \dot{\mathbf{R}}_G = \frac{m_1\dot{\mathbf{R}}_1 + m_2\dot{\mathbf{R}}_2}{m_1 + m_2}$$

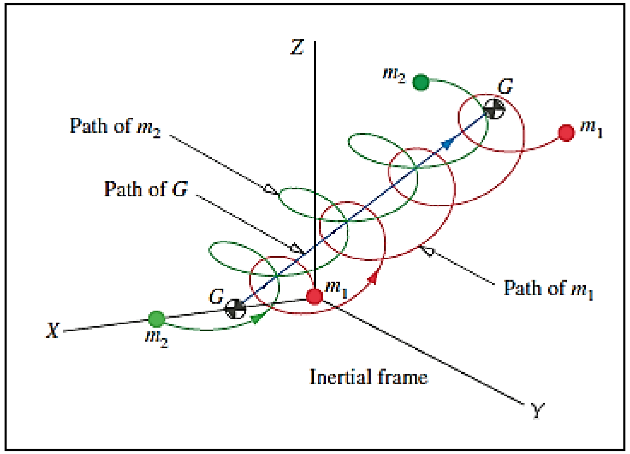
$$\mathbf{a}_G = \ddot{\mathbf{R}}_G = \frac{m_1\ddot{\mathbf{R}}_1 + m_2\ddot{\mathbf{R}}_2}{m_1 + m_2}$$

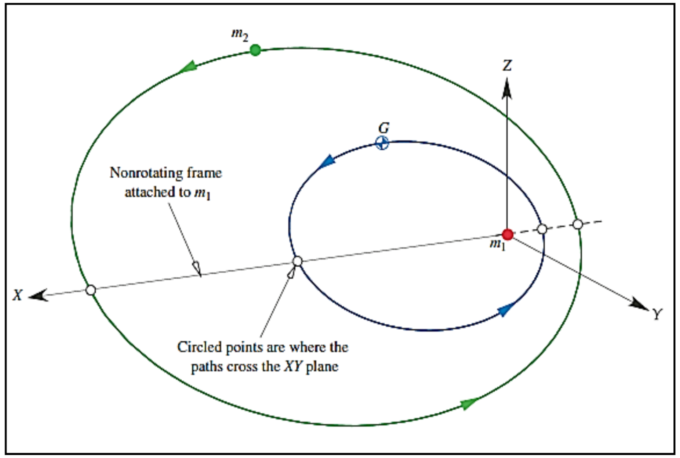
$$\boxed{\begin{aligned} m_1\ddot{\mathbf{R}}_1 + m_2\ddot{\mathbf{R}}_2 &= 0. \\ \mathbf{R}_G &= \mathbf{R}_{G_0} + \mathbf{v}_G t \end{aligned}}$$

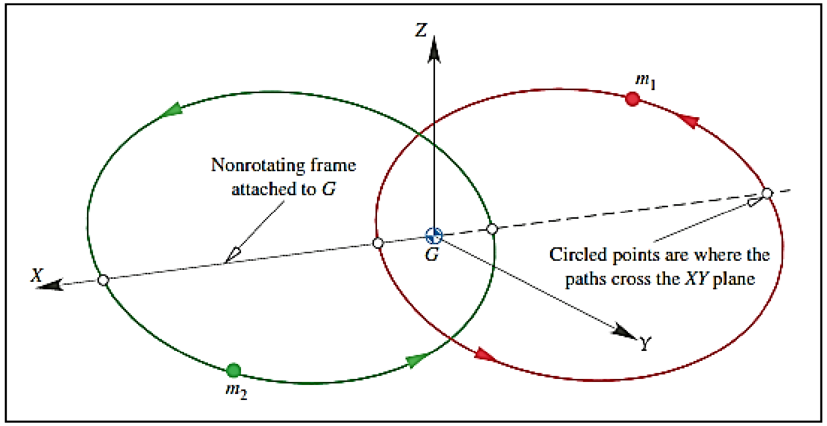
Restricted three Body Problem

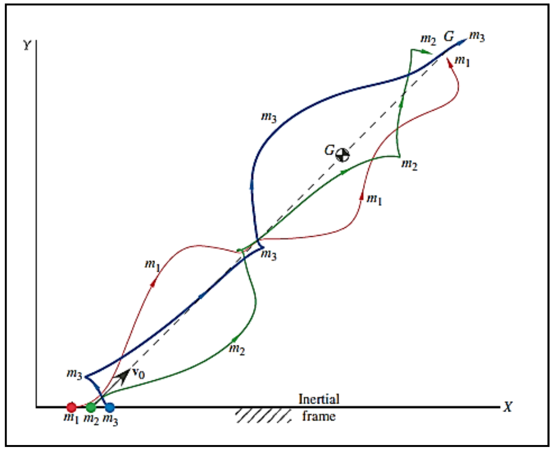
1. The center of mass moves in a straight line at constant speed, and can be used as origin of an inertial system
2. If $m_1 >> m_2$ the motion is called Central Force Motion and the mass m_1 can be considered as origin of an inertial system, i.e. $r = R_2 - R_1 \approx R_G$

$$\boxed{\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}}$$

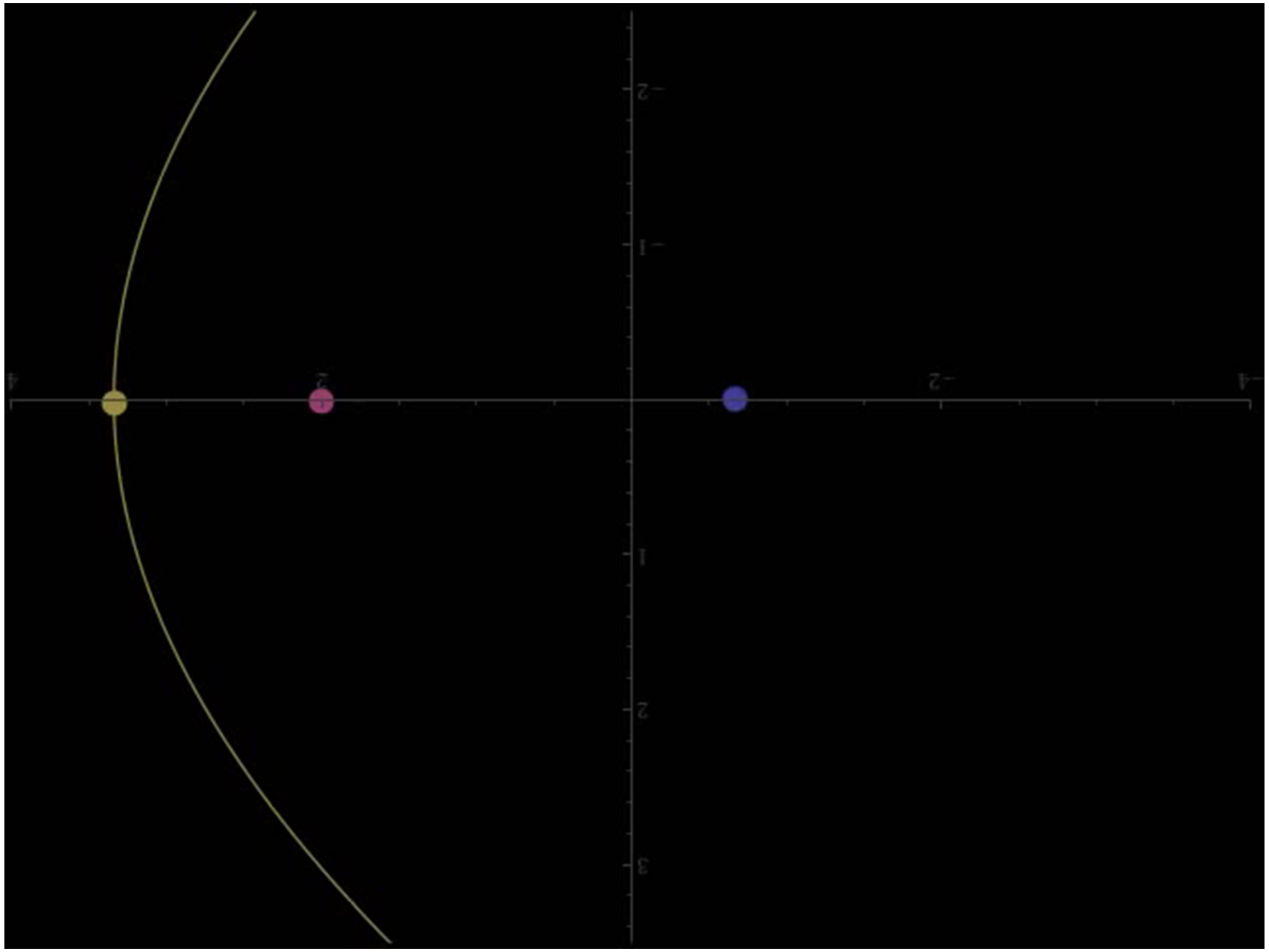






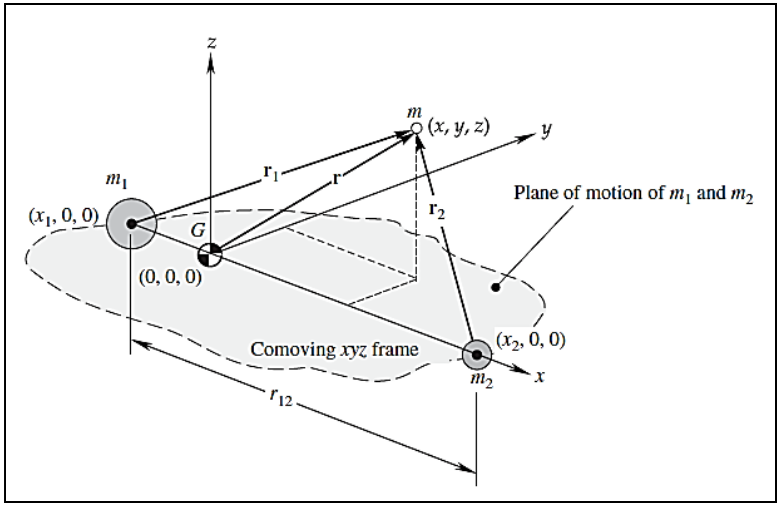


Restricted three Body Problem

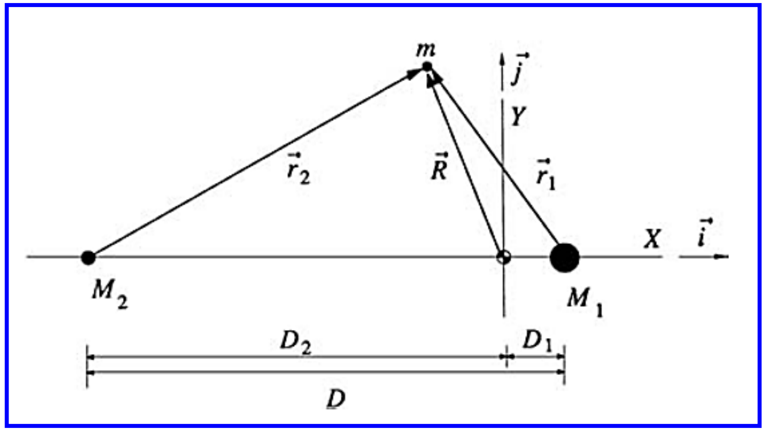


Restricted three Body Problem

- The general solution to the three-body problem must be obtained numerically, and even with these tools, the solution is chaotic: that is small changes in initial conditions lead to widely diverging behavior.
- **Circular restricted 3 – body problem:** two primary bodies moving in circular orbits about their common center of mass, plus a third small mass with negligible gravitational influence.



- Consider the motion in the moving reference frame and the Earth – Moon system (see Wie's text for complete derivation)



- **Constant Angular Velocity E-M system:**

$$n = \sqrt{G(M_1 + M_2)/D^3}$$

$$n = 2.661699 \times 10^{-6} \text{ rad/s}$$

$$n_{KEP} = 2.6944 \times 10^{-6}$$

- Earth - Moon Data:

$\mu_1 = GM_1 = 398{,}601 \text{ km}^3/\text{s}^2$ $\quad D_1 = 0.01215D = 4674 \text{ km}$

$\mu_2 = GM_2 = 4887 \text{ km}^3/\text{s}^2$ $\quad D_2 = 0.98785D = 380{,}073 \text{ km}$

$M_1 = 81.3045M_2$

$D = 384{,}748 \text{ km}$

Restricted three Body Problem

- Inertial acceleration of a mass m in a moving frame with angular velocity $\boldsymbol{\Omega}$ and center of mass G

$$\ddot{\boldsymbol{R}} = \boldsymbol{a}_G + \dot{\boldsymbol{\Omega}} \times \boldsymbol{R} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R}) + 2\boldsymbol{\Omega} \times \boldsymbol{v}_{REL} + \boldsymbol{a}_{REL}$$

- In our case, the center of mass has constant speed, the moving reference rotates at constant angular velocity, and the masses M_1 and M_2 move in circular orbits, thus:

$$\begin{cases} \boldsymbol{a}_G = 0 \\ \dot{\boldsymbol{\Omega}} = 0 \Rightarrow \boldsymbol{\Omega} = n\boldsymbol{k} = const \end{cases} \qquad \ddot{\boldsymbol{R}} = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R}) + 2\boldsymbol{\Omega} \times \boldsymbol{v}_{REL} + \boldsymbol{a}_{REL}$$

- In components:

$$\ddot{\boldsymbol{R}} = \left(\ddot{X} - 2n\dot{Y} - n^2 X\right)\boldsymbol{i} + (\ddot{Y} + 2n\dot{X} - n2Y)\boldsymbol{j} + \ddot{Z}\boldsymbol{k}$$

$$\begin{cases} \ddot{X} - 2n\dot{Y} - n^2 X \\ \ddot{Y} + 2n\dot{X} - n2Y \\ \ddot{Z} \end{cases}$$

- From Newton's Law:

$$m\ddot{\boldsymbol{R}} = -\frac{GM_1 m}{r_1^3}\boldsymbol{r}_1 - \frac{GM_2 m}{r_2^3}\boldsymbol{r}_2$$

$$\begin{cases} \boldsymbol{r}_1 = -D_1\boldsymbol{i} + R = (X - D_1)\boldsymbol{i} + Y\boldsymbol{j} + Z\boldsymbol{k} \\ \boldsymbol{r}_2 = D_2\boldsymbol{i} + R = (X + D_2)\boldsymbol{i} + Y\boldsymbol{j} + Zk \end{cases} \boldsymbol{k}$$

Restricted three Body Problem

- Equating the components:

$$\begin{cases} \ddot{X} - 2n\dot{Y} - n^2 X = -\dfrac{\mu_1 (X - D_1)}{r_1^3} - \dfrac{\mu_2 (X + D_2)}{r_2^3} \\ \ddot{Y} + 2n\dot{X} - n^2 Y = -\dfrac{\mu_1 Y}{r_1^3} - \dfrac{\mu_2 Y}{r_2^3} \\ \ddot{Z} = -\dfrac{\mu_1 Z}{r_1^3} - \dfrac{\mu_2 Z}{r_2^3} \end{cases}$$

- Multiply respectively by $\dot{X}, \dot{Y}, \dot{Z}$ and add, yields:

$$\begin{aligned} \ddot{X}\dot{X} + \ddot{Y}\dot{Y} + \ddot{Z}\dot{Z} - n^2 X\dot{X} - n^2 Y\dot{Y} &= -(\mu_1/r_1^3)[(X - D_1)\dot{X} + Y\dot{Y} + Z\dot{Z}] \\ &- (\mu_2/r_2^3)[(X + D_2)\dot{X} + Y\dot{Y} + Z\dot{Z}] \end{aligned}$$

- Integrating with respect to time, obtain a constant of motion called **<u>Jacobi Integral</u>**:

$$\frac{1}{2}(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - \frac{1}{2}n^2(X^2 + Y^2) - (\mu_1/r_1) - (\mu_2/r_2) = C$$

- We can define a pseudo-potential function (gravitational + centrifugal) as:

$$U = \frac{1}{2}n^2(X^2 + Y^2) + (\mu_1/r_1) + (\mu_2/r_2)$$

Restricted three Body Problem

- Then Jacobi's integral and the equations of motion can be written in compact form as:

$$C = \frac{1}{2}(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - U$$

$$\begin{cases} \ddot{X} - 2n\dot{Y} = \dfrac{\partial U}{\partial X} \\ \ddot{Y} + 2n\dot{X} = \dfrac{\partial U}{\partial Y} \\ \ddot{Z} = \dfrac{\partial U}{\partial Z} \end{cases}$$

- **Question: is there an equilibrium solution for the problem?**

- Introduce the mass ratio of Earth – Moon as

$$M_2/(M_1 + M_2) = \rho = 0.01215$$

$$M_1/(M_1 + M_2) = 1 - \rho = 0.98785$$

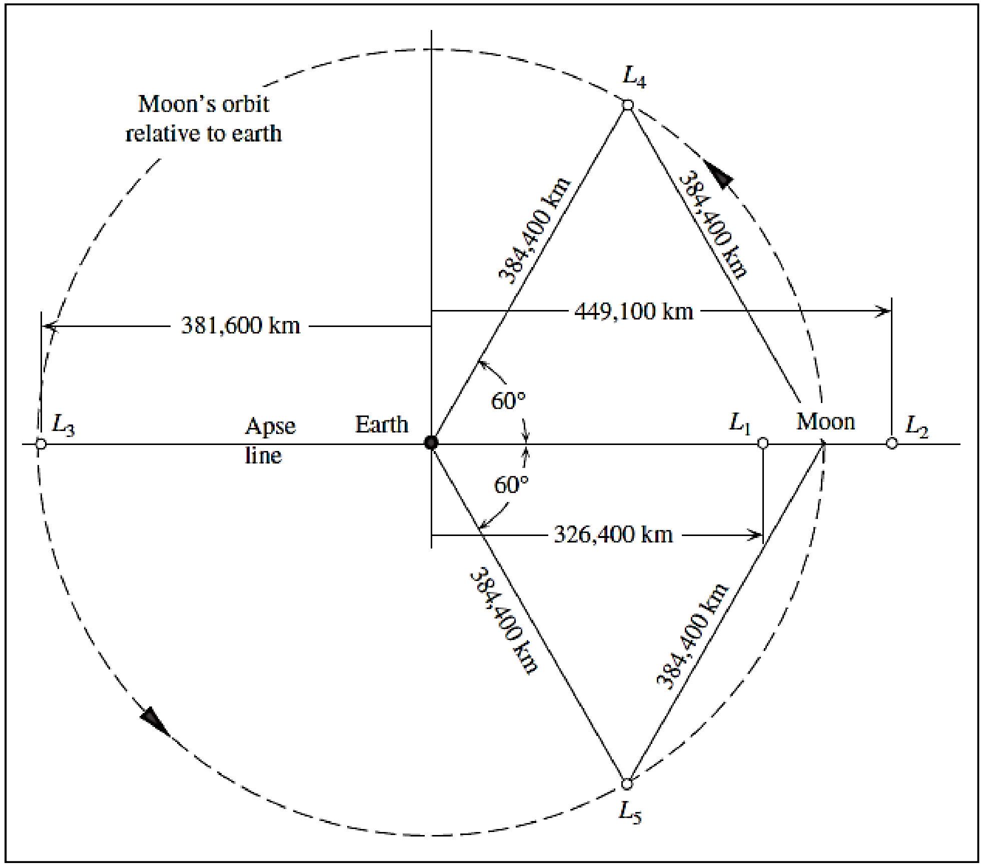
(*)

$$\begin{cases} \ddot{X} - 2\dot{Y} - X = -\dfrac{(1-\rho)(X-\rho)}{r_1^3} - \dfrac{\mu_2(X+1-\rho)}{r_2^3} \\ \ddot{Y} + 2\dot{X} - Y = -\dfrac{(1-\rho)Y}{r_1^3} - \dfrac{\rho Y}{r_2^3} \\ \ddot{Z} = -\dfrac{(1-\rho)Z}{r_1^3} - \dfrac{\rho Z}{r_2^3} \end{cases}$$

$$\begin{cases} r_1^2 = (X-\rho)^2 + Y^2 + Z^2 \\ r_2^2 = (X+1-\rho)^2 + Y^2 + Z^2 \end{cases}$$

Restricted three Body Problem

- By definition, at the equilibrium all the time derivatives in **(*)** must vanish, therefore:



- There are 5 equilibrium positions for the mass m called Lagrangian or Libration Points:
  - For all points $Z = 0$:
  - 3 of the equilibrium points have $Y = 0$, they are called collinear points L_1, L_2, L_3. They were discovered by Euler
  - There are 2 more equilibrium points located at the vertices of two equilateral triangles discovered by Lagrange and called L_4 and L_5.

Libration points	X	Y	Z
L_1	-0.83692	0	0
L_2	-1.15568	0	0
L_3	1.00506	0	0
L_4	-0.48785	$\sqrt{3}/2$	0
L_5	-0.48785	$-\sqrt{3}/2$	0

- **Stability of Lagrangian Points:** Of interest for spaceflight operations is the stability of the Lagrangian points

  - Example of procedure:
    - Linearize equations of motion about the equilibrium $X = X_0 + x; Y = Y_0 + y; Z = Z_0 + z = z$
    - Use Newton's binomial expansion to get approximate linear equations

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \quad (1+x)^n \approx 1 + nx; \forall x \ll 1$$

  - Stability of equilateral points $M_2/(M_1 + M_2) = \rho = 0.01215$

$$\ddot{x} - 2\dot{y} - \frac{3}{4}x - \frac{3\sqrt{3}}{2}\left(\rho - \frac{1}{2}\right)y = 0$$
$$\ddot{y} + 2\dot{x} - \frac{3\sqrt{3}}{2}\left(\rho - \frac{1}{2}\right)x - \frac{9}{4}y = 0$$

$$\lambda^4 + \lambda^2 + (27/4)\rho(1-\rho) = 0 \qquad \lambda = \pm\sqrt{\frac{-1 \pm \sqrt{1 - 27\rho(1-\rho)}}{2}}$$

For $\rho \leq 0.03852$ or $0.96148 \leq \rho$, the four eigenvalues become pure imaginary numbers, and the in-plane motion is said to be stable.

  - Stability of collinear points

$$\ddot{x} - 2\dot{y} - (2\sigma + 1)x = 0$$
$$\ddot{y} + 2\dot{x} + (\sigma - 1)y = 0$$
$$\ddot{z} + \sigma z = 0$$

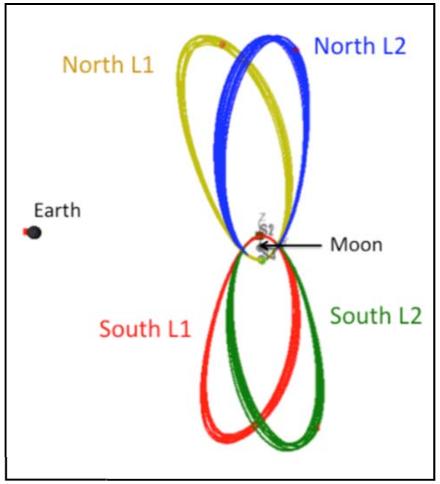
$$\lambda^4 - (\sigma - 2)\lambda^2 - (2\sigma + 1)(\sigma - 1) = 0$$

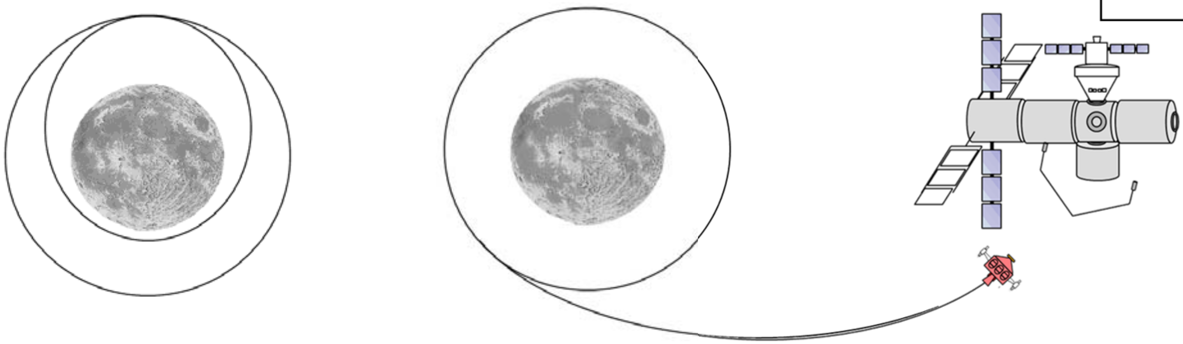
Two real (one positive) and two imaginary roots, the motion is unstable

Restricted three Body Problem

- Although collinear points are unstable in nature, the cislunar *L*1 and translunar *L*2 points are of practical importance for future space missions involving the stationing of a communication platform or a lunar space station. The cislunar *L*1 point could serve as a transportation node for lunar transfer trajectories. The translunar *L*2 point would be an excellent orbital location to station a satellite to provide a communications link to the far side of the moon.

  - Cis – lunar Transport Habitat (CTH) in orbit near L2 Lagrangian.
  - Lunar Ascent Element (LAE) Rendez Vous with CTH.
  - Dynamics and Control for Rendez Vous in near rectilinear orbits (NRO) based on 3 – Body problem not restricted to circular orbits.



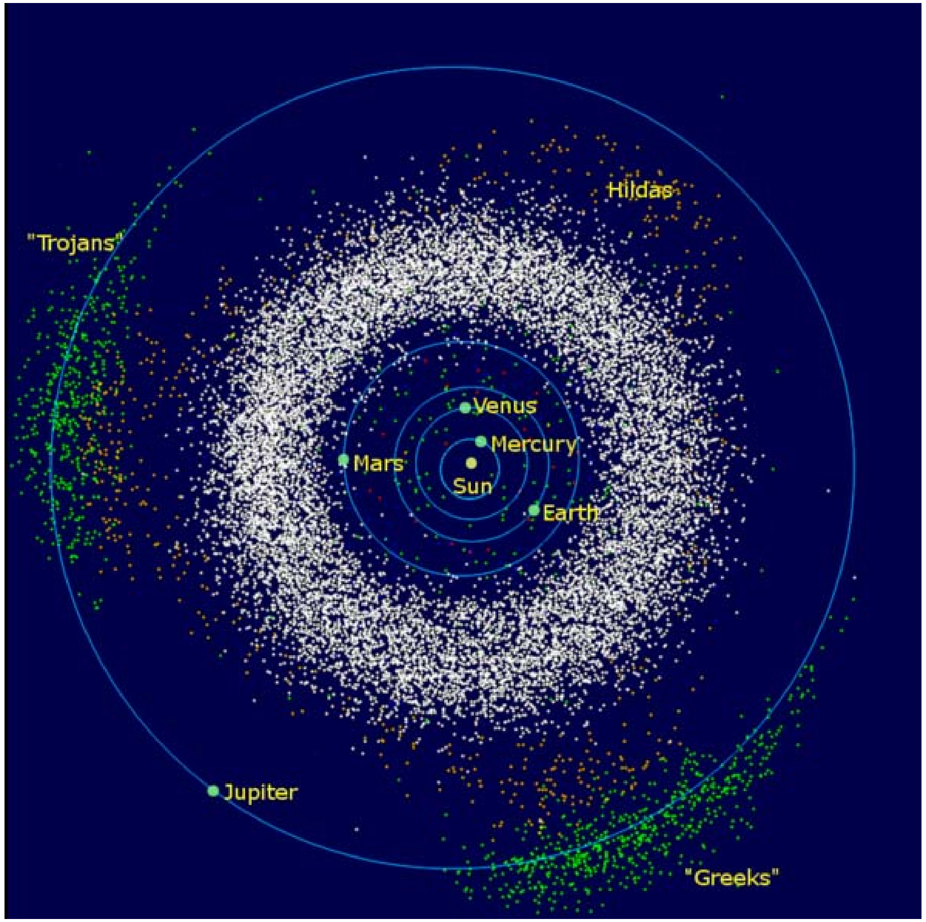


Phase 1
Circularization

Phase 2
from CRO to NRO

Phase 3
Rendez Vous LAE - CTH

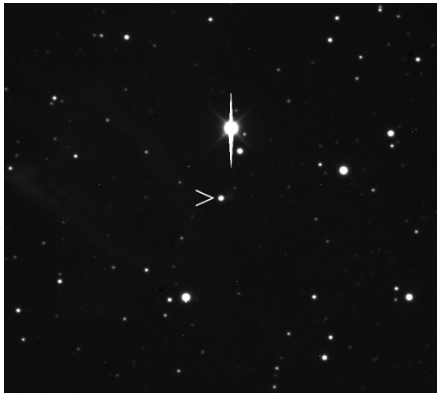
- **Planetary example:** Jupiter – Sun Trojans



The largest Jupiter trojans

Trojan	Diameter (km)
624 Hektor	225
911 Agamemnon	167
1437 Diomedes	164
1172 Äneas	143
617 Patroclus	141
588 Achilles	135
1173 Anchises	126
1143 Odysseus	126
Source: JPL Small-Body Database, IRAS data	

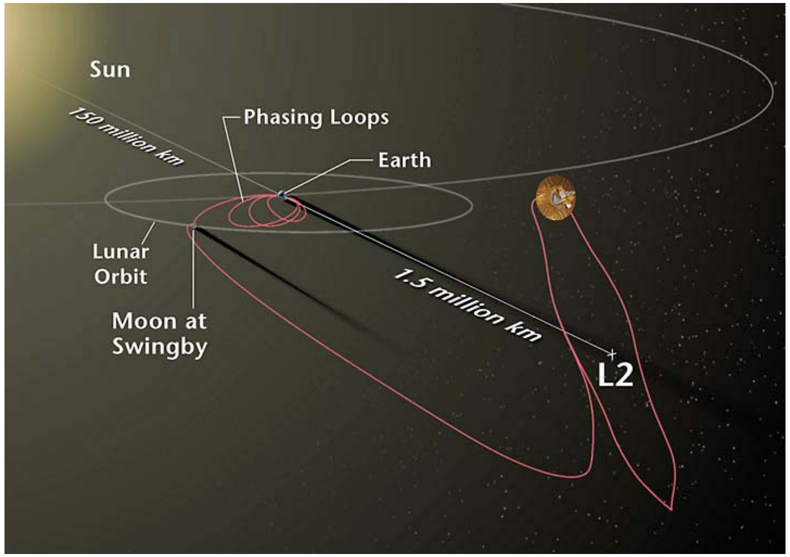
1906 Wolf



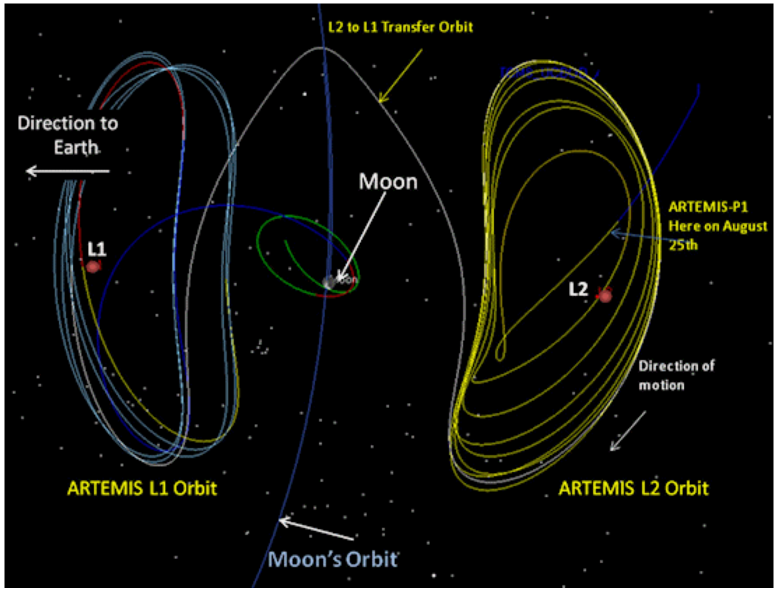
Trojan 624 Hektor (at center) is similar in brightness to dwarf planet Pluto.

Spacecraft Examples:

The Wilkinson Microwave Anisotropy Probe (WMAP), originally known as the Microwave Anisotropy Probe (MAP) was a spacecraft operating from 2001 to 2010 which measured differences across the sky in the temperature of the cosmic microwave background (CMB) – the radiant heat remaining from the Big Bang.[3][4]

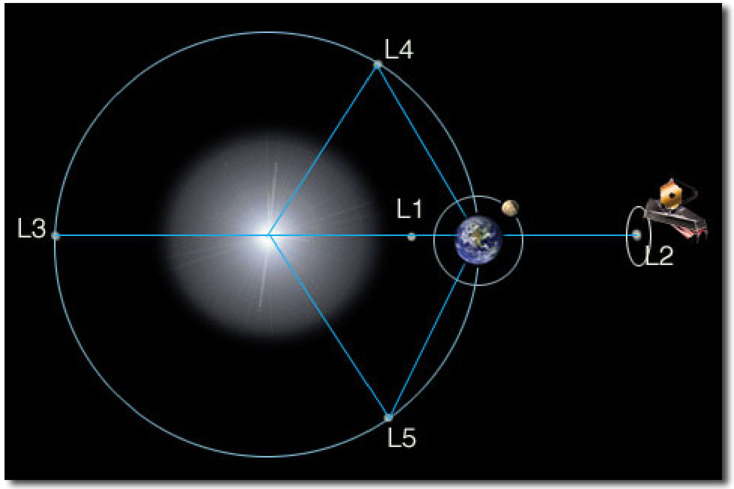


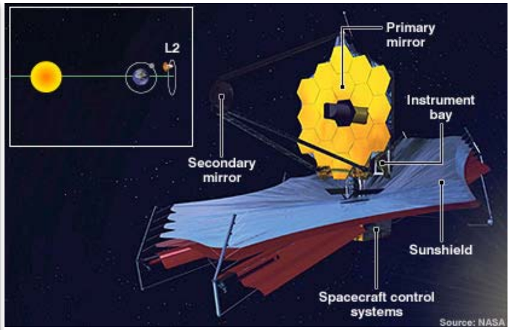
NASA has been studying the region around the Earth which protects us from the solar wind. A unique group of five satellites, called THEMIS, are helping scientist to unlock the mystery of how Earth's magnetosphere stores and releases energy from the sun. Three of these satellites continue to study sub storms that are visible in the Northern Hemisphere as a sudden brightening of the Northern Lights, or aurora borealis.
Two of the five spacecraft have been sent on a new mission to the moon. This mission is called ARTEMIS, or Acceleration, Reconnection, Turbulence and Electrodynamics of Moon's Interaction with the Sun. As the name suggests, the two spacecraft will measure what happens when the Sun's radiation hits our rocky moon, where there is no magnetic field to protect it. As the moon orbits the Earth, it moves into and out of our magnetic shield. What happens to the moon when it is within Earth's magnetic shield?

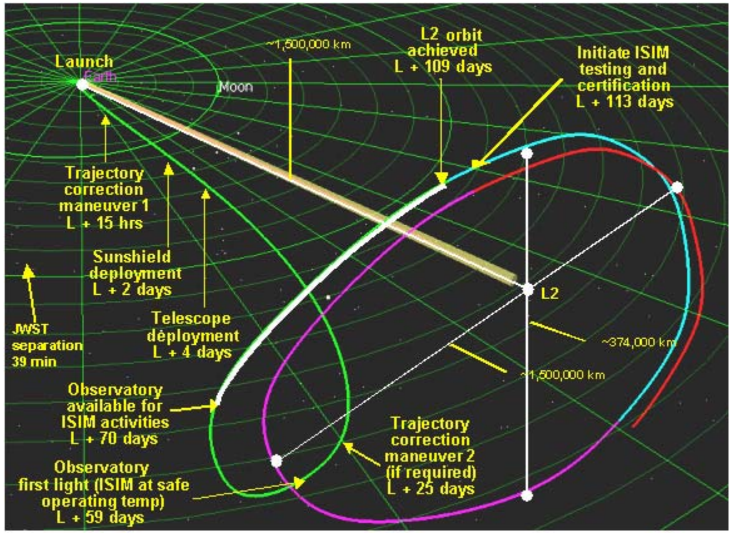


Restricted three Body Problem

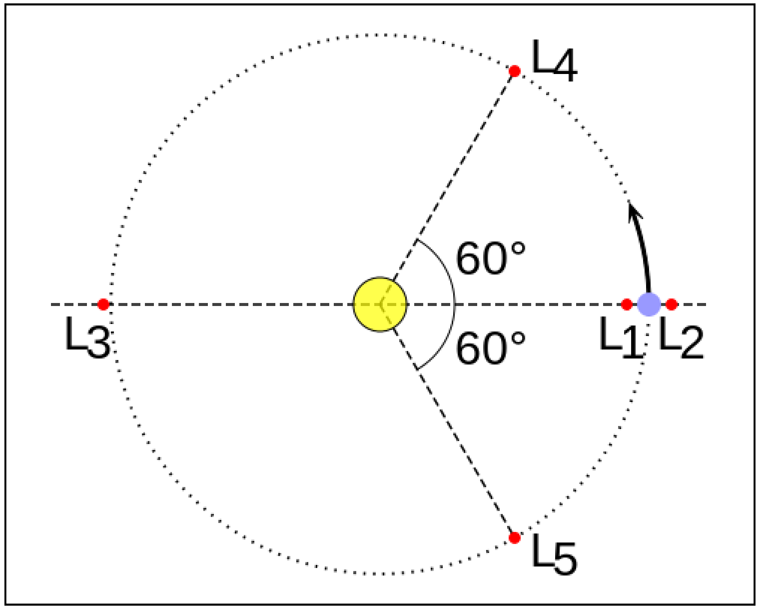
❑ Spacecraft Examples:







The **James Webb Space Telescope** (JWST), previously known as Next Generation Space Telescope (NGST), is a space observatory under construction and scheduled to launch in October 2018. The JWST will offer unprecedented resolution and sensitivity from long-wavelength visible to the mid-infrared, and is a successor instrument to the Hubble Space Telescope and the Spitzer Space Telescope. The telescope features a segmented 6.5-meter (21 ft) diameter primary mirror and will be located near the Earth–Sun L2 point. A large sunshield will keep its mirror and four science instruments below 50 K (−220 °C; −370 °F). **It Should launch end of December 2021!!!**



- **Sun – Earth L1**

- WIND (spacecraft) (At L1 since 2004)
- The Solar and Heliospheric Observatory (SOHO)
- The Advanced Composition Explorer (ACE)
- The Deep Space Climate Observatory (DSCOVR), designed to monitor global warming
- LISA Pathfinder

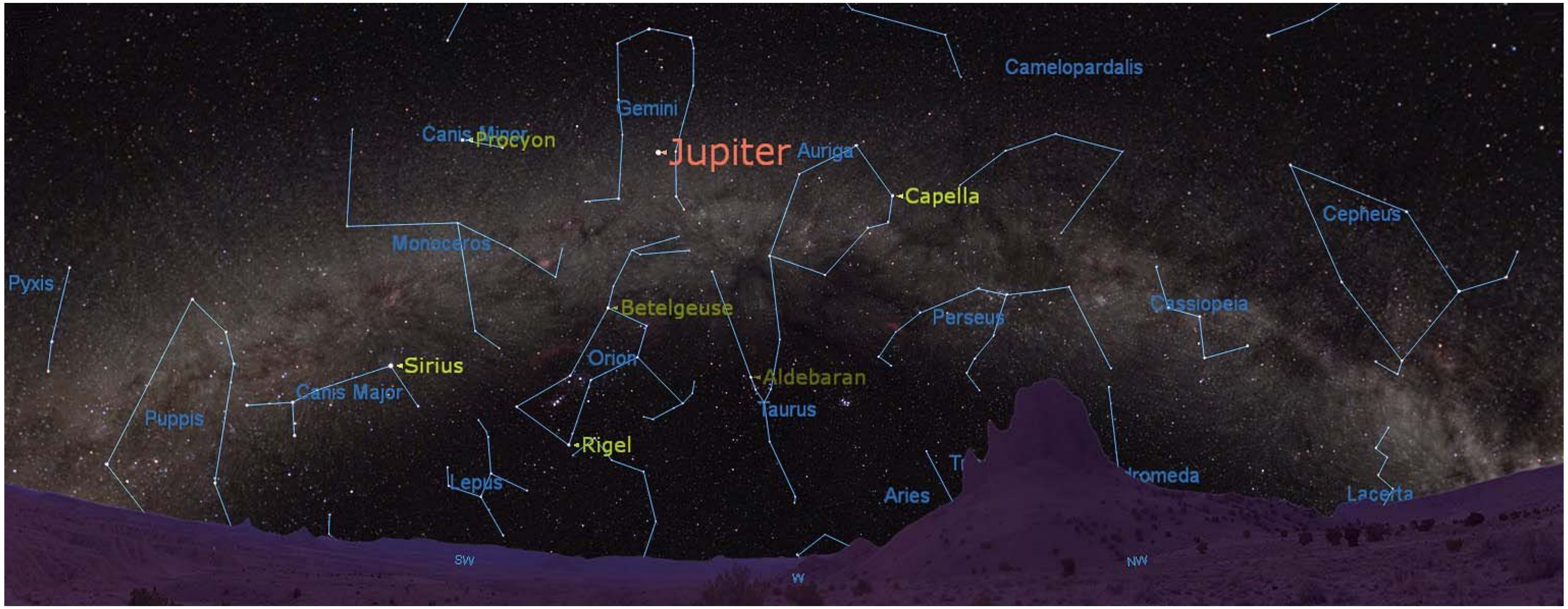
- **Sun – Earth L2**

- The ESA Gaia probe

Planned probes [edit]

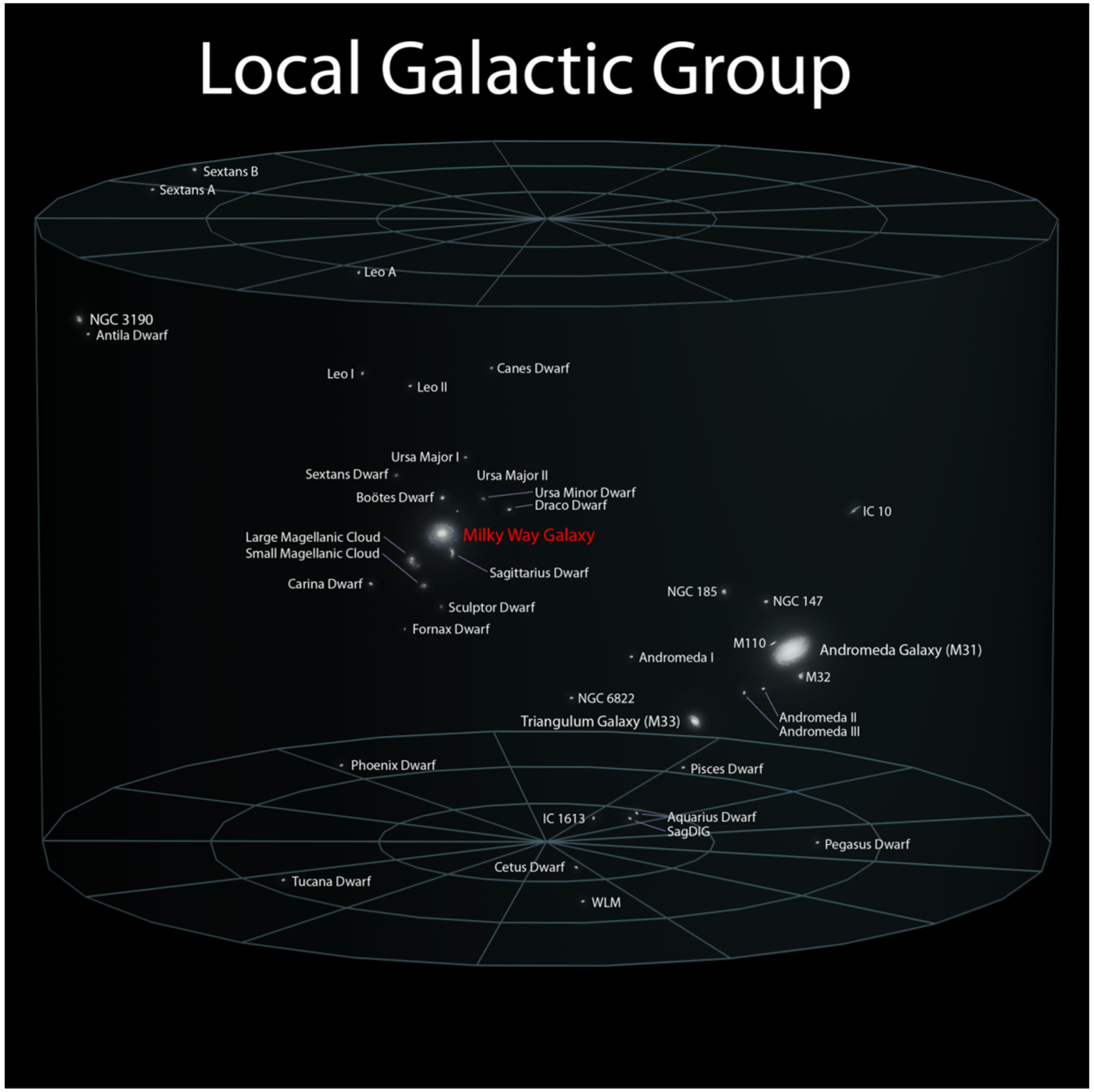
- The joint NASA, ESA and CSA James Webb Space Telescope (JWST), formerly known as the Next Generation Space Telescope (NGST)
- The ESA PLATO mission, which will find and characterize rocky exoplanets.
- The NASA Advanced Technology Large-Aperture Space Telescope, which would replace the Hubble Space Telescope and possibly the JWST.

A more general View

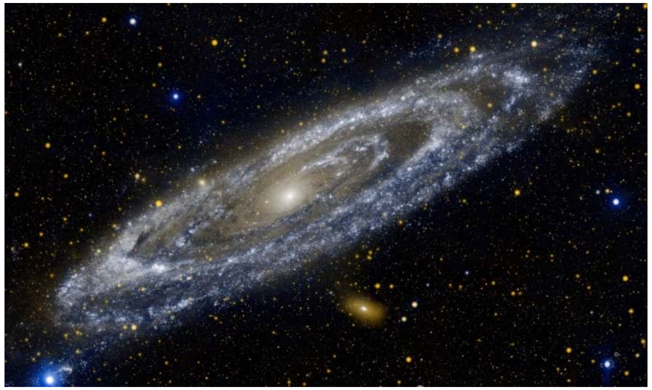


The Milky Way is a barred spiral galaxy that has a diameter usually considered to be about **100,000–120,000 light-years but may be 150,000–180,000 light-years**. The Milky Way is estimated to contain 200–400 billion stars **($2 – 4x\ 10^8$)**, although this number may be as high as one trillion. There are probably at least 100 billion planets in the Milky Way. The Solar System is located within the disk, about **27,000 light-years** from the Galactic Center, on the inner edge of one of the spiral-shaped concentrations of gas and dust called the Orion Arm. The stars in the inner ≈10,000 light-years form a bulge and one or more bars that radiate from the bulge. The very center is marked by an intense radio source, named **Sagittarius A***, which is likely to be a supermassive black hole.

A more general View



The Local Group comprises more than 54 galaxies, most of them dwarf galaxies. Its gravitational center is located somewhere between the Milky Way and the Andromeda Galaxy. **The Local Group covers a diameter of 10 Mly** and has a binary (dumbbell) distribution.

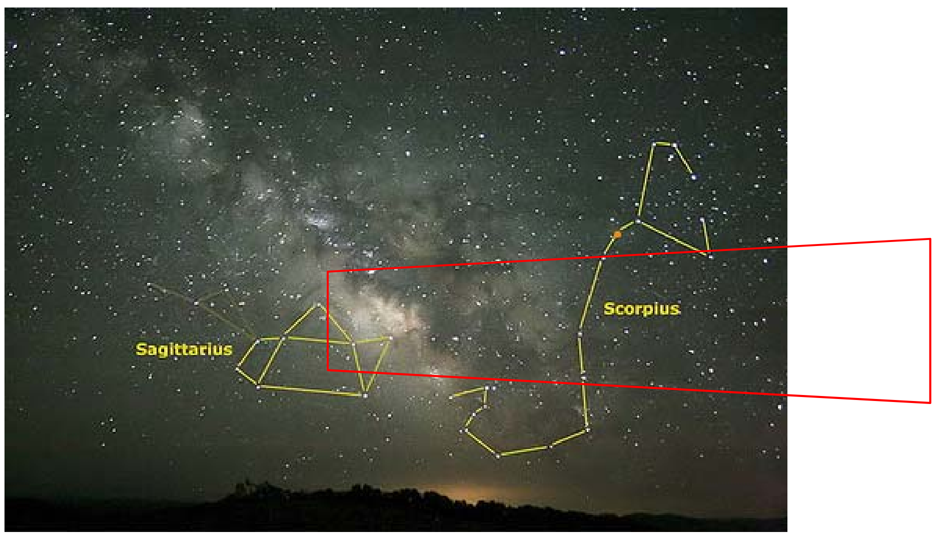


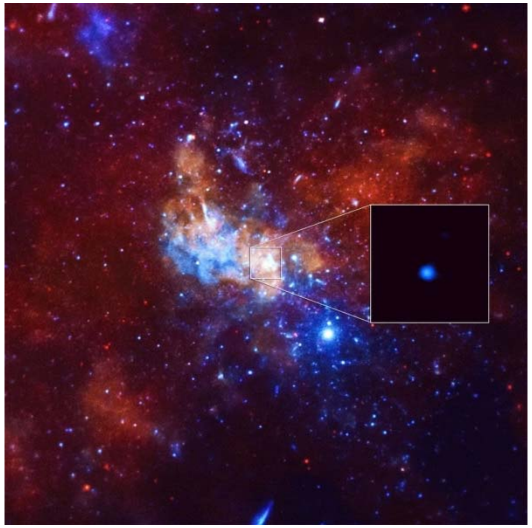
The Andromeda Galaxy , also known as Messier 31, M31, or NGC 224, is a spiral galaxy approximately **2.5 Mly from Earth**. It is the nearest major galaxy to the Milky Way. Being approximately **220,000 light years across**, it is the largest galaxy of the Local Group, which also contains the Milky Way, the Triangulum Galaxy, and about 44 other smaller galaxies.

The Triangulum Galaxy is a spiral galaxy approximately **3 Mly from Earth** in the constellation Triangulum. It is catalogued as Messier 33 or NGC 598. The Triangulum Galaxy is the third-largest member of the Local Group

Computation of position far away Objects

- Indirect Information on Milky Way's central Black Hole Sagittarius A*
  - Sagittarius A*: A radio signal was coming from a location at the center of the Milky Way, in the direction of the constellation of Sagittarius was discovered on February 13 and 15, 1974. It is part of a larger astronomical feature known as Sagittarius A. Sagittarius A* is thought to be the location of a supermassive black hole distant about 26,000 Ly.

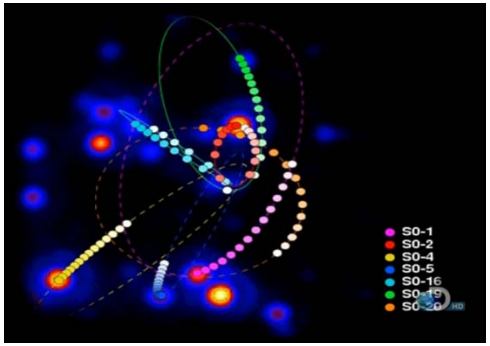


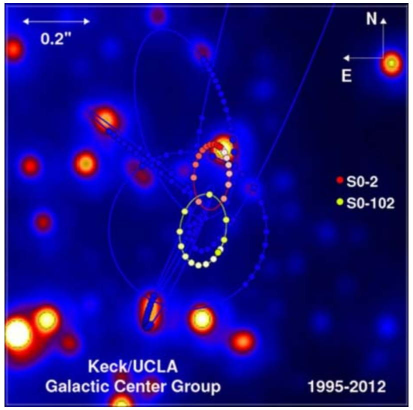


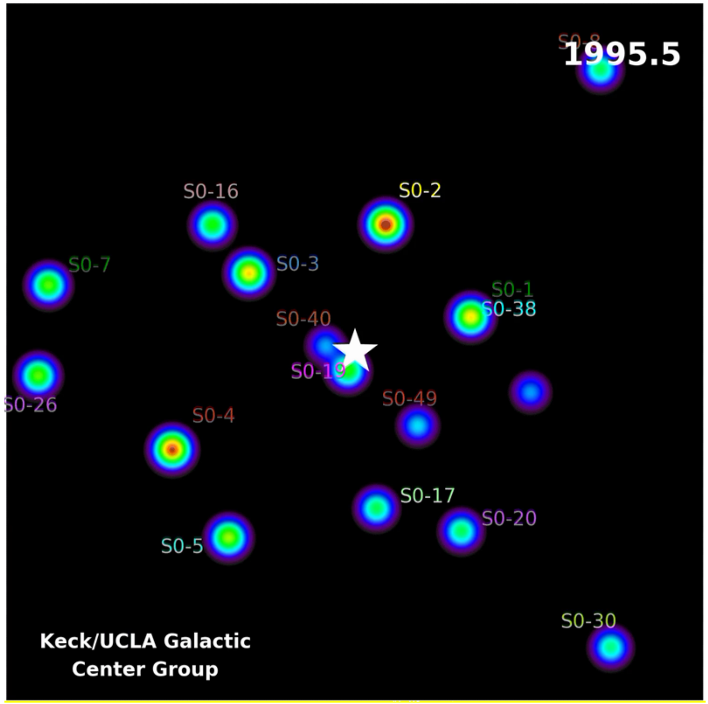
S2, also known as S0–2, is a star that is located close to the radio source Sagittarius A*, orbiting it with an orbital period of 15.56 ± 0.35 years. Its changing apparent position has been monitored since 1995 by two groups (at UCLA and at the Max Planck Institute for Extraterrestrial Physics). By 2008, S2 had been observed for one complete orbit. The mass of S2 is estimated to be 15 solar masses

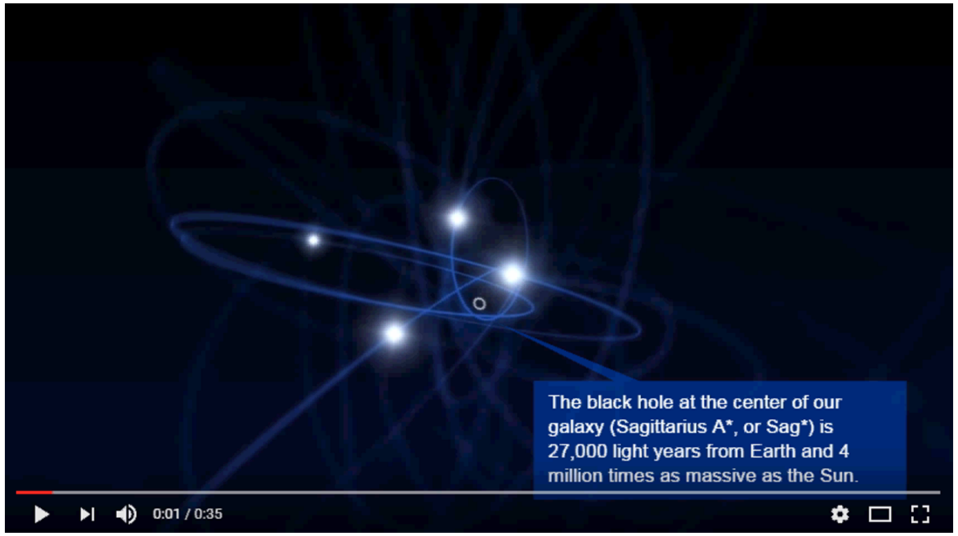
Computation of far away Objects

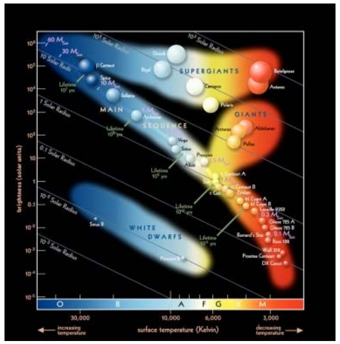
- Keck Observatory and La Silla Observatory











Computation of far away Objects

- Use Keplerian model from S2 star observations

$$\left(\frac{v^2}{2}-\frac{\mu}{r}\right)=-\frac{\mu}{2a} \qquad r=\frac{a(1-e^2)}{1+e\cos f} \qquad P^2=\left(4\frac{\pi^2}{\mu}\right)a^3 \qquad \begin{matrix} r_p=a(1-e) \\ r_a=a(1+e) \end{matrix} \qquad h=r_p v_p=r_a v_a \cong r^2\dot{f}$$

- Computation of Sag A* mass:

$$\begin{cases} \mu = GM_{SAGA*}m_{S2} \simeq GM_{SAGA*} \\ G=6.6262\times 10^{-11}\left[\dfrac{m^3}{s^2 Kg}\right] \end{cases}$$

- From observations:

$$\begin{cases} P \approx 15.6\, years \\ r_p \approx 120\, AU \\ r_a \approx 1800\, AU \end{cases} \qquad a=\frac{r_p+r_a}{2}=960\, AU \approx 1.44\times 10^{11}\, Km$$

$$\boxed{M_{SAGA*} \simeq \frac{4\pi^2 a^3}{GP^2}=4\times 10^6\, Suns = 7.292\times 10^{36}\, Kg}$$

$$\frac{v_p^2}{2}-\frac{\mu}{r_p}=\frac{v_a^2}{2}-\frac{\mu}{r_a}=-\frac{\mu}{2a} \qquad h=r_p v_p=r_a v_a \qquad \begin{cases} v_p \simeq 7.1\times 10^6\left[\dfrac{m}{s}\right]=25\times 10^6\left[\dfrac{Km}{h}\right]=2.4\%[c] \\ v_a \simeq 4.7\times 10^5\left[\dfrac{m}{s}\right]=1.692\times 10^6\left[\dfrac{Km}{h}\right]=0.16\%[c] \end{cases}$$

- S0–102 is a star that is located very close to the center of the Milky Way, orbiting it with an orbital period of 11.5 years. As of 2012 it is the star with the shortest known period orbiting the black hole. To get a sense of the enormity of the Milky Way's black hole, consider that Sedna orbits the Sun in about 11,700 years. Sagittarius A* pulls S0-102 through its orbit in only 11.5 years. S0-102 reaches over 1% the speed of light at periapsis.

Computation of far away Objects

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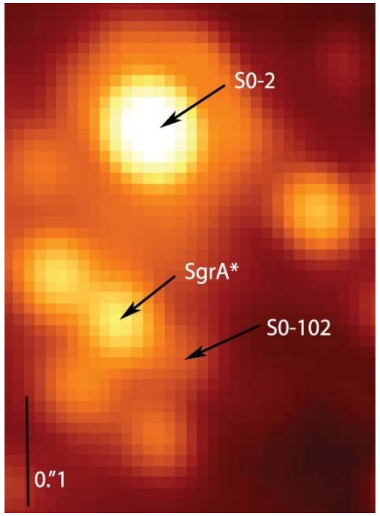
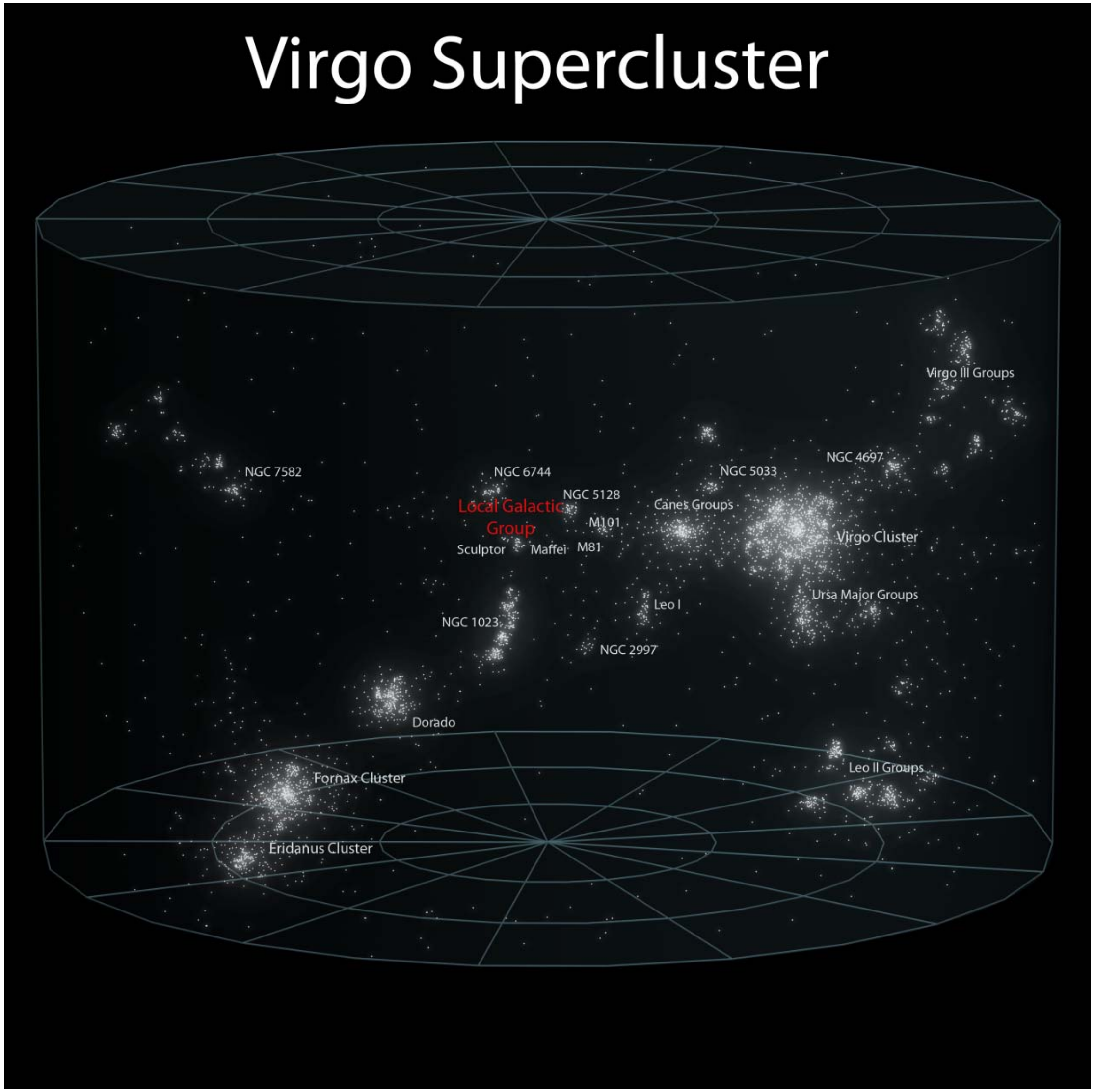


A more general View

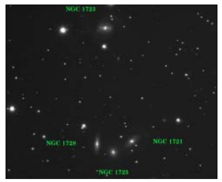


The Virgo Supercluster (Virgo SC) or the Local Supercluster (LSC or LS) is a mass concentration of galaxies that contains the Virgo Cluster in addition to the Local Group. At least 100 galaxy groups and clusters are located within its diameter of 33 megaparsecs (110 Mly). **It is one of millions of superclusters** in the observable universe.

The Virgo Cluster is a cluster of galaxies whose center is 53.8 ± 0.3 Mly away in the constellation Virgo. It comprises approximately of 1300 member galaxies,

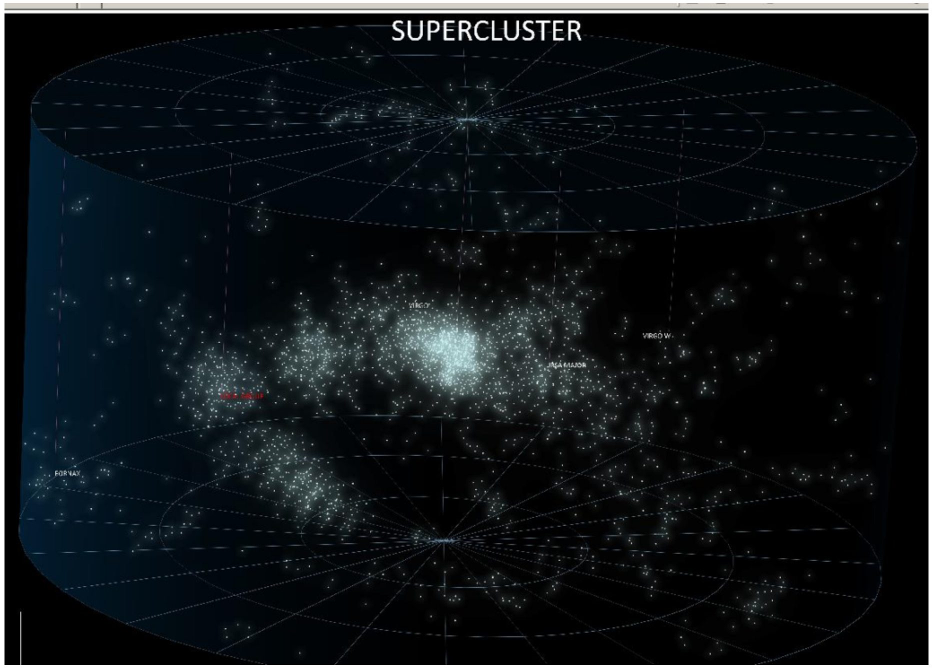
The Fornax Cluster is a cluster of galaxies lying at a distance of 62 Mly. It is the second richest galaxy cluster within 100 million light-years,

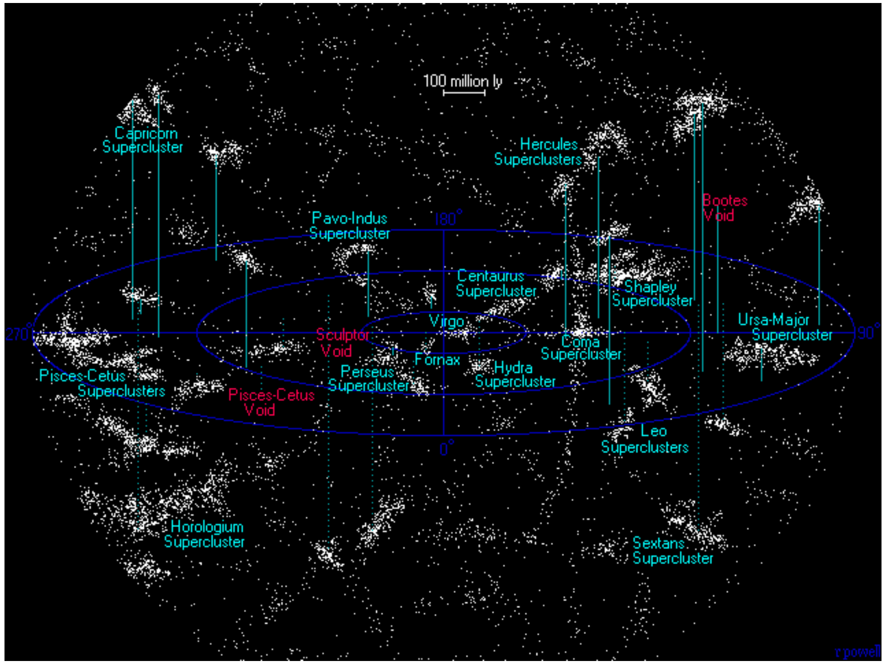
The Eridanus cluster contains about 73 galaxies at a distance of about 70 Mly

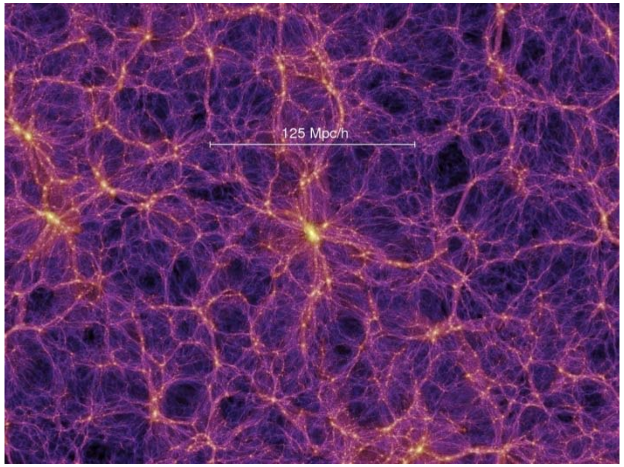


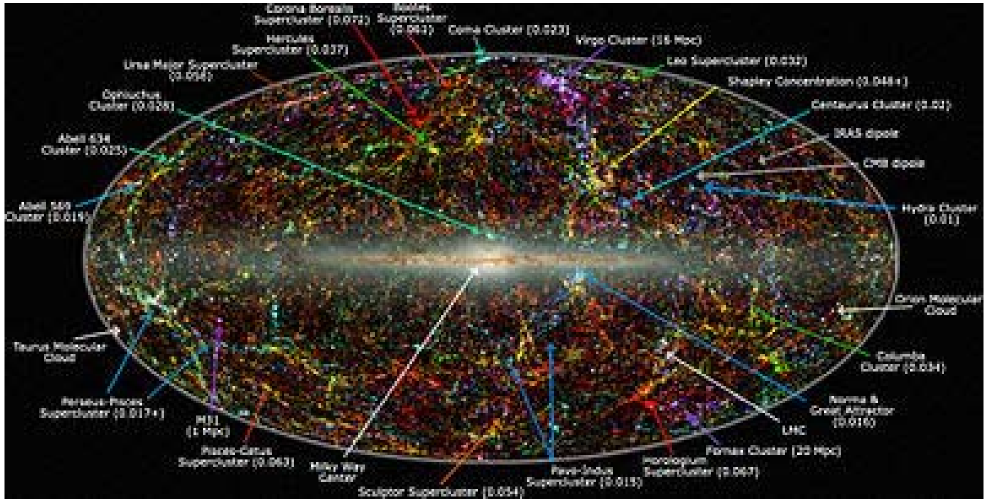
Automation and Robotics Laboratory

A more general View

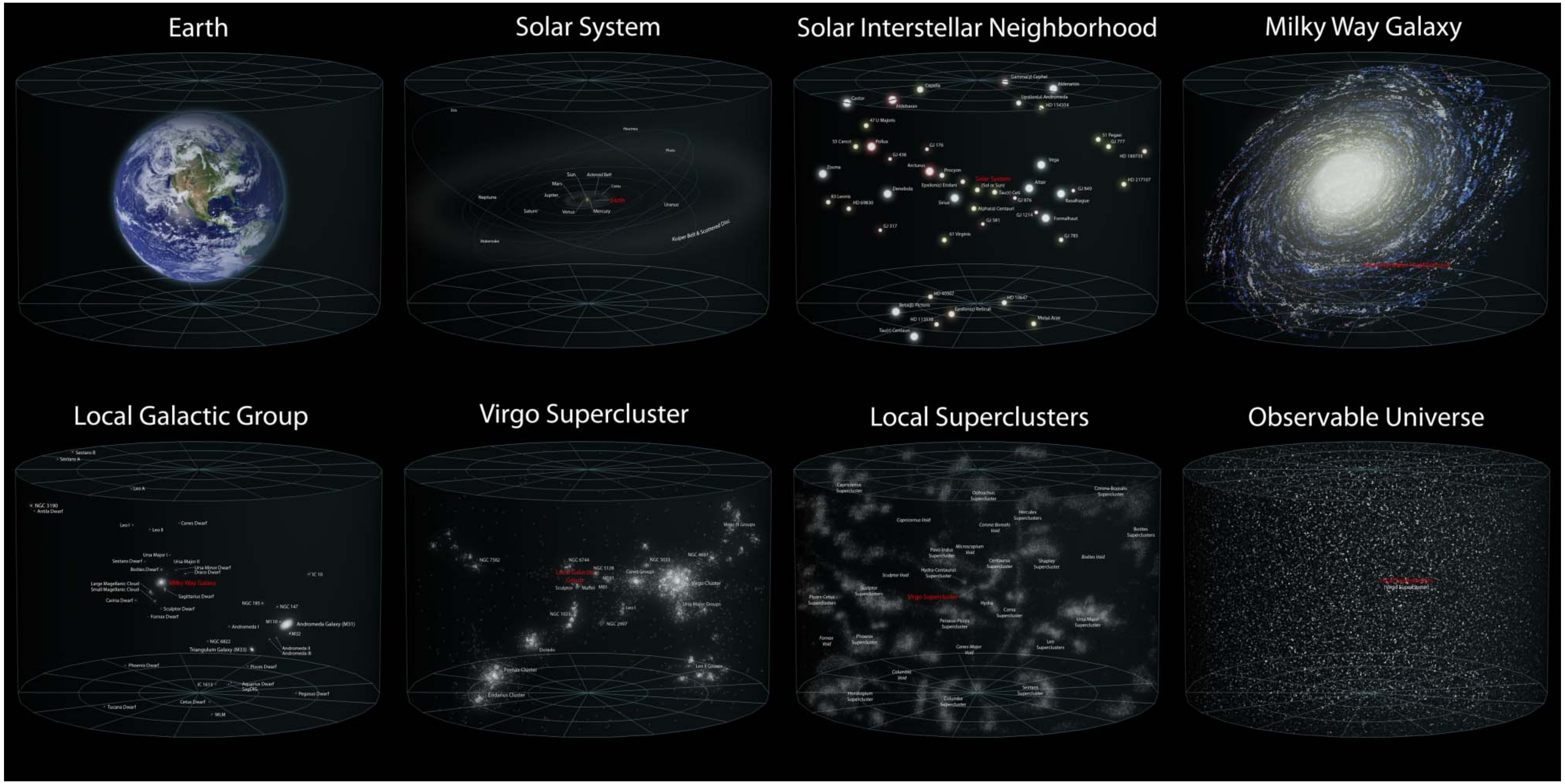








A more general View



A more general View

The distance from Earth to the edge of the observable universe is about 14.26 gigaparsecs (46.5 billion light years or 4.40×1026 meters) in any direction. The observable universe is thus a sphere with a diameter of about 28.5 gigaparsecs (93 Gly or 8.8×1026 m). Assuming that space is roughly flat, this size corresponds to a volume of about 1.16×1055 Gpc3 (3.8×1055 Gly3 or 3.57×1080 m^3).
The figures quoted above are distances now (in cosmological time), not distances at the time the light was emitted. For example, the cosmic microwave background radiation that we see right now was emitted at the time of photon decoupling, estimated to have occurred about 380,000 years after the Big Bang, which occurred around 13.8 billion years ago. This radiation was emitted by matter that has, in the intervening time, mostly condensed into galaxies, and those galaxies are now calculated to be about 46 billion light-years from us.

https://www.youtube.com/watch?v=74IsySs3RGU

https://www.youtube.com/watch?v=rENyyRwxpHo

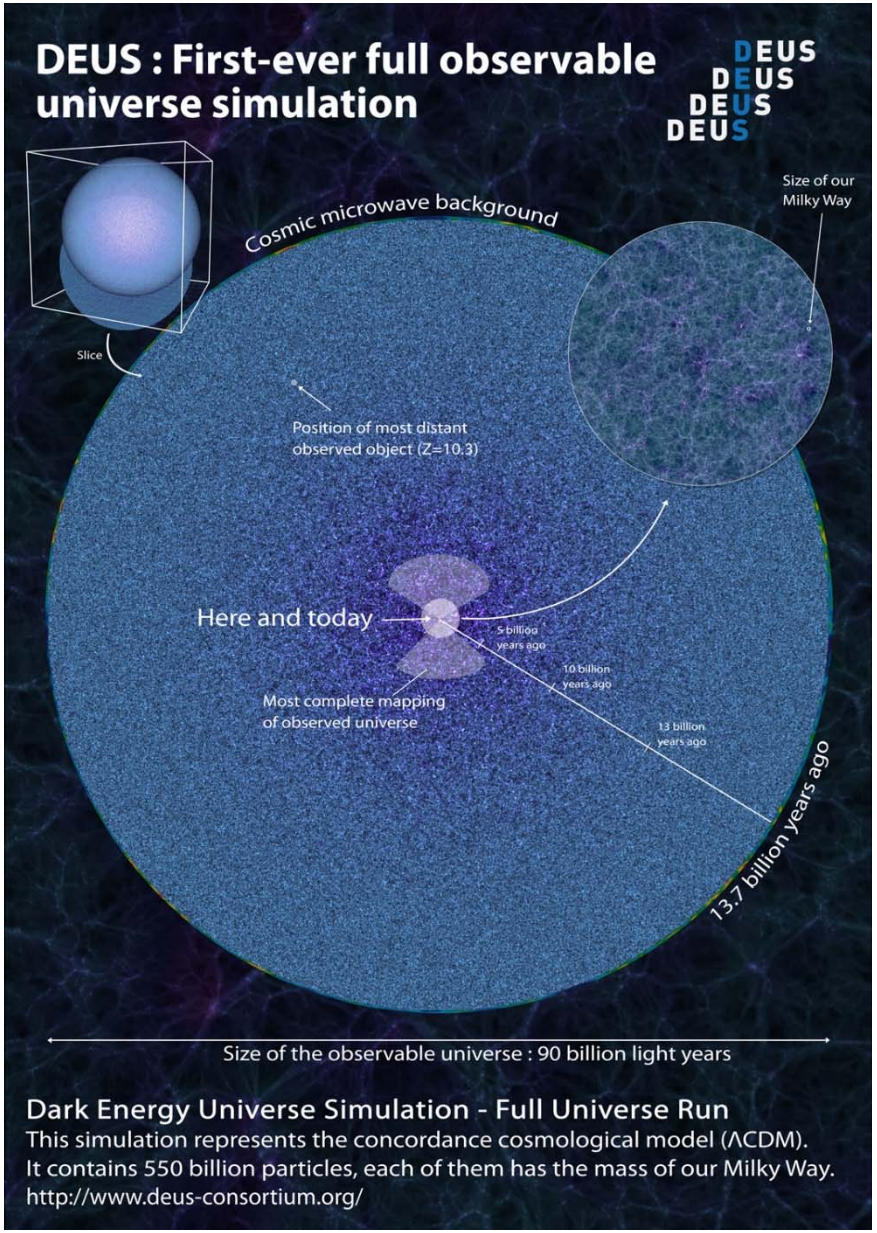
https://www.youtube.com/watch?v=0w4OTD4L0GQ

astronomical units	2.0626×10^5 AU 3.261 56 ly

The first indications of a deviation from uniform expansion of the universe were reported in 1973 and again in 1978. The location of the Great Attractor was finally determined in 1986, and is situated at a distance of somewhere between 150 and 250 Mly from the Milky Way, in the direction of the constellations Triangulum Australis (The Southern Triangle) and Norma (The Carpenter's Square)

The Great Attractor is a gravity anomaly in intergalactic space within the vicinity of the Hydra-Centaurus Supercluster at the center of the Laniakea Supercluster that reveals the existence of a localized concentration of mass tens of thousands of times more massive than the Milky Way. This mass is observable by its effect on the motion of galaxies and their associated clusters over a region hundreds of millions of light-years across. The Great Attractor is moving towards the Shapley Supercluster. These galaxies are all redshifted, in accordance with the Hubble Flow, indicating that they are receding relative to us and to each other, but the variations in their redshift are sufficient to reveal the existence of the anomaly. The variations in their redshifts are known as peculiar velocities, and cover a range from about +700 km/s to −700 km/s, depending on the angular deviation from the direction to the Great Attractor.

A more general View



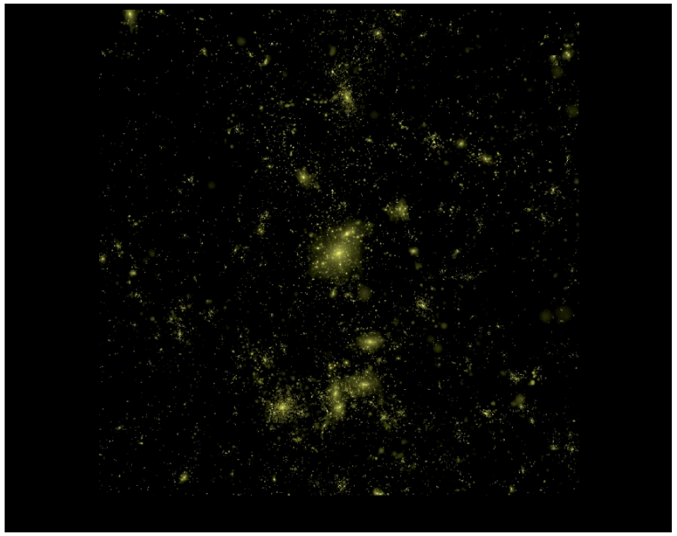
Carnegie Mellon University

Physics

Tiziana Di Matteo

Professor, Physics

[bluetides simulator]



5% Visible Matter
27% Dark Matter
68% Dark Energy

A more general View

Drake equation explained by Carl Sagan

SIZES

Alfonso Gonzalez - Astrodynamics & SE Podcast