Orbital Elements

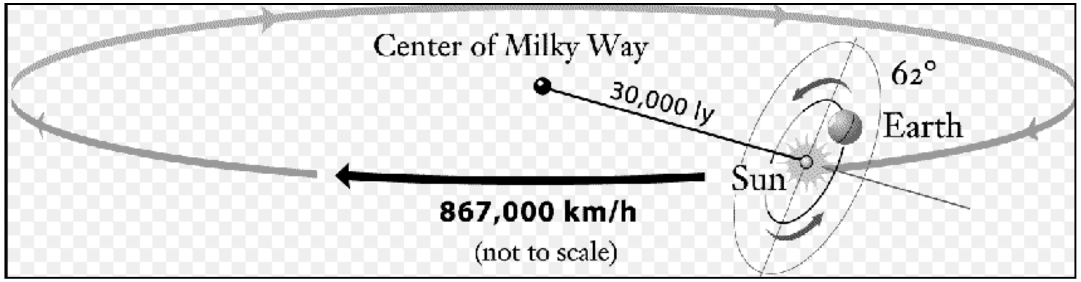
- Recall the main results of 2 – Body problem:

$$\ddot{\boldsymbol{r}} + \frac{\mu}{r^3}\boldsymbol{r} = 0 \qquad r = \frac{p = h^2 / \mu}{1 + e\cos f} \qquad M = nt = E - e\sin E \qquad \boldsymbol{n} = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}} \qquad (\frac{v^2}{2} - \frac{\mu}{r}) = -\frac{\mu}{2a}$$

- Complete solution requires 6 constants or integrals of motion (for instance initial position and initial velocity. In Orbital Mechanics we use a set of standard integrals of motion called **ORBITAL ELEMENTS**, the knowledge of which allows precise determination of the location of the body in the inertial space.

    - Coordinate Systems: Cartesian references used in Space Dynamics

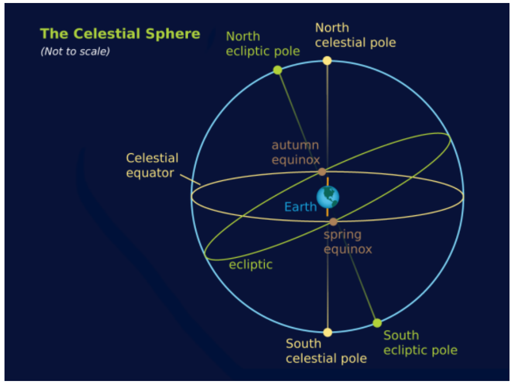
    - Description and computation of the 6 orbital elements



Coordinate Systems:

- From the 2 – Body assumption we obtain the type of orbit, and the fact that the trajectory of the orbiting body is planar.

- **Celestial Sphere**: The celestial sphere is an imaginary sphere of gigantic radius with the Earth located at its center. The poles of the celestial sphere are aligned with the poles of the Earth. The celestial equator lies along the celestial sphere in the same plane that includes the Earth's equator.



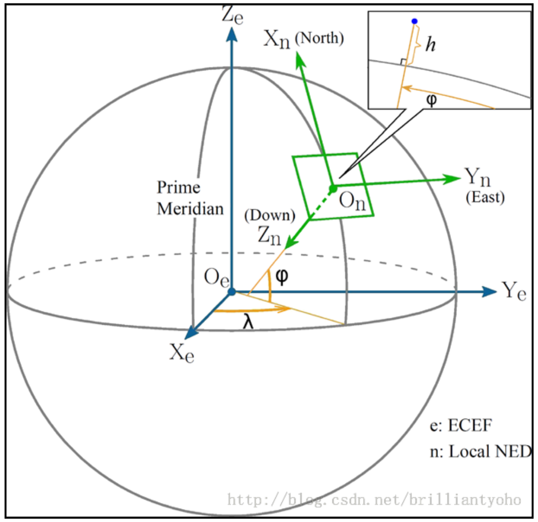
- **Ecliptic plane**: used for reference of near – Earth orbits, it is the plane of apparent motion of the Sun from West to East (and the majority of planets) with respect to the Earth's equatorial plane

- **Inclination:** The ecliptic has an inclination of about 23.4 degrees with respect to the celestial equator

- **Lines of Nodes:** The ecliptic plane and the celestial equator plane intersect in two points corresponding to the vernal (direction of Aries) and autumnal equinoxes (direction of Libra today)

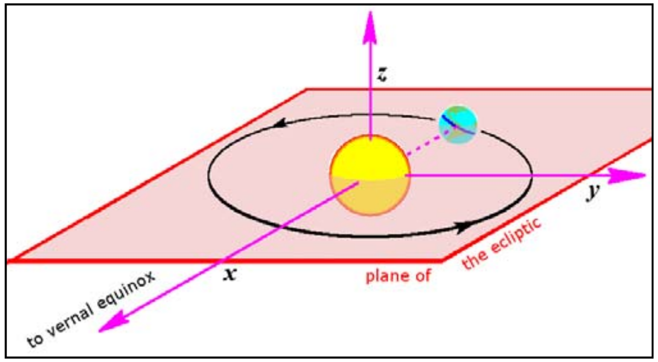
❑ Coordinate Systems:

- **Geocentric System**: used for reference of near – Earth orbits (ECI)



- XeOeYe is the equatorial plane.
- Oe is the center of the Earth (assumed spherical).
- Xe direction depends on the problem (usually the direction of the Vernal Equinox)
- **Orientation:** The motion of the body can be expressed in Cartesian coordinates (x, y, z), or in spherical coordinates (α, δ, r) or (λ, ϕ, h).

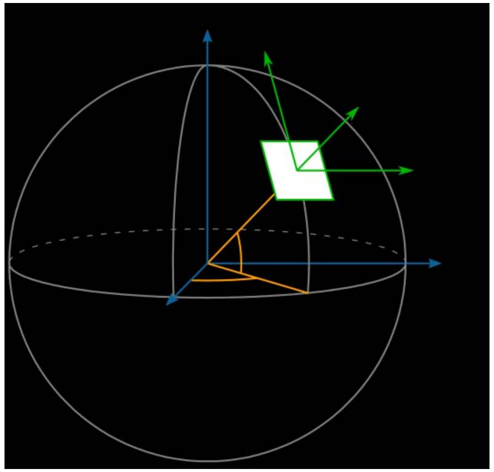
Orbital Elements

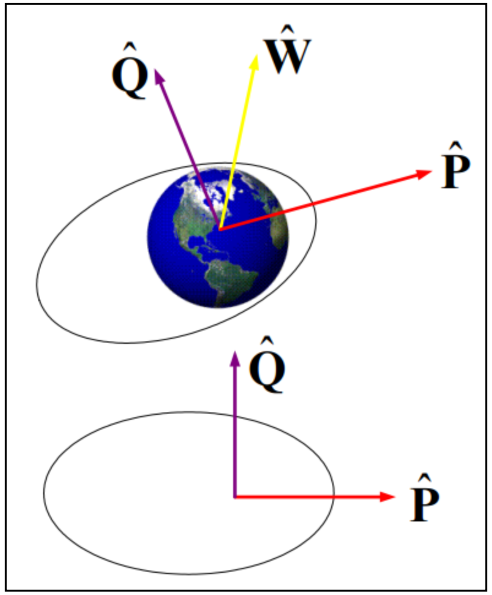


- **Heliocentric System:** In the case of orbits around the Sun the heliocentric-ecliptic coordinate system is convenient. As the name implies, the heliocentric-ecliptic system has its origin at the center of the Sun. The X-Y or fundamental plane coincides with the ecliptic, which is the plane of Earth's revolution around the Sun. The line-of-intersection of the ecliptic plane and Earth's equatorial plane defines the direction of the X-axis. On the first day of spring a line joining the center of Earth and the center of the Sun points in the direction of the positive X-axis. This is called the vernal equinox direction. The Y-axis forms a right-handed set of coordinate axes with the X-axis. The Z-axis is perpendicular to the fundamental plane and is positive in the north direction.

- **Precession and Nutation:** It is known that Earth wobbles slightly and its axis of rotation shifts in direction slowly over the centuries. This effect is known as precession and causes the line-of-intersection of Earth's equator and the ecliptic to shift slowly. As a result the heliocentric-ecliptic system is not really an inertial reference frame. Where extreme precision is required, it is necessary to specify that the XYZ coordinates of an object are based on the vernal equinox direction of a particular year or epoch.

Orbital Elements





- **Topocentric System**: Used for referring to the local horizon. It can be NED or SED depending on the direction of the X axis.

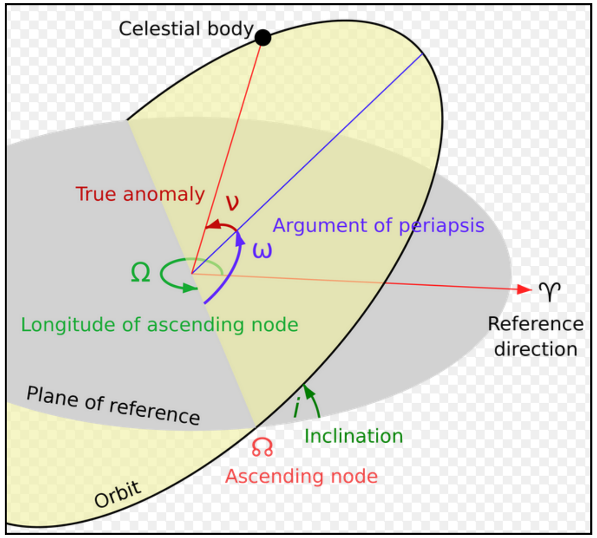
- **Coordinates:** Az (Azimut) is the angle from the X direction on the XY plane, El (Elevation) is the angle of rotation of the XZ plane, and distance r.

Perifocal System: is a frame of reference for an orbit. The frame is centered at the focus of the orbit. The X axis is directed towards the periapsis of the orbit, and the Y axis has a true anomaly *f* of 90 degrees past the periapsis. The Z axis is in the direction of the angular momentum vector and is orthogonal to the orbital plane

Time and Date (see later): Measure of time depends on the problem. Several measures exist, such as: Atomic time, Sidereal time, Solar time, Universal time (solar time w.r.t. Greenwich).
The Julian Date refers to January 1st 4713 B.C.
Currently we use J2000

Orbital Elements

- **Orbital elements:** are the parameters required to uniquely identify a specific orbit. In celestial mechanics these elements are generally considered in classical two-body systems, where a Kepler orbit is used. There are many different ways to mathematically describe the same orbit, but a set of six parameters is commonly used.



$$\ddot{\boldsymbol{r}} + \frac{\mu}{r^3}\boldsymbol{r} = 0$$

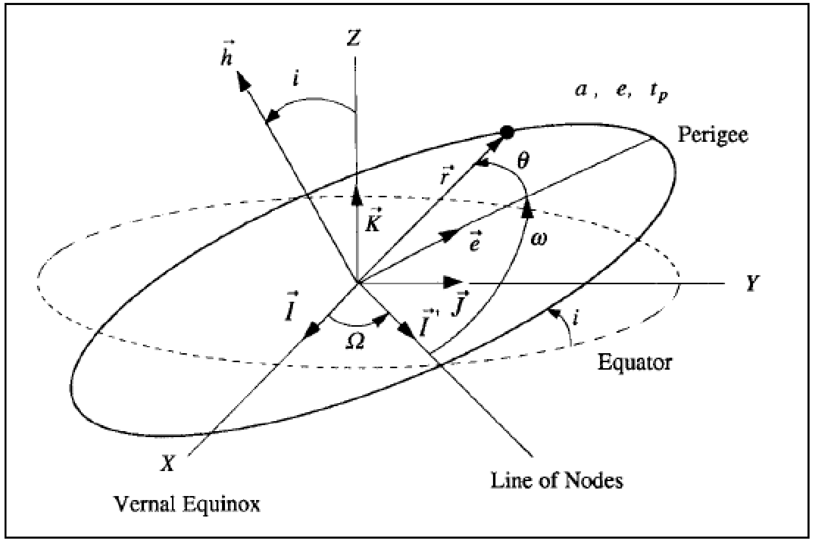
- The complete solution requires the knowledge of $\boldsymbol{r}(t)$ and $\boldsymbol{v}(t)$

- Position of the orbit in the inertial space (3)
- Type of orbit (2)
- Position of body in the orbit at current time (1)

Orbital Elements

Classical elements

- i = Inclination of the orbit
- Ω = Right Ascension of ascending node
- (longitudine del nodo ascendente)
- ω = Argument of periapsis
- a = Semi major axis (or semilatus rectum)
- e = Eccentricity of the orbit
- f = True anomaly (or time to periapsis)



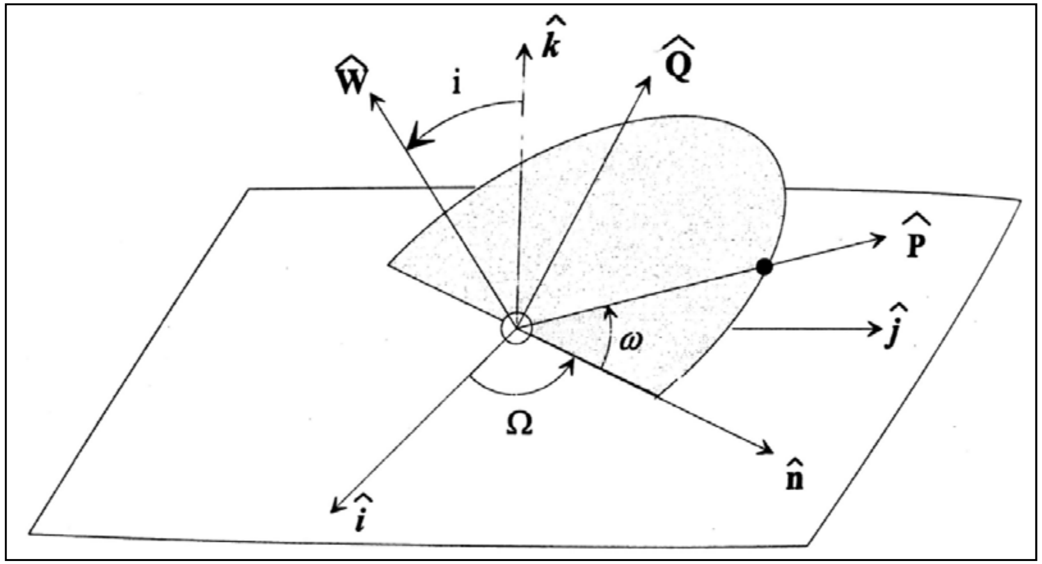
- With reference to a Earth orbiting satellite, and a Geocentric non rotating frame (ECI):

The elements a and e determine the size and shape of the elliptic orbit, respectively, and t_p relates position in orbit to time. The angles Ω and i specify the orientation of the orbit plane with respect to the geocentric-equatorial reference frame. The angle ω specifies the orientation of the orbit in its plane. Orbits with $i < 90$ deg are called *prograde orbits*, whereas orbits with $i > 90$ deg are called *retrograde orbits*. The term prograde means easterly direction in which the sun, Earth, and most of the planets and their moons rotate on their axes. The term retrograde means westerly direction, which is simply the opposite of prograde. An orbit whose inclination is near 90 deg is called a polar orbit. An equatorial orbit has zero inclination.

Orbital Elements

Orbital Elements

- **Coordinate Transformation between ECI and Perifocal Frame:** we can use a standard Euler transformation (3-1-3)



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{PQW} = \left[T(\omega)\right]^{Z} \cdot \left[T(i)\right]^{X} \cdot \left[T(\Omega)\right]^{Z} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{IJK}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{PQW} = \left[T(\omega, i, \Omega)\right]_{IJK}^{PQW} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{IJK}$$

- Since it is an ORTHOGONAL transformation: $\left[T(\Omega, i, \omega)\right]_{PQW}^{IJK} = \left\{\left[T(\omega, i, \Omega)\right]_{IJK}^{PQW}\right\}^{T}$

$$\left[T(\Omega, i, \omega)\right]_{PQW}^{IJK} = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i \\ \cos\omega\sin\Omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega - \cos\Omega\cos\omega\cos i & -\sin i\cos\Omega \\ \sin\omega\sin i & \sin i\cos\omega & \cos i \end{bmatrix}$$

Orbital Elements

- **Orbital Elements from position and velocity** $\begin{bmatrix} \boldsymbol{r} & \dot{\boldsymbol{r}} = \boldsymbol{v} \end{bmatrix} \Rightarrow \begin{bmatrix} i & \Omega & \omega & a & e & f \end{bmatrix}$ $\begin{cases} \boldsymbol{r}(t) = r_x \hat{\boldsymbol{P}} + r_y \hat{\boldsymbol{Q}} \\ \boldsymbol{v}(t) = r_x \hat{\boldsymbol{P}} + r_y \hat{\boldsymbol{Q}} \end{cases}$

1. From energy we find the semi major axis:

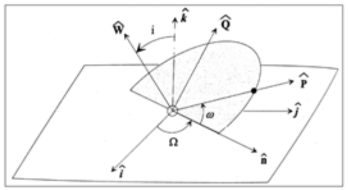
$$E = \frac{1}{2}\left|\boldsymbol{v}\right|^2 - \frac{\mu}{\left|\boldsymbol{r}\right|} \qquad \boxed{a = -\frac{\mu}{2E}}$$

2. From polar equation we can find the eccentricity:

$$r = \frac{p = \dfrac{h^2}{\mu}}{1 + e\cos f}; h = \left|\boldsymbol{r} \times \dot{\boldsymbol{r}}\right| \qquad \boxed{e = \sqrt{1 + \frac{2Eh^2}{\mu^2}}}$$

3. The inclination angle i can be found from the frames relations (is zero for Equatorial orbits):

$$\boxed{\cos i = \frac{\hat{\boldsymbol{k}} \cdot \boldsymbol{h}}{h} \Rightarrow i = \cos^{-1}\left(\frac{\hat{\boldsymbol{k}} \cdot \boldsymbol{h}}{h}\right)}$$



- The line of nodes (Intersection between ECI and PQW) unit vector can be found as:

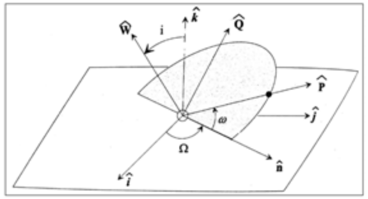
$$\hat{\boldsymbol{n}} = \frac{\hat{\boldsymbol{k}} \times \boldsymbol{h}}{\left|\hat{\boldsymbol{k}} \times \boldsymbol{h}\right|}$$

Orbital Elements

4. Right Ascension of ascending node (RAAN) Ω:

$$\cos\Omega = \hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{n}} \Rightarrow \Omega = \begin{cases} \cos^{-1}(\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{n}}) & if \quad \hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{j}} > 0 \\ 360^o - \cos^{-1}(\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{n}}) & if \quad \hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{j}} < 0 \end{cases}$$

- For Equatorial orbits, the line of nodes is undefined so is Ω.



5. Argument of Periapsis ω:

$$\cos\omega = \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{e}}{e} \Rightarrow \omega = \begin{cases} \cos^{-1}(\frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{e}}{e}) & if \quad \boldsymbol{e}\cdot\hat{\boldsymbol{k}} > 0 \\ 360^o - \cos^{-1}(\frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{e}}{e}) & if \quad \boldsymbol{e}\cdot\hat{\boldsymbol{k}} < 0 \end{cases}$$

- For Equatorial orbits, and circular orbits, ω is undefined.

6. True anomaly f:

$$\cos f = \frac{\boldsymbol{e}\cdot\boldsymbol{r}}{er} \Rightarrow f = \begin{cases} \cos^{-1}(\frac{\boldsymbol{e}\cdot\boldsymbol{r}}{er}) & if \quad \boldsymbol{r}\cdot\boldsymbol{v} > 0 \\ 360^o - \cos^{-1}(\frac{\boldsymbol{e}\cdot\boldsymbol{r}}{er}) & if \quad \boldsymbol{r}\cdot\boldsymbol{v} < 0 \end{cases}$$

$$\boldsymbol{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\boldsymbol{r} - (\boldsymbol{r}\cdot\boldsymbol{v})\boldsymbol{v}\right]$$

Problem: Suppose we observe an object in the ECI frame at position

$$\vec{r} = \begin{bmatrix} 6524.8 \\ 6862.8 \\ 6448.3 \end{bmatrix} km \quad \text{moving with velocity} \quad \vec{v} = \begin{bmatrix} 4.901 \\ 5.534 \\ -1.976 \end{bmatrix} km/s$$

Determine the orbital elements.

- Although not necessary, lets first convert to canonical units (1ER = 6378.14km, 1TU = 806.3s).

$$\begin{cases} DU_{\oplus} = 1R_{\oplus} = 6378\,km \\ TU_{\oplus} = \sqrt{\dfrac{R_{\oplus}^3}{\mu_{\oplus}}} = 806.78\,\text{sec} \end{cases}$$

$$\vec{r}' = \frac{\vec{r}}{6378.14km} = \begin{bmatrix} 1.023 \\ 1.076 \\ 1.011 \end{bmatrix}$$

$$\vec{v}' = \vec{r}\frac{806.8s}{6378.14km} = \begin{bmatrix} .62 \\ .7 \\ -.25 \end{bmatrix}$$

$$\mu_{EARTH} = 1$$

- Compute specific angular momentum, line of nodes, and eccentricity vectors:

$$\vec{h} = \vec{r} \times \vec{v} = \begin{bmatrix} -.9767 \\ .882 \\ .049 \end{bmatrix} \frac{ER^2}{TU}$$

Orbital Elements

Since $\vec{r}$ and $\vec{v}$ are in ECI coordinates,

$$\vec{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \vec{h} = \begin{bmatrix} -.882 \\ -.9767 \\ 0 \end{bmatrix} \frac{ER^2}{TU}.$$

$$\vec{e} = \frac{1}{\mu}\vec{v} \times \vec{h} - \frac{\vec{r}}{r} = \begin{bmatrix} -.315 \\ -.385 \\ .668 \end{bmatrix}$$

- The eccentricity is found from the above: $e = \|\vec{e}\| = .8328$

- From the specific energy, we compute semi major axis and semilatus rectum:

$$E = \frac{v^2}{2} - \frac{\mu}{r} = -.088$$

$$a = -\frac{\mu}{2E} = 5.664ER$$

$$p = \frac{h^2}{\mu} = 1.735ER$$

Orbital Elements

- The orbit inclination is given by (with no quadrant ambiguity):

$$i = \cos^{-1}\left(\frac{\vec{h}}{h} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = 87.9\,\mathrm{deg}$$

- The Right Ascension has quadrant ambiguity:

$$\Omega = \cos^{-1}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \frac{\vec{n}}{\|\vec{n}\|}\right) = \pm 132.10\,\mathrm{deg}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \vec{n} = -.9767 < 0$$ The line of nodes is in the third quadrant, therefore:

$$\Omega = 360 - 132.10 = 227.9\,\mathrm{deg}$$

Orbital Elements

- The argument of periapsis (perigee):

$$\omega = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{\|\vec{n}\|e}\right) = \pm 53.4\,\deg$$

  - Here there is a quadrant ambiguity as well:

$$\begin{bmatrix}0\\0\\1\end{bmatrix}\cdot\vec{e} = .668 > 0$$ This implies the right quadrant $\omega = 53.4\,\deg$

- Computation of true anomaly (angle between position vector and eccentricity vector):

$$f = \cos^{-1}\left(\frac{\vec{r}\cdot\vec{e}}{re}\right) = \pm 92.3\,\deg$$

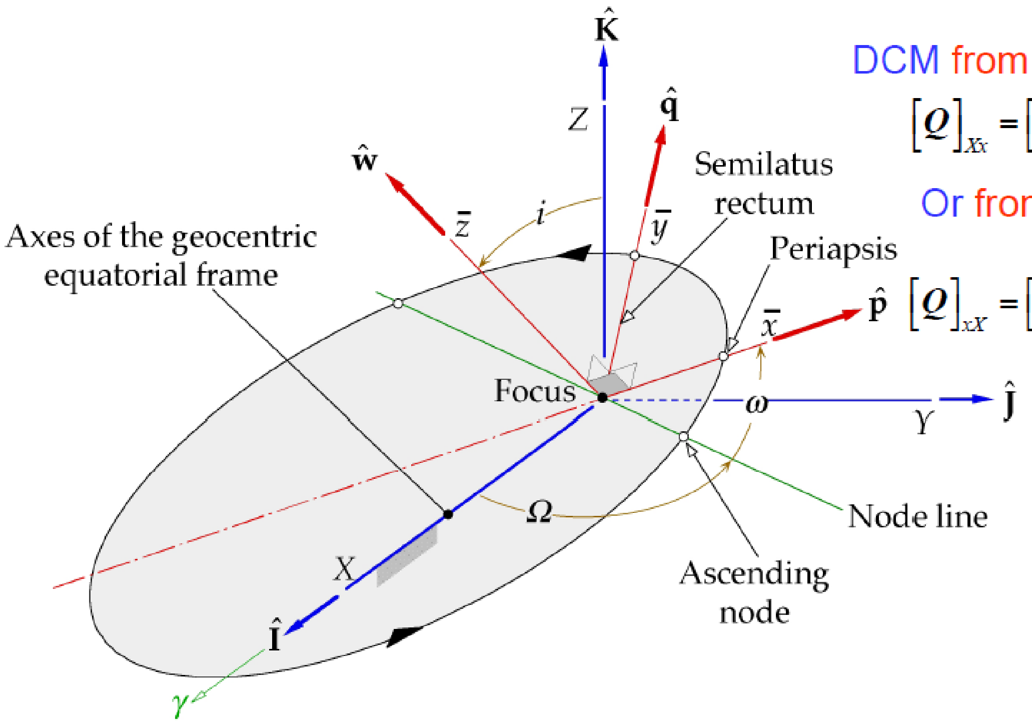
$\vec{r}\cdot\vec{v} > 0$ So we are in the right quadrant

$$f = 92.3\,\deg$$

- **NOTE:** The time propagation (rather than true anomaly) can be found using Kepler's equation

Position and velocity from Orbital Elements $\begin{bmatrix} i & \Omega & \omega & a & e & f \end{bmatrix} \Rightarrow \begin{bmatrix} \boldsymbol{r} & \dot{\boldsymbol{r}} = \boldsymbol{v} \end{bmatrix}$

- Important problem. Suppose we know position and velocity at some time t_0. How can we find the position and velocity at some time t_1?



DCM from XYZ to $\bar{x}\,\bar{y}\,\bar{z}$:

$$\left[\boldsymbol{Q}\right]_{Xx} = \left[\boldsymbol{R}_3(\omega)\right]\left[\boldsymbol{R}_1(i)\right]\left[\boldsymbol{R}_3(\Omega)\right]$$

Or from xyz to XYZ:

$$\left[\boldsymbol{Q}\right]_{xX} = \left[\boldsymbol{R}_3(-\Omega)\right]\left[\boldsymbol{R}_1(-i)\right]\left[\boldsymbol{R}_3(-\omega)\right]$$

Orbital Elements

- Recall the transformation matrices between ECI and Perifocal
- Assume Perifocal as Inertial Reference as well

$$\boldsymbol{r} = r\cos f\hat{\boldsymbol{P}} + r\sin f\hat{\boldsymbol{Q}} \qquad r = \frac{p = a(1-e^2)}{1+e\cos f}$$

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = (\dot{r}\cos f - r\dot{f}\sin f)\hat{\boldsymbol{P}} + (\dot{r}\sin f - r\dot{f}\cos f)\hat{\boldsymbol{Q}}$$

- We need to find the speed components as functions of the orbital elements:

$$r(1+e\cos f) = p = \text{const.} \quad \Rightarrow \dot{r}(1+e\cos f) - er\dot{f}\sin f = 0 \Rightarrow \dot{r} = \frac{er\dot{f}\sin f}{1+e\cos f}$$

$$\dot{r} = \frac{er^2\dot{f}\sin f}{r(1+e\cos f)}$$

- Recall previous results: $\begin{cases} h = \left|\boldsymbol{h}\right| = r(r\dot{f}) \\ p = h^2 / \mu \end{cases}$

$$r\dot{f} = \sqrt{\frac{\mu}{p}}(1+e\cos f)$$

Orbital Elements

- Substituting in the expression of velocity:

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \sqrt{\frac{\mu}{p}}[-\sin f\hat{\boldsymbol{P}} + (e+\cos f)\hat{\boldsymbol{Q}}]$$

- **NOTE 1**: position and velocity vectors depend on a, e, f only since we used the Perifocal reference frame

- **NOTE 2**: position and velocity vectors components in the ECI reference include functionality of all 6 orbital elements through the transformation matrix

$$\left[T(\Omega,i,\omega)\right]_{PQW}^{IJK} = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i \\ \cos\omega\sin\Omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega - \cos\Omega\cos\omega\cos i & -\sin i\cos\Omega \\ \sin\omega\sin i & \sin ic\omega & \cos i \end{bmatrix}$$

$$\boxed{r_{ECI} = \frac{p}{1+e\cos f}\left[T(\Omega,i,\omega)\right]_{PQW}^{IJK}\begin{bmatrix}\cos f \\ \sin f \\ 0\end{bmatrix}}$$

$$\boxed{v_{ECI} = \left[T(\Omega,i,\omega)\right]_{PQW}^{IJK}\begin{bmatrix} -\sqrt{\frac{\mu}{p}}\sin f \\ \sqrt{\frac{\mu}{p}}(e+\cos f) \\ 0\end{bmatrix}}$$

Problem: Given the following orbital elements, find $\vec{r}$ and $\vec{v}$.

$$a = 35,960km = 5.64ER \qquad e = .832 \qquad f = 92.335\,\text{deg}$$
$$i = 87.87\,\text{deg} \qquad \Omega = 227.9\,\text{deg} \qquad \omega = 53.39\,\text{deg}$$

Solution: First solve for r and h.

$$p = a(1 - e^2) = 1.735ER$$

$$r = \frac{p}{1 + e\cos f} = 1.7947$$

$p = h^2/\mu$, so

$$h = \sqrt{p} = 1.3172.$$

Now in perifocal coordinates

$$\vec{r}_{PQW} = \begin{bmatrix} -.07319 \\ 1.7947 \\ 0 \end{bmatrix} \qquad \vec{v}_{PQW} = \begin{bmatrix} -.7585 \\ .6013 \\ 0 \end{bmatrix}$$

- Using Matlab

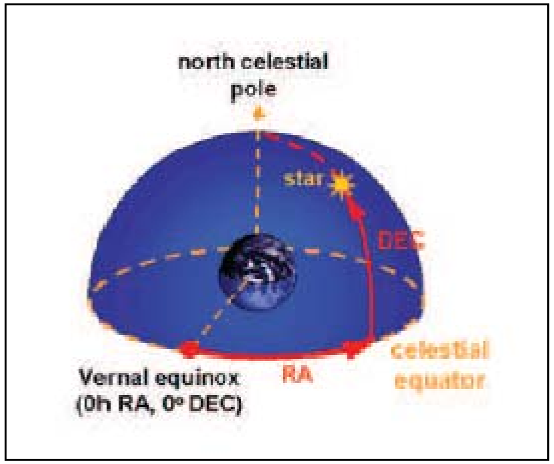
$$\vec{r}_{ECI} = \begin{bmatrix} 1.023 \\ 1.076 \\ 1.011 \end{bmatrix} ER \qquad \vec{v}_{ECI} = \begin{bmatrix} .62 \\ .7 \\ -.25 \end{bmatrix} ER/TU$$

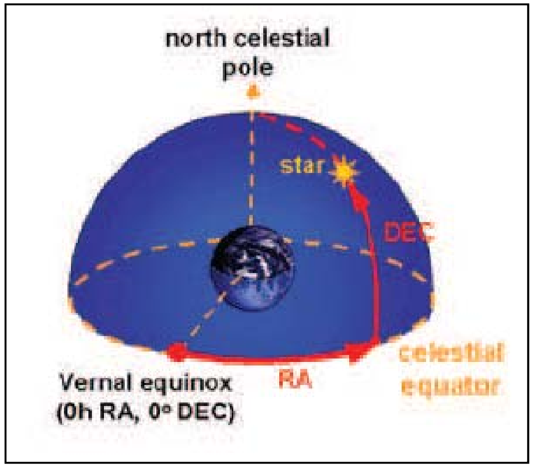
Orbital Elements

- Knowing position and velocity, several operations can be performed:

1. Tracking
2. Communications
3. Intercept
4. Observations

- Tracking: Position and velocity must be translated into appropriate coordinates to be used by radio telescopes/ground antennas





$$r_{ECI} = \sqrt{\sum_{i=1}^{3} \left(r_{ECI}^{i}\right)^2}$$

$$\alpha = \tan^{-1}\left(\frac{r_{ECI}^{y}}{r_{ECI}^{x}}\right)$$

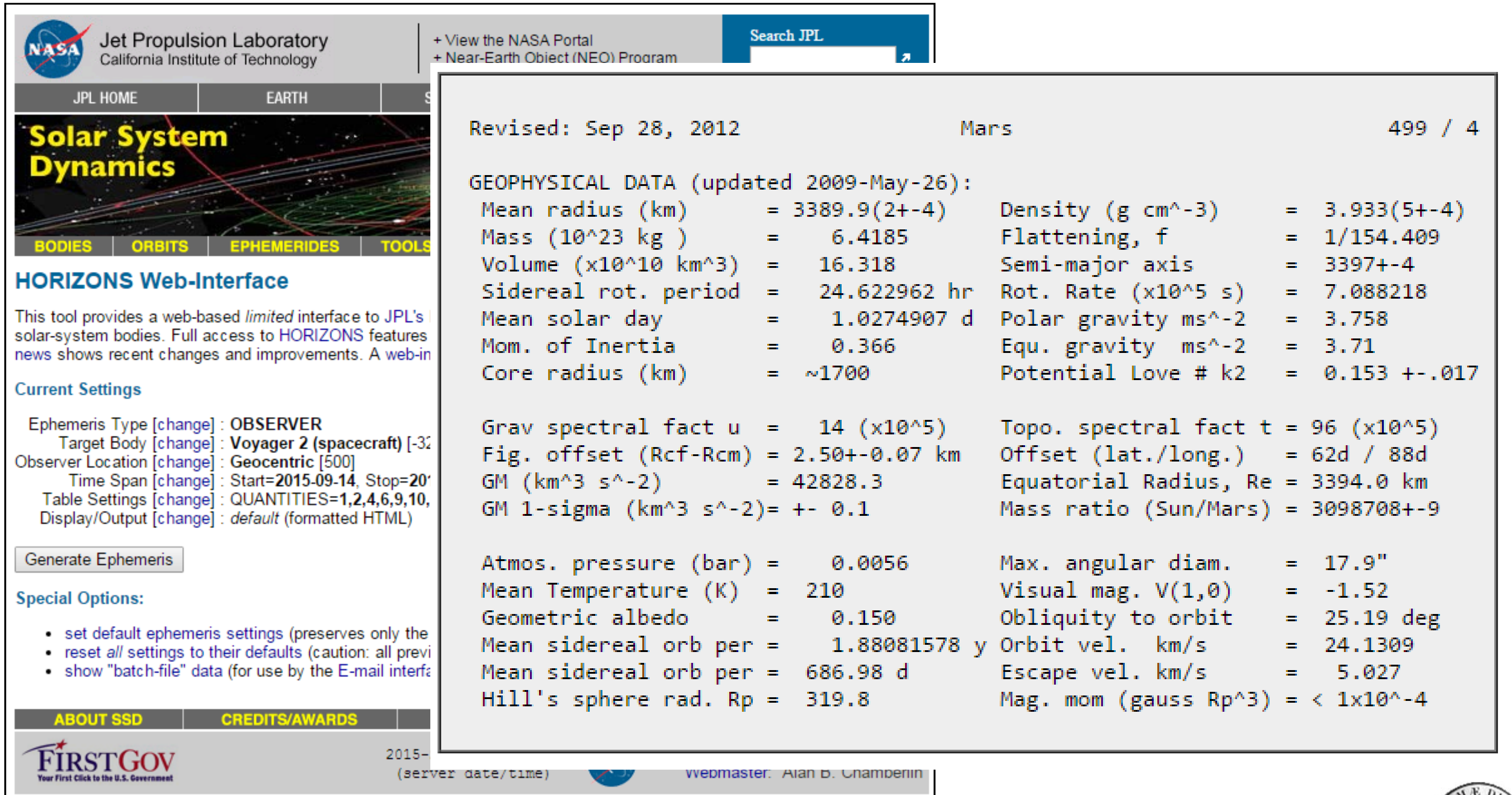
$$\delta = \tan -1 \left(\frac{r_{ECI}^{z}}{\sqrt{(r_{ECI}^{x})^2 + (r_{ECI}^{y})^2}}\right)$$

α = Right Ascension
δ = Declination
r = Distance

$$\begin{aligned} r_{ECI}^{x} &= r\cos\delta\cos\alpha \\ r_{ECI}^{y} &= r\cos\delta\sin\alpha \\ r_{ECI}^{z} &= r\sin\delta \end{aligned}$$

Orbital Elements

- **Ephemerides**: they give the positions of naturally occurring astronomical objects as well as artificial satellites in the sky at a given time or times. Solar system ephemerides are essential for the navigation of spacecraft and for all kinds of space observations of the planets, their natural satellites, stars, and galaxies.




```
Revised: Sep 28, 2012                 Mars                              499 / 4

GEOPHYSICAL DATA (updated 2009-May-26):
 Mean radius (km)        = 3389.9(2+-4)    Density (g cm^-3)       =  3.933(5+-4)
 Mass (10^23 kg )        =    6.4185       Flattening, f           =  1/154.409
 Volume (x10^10 km^3)    =   16.318        Semi-major axis         =  3397+-4
 Sidereal rot. period    =   24.622962 hr  Rot. Rate (x10^5 s)     =  7.088218
 Mean solar day          =    1.0274907 d  Polar gravity ms^-2     =  3.758
 Mom. of Inertia         =    0.366        Equ. gravity  ms^-2     =  3.71
 Core radius (km)        =  ~1700          Potential Love # k2     =  0.153 +-.017

 Grav spectral fact u    =   14 (x10^5)    Topo. spectral fact t = 96 (x10^5)
 Fig. offset (Rcf-Rcm) = 2.50+-0.07 km     Offset (lat./long.)     = 62d / 88d
 GM (km^3 s^-2)          = 42828.3         Equatorial Radius, Re = 3394.0 km
 GM 1-sigma (km^3 s^-2)= +- 0.1            Mass ratio (Sun/Mars) = 3098708+-9

 Atmos. pressure (bar) =    0.0056         Max. angular diam.      =  17.9"
 Mean Temperature (K)  =  210              Visual mag. V(1,0)      =  -1.52
 Geometric albedo        =    0.150        Obliquity to orbit      =  25.19 deg
 Mean sidereal orb per =    1.88081578 y   Orbit vel.  km/s        =  24.1309
 Mean sidereal orb per =  686.98 d         Escape vel. km/s        =   5.027
 Hill's sphere rad. Rp =  319.8            Mag. mom (gauss Rp^3) = < 1x10^-4
```

https://ssd.jpl.nasa.gov/

Orbital Elements

```
*******************************************************************************
Ephemeris / WWW_USER Wed Sep 16 02:11:39 2015 Pasadena, USA      / Horizons
*******************************************************************************
Target body name: Mars (499)                      {source: MAR097}
Center body name: Earth (399)                     {source: MAR097}
Center-site name: GEOCENTRIC
*******************************************************************************
Start time      : A.D. 2015-Sep-14 00:00:00.0000 UT
Stop  time      : A.D. 2015-Oct-16 00:00:00.0000 UT
Step-size       : 1440 minutes
*******************************************************************************
Target pole/equ : IAU_MARS                        {East-longitude -}
Target radii    : 3396.2 x 3396.2 x 3376.2 km     {Equator, meridian, pole}
Center geodetic : 0.00000000,0.00000000,0.0000000 {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.00000000,0.00000000,0.0000000 {E-lon(deg),Dxy(km),Dz(km)}
Center pole/equ : High-precision EOP model        {East-longitude +}
Center radii    : 6378.1 x 6378.1 x 6356.8 km     {Equator, meridian, pole}
Target primary  : Sun
Vis. interferer : MOON (R_eq= 1737.400) km        {source: MAR097}
Rel. light bend : Sun, EARTH                      {source: MAR097}
Rel. lght bnd GM: 1.3271E+11, 3.9860E+05 km^3/s^2
Atmos refraction: NO (AIRLESS)
RA format       : HMS
Time format     : CAL
EOP file        : eop.150915.p151207
EOP coverage    : DATA-BASED 1962-JAN-20 TO 2015-SEP-15. PREDICTS-> 2015-DEC-06
Units conversion: 1 au= 149597870.700 km, c= 299792.458 km/s, 1 day= 86400.0 s
Table cut-offs 1: Elevation (-90.0deg=NO ),Airmass (>38.000=NO), Daylight (NO )
Table cut-offs 2: Solar Elongation (  0.0,180.0=NO ),Local Hour Angle( 0.0=NO )
**************************************************************************************************************************************************************************************************
 Date__(UT)__HR:MN     R.A._(ICRF/J2000.0)_DEC R.A.__(a-apparent)__DEC Azi_(a-appr)_Elev  APmag  S-brt   Illu% Ang-diam Ob-lon Ob-lat                r         rdot            delta      deldot    1-way_LT
**************************************************************************************************************************************************************************************************
$$SOE
 2015-Sep-14 00:00     09 42 15.00 +15 02 44.3 09 43 05.20 +14 58 25.1   .n.a.     .n.a.   1.78   4.40  97.907 3.800626  43.77  21.97  1.643503972273   1.1283575 2.46414648812173  -6.7878488  20.493681
 2015-Sep-15 00:00     09 44 43.32 +14 50 32.2 09 45 33.42 +14 46 11.2   .n.a.     .n.a.   1.78   4.40  97.861 3.806733  34.01  22.12  1.644151229477   1.1129778 2.46019344277821  -6.9015062  20.460805
 2015-Sep-16 00:00     09 47 11.26 +14 38 15.1 09 48 01.27 +14 33 52.2   .n.a.     .n.a.   1.78   4.40  97.815 3.812961  24.25  22.26  1.644789588073   1.0975424 2.45617483493000  -7.0148835  20.427383
 2015-Sep-17 00:00     09 49 38.82 +14 25 53.0 09 50 28.75 +14 21 28.5   .n.a.     .n.a.   1.78   4.41  97.769 3.819312  14.49  22.41  1.645419016169   1.0820523 2.45209084394190  -7.1279210  20.393418
 2015-Sep-18 00:00     09 52 06.00 +14 13 26.3 09 52 55.85 +14 09 00.0   .n.a.     .n.a.   1.78   4.41  97.722 3.825786   4.73  22.55  1.646039482404   1.0665084 2.44794168158063  -7.2405662  20.358910
 2015-Sep-19 00:00     09 54 32.82 +14 00 54.8 09 55 22.59 +13 56 26.8   .n.a.     .n.a.   1.78   4.41  97.674 3.832383 354.96  22.68  1.646650955942   1.0509114 2.44372758806673  -7.3527730  20.323863
 2015-Sep-20 00:00     09 56 59.27 +13 48 18.8 09 57 48.96 +13 43 49.1   .n.a.     .n.a.   1.78   4.42  97.626 3.839105 345.20  22.82  1.647253406474   1.0352625 2.43944882823823  -7.4645023  20.288277
 2015-Sep-21 00:00     09 59 25.35 +13 35 38.3 10 00 14.97 +13 31 07.0   .n.a.     .n.a.   1.78   4.42  97.578 3.845952 335.43  22.95  1.647846804210   1.0195625 2.43510568720539  -7.5757236  20.252156
 2015-Sep-22 00:00     10 01 51.07 +13 22 53.5 10 02 40.63 +13 18 20.6   .n.a.     .n.a.   1.78   4.42  97.529 3.852925 325.67  23.07  1.648431119884   1.0038124 2.43069846431472  -7.6864190  20.215503
 2015-Sep-23 00:00     10 04 16.44 +13 10 04.5 10 05 05.92 +13 05 29.9   .n.a.     .n.a.   1.78   4.43  97.479 3.860026 315.90  23.20  1.649006324747   0.9880129 2.42622746370478  -7.7965912  20.178319
 2015-Sep-24 00:00     10 06 41.45 +12 57 11.3 10 07 30.87 +12 52 35.2   .n.a.     .n.a.   1.78   4.43  97.430 3.867253 306.13  23.32  1.649572390566   0.9721649 2.42169297949965  -7.9062743  20.140606
 2015-Sep-25 00:00     10 09 06.12 +12 44 14.0 10 09 55.47 +12 39 36.5   .n.a.     .n.a.   1.78   4.43  97.380 3.874609 296.37  23.44  1.650129289622   0.9562694 2.41709527423888  -8.0155456  20.102368
```

Lagrange Universal Coefficients (Curtis Chapter 2, Sect. 11)

- A fundamental problem in interplanetary mission planning is to determine position and velocity at some specific time, given the position and velocity at some known time (usually initial time), that is:

$$\left[\boldsymbol{r}(t_0), \boldsymbol{v}(t_0), t_0\right] \Rightarrow \left[\boldsymbol{r}(t), \boldsymbol{v}(t)\right]$$

- This requires the numerical solution of Kepler's equation, the relationship between time and orbital parameters, and finally the computation of position and velocity in some reference frame.

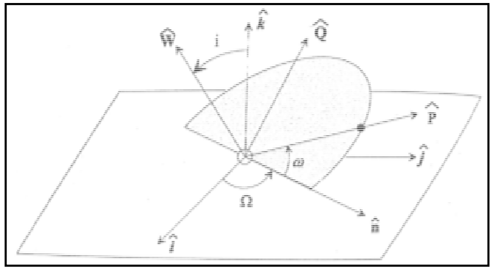
- If we consider the Keplerian motion approximation, we can express the above relationship in compact form using the **Lagrange Coefficients** as:

$$\boldsymbol{r}(t) = F\boldsymbol{r}(t_0) + G\boldsymbol{v}(t_0)$$
$$\boldsymbol{v}(t) = \dot{F}\boldsymbol{r}(t_0) + \dot{G}\boldsymbol{v}(t_0)$$

- Computation of Lagrange coefficients (trace):

  1. Position and velocity in Perifocal System coordinates:

$$\boldsymbol{r} = x\hat{\boldsymbol{P}} + y\hat{\boldsymbol{Q}}$$
$$\boldsymbol{v} = \dot{x}\hat{\boldsymbol{P}} + \dot{y}\hat{\boldsymbol{Q}}$$



2. Computation of angular momentum (invariant of the orbit):

$$\boldsymbol{r} \times \boldsymbol{v} = \boldsymbol{h} = \boldsymbol{h}_0 = \boldsymbol{r}_0 \times \boldsymbol{v}_0 = \left(x_0 \dot{y}_0 - y_0 \dot{x}_0\right) \hat{\boldsymbol{W}}$$

$$\mathbf{r} = \overline{x}\left(\frac{\dot{\overline{y}}_0}{h}\mathbf{r}_0 - \frac{\overline{y}_0}{h}\mathbf{v}_0\right) + \overline{y}\left(-\frac{\dot{\overline{x}}_0}{h}\mathbf{r}_0 + \frac{\overline{x}_0}{h}\mathbf{v}_0\right) = \frac{\overline{x}\,\dot{\overline{y}}_0 - \overline{y}\,\dot{\overline{x}}_0}{h}\mathbf{r}_0 + \frac{-\overline{x}\,\overline{y}_0 + \overline{y}\,\overline{x}_0}{h}\mathbf{v}_0$$

$$\mathbf{v} = \dot{\overline{x}}\left(\frac{\dot{\overline{y}}_0}{h}\mathbf{r}_0 - \frac{\overline{y}_0}{h}\mathbf{v}_0\right) + \dot{\overline{y}}\left(-\frac{\dot{\overline{x}}_0}{h}\mathbf{r}_0 + \frac{\overline{x}_0}{h}\mathbf{v}_0\right) = \frac{\dot{\overline{x}}\,\dot{\overline{y}}_0 - \dot{\overline{y}}\,\dot{\overline{x}}_0}{h}\mathbf{r}_0 + \frac{-\dot{\overline{x}}\,\overline{y}_0 + \dot{\overline{y}}\,\overline{x}_0}{h}\mathbf{v}_0$$

$$\boxed{F\dot{G} - \dot{F}G = 1}$$

3. Orbit properties are not necessary, although eccentricity and true anomaly can be found from the angular momentum. Numerical computation of Lagrange coefficients can be found from available Matlab code.

$$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}} \qquad r = \frac{\dfrac{h^2}{\mu}}{1 + e\cos\theta}$$

- Consider for example the computation of Lagrange coefficients as function of true Anomaly. It can be shown that:

$$\boxed{\begin{aligned} f &= 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\theta) \\ g &= \frac{r r_0}{h}\sin\Delta\theta \\ \dot{f} &= \frac{\mu}{h}\frac{1 - \cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1 - \cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right] \\ \dot{g} &= 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\theta) \end{aligned}}$$

D.8 ALGORITHM 2.3: CALCULATE THE STATE VECTOR FROM THE INITIAL STATE VECTOR AND THE CHANGE IN TRUE ANOMALY

FUNCTION FILE: `rv_from_r0v0_ta.m`

Algorithm:

- Let $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ initial position and velocity at time t_0. Define the change in true anomaly as: $\Delta\theta = \theta - \theta_0$. The objective is to find $\boldsymbol{r}$ and $\boldsymbol{v}$ at time t.

a) Compute the magnitude of initial position and velocity :

$$r_0 = \sqrt{\boldsymbol{r}(t_0)\cdot\boldsymbol{r}(t_0)};\, v_0 = \sqrt{\boldsymbol{v}(t_0)\cdot\boldsymbol{v}(t_0)}$$

b) Compute the radial component of initial velocity:

$$v_{r0} = \frac{\boldsymbol{r}(t_0)\cdot\boldsymbol{v}(t_0)}{r_0}$$

c) Compute the magnitude of the angular momentum:

$$|\boldsymbol{h}| = |\boldsymbol{h}_0| = r_0 v_{\perp 0} = r_0\sqrt{v_0^2 - v_{r0}^2}$$

d) Compute the position r at current time t from:

$$r = \frac{h^2}{\mu}\frac{1}{1 + \left(\frac{h^2}{\mu r_0} - 1\right)\cos\Delta\theta - \frac{h v_{r0}}{\mu}\sin\Delta\theta}$$

e) Use the previous values to compute the Lagrange coefficients from:

$$f = 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\theta)$$

$$g = \frac{r r_0}{h}\sin\Delta\theta$$

$$\dot{f} = \frac{\mu}{h}\,\frac{1-\cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1-\cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right]$$

$$\dot{g} = 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\theta)$$

- Example #1:
  - An Earth satellite moves in the xy plane of an inertial frame with the origin at the Earth's center. Relative to that frame, the position and velocity of the satellite at time t_0 are:

$$\mathbf{r}_0 = 8182.4\hat{\mathbf{i}} - 6865.9\hat{\mathbf{j}}\ (\text{km})$$

$$\mathbf{v}_0 = 0.47572\hat{\mathbf{i}} + 8.8116\hat{\mathbf{j}}\ (\text{km/s})$$

  - Compute position and velocity vectors after the satellite has traveled of a true anomaly of 120 degrees.

Orbital Elements

(a) $r_0 = \sqrt{\mathbf{r}_0 \cdot \mathbf{r}_0} = 10{,}861 \text{ km} \quad v_0 = \sqrt{\mathbf{v}_0 \cdot \mathbf{v}_0} = 8.8244 \text{ km/s}$

(b) $$v_{r0} = \mathbf{v}_0 \cdot \frac{\mathbf{r}_0}{r_0} = \frac{\left(0.47572\hat{\mathbf{i}} + 8.8116\hat{\mathbf{j}}\right) \cdot \left(8182.4\hat{\mathbf{i}} - 6865.9\hat{\mathbf{j}}\right)}{10.681} = -5.2996 \text{ km/s}$$

(c) $$h = r_0\sqrt{v_0^2 - v_{r0}^2} = 10{,}861\sqrt{8.8244^2 - (-5.2996)^2} = 75{,}366 \text{ km}^2/\text{s}$$

$$r = \frac{h^2}{\mu} \frac{1}{1 + \left(\frac{h^2}{\mu r_0} - 1\right)\cos\Delta\theta - \frac{h v_{r0}}{\mu}\sin\Delta\theta}$$

$$= \frac{75{,}366^2}{398{,}600} \frac{1}{1 + \left(\frac{75366^2}{398600 \cdot 10681} - 1\right)\cos 120^\circ - \frac{75366 \cdot (-5.2996)}{398600}\sin 120^\circ}$$

$$= 8378.8 \text{ km}$$

$$f = 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\theta) = 1 - \frac{398{,}600 \cdot 8378.8}{75{,}366^2}(1 - \cos 120^\circ) = 0.11802 \,(\text{dimensionless})$$

$$g = \frac{r r_0}{h}\sin(\Delta\theta) = \frac{8378.8 \cdot 10{,}681}{75{,}366}\sin(120^\circ) = 1028.4 \text{ s}$$

$$\dot{f} = \frac{\mu}{h}\frac{1 - \cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1 - \cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right]$$

$$= \frac{398{,}600}{75{,}366}\frac{1 - \cos 120^\circ}{\sin 120^\circ}\left[\frac{398{,}600}{75{,}366^2}(1 - \cos 120^\circ) - \frac{1}{10{,}681} - \frac{1}{8378.9}\right]$$

$$= -9.8666 \times 10^{-4} \,(\text{dimensionless})$$

$$\dot{g} = 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\theta) = 1 - \frac{398{,}600 \cdot 10{,}681}{75{,}366^2}(1 - \cos 120^\circ) = -0.12435 \,(\text{dimensionless})$$

- At this point we can compute the position and velocity at $\theta = 120$ degrees:

$$\begin{aligned}
\mathbf{r} &= f\mathbf{r}_0 + g\mathbf{v}_0 \\
&= 0.11802\left(8182.4\hat{\mathbf{i}} - 6865.9\hat{\mathbf{j}}\right) + 1028.4\left(0.47572\hat{\mathbf{i}} + 8.8116\hat{\mathbf{j}}\right) \\
&= \boxed{1454.9\hat{\mathbf{i}} + 8251.6\hat{\mathbf{j}}\ (\text{km})} \\
\mathbf{v} &= \dot{f}\mathbf{r}_0 + \dot{g}\mathbf{v}_0 \\
&= \left(-9.8666 \times 10^{-4}\right)\left(8182.4\hat{\mathbf{i}} - 6865.9\hat{\mathbf{j}}\right) + (-0.12435)\left(0.47572\hat{\mathbf{i}} + 8.8116\hat{\mathbf{j}}\right) \\
&= \boxed{-8.1323\hat{\mathbf{i}} + 5.6785\hat{\mathbf{j}}\ (\text{km/s})}
\end{aligned}$$

❑ Example #2: Find the eccentricity of the orbit as well as the true anomaly at the initial time t_0 and, hence, the location of the perigee for this orbit.

- Recall the previous results:

(a)
$$r_0 = 10{,}861 \text{ km}$$
$$v_{r0} = -5.2996 \text{ km/s}$$
$$h = 75{,}366 \text{ km}^2/\text{s}$$

- Since the radial speed is negative, the satellite is approaching the perigee, thus [180° < θ<360°]. Using the orbit formulas:

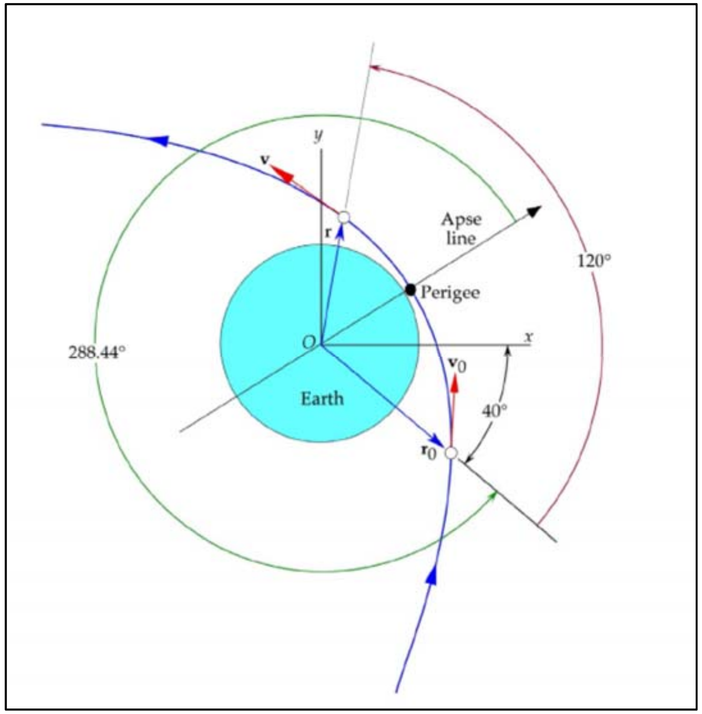
$$r_0 = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta_0} \qquad v_{r0} = \frac{\mu}{h} e \sin\theta_0$$

$$10{,}861 = \frac{75{,}366^3}{398{,}600}\frac{1}{1 + e\cos\theta_0} \qquad -5.2996 = \frac{398{,}600}{75{,}366} e \sin\theta_0$$

Orbital Elements

- Now we can compute the eccentricity **e = 1.0563**, therefore the orbit is a Hyperbola.
- For the true anomaly we have. $\cos\theta_0 = \frac{0.3341}{1.0563} = 0.3163$

  This means either θ_0 = 71.56 deg. or $\boldsymbol{\theta_0}$ **= 288.44 deg.** The latter is the correct one



- If the time interval is not large, an approximate procedure can be used (see. Mengali, page 123.)
- Example #3: We wish to find the time elapsed to go from the initial position to the final position (true anomaly of 120 degrees)
- Compute the orbit parameters:

$$e = 1.0563$$
$$a = \frac{\frac{h^2}{\mu}}{e^2 - 1} = 123{,}088.65\,Km$$
$$r_p = a(e-1) = 6{,}929.9\,Km$$
$$r_a = -a(1+e) = -253{,}107.63\,Km$$

- Use Kepler's equation for Hyperbolic orbits:

$$\begin{cases} t - t_p = \sqrt{\dfrac{(-a^3)}{\mu}}(e\sinh F - F) \\ F = \ln(\cosh F + \sqrt{(\cosh F)^2} - 1 \end{cases} \qquad \cosh F = \frac{e + \cos\theta}{1 + e\cos\theta} \qquad \begin{matrix} \theta_1 = 71.56\,\deg \\ e = 1.0563 \end{matrix}$$

- First find the time needed to reach perigee from initial true anomaly, then apply Kepler's equation

$$\cosh F_1 = \frac{e + \cos\theta_1}{1 + e\cos\theta_1} = \frac{1.0563 + 0.3163}{1 + 1.0563 \cdot 0.3163} = 1.0289 \Rightarrow F_1 = 0.2398\,rad$$

$$t - t_p = -\sqrt{\frac{(-a^3)}{\mu}}(e\sinh F_1 - F_1) = -1{,}089.62\,\sec$$

- Compute the true anomaly from perigee: $\theta_2 = 120^{\circ} - 71.56^{\circ} = 48.44^{\circ}$. Then apply Kepler's equation again

$$\cos\theta_2 = 0.6634 \Rightarrow \cosh F_2 = \frac{e + \cos\theta_2}{1 + e\cos\theta_2} = 1.0111 \Rightarrow F_2 = 0.1489\,rad$$

$$t - t_p = -\sqrt{\frac{(-a^3)}{\mu}}(e\sinh F_2 - F_2) = 616.7568\,\sec$$

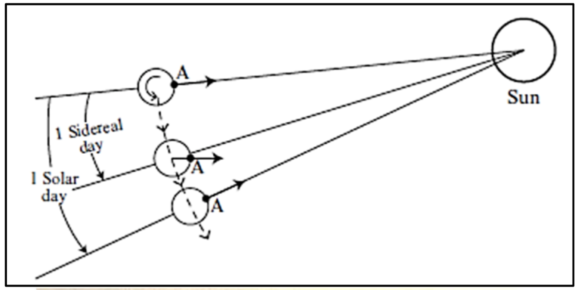
- Total time
- 1,706.3768 sec = 0.474 h

Orbital Elements

- **Measure of Time**: Time is a fundamental parameter in the equations of motion. To establish a system of time, one must define two quantities: the unit of duration (e.g., the second or day), and the epoch, or zero, of the chosen time. In physics and astronomy, there are four types of time systems in common use. Broadly speaking, they are the following:

1. Universal time (UT), in which the unit of duration is the (mean) solar day, defined to be as uniform as possible, despite variations in the rotation of the Earth. Universal time is the mean solar time at Greenwich, England, defining the zero epoch from which UT is measured.
2. Atomic time (TAI), in which the unit of duration corresponds to a defined number of wavelengths of radiation of a specified atomic transition of a chosen isotope. Atomic time is based on the Systeme International (SI) second (atomic second), which is defined as the elapsed time of 9,192,631,770 “oscillations of the undisturbed cesium atom.” Atomic time is regular. So the irregularity of the earth’s rotation rate implies that UT and TAI will get out of step. Therefore, the “leap second” was invented in 1972.
3. Dynamical time, in which the unit of duration is based on the orbital motion of the Earth, Moon, and planets. In 1952, the International Astronomical Union (IAU) introduced Ephemeris Time (ET), based on the occurrence of astronomical events, to cope with the irregularities in the earth’s rotation that affected the flow of mean solar time. The uniform time scale ET is the independent variable (dynamical time) in the differential equations of motion of the planets, the sun, and moon.
4. Sidereal time, in which the unit of duration is the period of the Earth’s rotation with respect to a point nearly fixed with respect to the stars. Apart from the inherent motion of the equinox due to precession and nutation, sidereal time is a direct measure of the diurnal rotation of the earth. In general terms, sidereal time is the hour angle of the vernal equinox, i.e., the angle between the mean vernal equinox of date and the Greenwich meridian.

Orbital Elements



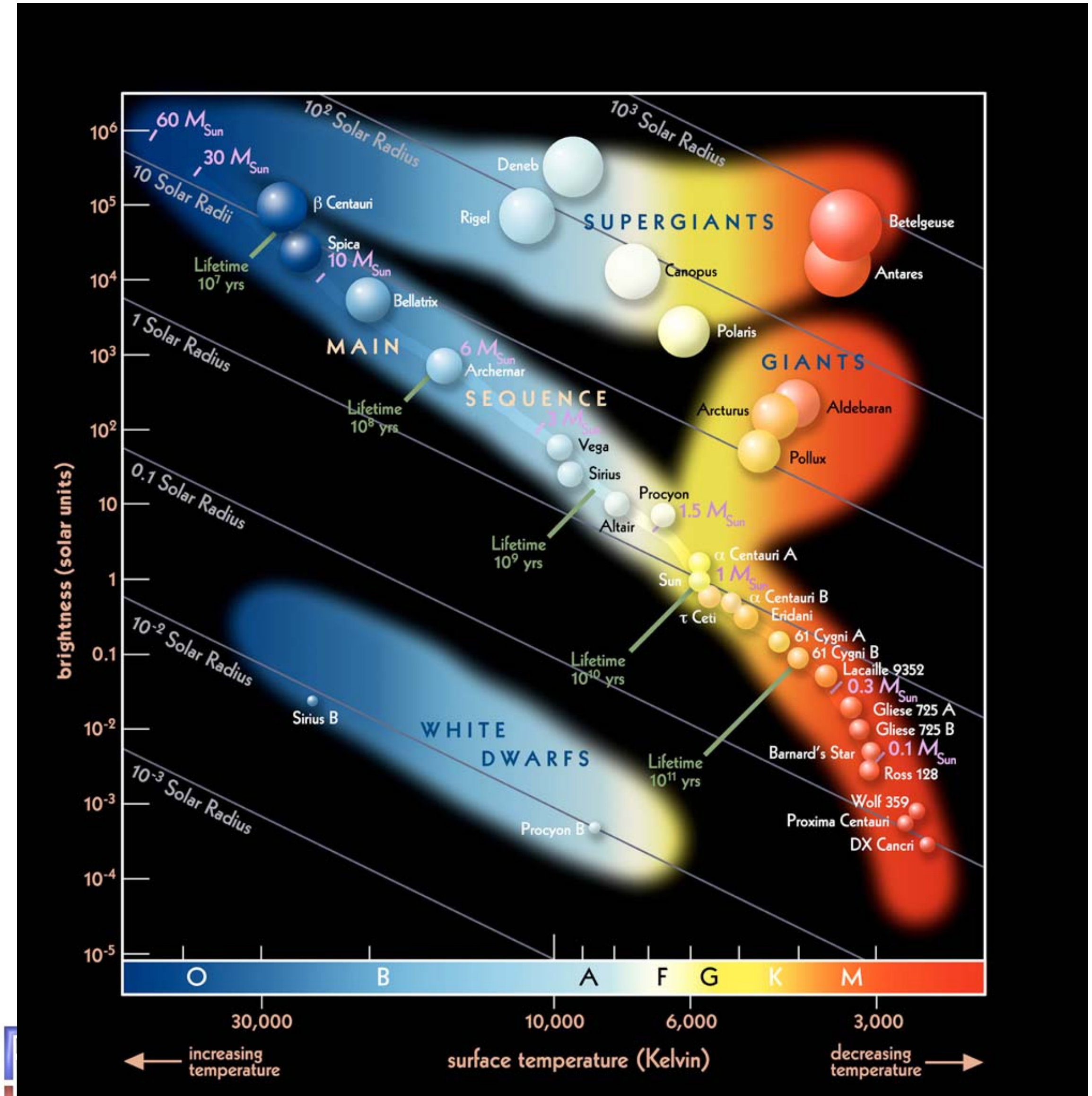
1 giorno solare medio	86400 s
1 giorno siderale medio	86164.09054 s
1 anno tropicale	365.2421988 giorni solari medi
1 anno giuliano	365 (366 se bisestile) giorni solari medi
1 secondo solare medio	1.1574074×10^{-5} giorni solari medi 1.1605763×10^{-5} giorni siderali 1.002737909 secondi siderali

- Def.: <u>A solar day</u> is the time interval between two successive "high noons" or upper transits of the sun.
- Def.: <u>A mean solar day</u> is the average length of the individual solar days throughout the year. We break up a mean solar day into 24 h.
- Def.: <u>A sidereal day</u> is the time interval between two successive high transits of a star. That is, a sidereal day is the time required for the earth to rotate once on its axis relative to the stars.

- **Julian Date (day)**: <u>The Julian day system provides a unique number to all days that have elapsed since a selected standard reference day, January 1, 4713 B.C.</u>, in the Julian calendar. The days are in mean solar measure. The Julian day (JD) numbers are never repeated and are not partitioned into weeks or months. Therefore, the number of day between two dates can be obtained by subtracting Julian day numbers. There are 36,525 mean solar days in a Julian century and 86,400 s in a day. The Julian century does not refer to a particular time system; but rather is merely a count of a fixed number of days. <u>Ephemeris calculations are done in Julian days and Julian centuries.</u> A Julian day starts at noon UT instead of midnight, an astronomical custom that is followed to avoid having the day number change during a night's observations. Julian days can be calculated from calendar dates between the years 1901 A.D. and 2099 A.D. as follows:

$$J = 367Y - \mathrm{int}\left[7\left(\frac{Y + \mathrm{int}((M+9)/12)}{4}\right)\right] + \mathrm{int}\left(\frac{275M}{9}\right) + D + 1{,}721{,}013.5$$

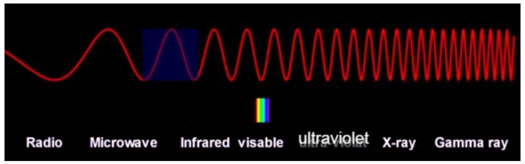
Hertzsprung–Russell diagram



Relationship between the stars' absolute magnitudes or luminosities versus their stellar classifications or effective temperatures. Based on early Spectral analysis (c.a. 1800), it uses the amount of EM radiation emitted by the star at a specified apparent distance of 30 parsec (apparent brightness).

Elements needed for determining distance:

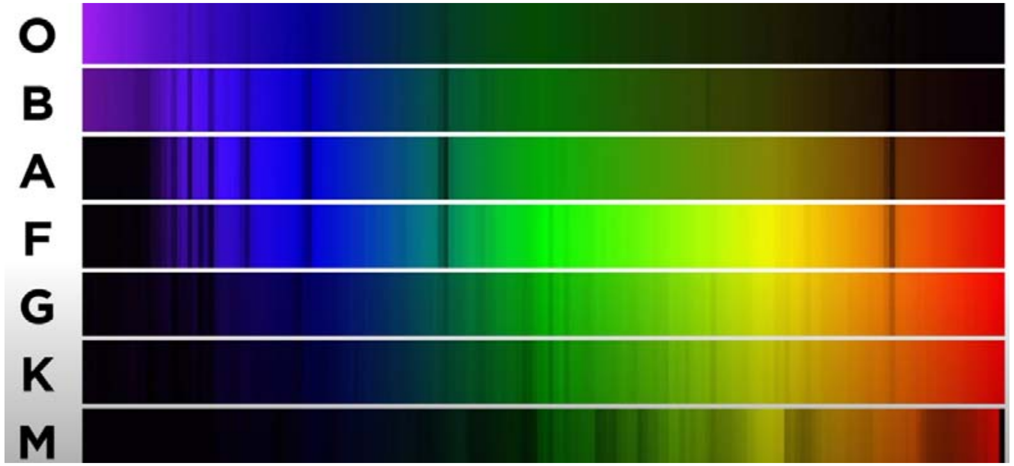
1. Speed of Light (light to get to Earth from Io)
2. Light is a EM radiation
3. Color (light as wave), E = hv, red = low frequency and Temperature
4. Inverse square Law

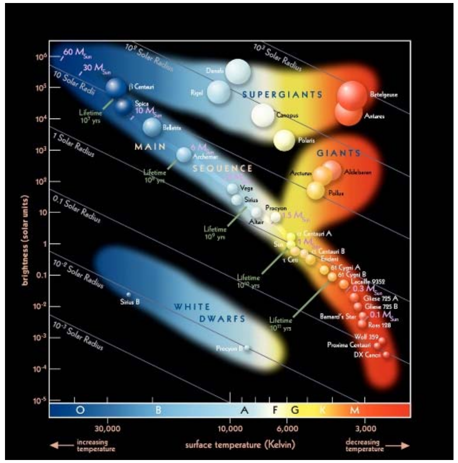


Luminosity = Luminosità, amount of energy proper of a star (in Watts) also True Magnitude
Brightness = Splendor that depends on a specific distance (Earth or 10 parsec) also Apparent Magnitude

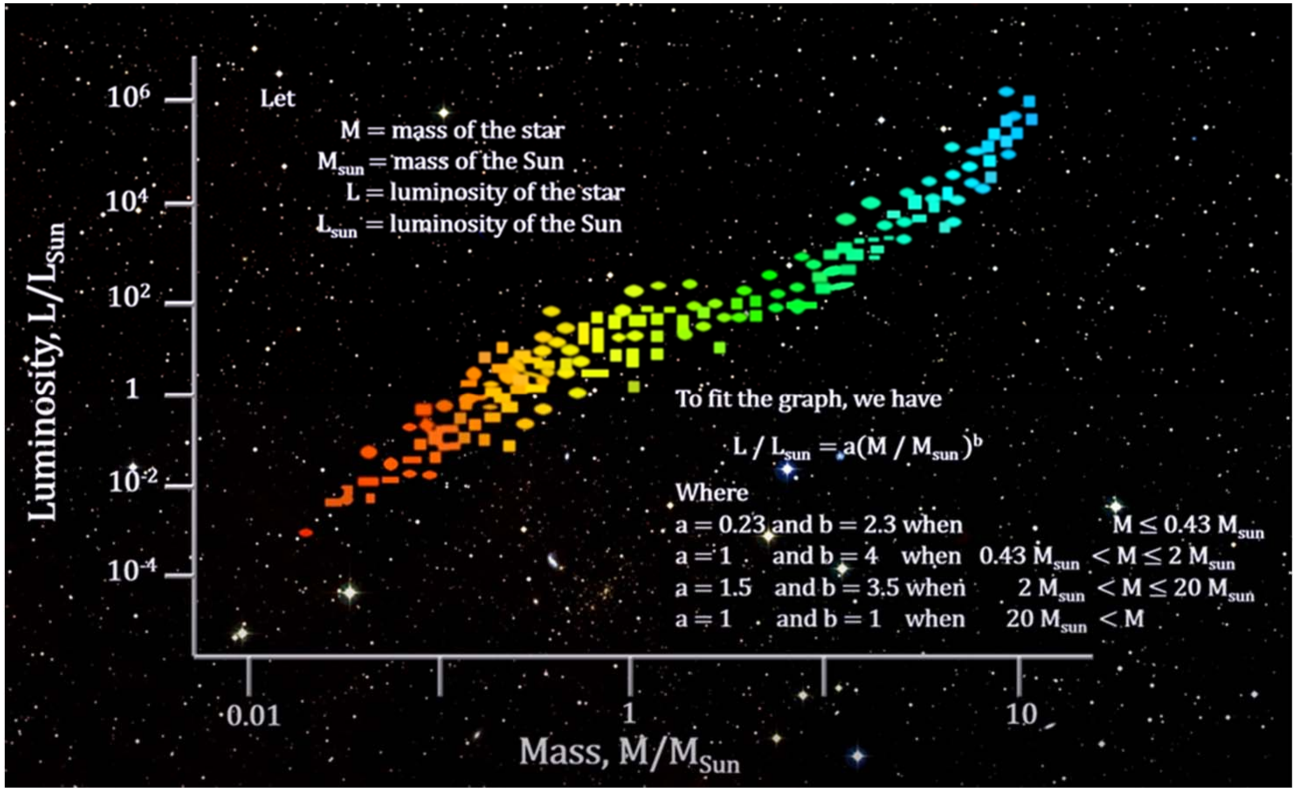
Star Distance, Size, Temperature Measures

- Waves,
- EM radiation, Energy, Photons, Temperature,
- Blackbody Radiation
- Color, Doppler shift,
- Luminosity, Magnitude,…
- Wien's Law, Stefan - Boltzman Law, Plank's law



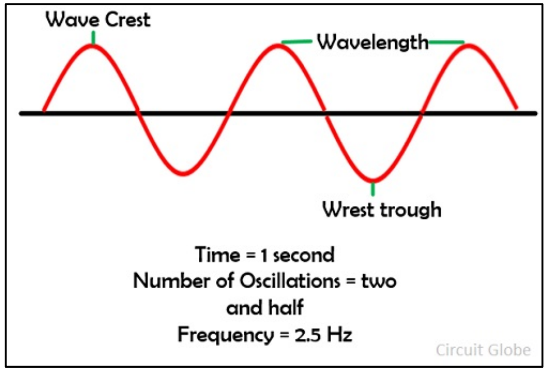


- ❑ Star spectral content provides information on:
  - ❑ Color → Temperature
  - ❑ Chemical composition (black lines) → Age



Cepheid Period-Distance Computation

There exists a well-defined relationship between a classical Cepheid variable's luminosity and pulsation period, yielding to their use as standard candles



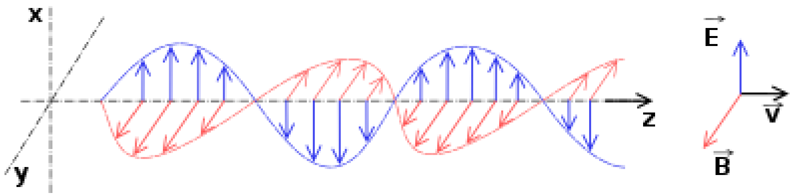
- A wave is an ondulatory motion characterized by 3 elements:
  - Speed
  - Wavelength **λ** (distance at which the wave repeats itself) measured in meters
  - frequency **f** (how many times the wave repeats itself in the unit time) measured in cycles/sec

$$v = f\lambda$$

od T

Note: According to the molecular collision theory of chemical reactions, Frequency is directly proportional to the average velocity of the molecules, and according to the Kinetic Theory of Gases, the average velocity is directly proportional to the square root of the temperature. So we can say that Frequency is directly proportional to the square root of temperature. So, on increasing the temperature, the collision frequency increases and hence, the fraction of effective collisions increases, so these collisions lead to and increase in the energy.

- **Electromagnetic radiation** (EM radiation) refers to the waves (or their quanta, photons) of the electromagnetic field, propagating (radiating) through space, carrying electromagnetic radiant energy. It includes radio waves, microwaves, infrared, (visible) light, ultraviolet, X-rays, and gamma rays.

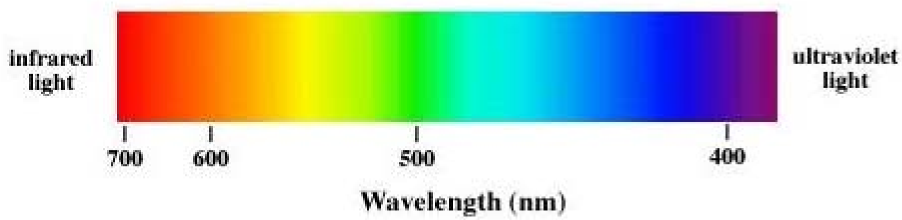


$$c = f\lambda = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

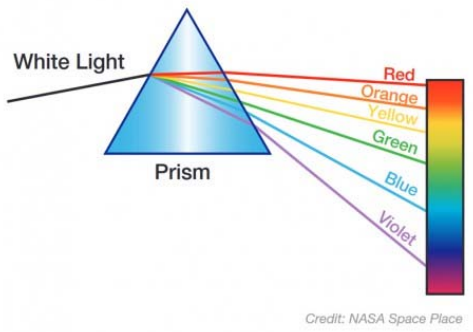
c = 299,792.458 Km / s

EM Radiation

$$c = f\lambda$$



Type of Radiation	Frequency Range (Hz)	Wavelength Range
gamma-rays	10^{20} - 10^{24}	< 10^{-12} m
x-rays	10^{17} - 10^{20}	1 nm - 1 pm
ultraviolet	10^{15} - 10^{17}	400 nm - 1 nm
visible	4 - 7.5*10^{14}	750 nm - 400 nm
near-infrared	1*10^{14} - 4*10^{14}	2.5 μm - 750 nm
infrared	10^{13} - 10^{14}	25 μm - 2.5 μm
microwaves	3*10^{11} - 10^{13}	1 mm - 25 μm
radio waves	< 3*10^{11}	> 1 mm

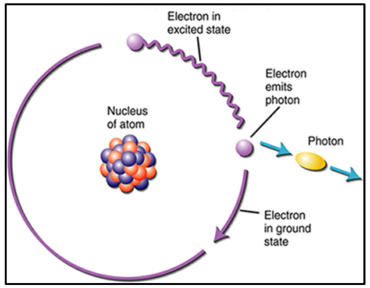


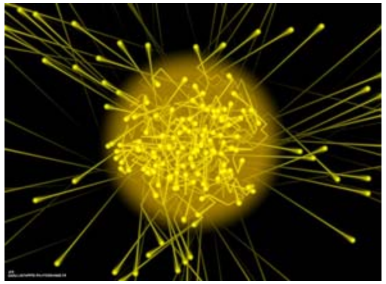
$$\frac{c}{\lambda} = f = \frac{3\times10^{8}\,m/s}{500\,nm} = \frac{3\times10^{8}}{500\times10^{-9}} = 6\times10^{14}\,Hz$$

Frequency of color green

Note: All bodies emit EM radiation since it is the energy obtained from the molecular motion, except for those at 0 degrees Kelvin (absolute zero)

❑ EM radiation as quantized representation





- According to quantum physics a beam of light is made of zillions of tiny packets of light, called **photons**, streaming through the air.
- A <u>photon is the smallest discrete amount or quantum of electromagnetic radiation</u>. It is the basic unit of all light.

- Photons are always in motion and, in a vacuum, travel at a constant speed of 2.998×10^8 m/s, which is the speed of light and have no mass and no charge.

- Photons have energy equal to their oscillation frequency times Planck's constant (Einstein). The energy of these photons is the height of their oscillation frequency, and the intensity of the light corresponds to the number of photons.

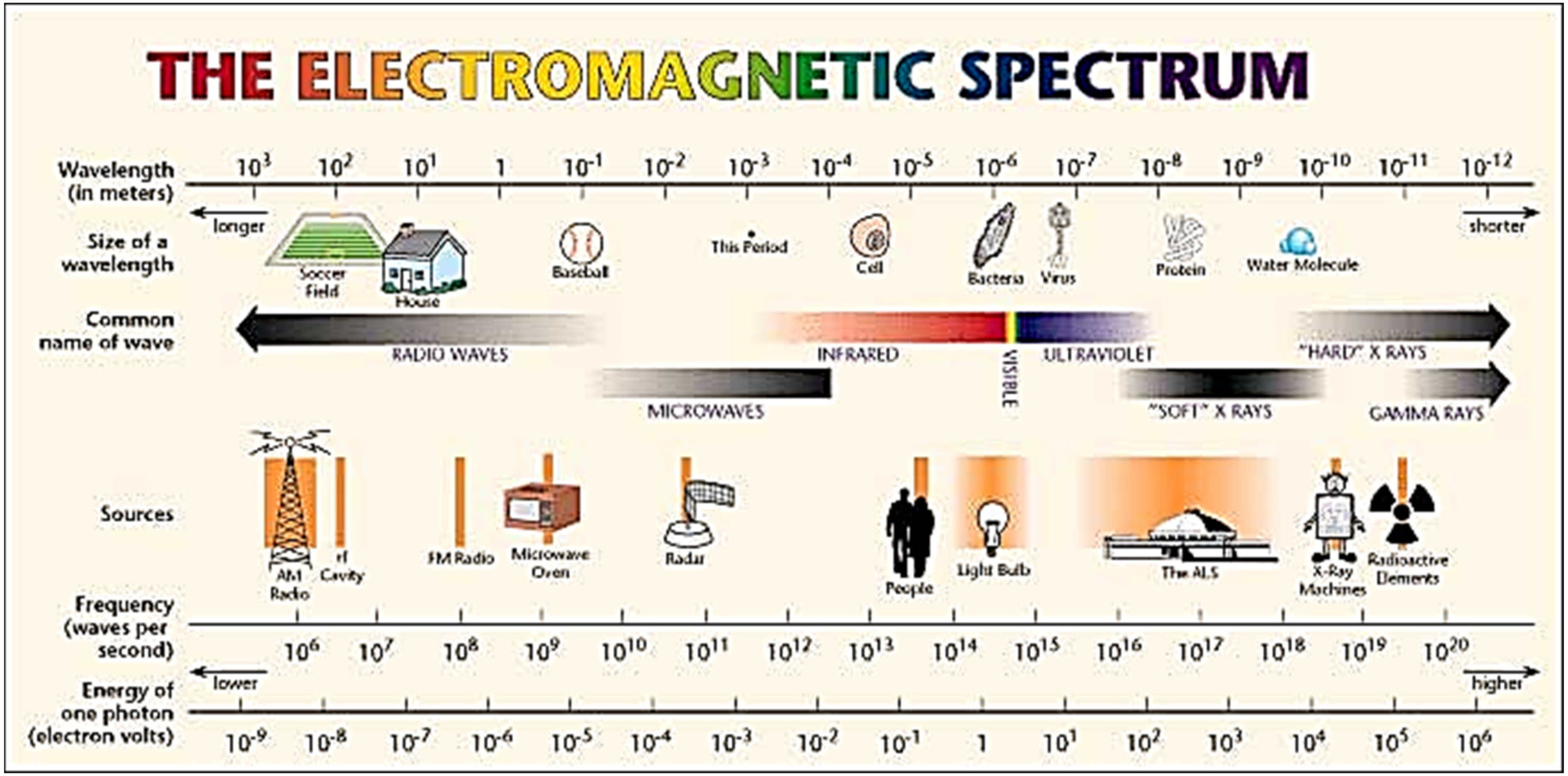
E = (n)**hf**, **Plank Constant (6.62607015×10^{-34} J.s)**xfrequency of the EM radiation (n = 0, 1, 2, 3, ..)

E = (1)(6.62607015×10^{-34} J.s)x(6×10^{14} Hz) = 3.97×10^{-19} J = 2.48 eV (visible light (green))

$$E = hf = h\frac{c}{\lambda} = mc^2 \Rightarrow \frac{h}{\lambda} = mc = hf$$

$$\lambda = \frac{h}{p = mv}$$

Electron as a wave (de Broglie)



Dual Nature of EM Radiation

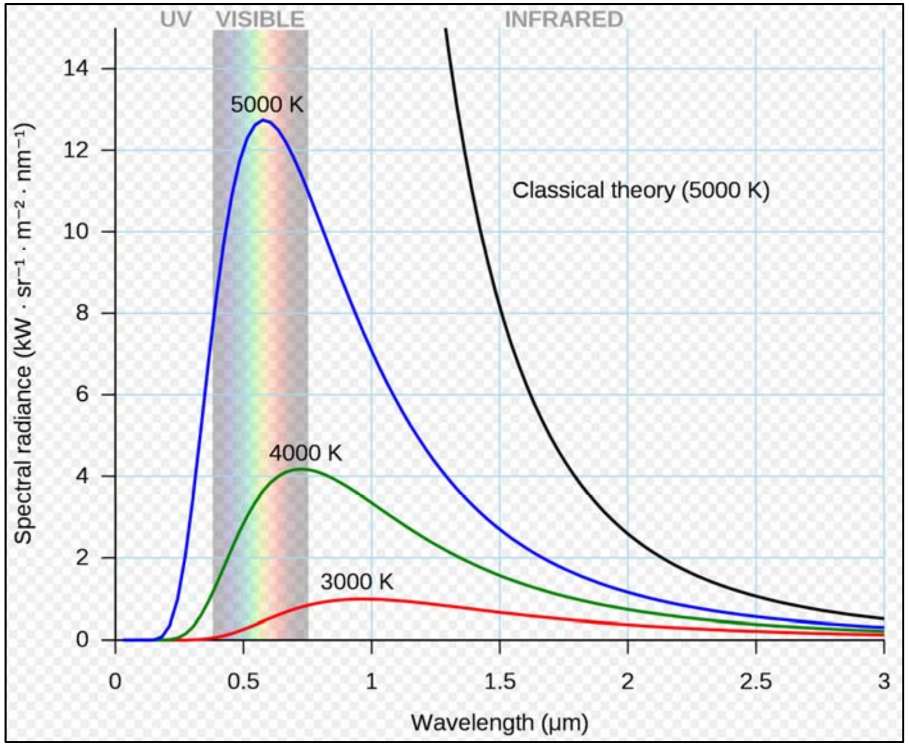
Dual Nature of Matter

❑ EM radiation propagation defines matter blocking properties

- **Transparent**: the EM radiation passes through completely. For instance, the atmosphere is transparent to visible light
- **Opaque**: the EM radiation is blocked. For instance, a wall does not allow visible light to go through. The atmosphere is opaque to x- rays and γ-rays

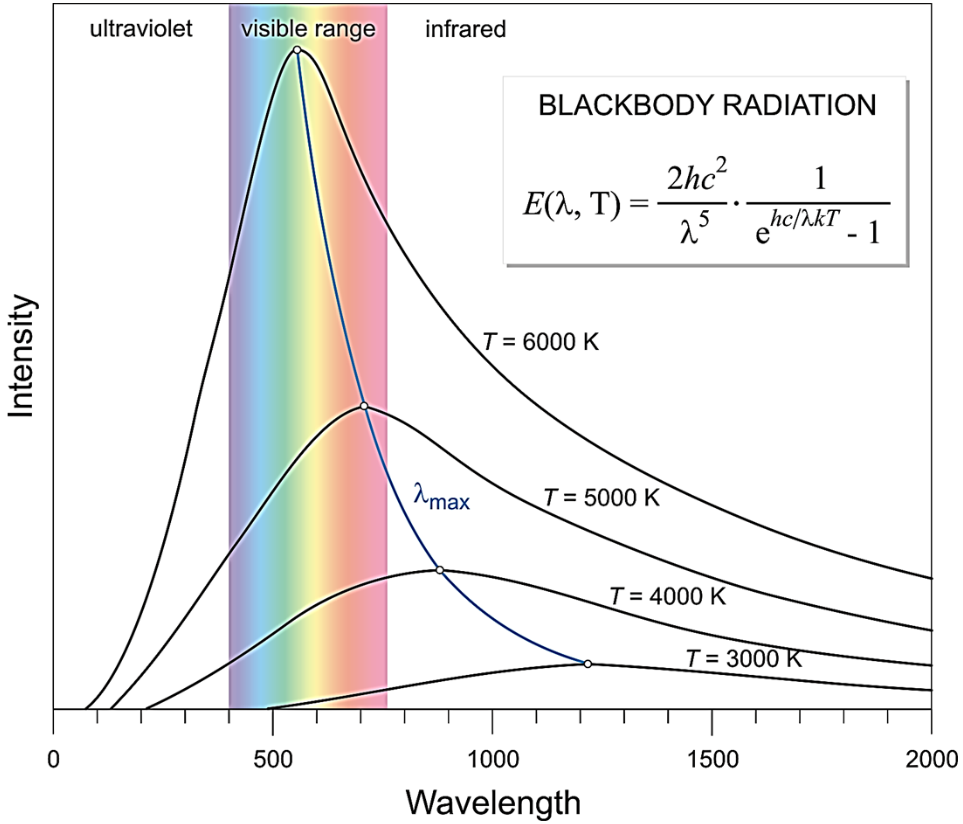
- **Blackbody**: is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence.

- A black body in thermal equilibrium (at a constant temperature) emits electromagnetic radiation called black-body radiation. The radiation is emitted according to Planck's law, meaning that it has a spectrum that is determined by the temperature alone. Stars temperature is measured as represented by blackbodies.



- **Plank's Law:** describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T, when there is no net flow of matter or energy between the body and its environment.
  - Contains quantum energy E = hf
  - The energy radiated by a blackbody is emitted in quanta (discrete amounts) as function of the temperature

  - Obtain Wien's law by differentiation
  - Obtain Stefan-Boltzman law by integration
  - It solves the UV catastrophe



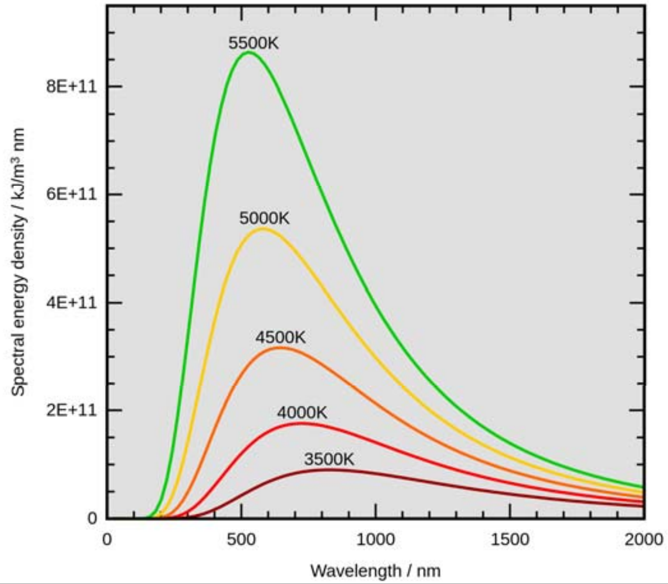
$$B(f,T) = \frac{2hf^3}{c^2} \cdot \frac{1}{e^{\frac{hf}{K_B T}} - 1} = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda K_B T}} - 1} = B(\lambda, T)$$

Blackbody radiation   Plank's Law 1   Plank's Law 2

- **Wien's Law:** the spectral radiance (energy) of black-body radiation per unit wavelength, peaks at the wavelength λ_{peak} :

$$\lambda_{peak} = \frac{0.0029}{T}$$

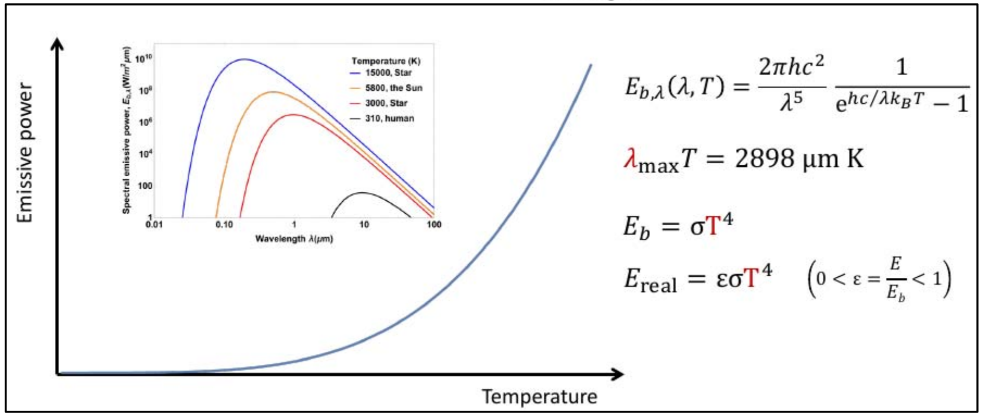
- From the EM spectrum of the Sun, the surface temperature can be found to be 5,800 °K
- It is used to measure the body's temperature



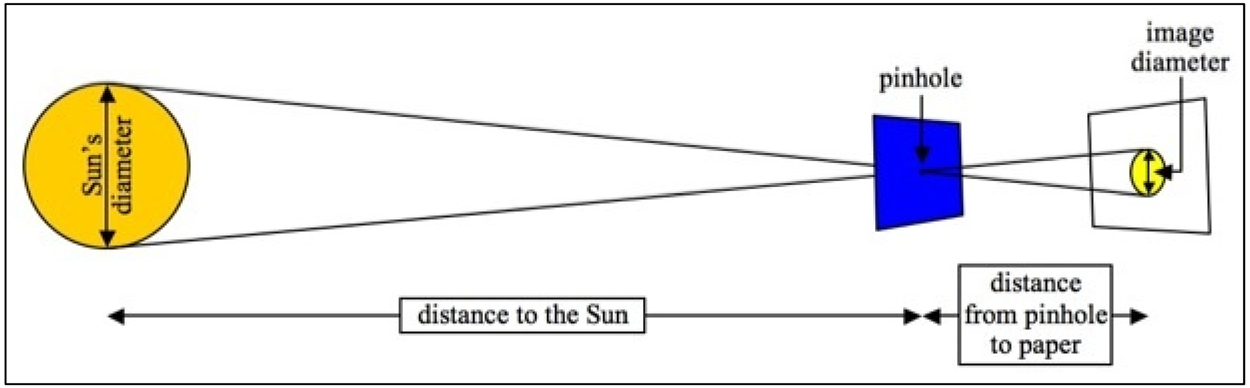
- **Stefan-Boltzman Law:** relationship between power per unit area and temperature

$$P = e\sigma A T^4$$

- Human body emits about 238 Cal/sec



❑ Intensity of Sun's radiation on Earth



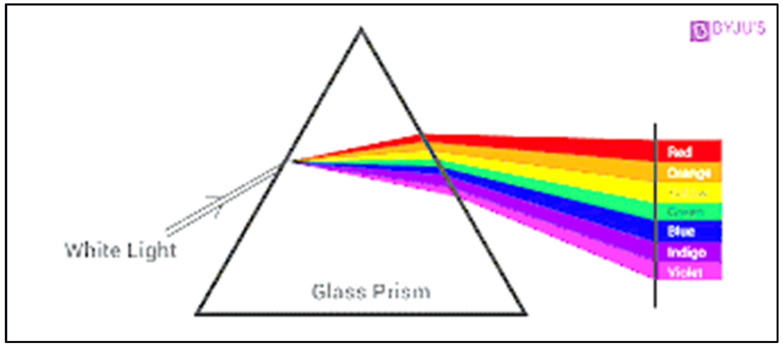
$$\begin{cases} P = \dfrac{dQ}{dT} = ? \\ R_{\odot} = 6.96 \times 10^{8} m \end{cases} \qquad P = e\sigma A_{\odot} T^{4} \qquad T_{\odot} = \frac{0.0029}{\lambda_{0} = 500nm} \simeq 5,800^{o} K$$

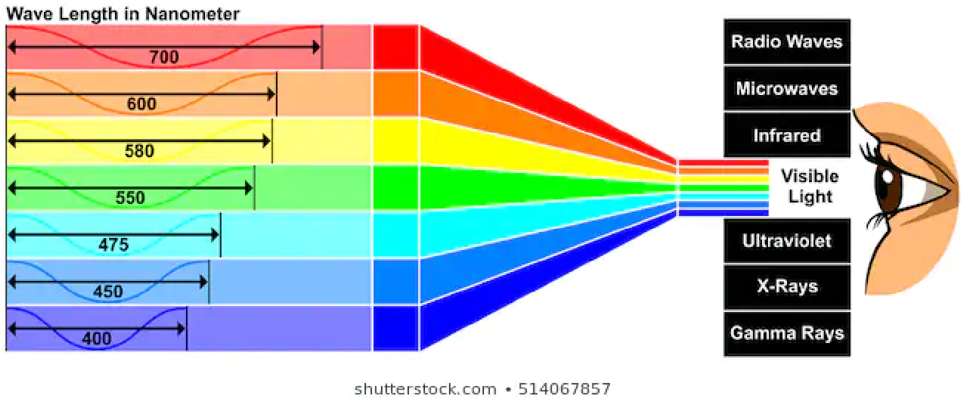
$$P = (1)(5.67 \times 10^{-9})(4\pi)(6.96 \times 10^{8})^{2}(5800)^{4} = 3.6 \times 10^{26} W$$

- Intensity = Power/Area; at Earth: $I = \dfrac{3.6 \times 10^{26} W}{4\pi(1.5 \times 10^{9})} = 1280 \dfrac{W}{m^{2}}$

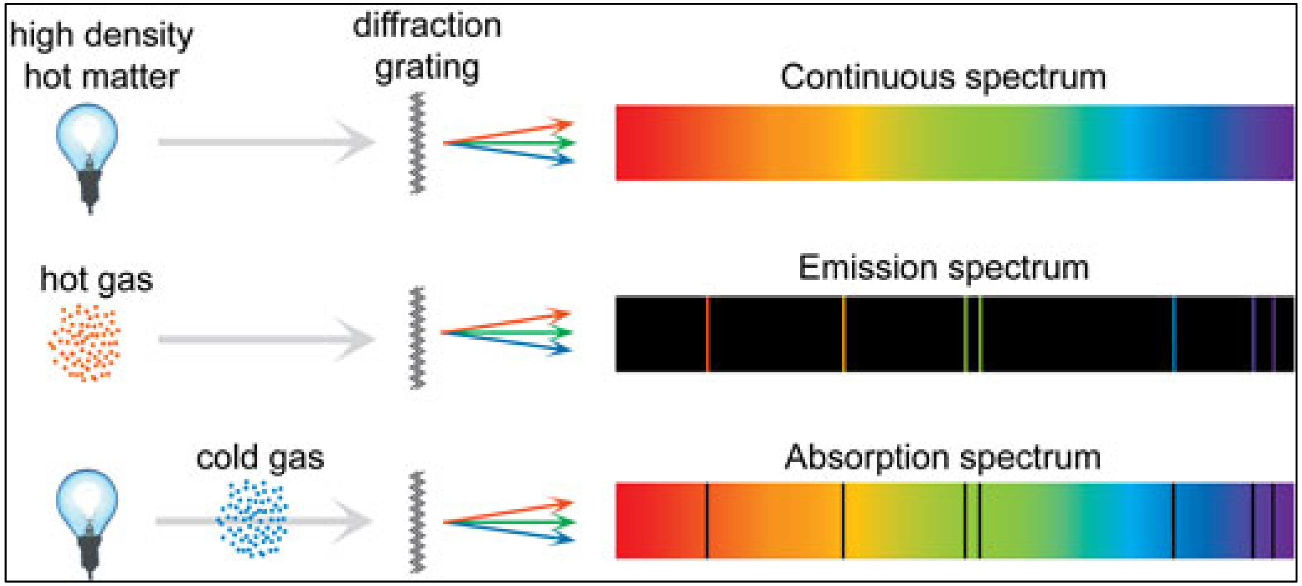
❑ About the Light Spectrum

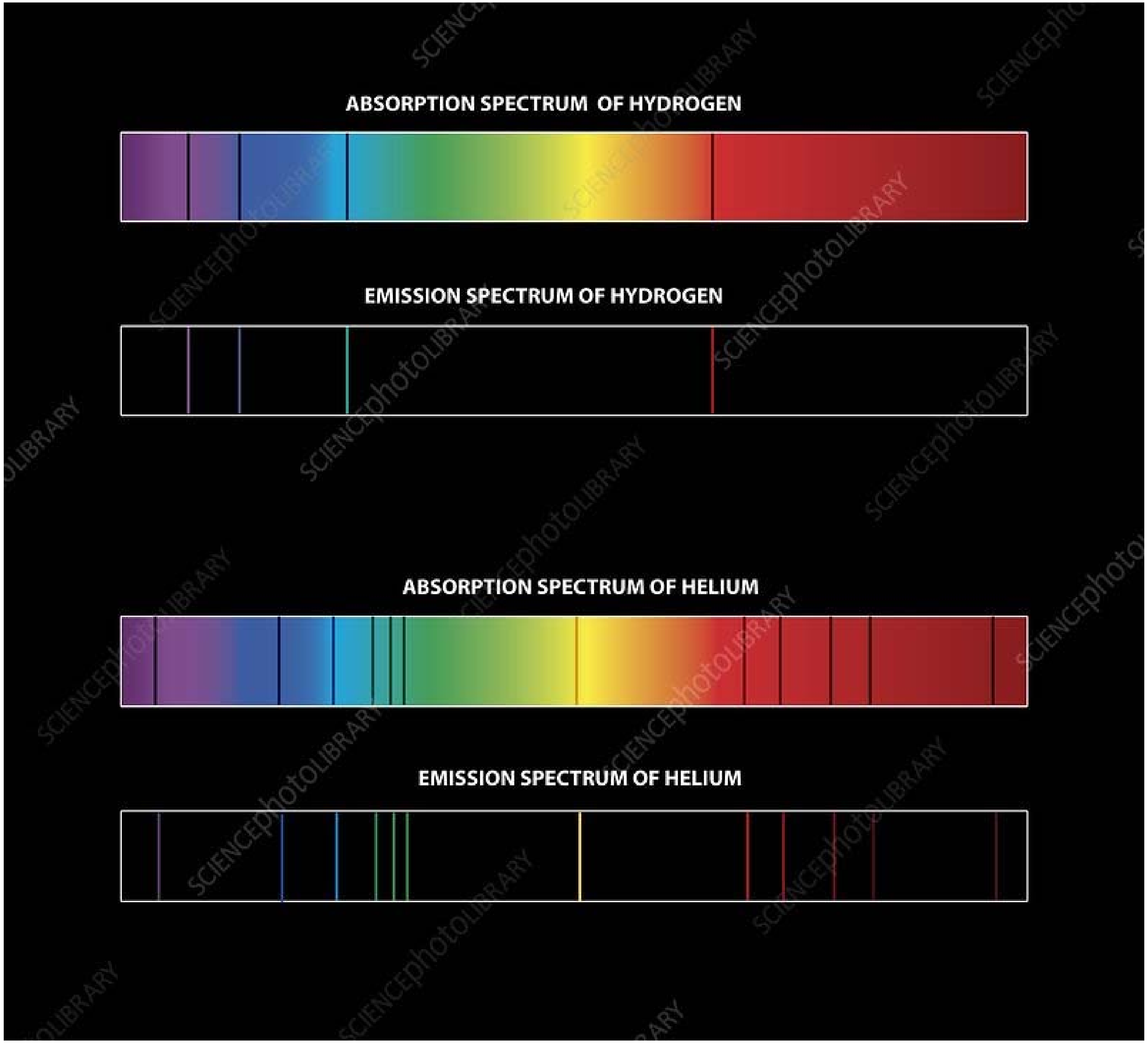
- Continuous Spectrum





- Emission and Absorption Spectra



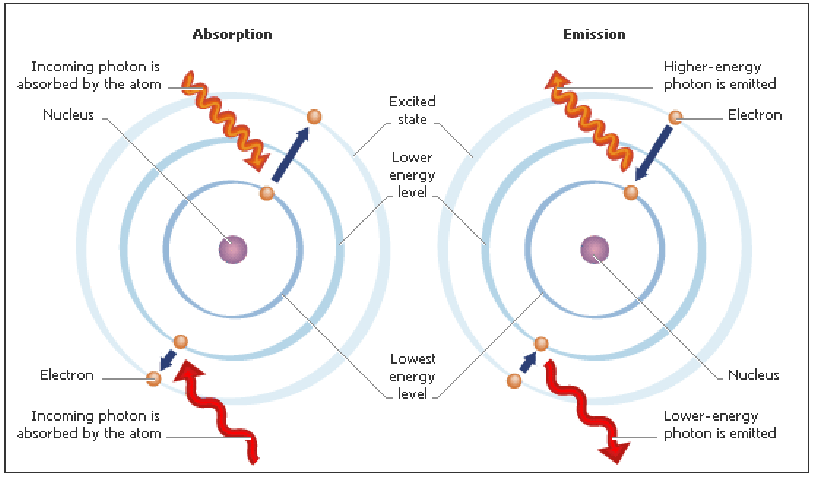

ABSORPTION SPECTRUM OF HYDROGEN
EMISSION SPECTRUM OF HYDROGEN
ABSORPTION SPECTRUM OF HELIUM
EMISSION SPECTRUM OF HELIUM

- An atom can absorb or emit one photon when an electron makes a transition from one stationary state, or energy level, to another. Conservation of energy determines the energy of the photon and thus the frequency of the emitted or absorbed light.

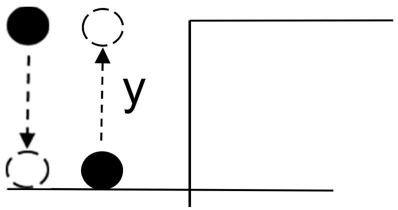
$$E = hf = h\frac{c}{\lambda} = mc^2 \Rightarrow \frac{h}{\lambda} = mc$$

❑ Hydrogen

- When the electron changes from n=3 or above to n=2, the photons emitted fall in the Visible Light region of the spectra.
- When the electron changes from n=2 or above to n=1, the photons emitted fall in the UltraViolet region of the spectra



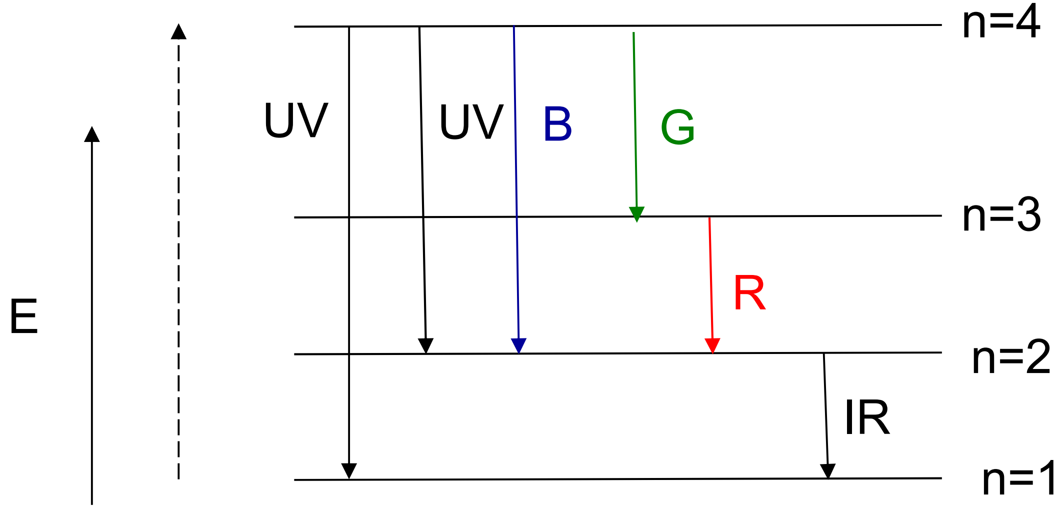
$PE \Rightarrow KE$ $\quad$ $W \Rightarrow mgy = PE$



- From previous results, using Planck-Einstein results

$$E = hf = \frac{hc}{\lambda} \Rightarrow E \uparrow = \lambda \downarrow$$

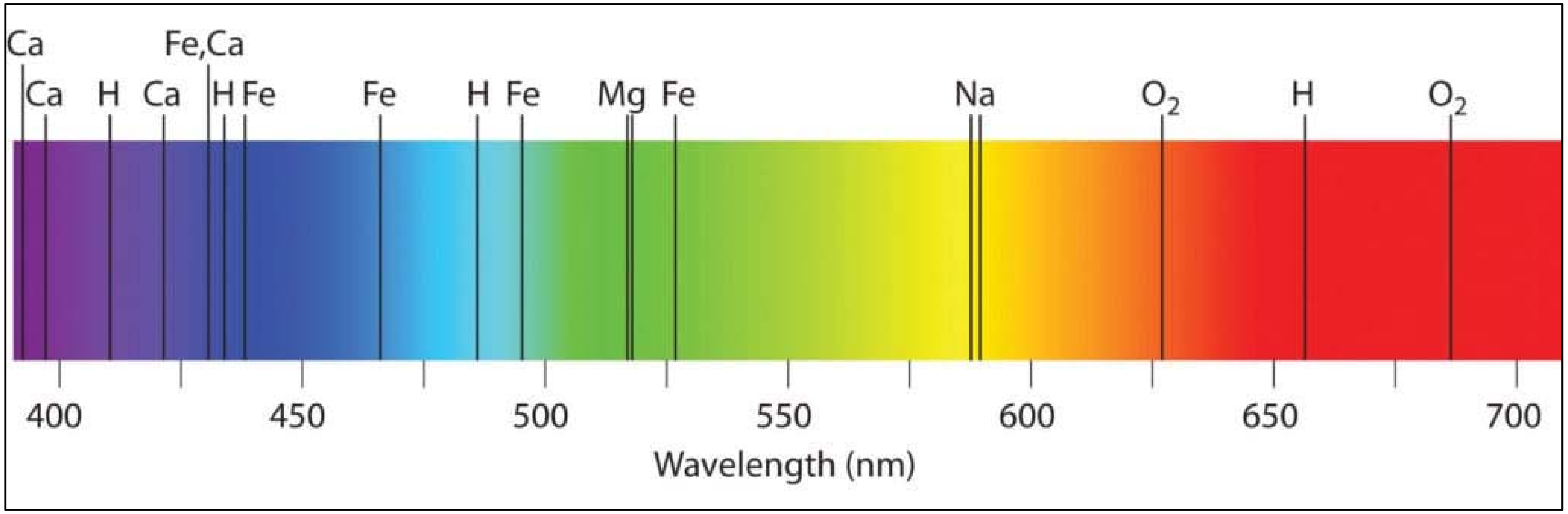
$hc = (6.62607015\times10^{-34})x(3.0x10^{11})\approx\approx1.986x10^{-24}$ J.m



- The type of color gives the emission spectrum proper of each element

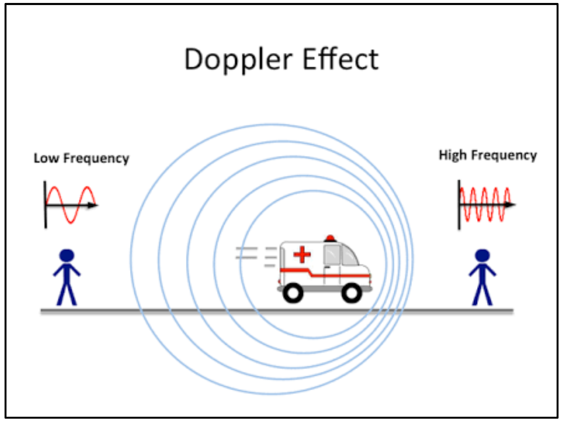
- It easier to look at the spectrum than compute the appropriate lines

- Spectrum of the Sun

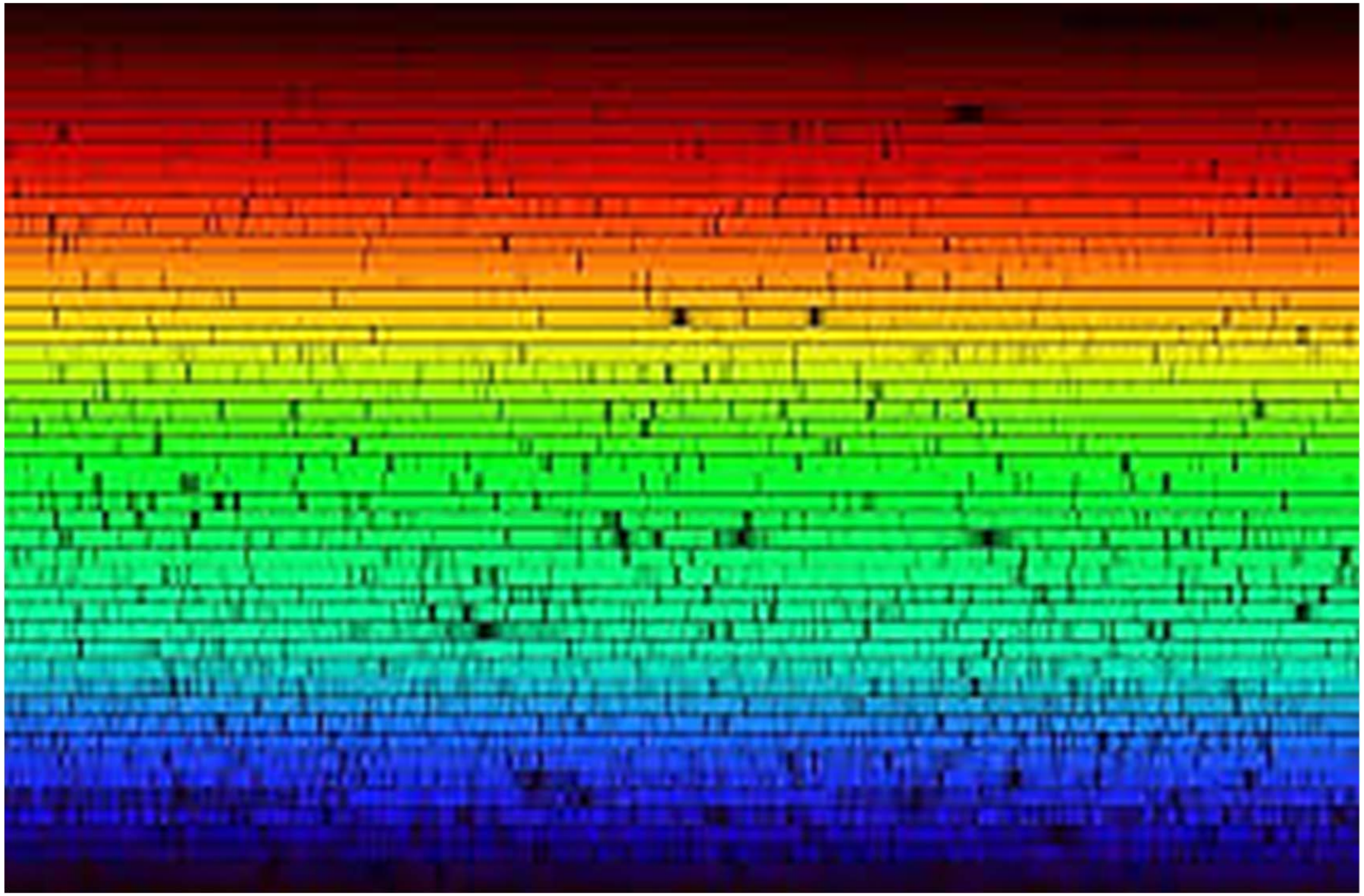


- Doppler Shift

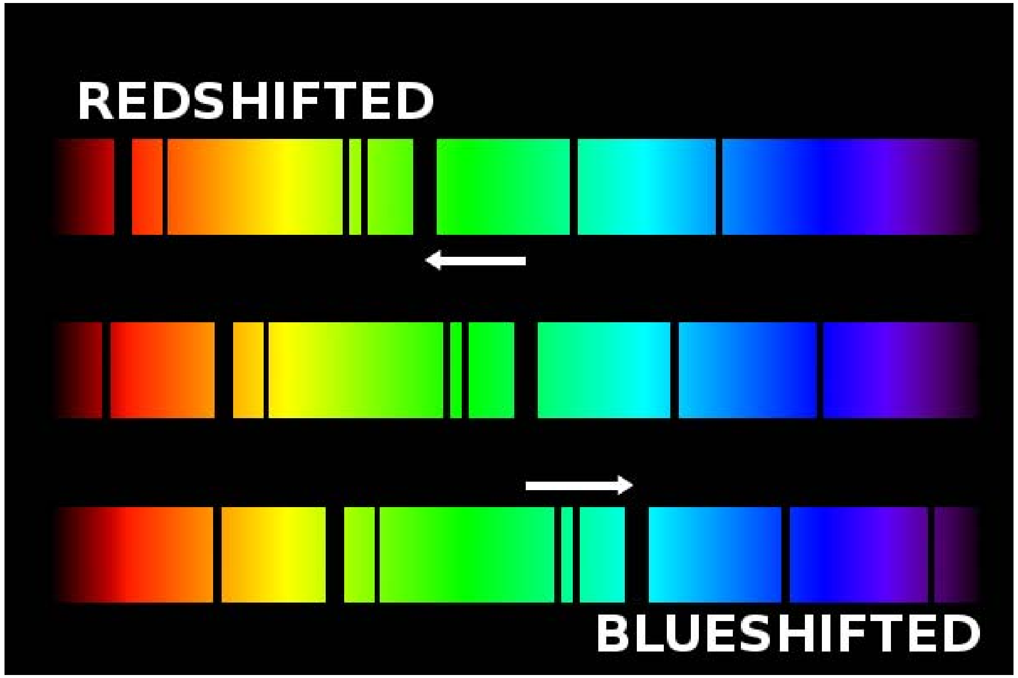
- The Doppler effect (or the Doppler shift) is the change in frequency of a wave in relation to an observer who is moving relative to the wave source. Usually associated with the sound wave

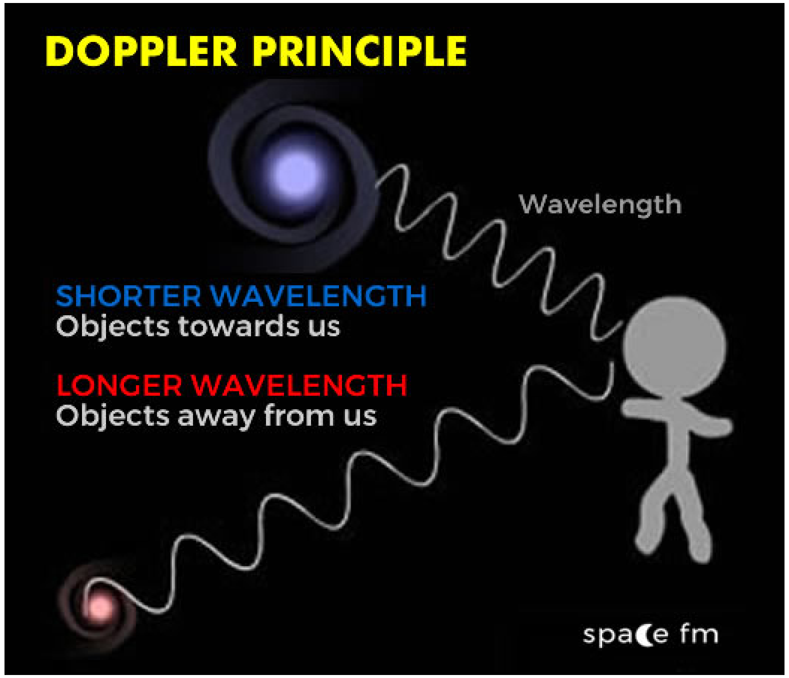


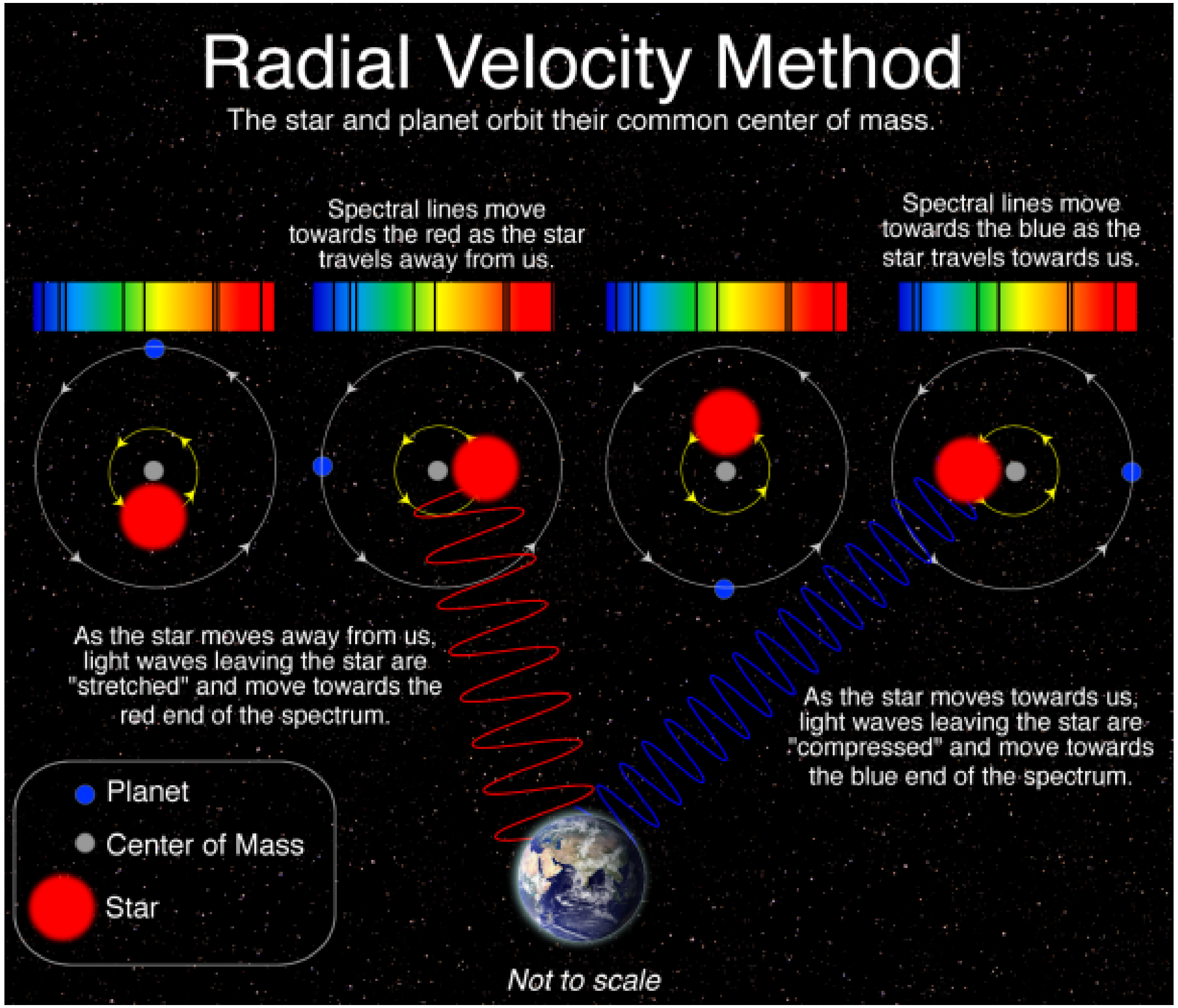
- Relativistic Doppler Shift



- EM radiation is also subjected to Doppler Shift

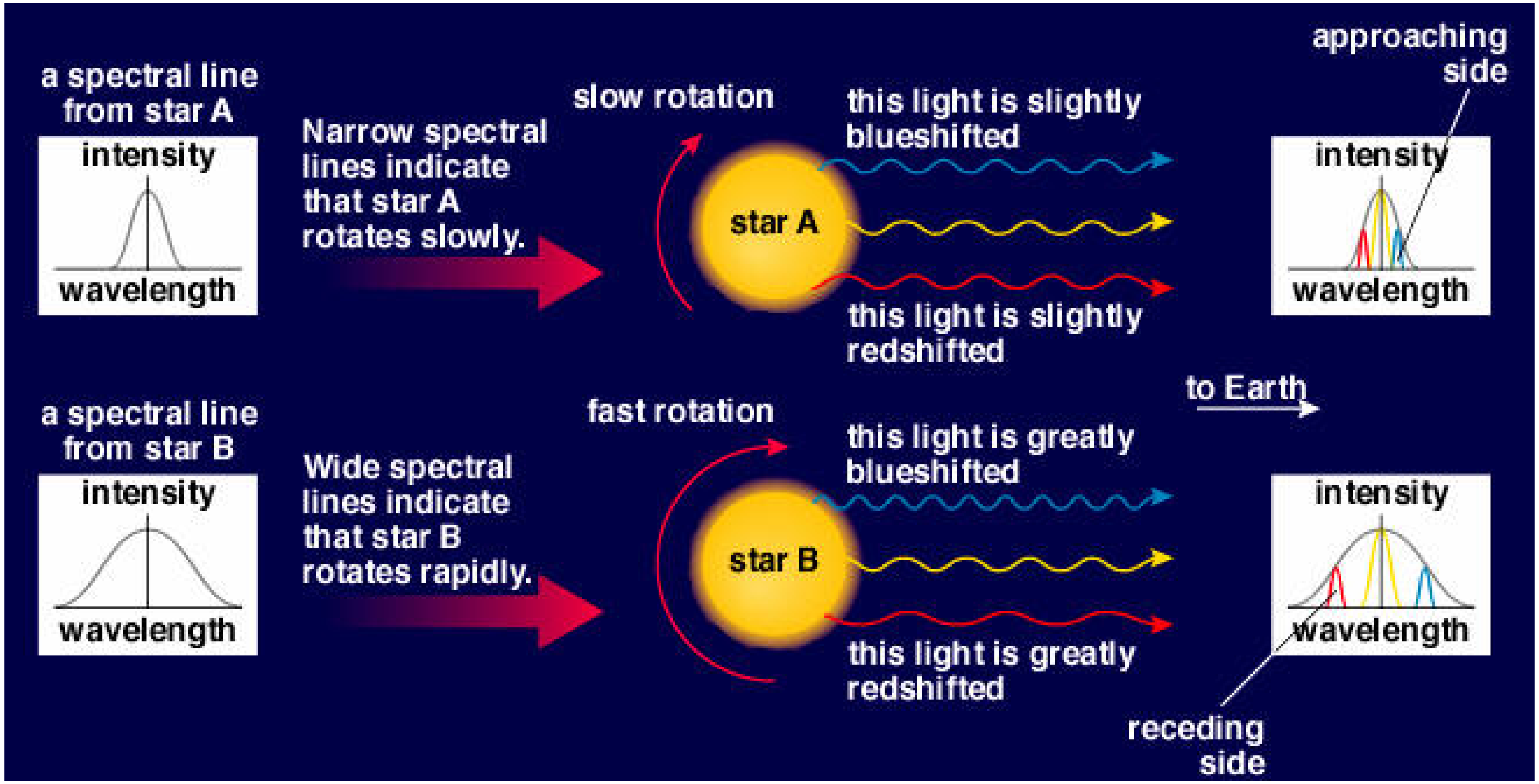





Radial Velocity Method
The star and planet orbit their common center of mass.
Spectral lines move towards the red as the star travels away from us.
Spectral lines move towards the blue as the star travels towards us.
As the star moves away from us, light waves leaving the star are "stretched" and move towards the red end of the spectrum.
As the star moves towards us, light waves leaving the star are "compressed" and move towards the blue end of the spectrum.
Planet
Center of Mass
Star
Not to scale

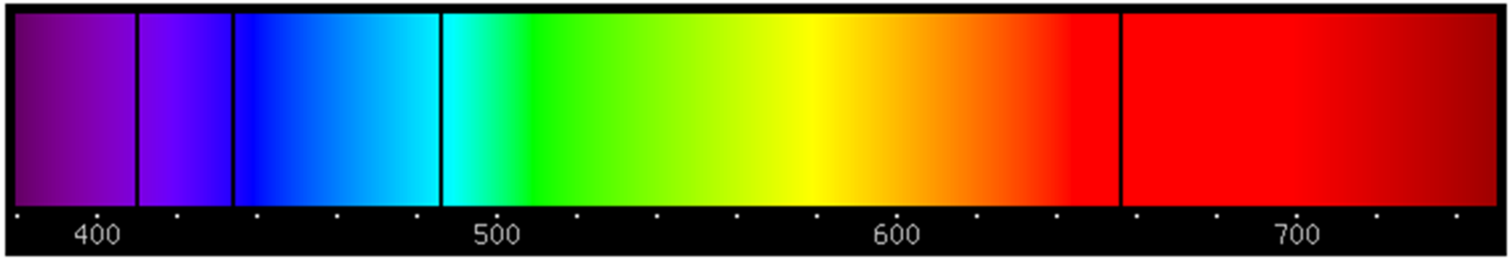
[Star wobble]

- Rotation of a Star



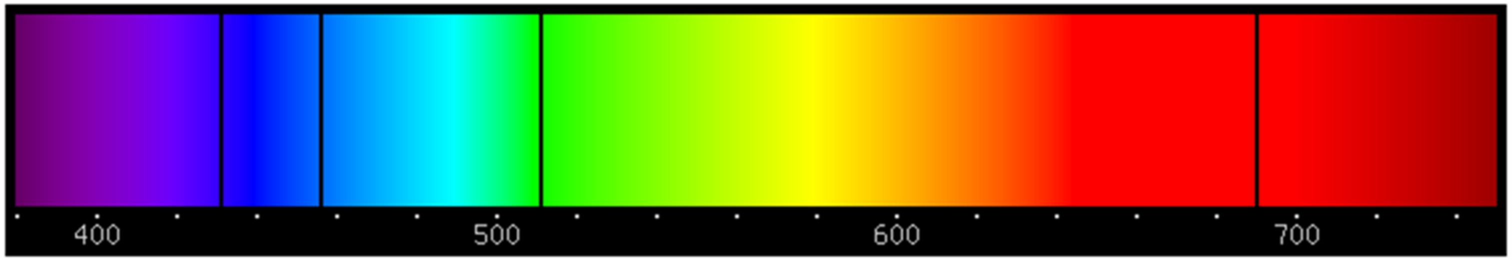
- Velocity (Radial) of a Star

434.0 nm 486.1 nm 656.3 nm



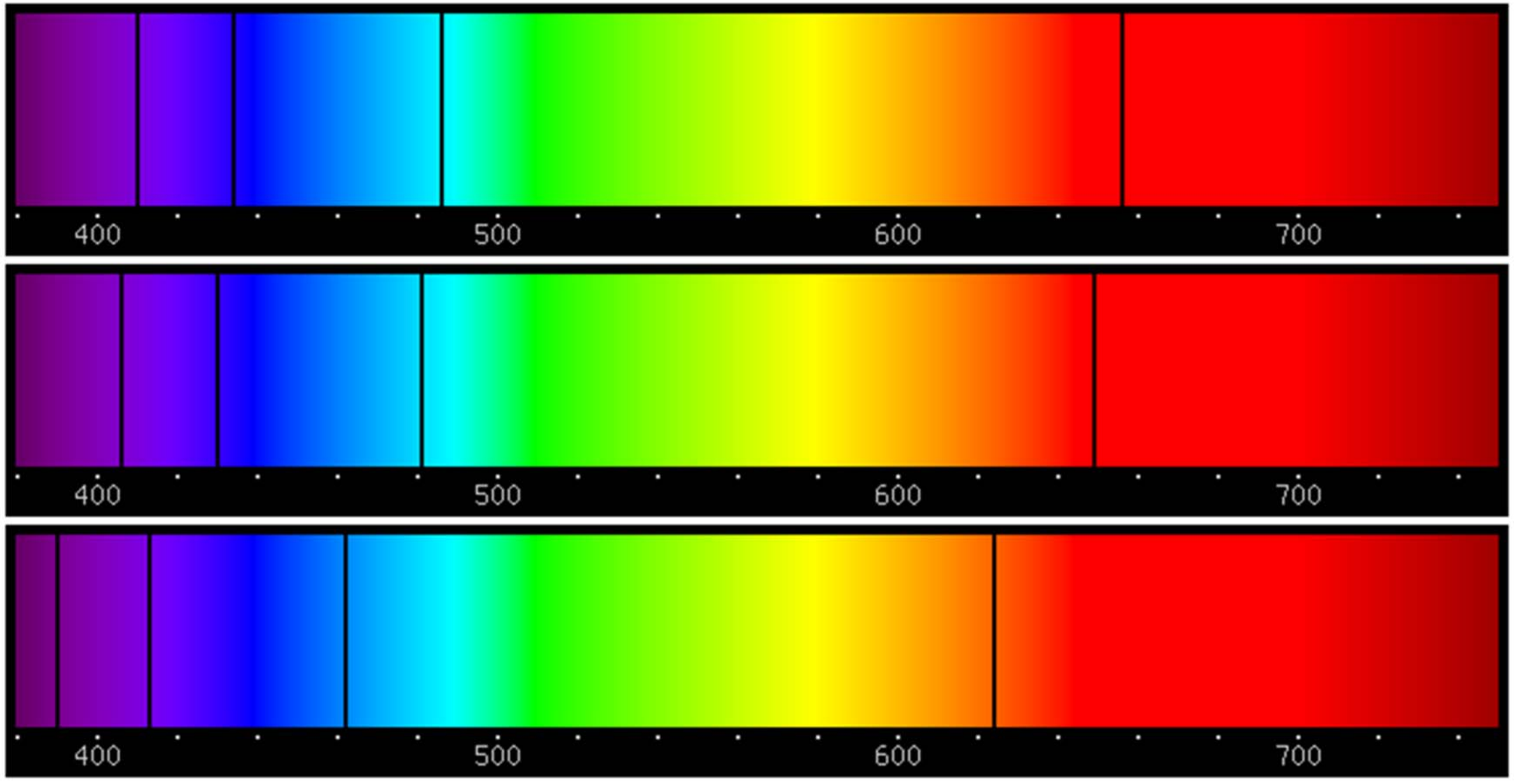
Velocity (Radial) of a Star →

657.3 nm



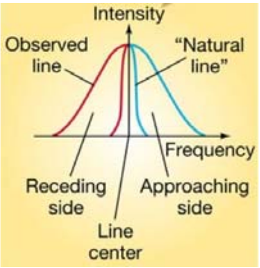
$$v = \frac{\Delta\lambda}{\lambda_{REST}} \times c = 457.1 \frac{Km}{\sec}$$ Doppler Effect equation

- Blue shifted Hydrogen

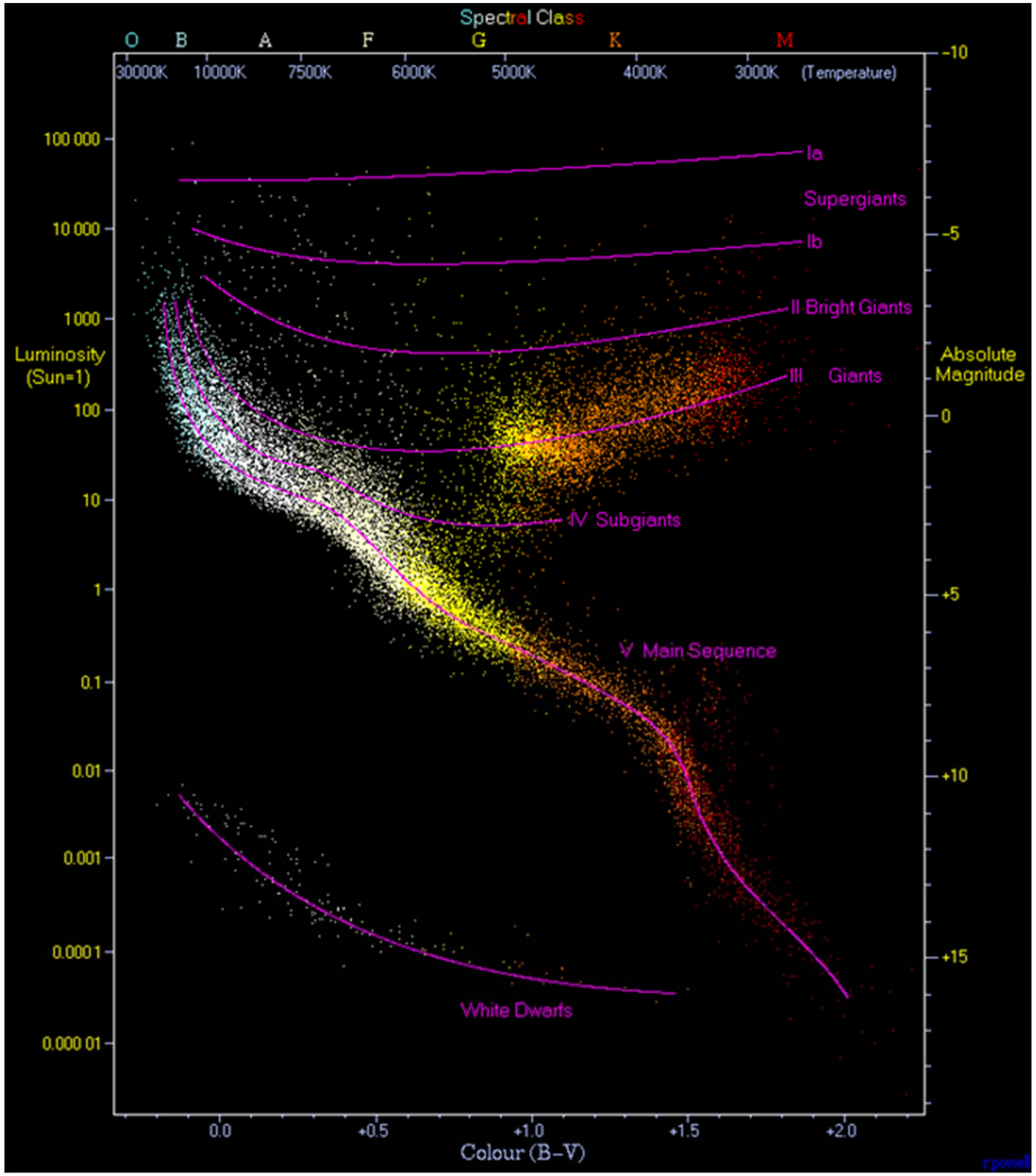


SUMMARY

- **Temperature**: using Wien's law $0.0029/\lambda_0$
- **Velocity and Rotation**: from Doppler shift $v = \dfrac{\Delta\lambda}{\lambda_{REST}} \times c$



- **Size**: from SB Law: $P = e\sigma A T^4$
- **Matter and Amount**: from Emission of Absorption Spectrum
- **Direction of Motion**: Blue shift or Red shift
- **Type of object**: Star → absorption, Nebula → emission, Planet → continuous
- **Color**: Temperature



- ❑ STARS

- **Size**
- **Color**
- **Temperature**
- Distance
- Luminosity
- Brightness
- **Motion**

https://theskylive.com/

- Example of relationship among luminosity, distance, size and the use of the HR diagram

- **Definitions**:
  - <u>Temperature</u>: motion of atoms (higher → more vibration)
  - <u>Heat</u>: energy due to motion of atoms (Joules or Watts)

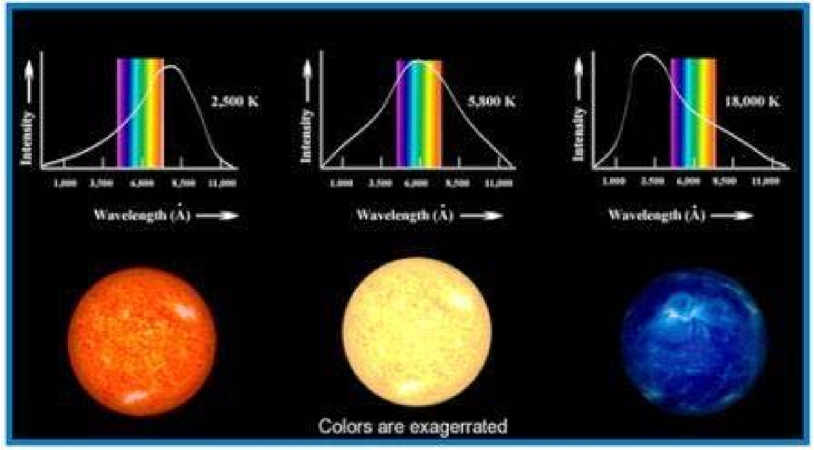
  - A higher temperature T↑ corresponds to a higher energy E↑ and viceversa a lower T↓ corresponds to a lower E↓
  - However the relationship <u>depends on the mass as well</u>

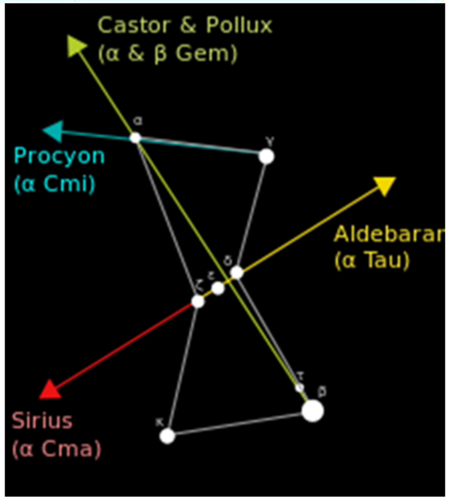
T1↑, E1↑

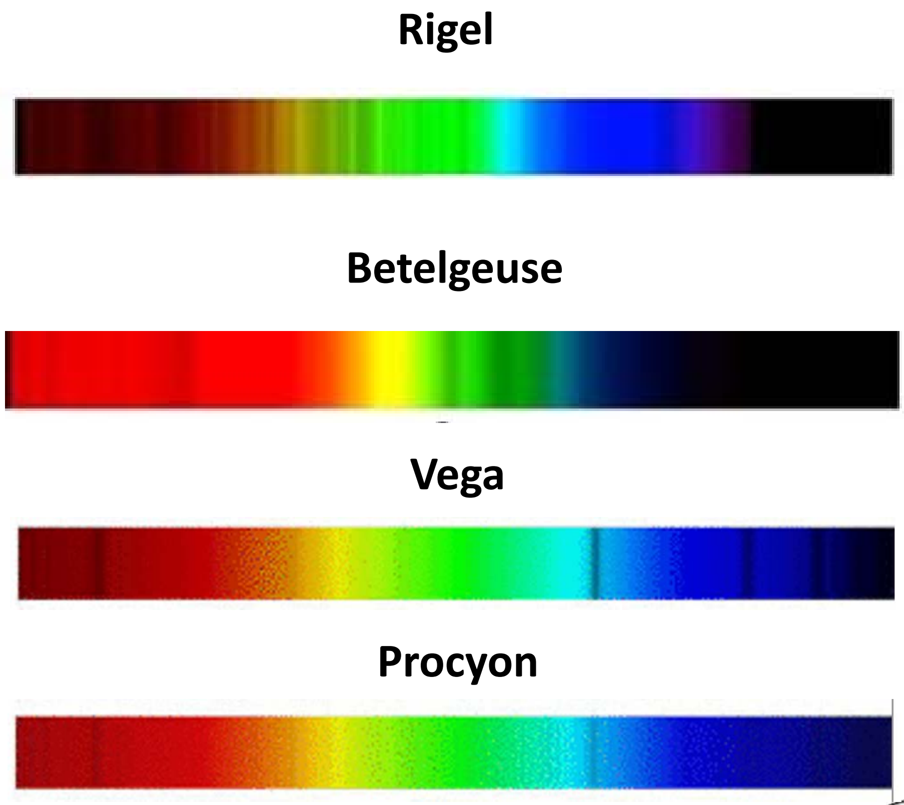
T2↓, E2↓
E1 > E2

T2↓, E3↑
E3 > E1

- The star **temperature** is computed from its color and its spectrum
- The star **luminosity** is the energy emitted by the star measured as power (Watts). It is usually measured with respect to the Sun ($1L_{\odot} = 3.83\times10^{17}$ GW)

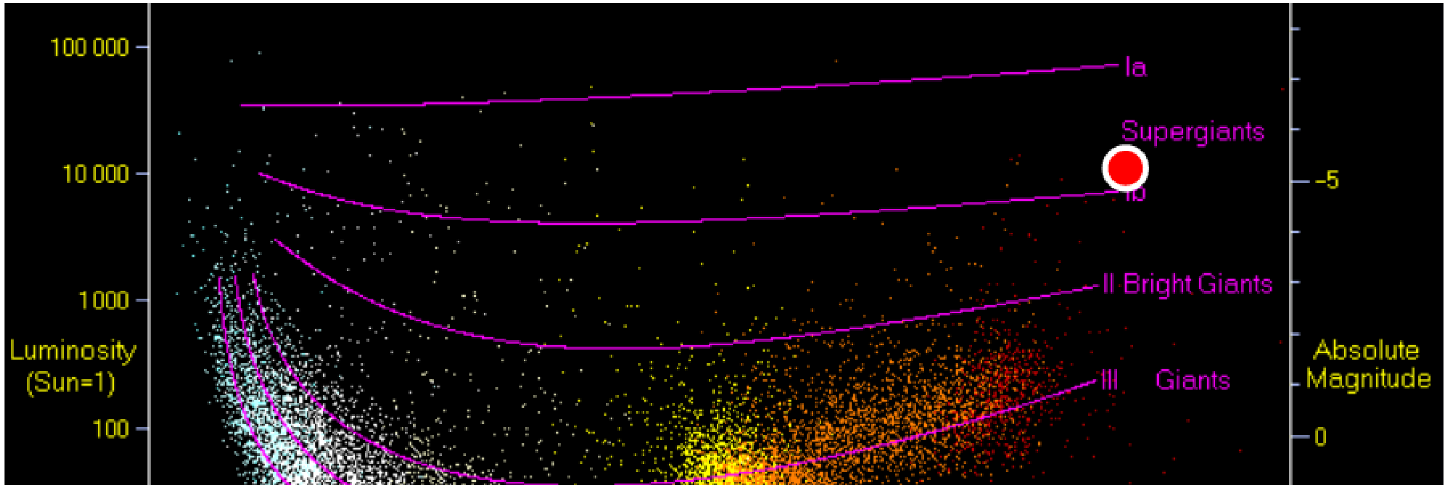




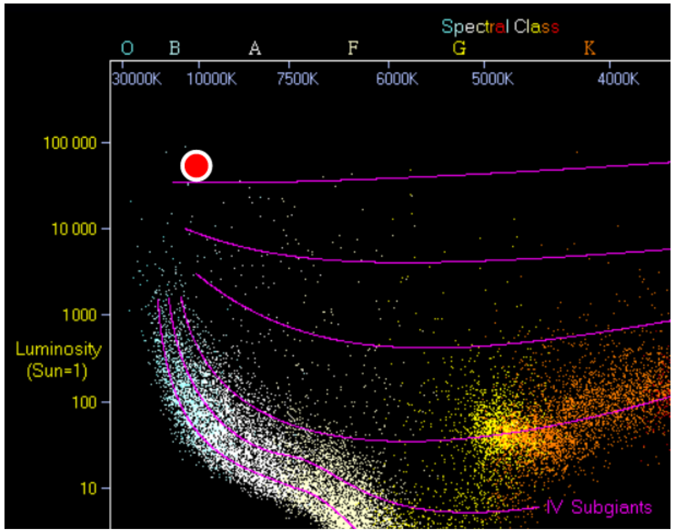


- Procyon (White Yellow Main)
  - Distance: 11.46 LY
  - Mass: 1.5 M⊙
  - Radius: 2.05 R⊙
  - Luminosity: 6.9 L⊙
  - Temp.: 6,500°K
- Betelgeuse (Red SG)
  - Distance: 650 LY
  - Mass: 15 M⊙
  - Radius: 887 R⊙
  - Luminosity: 1.2×10^5 L⊙
  - Temp.: 3,300°K
- Sirius (white Blue Main + white dwarf)
  - Distance: 8.6 LY
  - Mass: 2.06 M⊙
  - Radius: 1.7 R⊙
  - Luminosity: 25,4 L⊙
  - Temp.: 9,940°K

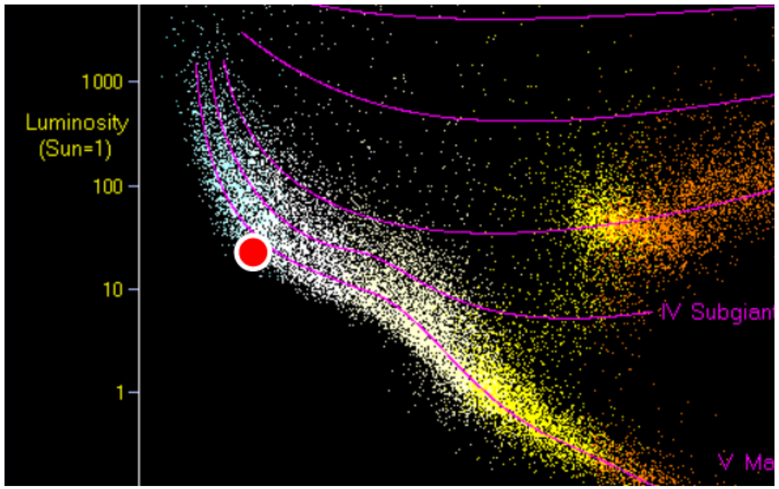
Betelgeuse (Red SG)

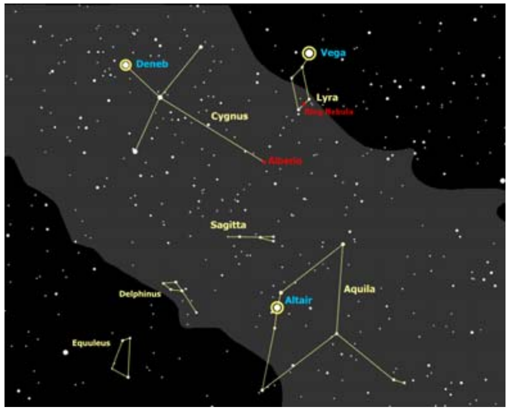


Rigel (Variable White Blue SG)

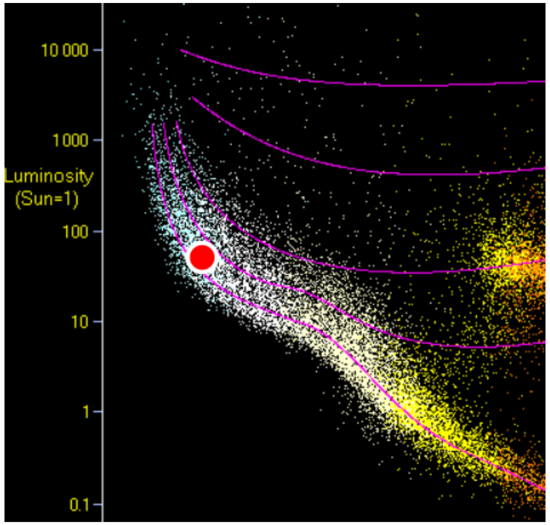


Sirius (Multiple White SG)





- Vega (white Blue Main + white dwarf)
  - Distance: 25.04 LY
  - Mass: 2.135 $M_\odot$
  - Radius: 2.4 $R_\odot$
  - Luminosity: 40 $L_\odot$
  - Temp.: 9,600°K



- Recall:

$$\lambda_{peak} = \frac{0.0029}{T}$$

Wien's law allows the computation of the star temperature from its spectrum and color

$$\frac{dQ}{dt} = P = L = e\sigma A T^4 = \propto A T^4$$

S-B law relates the energy emitted (power or <u>true luminosity</u>) with temperature and size of a star

$$L \propto \frac{1}{D^2}$$

Inverse square law relates luminosity with distance

- Stars Colors:

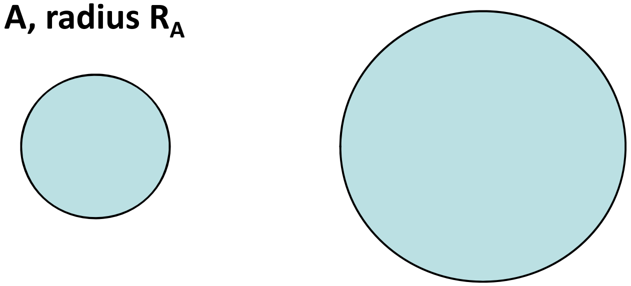
  - Red (coolest)
  - Red - orange
  - Yellow
  - White
  - Blue (hottest)

Color	Example	Surface Temperature (°C)
	Rigel (Orion)	28,000–11,000
	Sirius (Canis Major)	11,000–7,500
	Sun & Capella (Auriga)	6,000–5000
	Aldebaran (Taurus)	5,000–3,600
	Betelgeuse (Orion)	3,600–2,000

- Example:

A, radius R_A

B, radius R_B=2R_A



Area B = 4 Area A → 4 times the energy

$$A = 4\pi R^2$$

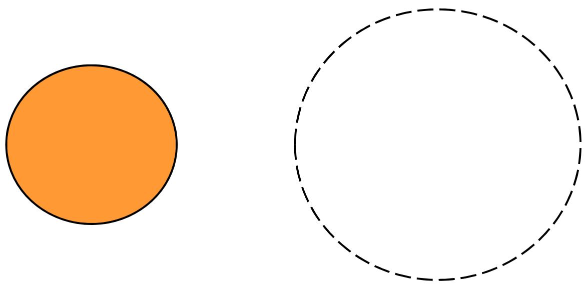
T_O = 5,000 K

T_W = 10,000 K

Note: not possible in reality since hotter stars are larger

$$\frac{T_W}{T_O} = \frac{10,000}{5,000} = 2 \Rightarrow (2)^4 = 16$$

The white star emits 16 times the energy based on the temperature ratio



Assume the white star having double the radius of the orange star

$$L_W = 16 \cdot 4L_O = 48L_O$$

- The previous computation allows finding the size of a star

T_R = 3,000 K T_Y = 6,000 K

- If the have the same size, due to temperature difference L_Y= 16 L_R
- We know that yellow star is bigger. How much bigger? Assume we know the relative luminosity: L_Y = 48 L_R

The difference in luminosity <u>due to size</u> is then 48/16 = 3

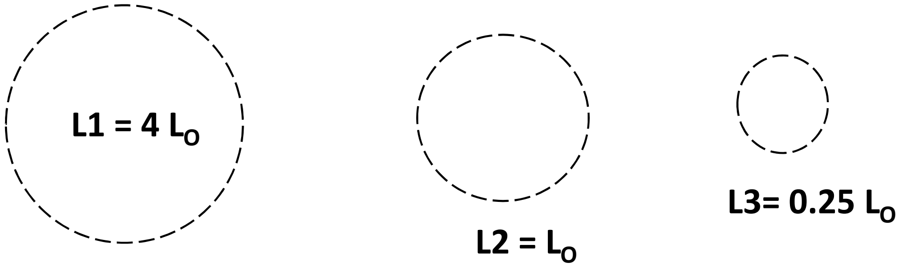
$$L_Y^{SIZE} = 3L_R^{SIZE} \Rightarrow 4\pi R_Y^2 = 3 \cdot 4\pi R_R^2 \Rightarrow R_Y = \sqrt{3}R_R \approx 1.7R_R$$

- **Question**: How do we know the luminosity if we do not know the distance?

- **Answer**: we can use the inverse square law

- The previous computation allows finding the relative distance between stars

- Consider 3 white stars with the same size and temperature:



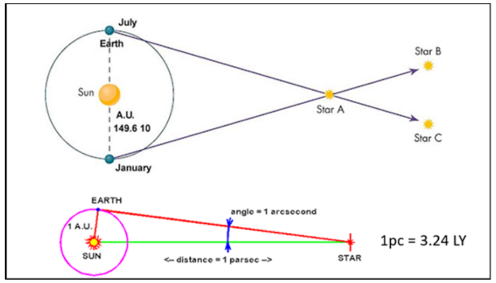
$$L \propto \frac{1}{D^2} \Leftrightarrow D \propto \frac{1}{\sqrt{L}}$$

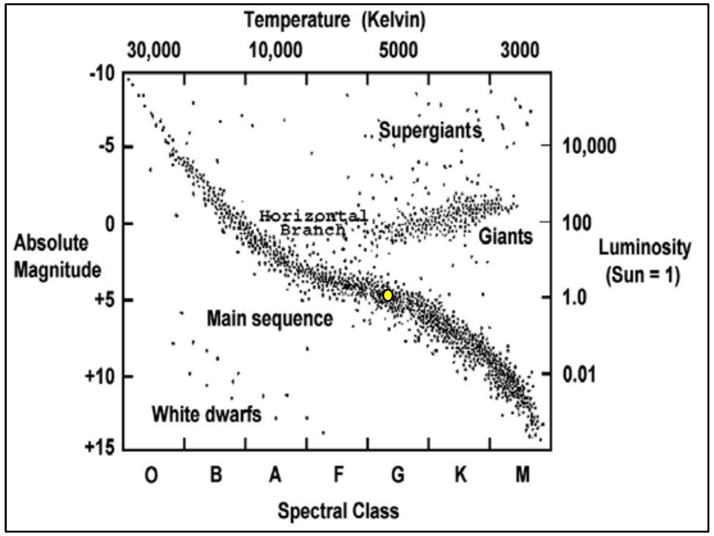
- If they have same temperature and same size, then their distance must be different !

$$D_1 = \frac{1}{2} D_2; D_2 = \frac{1}{2} D_3; D_1 = \frac{1}{4} D_3$$

- We know now:
  - Actual temperature
  - Relative size
  - Relative distance

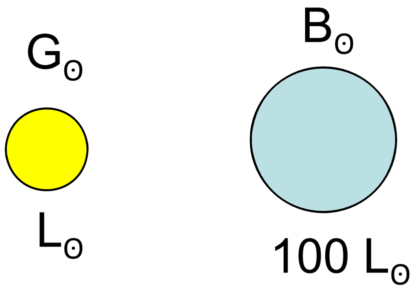
- We need:
  - Absolute size or
  - Absolute distance

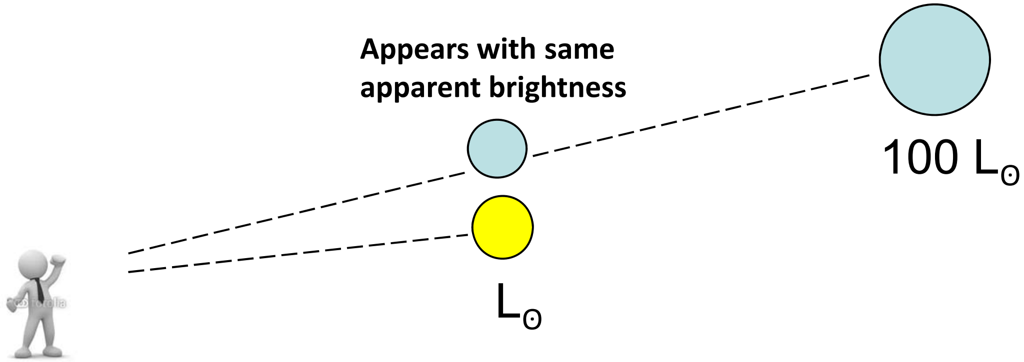




- The Hertzsprung–Russell diagram (≈ 1910) is a scatter plot of stars showing the relationship between the stars' absolute magnitudes or luminosities versus their stellar classifications or effective temperatures.

- Stars were populated using parallax measure or Cepheid variables measures.





- From HR diagram, knowing temperature and true luminosity, the B star is farther away

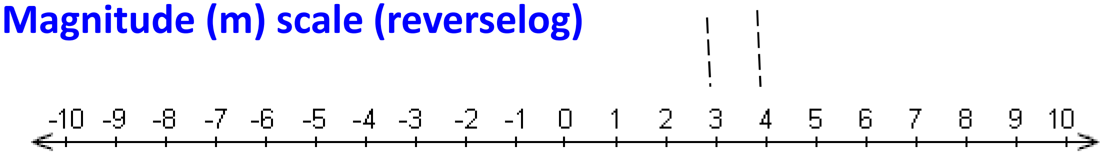
- From inverse square law we have: $D_B = 10\ D_G$

- We have the relative distance, but still we need the absolute distance → <u>absolute Magnitude (brightness) and Apparent Magnitude (brightness, luminosity)</u>

  - **The apparent magnitude (m)** is the brightness of an object as it appears in the night sky from Earth. Apparent magnitude depends on an object's intrinsic luminosity, its distance, and the extinction reducing its brightness.
  - **The absolute magnitude (M)** describes the intrinsic luminosity emitted by an object and is defined to be equal to the apparent magnitude that the object would have if it were placed at a certain distance from Earth, <u>10 parsecs for stars (32.6 LY)</u>.
  - **Parsec** is the distance covered by 1 arcsecond

$\sqrt[5]{100} \simeq 2.512$ **Brightness difference**

- **Apparent Magnitude (m) scale (reverselog)**



$$m_1 - m_{REF} = -2.5\log_{10}\left(\frac{b_1}{b_{REF}}\right)$$

$$\frac{b_1^{-2}}{b_2^{+3}} = 2.512^{(\Delta m)} = 2.512^{(5)} = 100$$

Sun: -26.7 1 AU
Moon: -12.6 2.57x10^{-3} AU
Venus: -4.4
Sirius: -1.4 8.6 LY

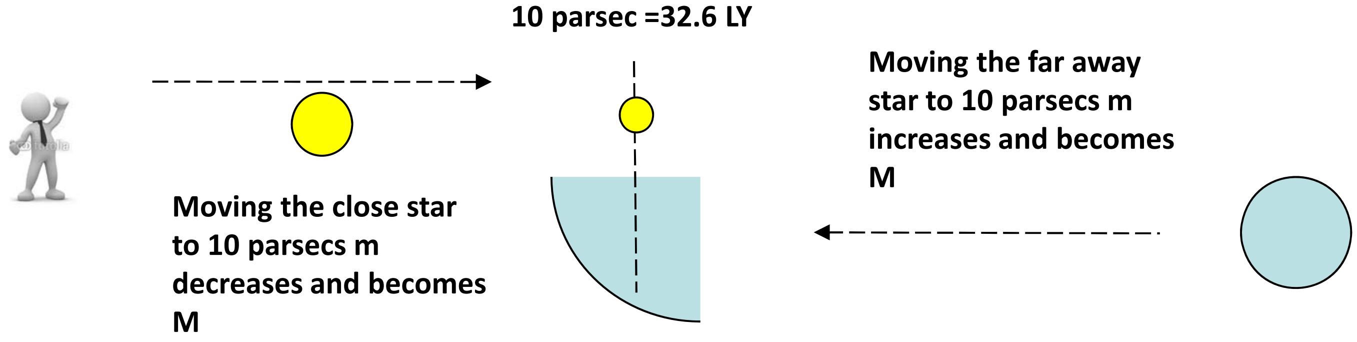
Arcturus: -1.4 37 LY
Canopus: -0.72 310 LY
Vega: +0.04 25.6 LY
Rigel: +0.18 860 LY

Procyon : +0.34 11.46 LY
Betelgeuse: 0.42 650 LY
Eye Limit: +6
Large Telescope limit: +21
Hubble telescope limit: +30

Absolute Magnitude (M) scale (log)

$$m - M = 5(\log_{10} pc - 1)$$

- Absolute Magnitude is found relative to apparent magnitude



Sun: +4.83 M
Betelgeuse: -5.85 M
Rigel: -7.84 M
Vega: +0.6 M
Procyon: +2.68 M
Sirius: +1.47 M
1A supernova: -19 MX

$$\frac{b_1^{SIRIUS}}{b_2^{SUN}} = 2.512^{(\Delta M)} = 2.512^{(3.36)} = 22$$

Sirius is 22 times brighter than the Sun

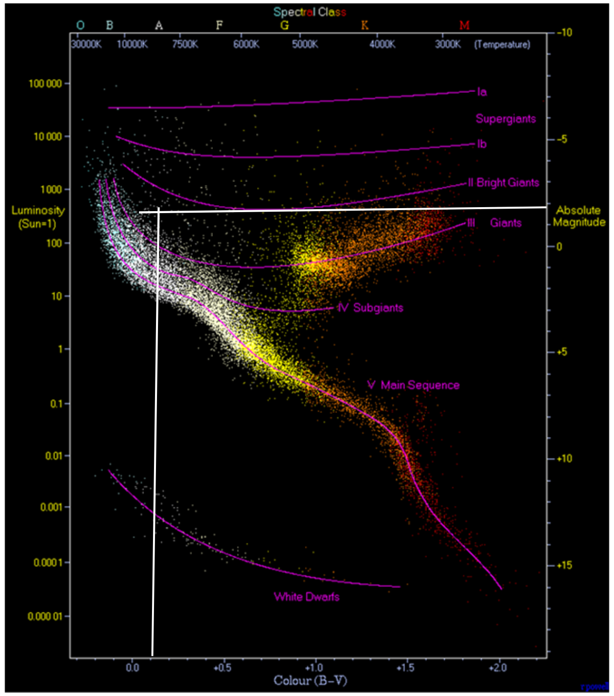
$$\frac{b_1^{SUPNOV}}{b_2^{SUN}} = 2.512^{(\Delta M)} = 2.512^{(23.83)} = 3.4 \cdot 10^9$$

Supernova explosion is 3.4 billion times brighter than the Sun

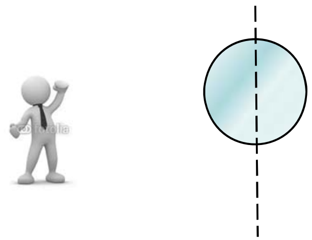
- Distance using apparent and absolute magnitudes

- **Example**: consider a star with m = +8.0 (measured with telescope which gives brightness and color)

- From HR diagram find M knowing the color as an example -2.0



10 parsec =32.6 LY



M = -2.0

m = +8.0

The star looks much dimmer to an Earth viewer

- Use the following relationships:

$$D = 10pc\sqrt{\Delta I}; D > 10pc \qquad D = \frac{10pc}{\sqrt{\Delta I}}; D < 10pc$$

$$\Delta M = m - M = 10 \Rightarrow \Delta I = 2.512^{\Delta M} = 10,000$$

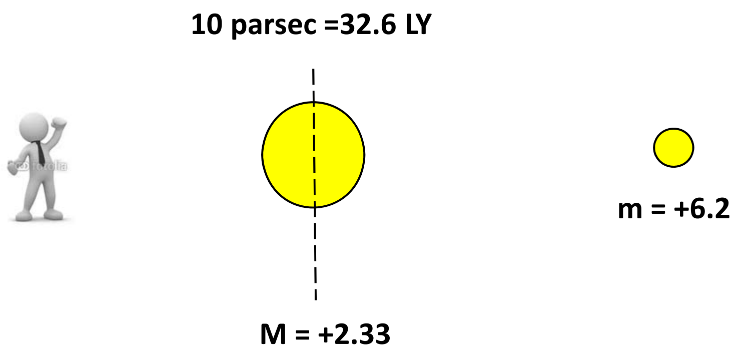
$$\color{red}{D} = 10pc\sqrt{10,000} = 1,000pc = \color{red}{\mathit{3,260LY}}$$

- **Example**: a star has luminosity 10 $L_\odot$ and an apparent magnitude of +6.2. How far is the star?

- First compute the absolute magnitude of the star knowing that $M_\odot$ = +4.83

$$\Delta I = 2.512^{\Delta M}$$

$$10 = 2.512^{\Delta M} \Rightarrow \log(10) = \Delta M \log(2.512) \Rightarrow \Delta M = 2.5 \Rightarrow M = +4.83 - 2.5 = +2.33$$

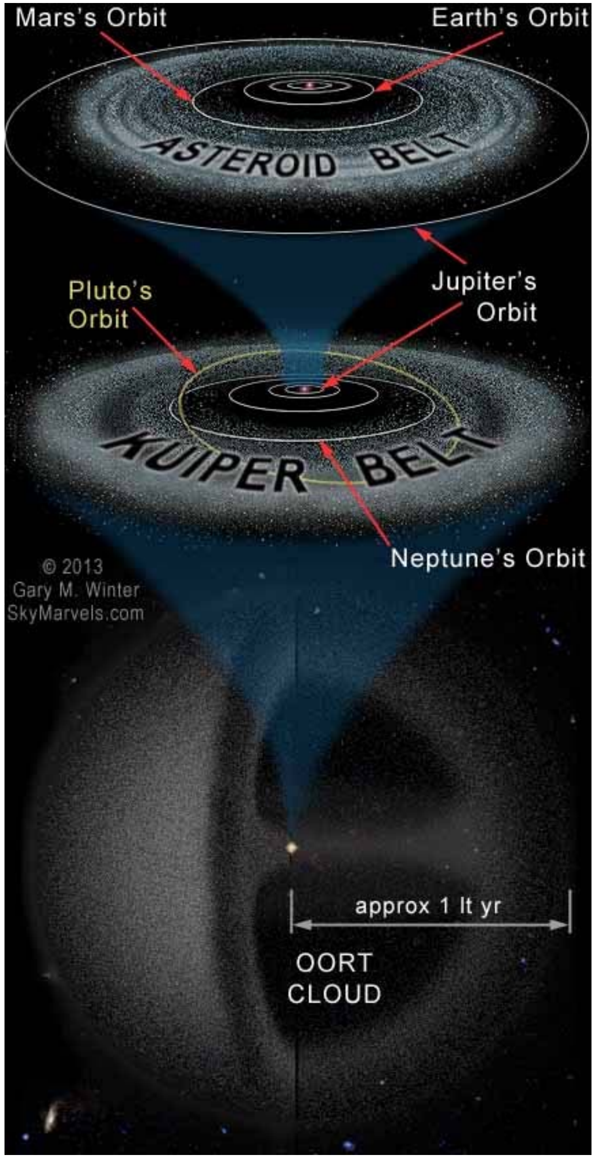


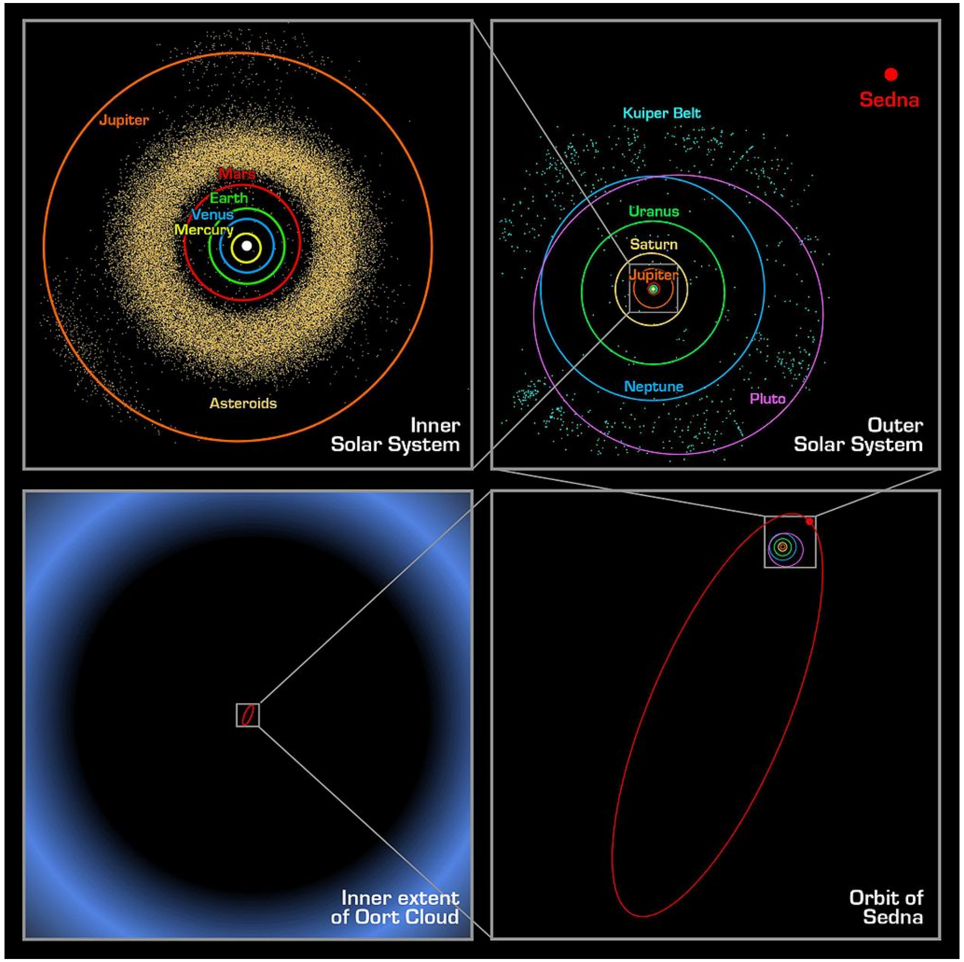
$$\Delta I = 2.512^{\Delta M} = 2.512^{(6.2-2.33)} = 35.3$$

$$D = 10pc\sqrt{35.3} = 59.4pc = \boldsymbol{193.6LY}$$

Star Life Cycle

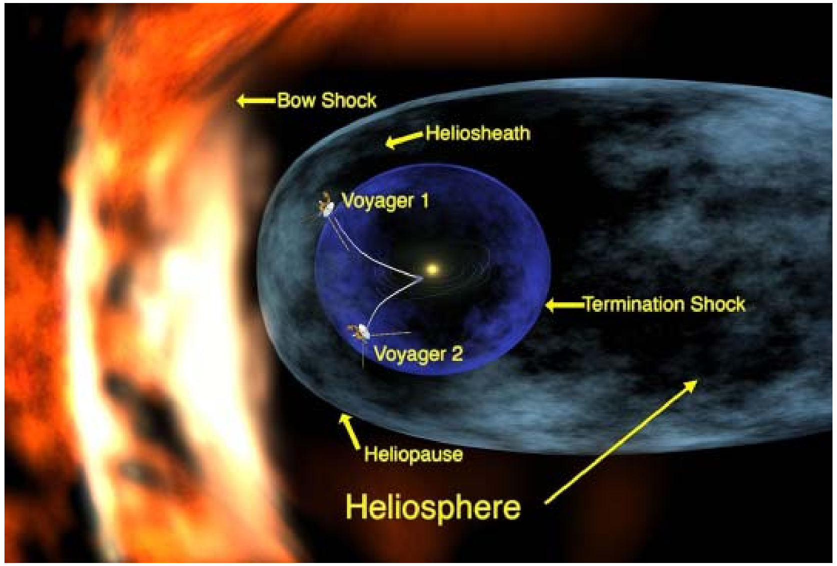
Boundaries of Solar System

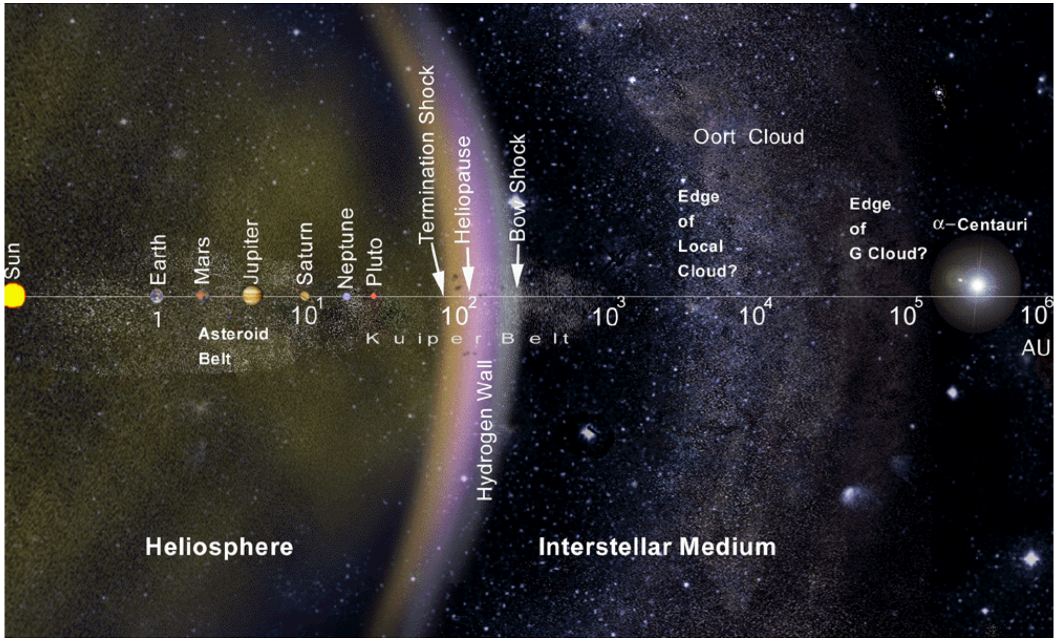




The Oort cloud is a theoretical cloud of predominantly icy planetesimals believed to surround the Sun to as far as somewhere between 50,000 and 200,000 AU (up to about 3.2 LY)

Boundaries of Solar System





- 1 LY =63240 AU

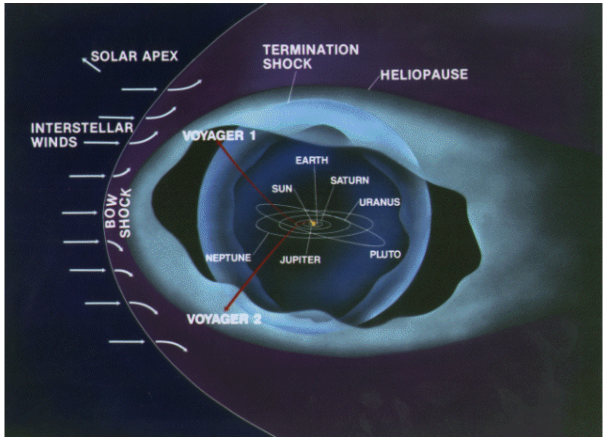
The solar wind is a stream of charged particles released from the upper atmosphere of the Sun, called the corona. This plasma consists of mostly electrons, protons and alpha particles.
The interstellar medium is the matter and radiation that exists in the space between the star systems in a galaxy. This matter includes gas in ionic, atomic, and molecular form, as well as dust and cosmic rays.

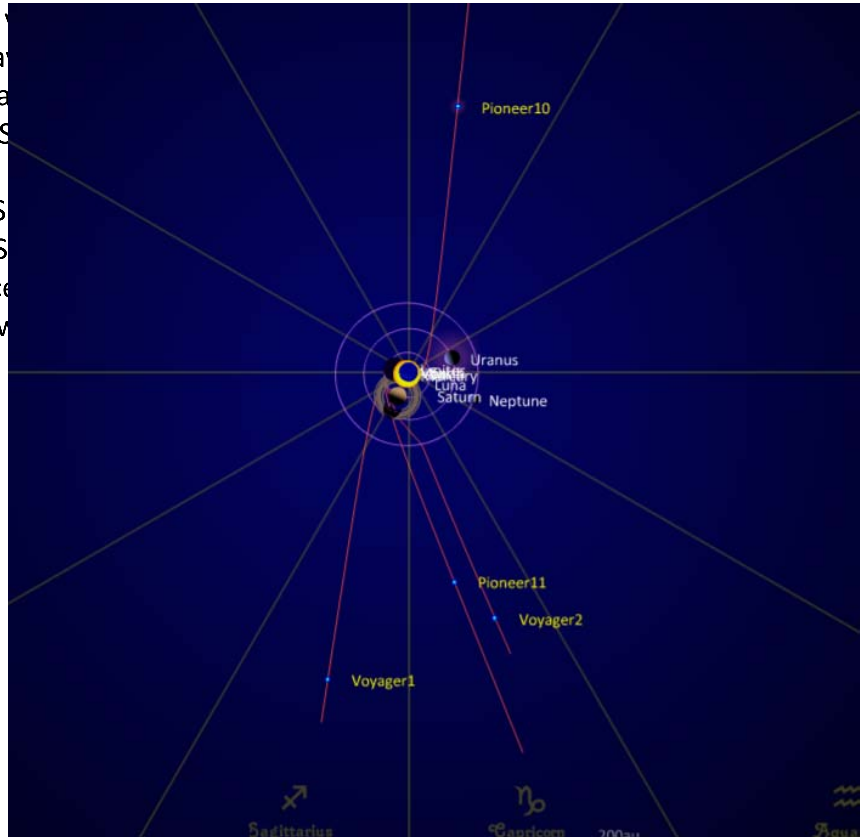
- **Heliosphere**: Volume where the solar wind density is higher than the interstellar wind density.
- **Heliopause**: Border at which the solar wind is stopped by the interstellar medium. Estimated size 120 AU
- **Termination Shock**: Border at which the solar wind becomes subsonic. Estimated size 75 – 90 AU
- **Heliosheath**: Area between Termination Shock and Heliosphere. Solar wind motion becomes compressed and turbulent, due to interaction with interstellar wind. Estimated size 80 – 100 AU

Boundaries of Solar System

The twin Voyager 1 and 2 spacecraft are exploring where nothing from Earth has flown before. Continuing on their more-than-37-year journey since their 1977 launches, they each are much farther away from Earth and the sun than Pluto. In August 2012, Voyager 1 made the historic entry into interstellar space, the region between stars, filled with material ejected by the death of nearby stars millions of
when Voyager 2, in the “heliosheath" -- the outermost la
pressure of interstellar medium -- also reaches interstella
information about their surroundings through the Deep S

The primary mission was the exploration of Jupiter and S
active volcanoes on Jupiter's moon Io and intricacies of S
to explore Uranus and Neptune, and is still the only space
current mission, the Voyager Interstellar Mission (VIM), w
beyond.





2004 Dec. 15	Voyager 1 crosses Termination Shock
2007 Sep. 5	Voyager 2 crosses Termination Shock
2012 Aug. 25	Voyager 1 enters Interstellar Space

Boundaries of Solar System

https://voyager.jpl.nasa.gov/mission/status/

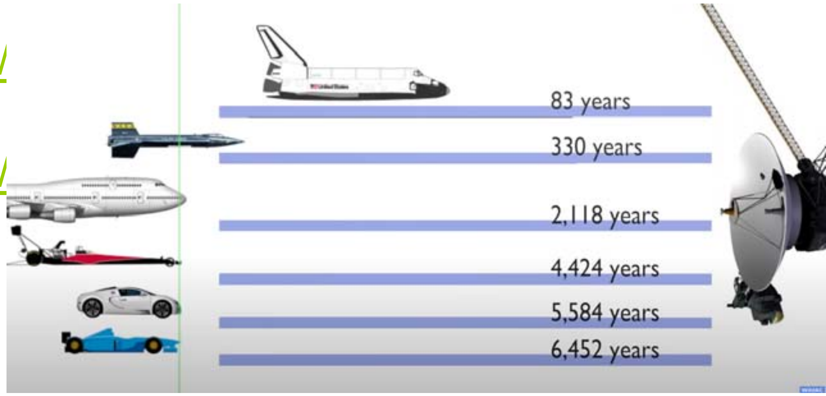
DSN Network

Voyager 1 will reach the Oort cloud in about 300 years and take about 30,000 years to pass through it. Though it is not heading towards any particular star, in about 40,000 years, it will pass within 1.6 light-years of the star Gliese 445, which is at present in the constellation Camelopardalis.

Voyager 2. On November 5, 2018, at a distance of 122 AU (1.83×1010 km) (about 16:58 light-hours)[6] from the Sun,[7] moving at a velocity of 15.341 km/s (55,230 km/h)[8] relative to the Sun, Voyager 2 left the heliosphere, and entered the interstellar medium (ISM), a region of outer space beyond the influence of the Solar System. Voyager 2 is not headed toward any particular star, although in roughly 40,000 years it should pass 1.7 light-years from the star Ross 248. And if undisturbed for 296,000 years, Voyager 2 should pass by the star Sirius at a distance of 4.3 light-years.

https:// Pg

https:// _Fzc

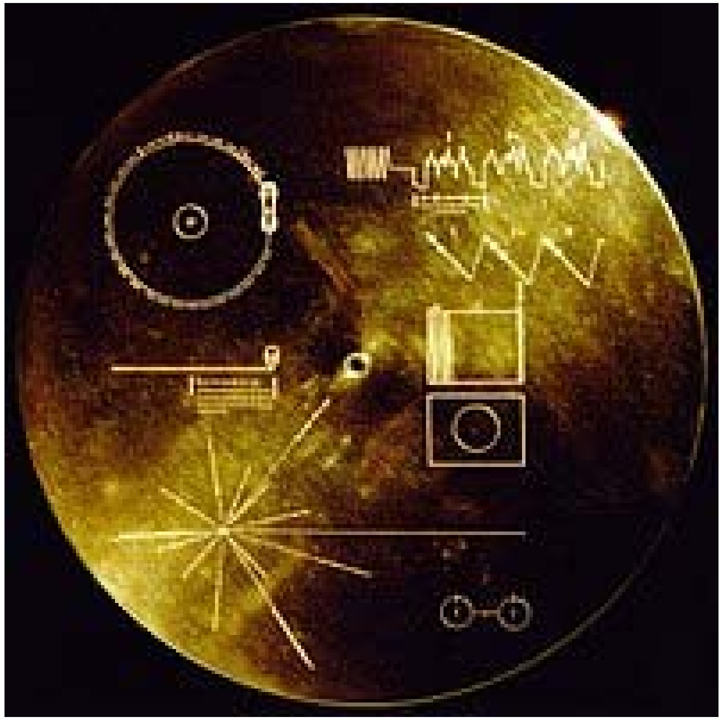


- Pictures taken from Voyager 1, in 1990 by request from Carl Sagan (1934 – 1996), Cornell University

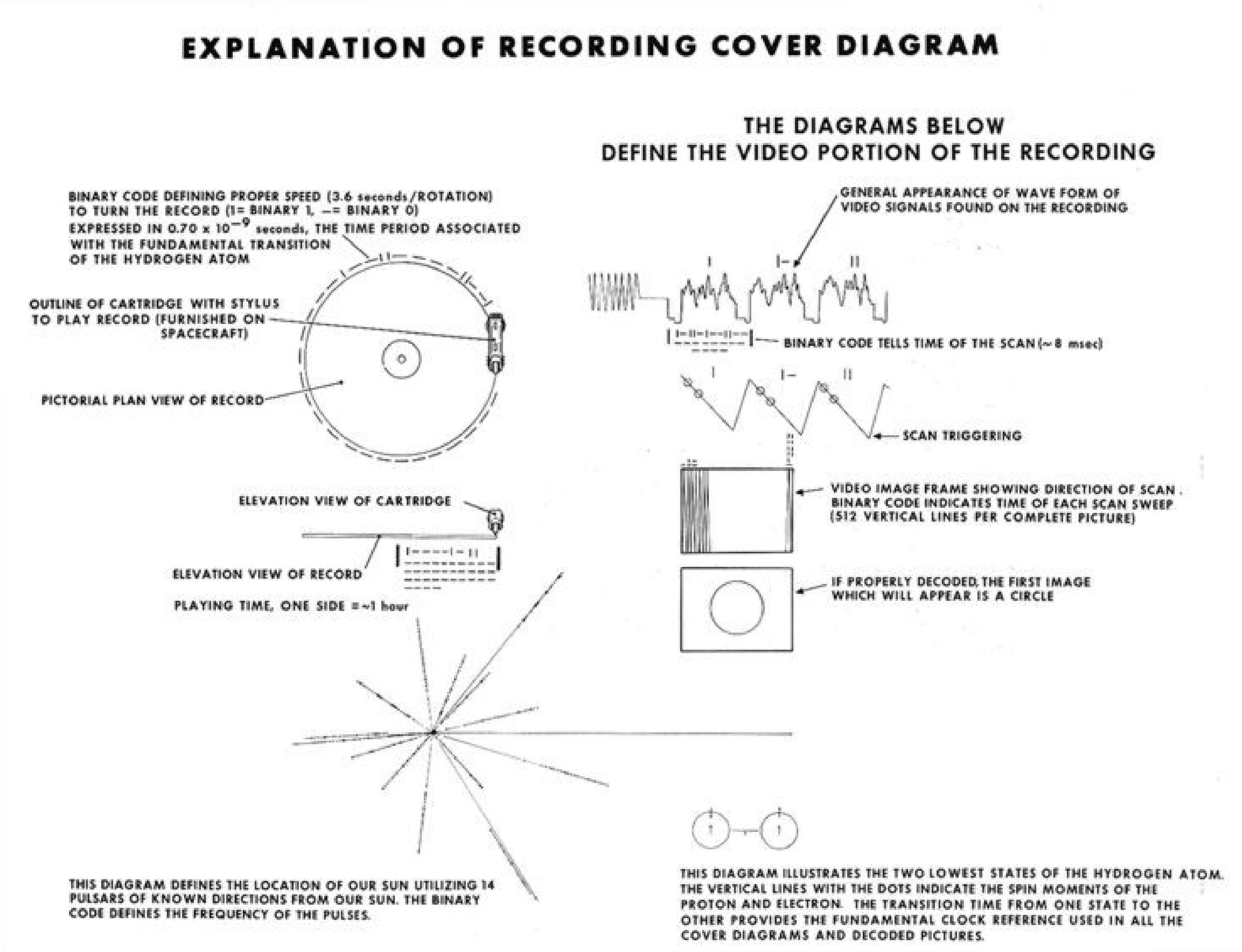
VGER 1 Orientation 1990

Carl Sagan's Comments

https://voyager.jpl.nasa.gov/golden-record/

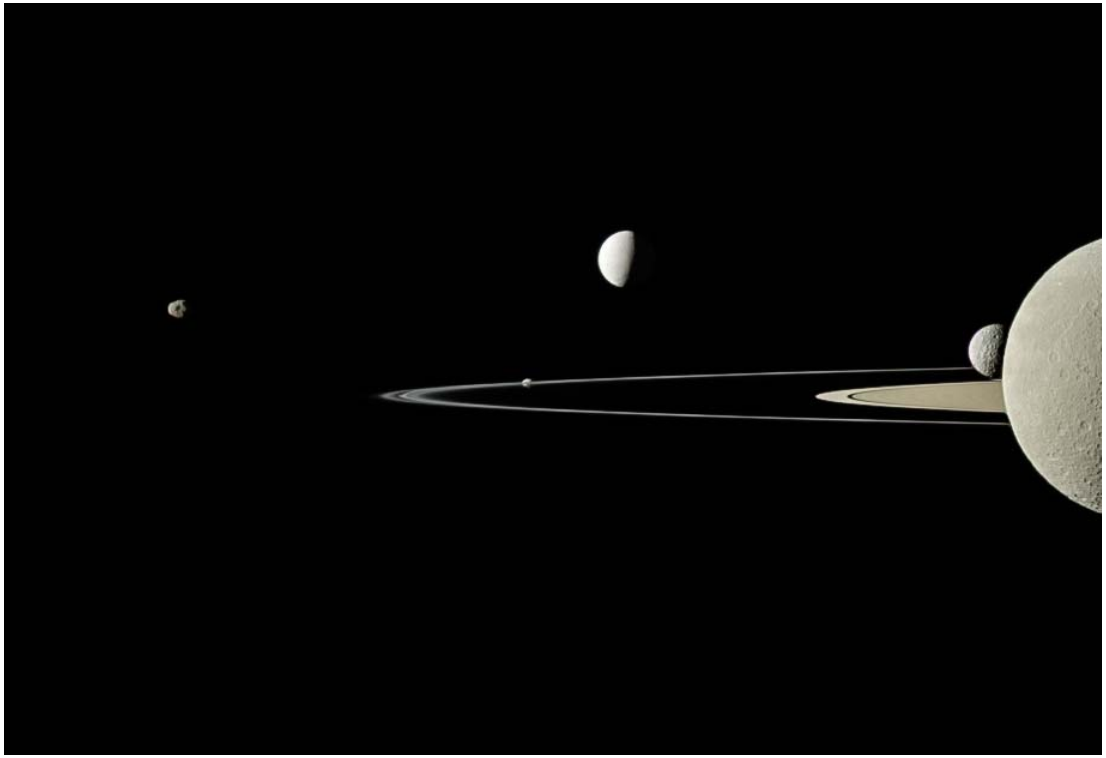


Boundaries of Solar System



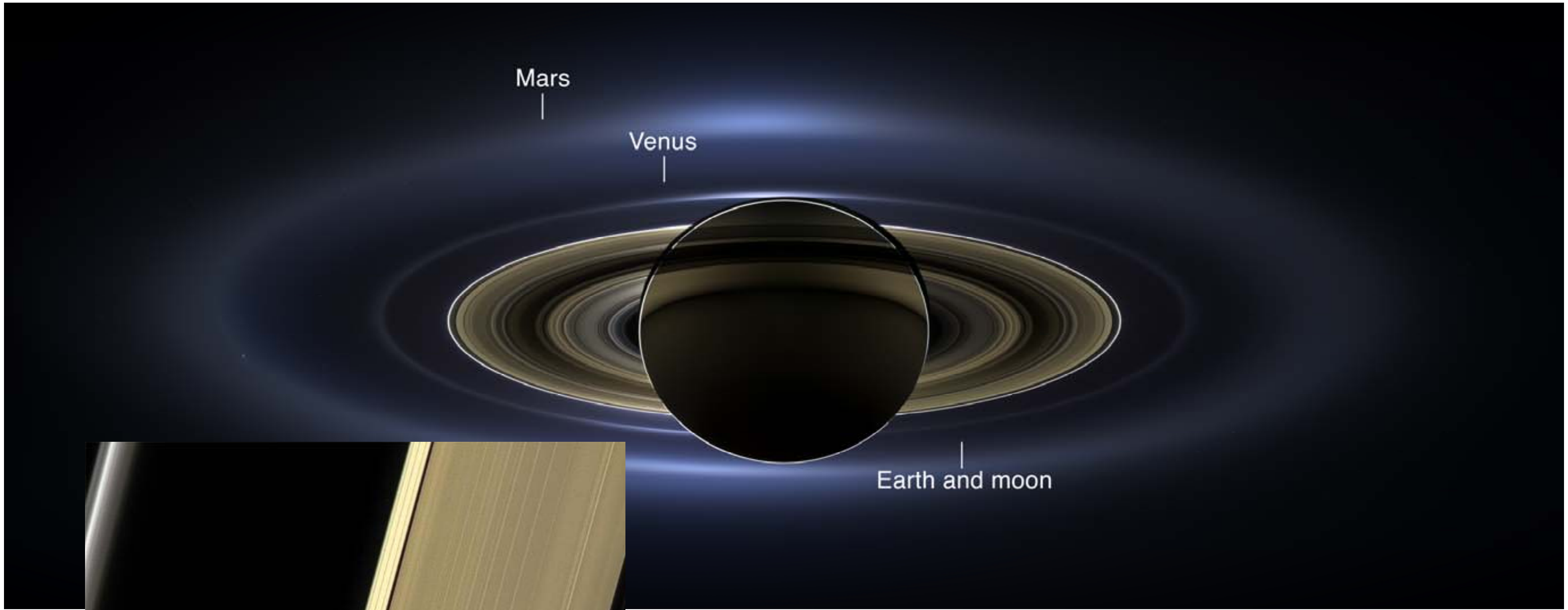
https://solarsystem.nasa.gov/resources/856/earth-from-afar/

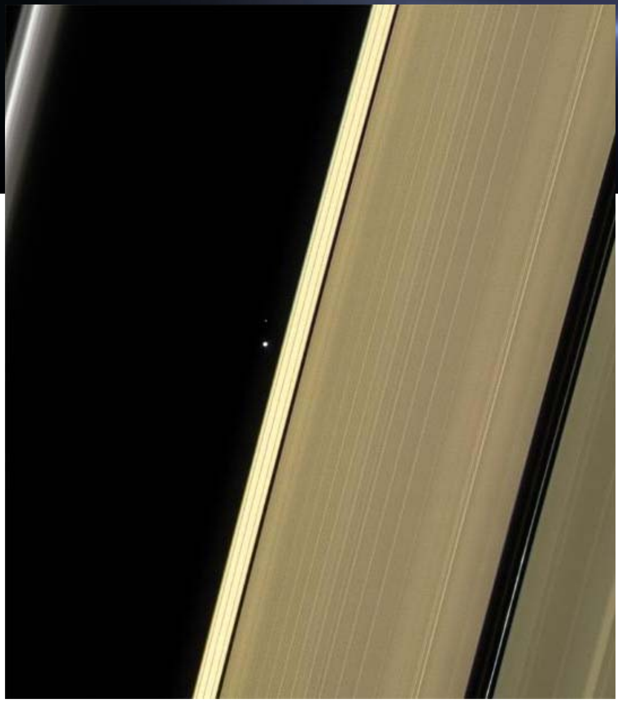
https://solarsystem.nasa.gov/resources/17841/grand-finale-orbits-begin-and-end-with-titan/



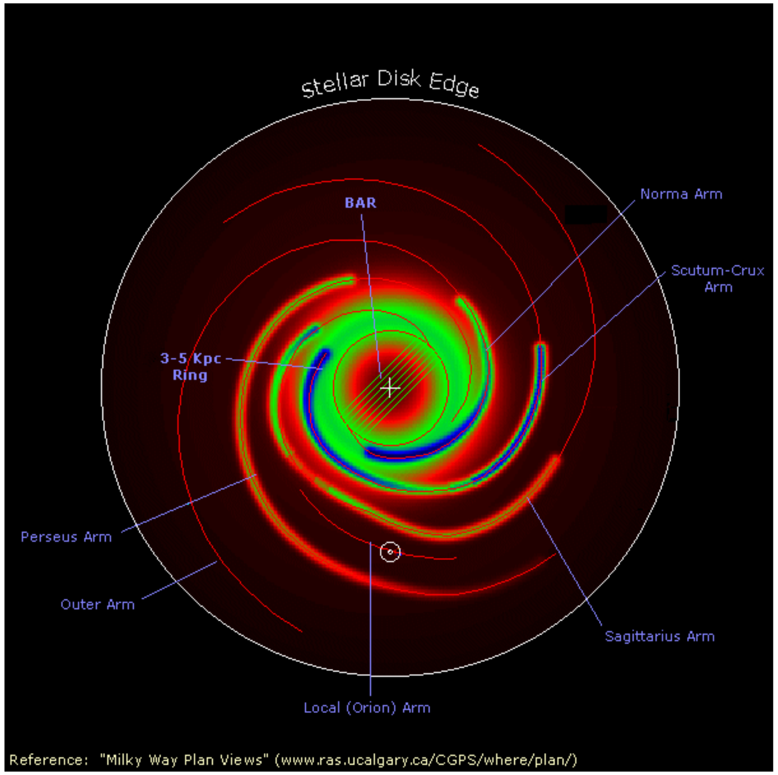
Moons visible in this view: **Janus** (111 miles, or 179 kilometers across) is on the far left; **Pandora** (50 miles, or 81 kilometers across) orbits just beyond the thin F ring near the center of the image; brightly reflective **Enceladu**s (313 miles, or 504 kilometers across) appears above center; Saturn's second largest moon, **Rhea** (949 miles, or 1,528 kilometers across), is bisected by the right edge of the image; and the smaller moon **Mimas** (246 miles, or 396 kilometers across) is seen just to the left of Rhea.

Boundaries of Solar System

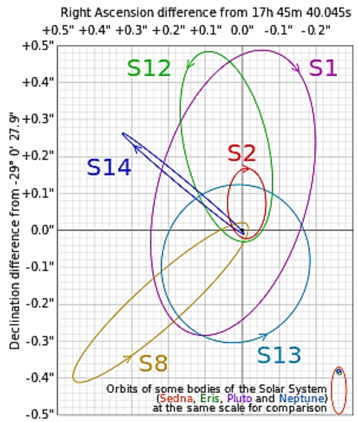




Location of Solar System



Sagittarius A* is a bright and very compact astronomical radio source at the center of the Milky Way, near the border of the constellations Sagittarius and Scorpius. Sagittarius A* is thought to be the location of a supermassive black hole (1974)



$$3.6^{+0.2}_{-0.4} \times 10^{6}\, M_{\odot}$$

- Milky Way Size about 120,000 – 180,000 LY
- Milky Way moves about 600 Km/sec
- Milky Way rotates counterclockwise with speed not following Newton's Law Distance

The Sun is 26,000–28,000 LY from the Galactic Center and 16–98 LY from the central plane of the Galaxy. It takes the Solar System about 240 million years to complete one orbit of the Milky Way. The orbital speed of the Solar System about the center of the Milky Way is approximately 220 km/s or 0.073% of the speed of light. The Sun moves through the heliosphere at 84,000 km/h (52,000 mph). The Solar System is headed in the direction of the zodiacal constellation Scorpius.

SAG A*

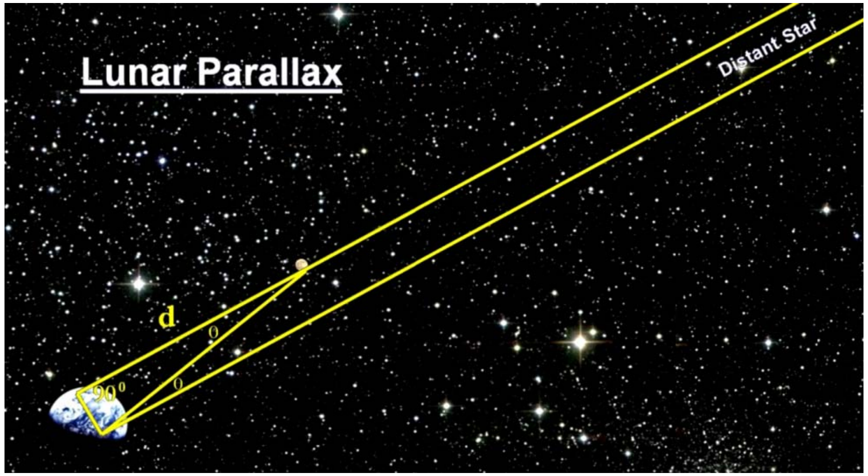
Earth North

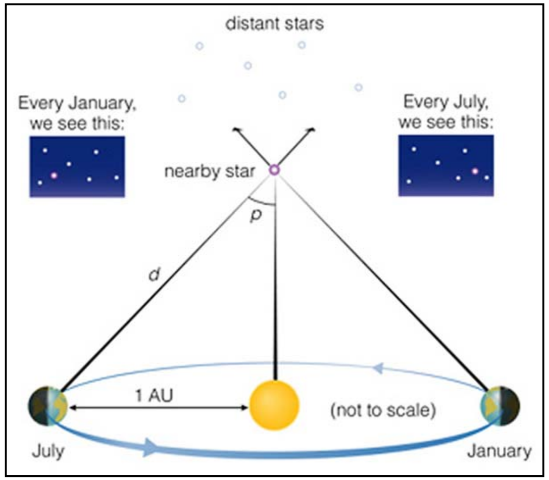
Distance Measurements

- Units of distance vary depending on how far away the object is

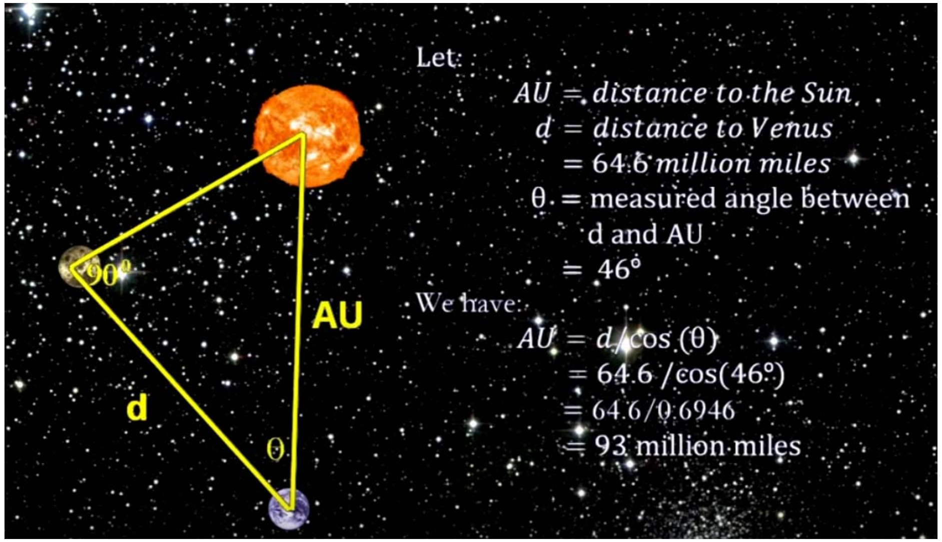
    - **Astronomical Unit (AU):** Average distance between the Earth and the Sun. 1 AU = 1.5×10^6 Km
    - **Light Year (LY):** Distance travelled by the light in vacuum in one Julian year. 1 LY ≈ 9.461×10^{12} Km ≈ 63241.077 AU

- **Parallax:** is a displacement or difference in the apparent position of an object viewed along two different lines of sight, and is measured by the angle or semi-angle of inclination between those two lines. the principle of parallax is used to measure distances to the closer stars. Here, the term "parallax" is the semi-angle of inclination between two sight-lines to the star, as observed when the Earth is on opposite sides of the Sun in its orbit

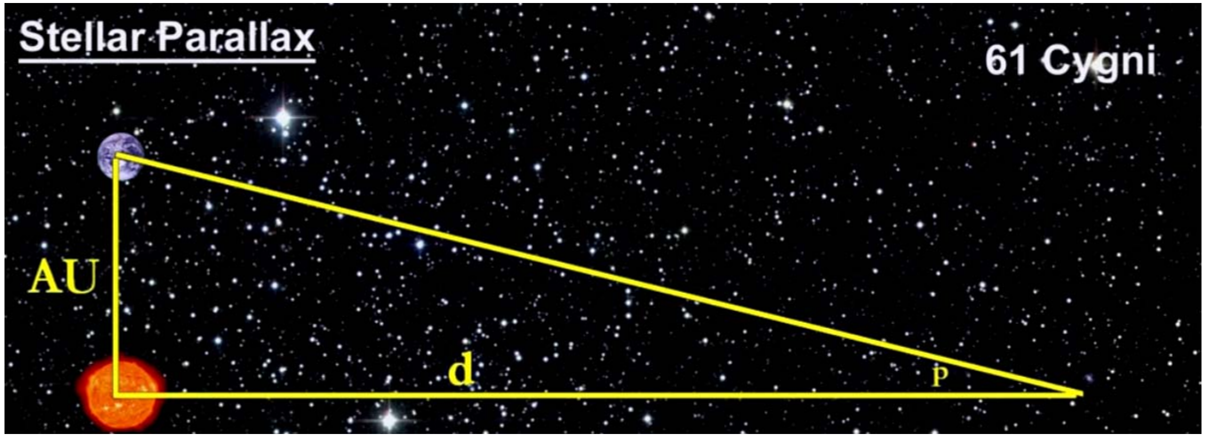




Distance Measurements



- Measurement of distance to the Sun using parallax



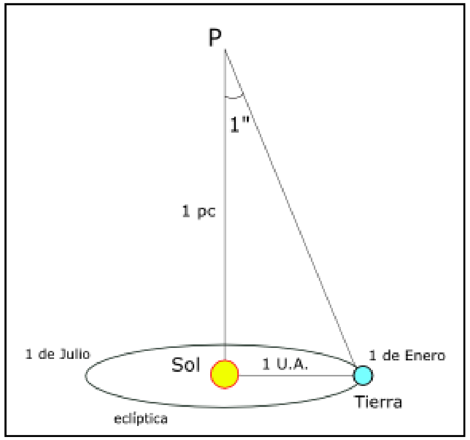
d = *distance to 61 Cgni*
baseline = AU
= 93 *million miles*
p = measured parallax
= 0.314"

$d = AU/\tan(p)$
d = 93 *million miles*/tan (0.314")
= 93 million miles/0.000001522
= 61,091,169 million miles
= 10.4 light years

- First measured distance to a star (Bessel 1838) about 11 LY

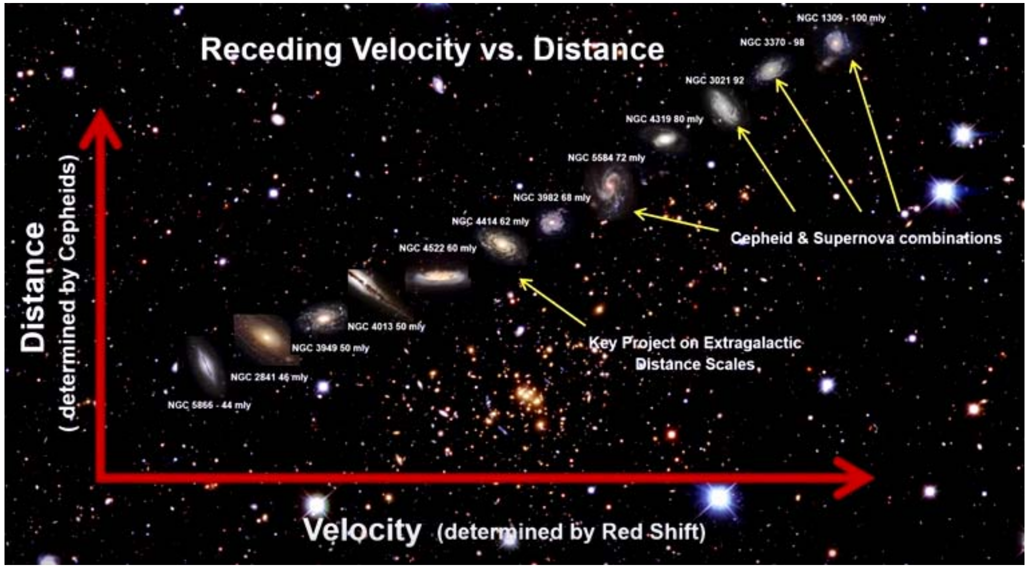
Distance Measurements

- **Parsec (pc):** is a unit of length used to measure large distances to objects outside our Solar System. One parsec is the distance at which one astronomical unit subtends an angle of one arcsecond. 1 parsec ≈ 206264.81 AU ≈ 3.0856×10^{13} Km ≈ 3.26 LY

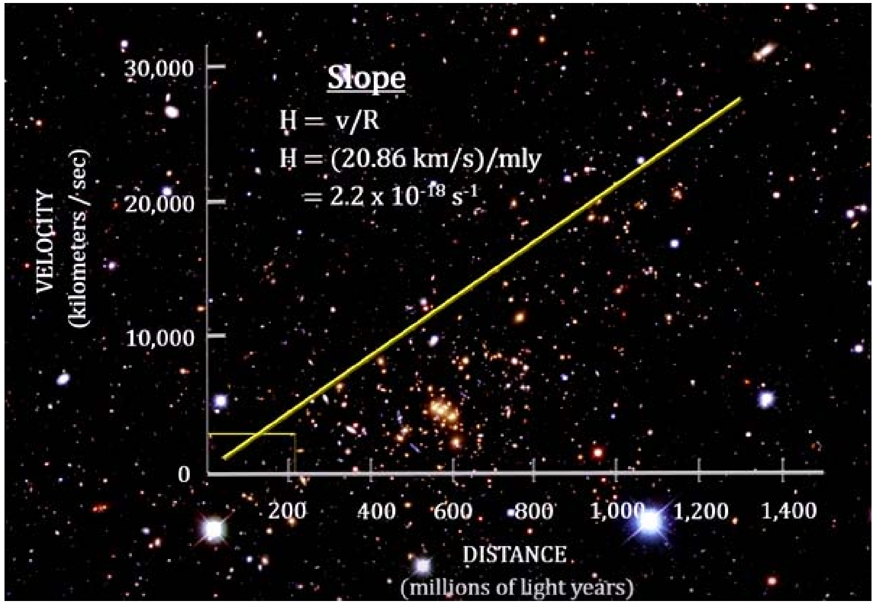


- Voyager1, was 0.00066 parsecs from Earth as of August 2016. It took Voyager1 39 years to cover that distance.
- The Oort cloud is estimated to be approximately 0.6 parsecs in diameter.
- The center of the Milky Way is more than 8 kpc (26000 ly) from the Earth, and the Milky Way is roughly 34 kpc (110000 ly) across.
- The Andromeda Galaxy is about 0.78 Mpc (2.5 million light-years) from the Earth.
- The nearest large galaxy cluster, the Virgo Cluster, is about 16.5 Mpc (54 million light-years) from the Earth.
- The galaxy filament Hercules–Corona Borealis Great Wall, currently the largest known structure in the universe, is about 3 Gpc (10 billion light-years) across.
- The particle horizon (the boundary of the observable universe) has a radius of about 14.0 Gpc (46 billion light-years).

- Edwin Hubble discovers that ALL galaxies are red shifted, and the universe is expanding



$$v = H \cdot R$$



$$H = 20.86 \, \frac{Km}{\sec} \Big/ 1Mpc$$

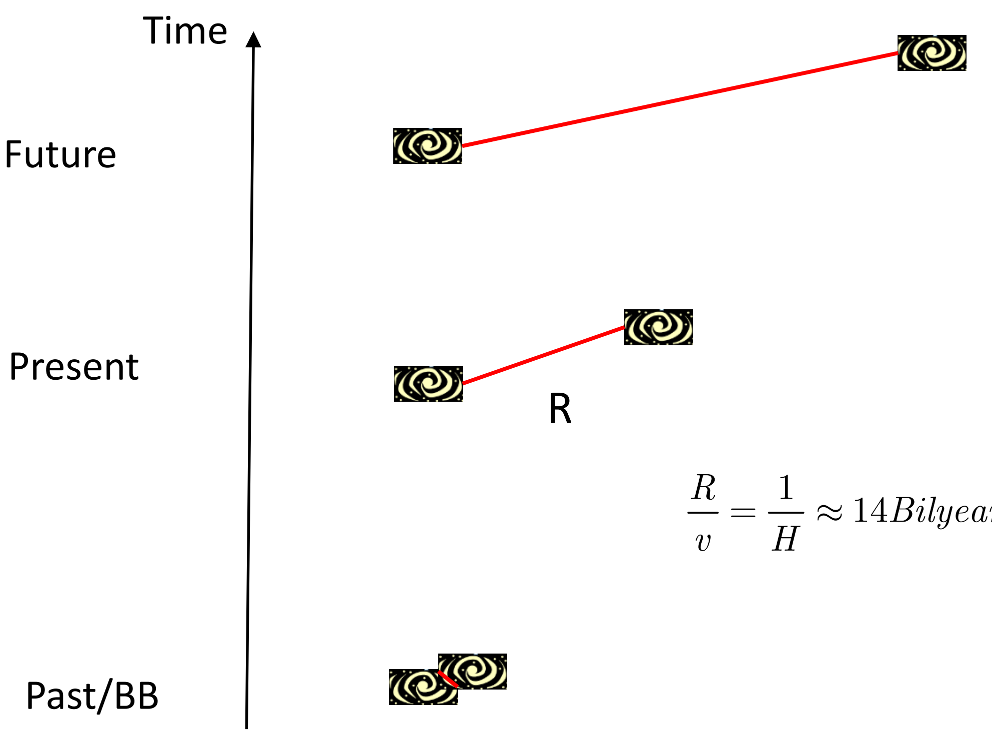
$$or: \; 67.4; 74; 82.4$$

Astronomers are not in agreement with the actual value of the constant

Extra Material

- Main Cosmological Assumptions:
  - Universe is Isotropic (same distribution of galaxies by any observer)
  - Universe is Homogeneous (Galaxy density is the same given a large enough distance)

- Measure today's distance from a galaxy and find relative distance in the future and in the past



$$\frac{R}{v} = \frac{1}{H} \approx 14 Bilyears$$ Assuming constant relative velocity

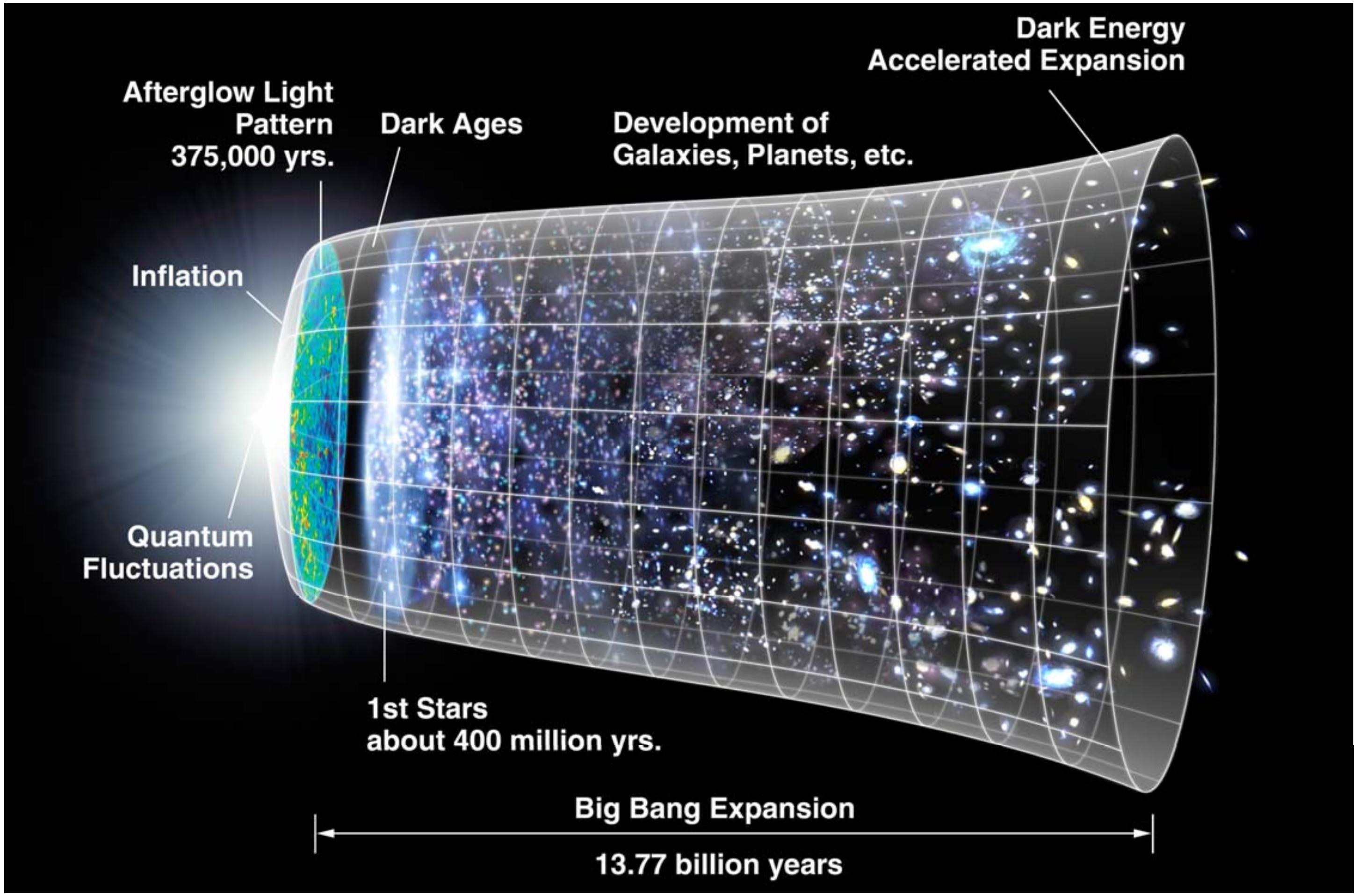
- **Problem**: The movement of galaxies away from each other cannot happen in a finite volume! Basic assumption of Big Bang Theory

Extra Material

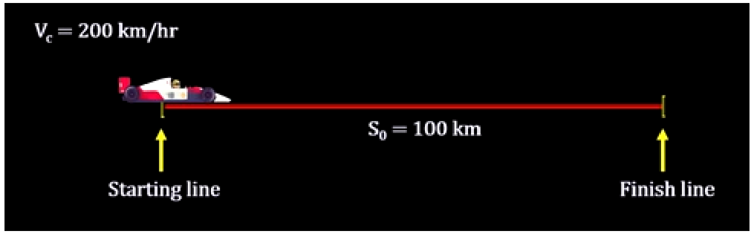
https://www.youtube.com/watch?v=dr6nNvw55C4

https://www.youtube.com/watch?v=u23vZsJbrjE

https://www.youtube.com/watch?v=tVJRS4oZ5ak

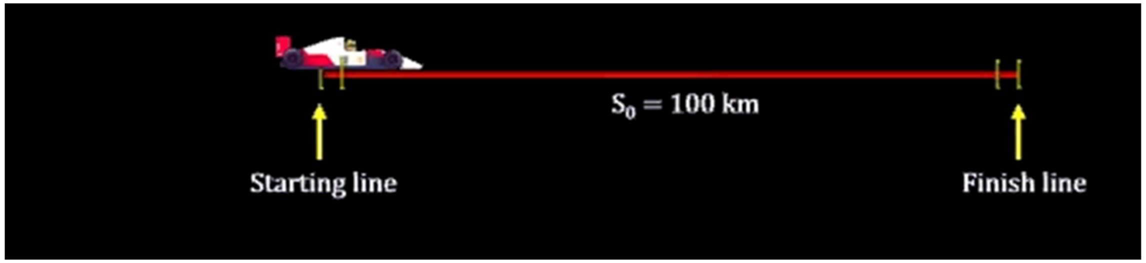


Extra Material



- The Odometer reads 100 Km after 0.5 hours

- Consider an expansion of the track of 50 Km/hr



1. How long would it take to reach the finish line?
2. How far would the car have travelled?

$$S_0 = 100km$$
$$V_C = 200km / h$$
$$V_X = 50km / h$$

Distance travelled by the car $S = S_0 + S_x/2$

$V_C t = S_0 + V_x t \rightarrow t = S_0/(V_C - V_X/2) \Rightarrow$ **$t = 0.571$ hr**

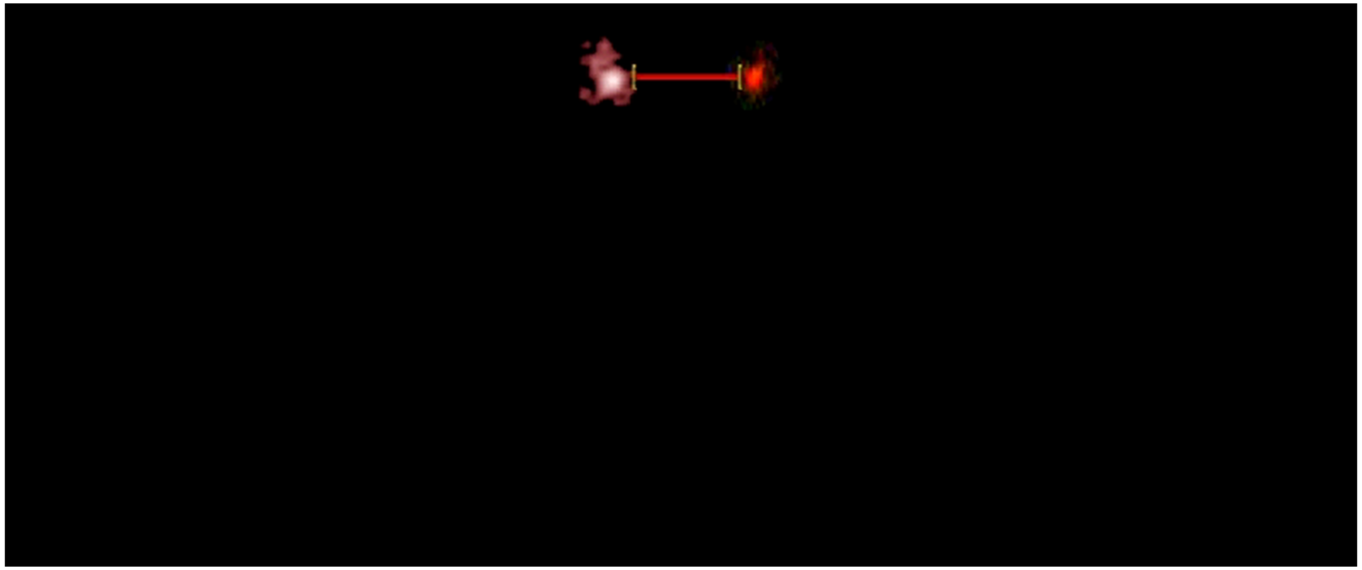
Space expansion $S_X = V_x t = 50 \times 0.571 = 28.57$ km

Total Distance $S = 100 + 28.57/2 \Rightarrow$ **$S = 114.286$ km**

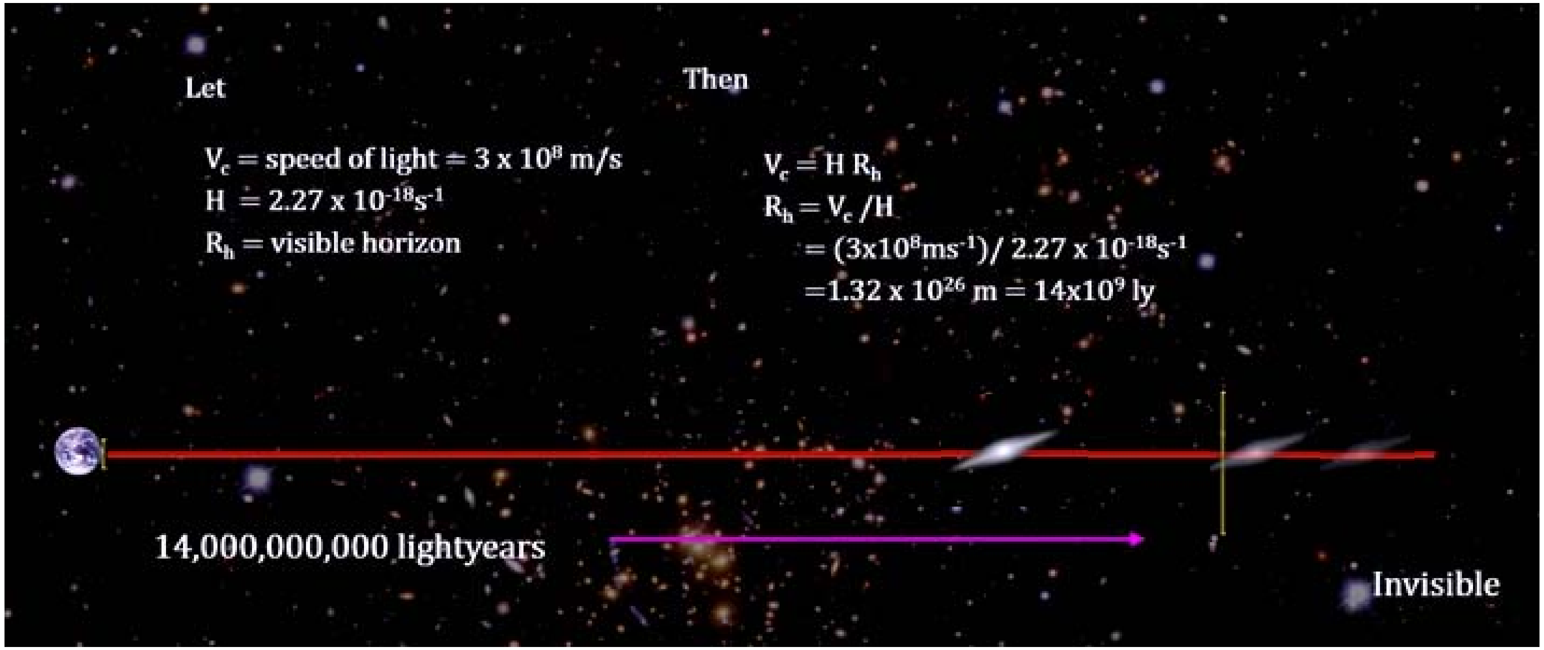
Ending track distance $S_E = S_0 + S_X \Rightarrow S_E = 128.57$ km

Extra Material

- GN-z11 is a high-redshift galaxy found in the constellation Ursa Major. GN-z11 is currently the oldest and most distant known galaxy in the observable universe (March 2016). GN-z11 has a spectroscopic redshift of $z = 11.09$, which corresponds to a proper distance of approximately 32 billion light-years (9.8 billion parsecs) from Earth.
- The object's name is derived from its location in the GOODS-North field of galaxies and its high Doppler z-scale redshift number (GN + z11). GN-z11 is observed as it existed 13.4 billion years ago, just 400 million years after the Big Bang; as a result, GN-z11's distance is inappropriately reported as 13.4 billion light years, its light travel distance measurement.

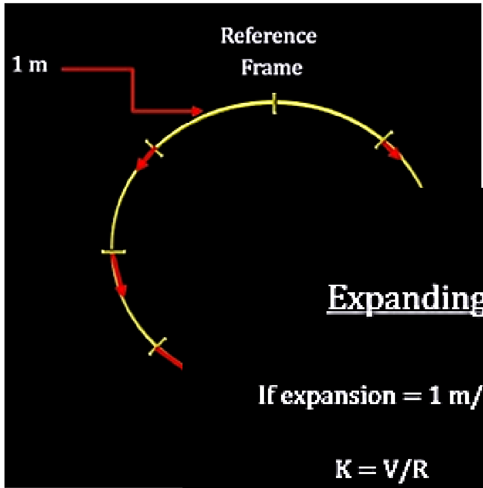


- **Conclusion**: if the light source is far enough its apparent velocity is higher than the speed of light. The paradox is due to the expansion of the Universe. The light emitted NOW by GN – z11 will NEVER reach us, nor the light emitted by galaxies further than 14 BLY

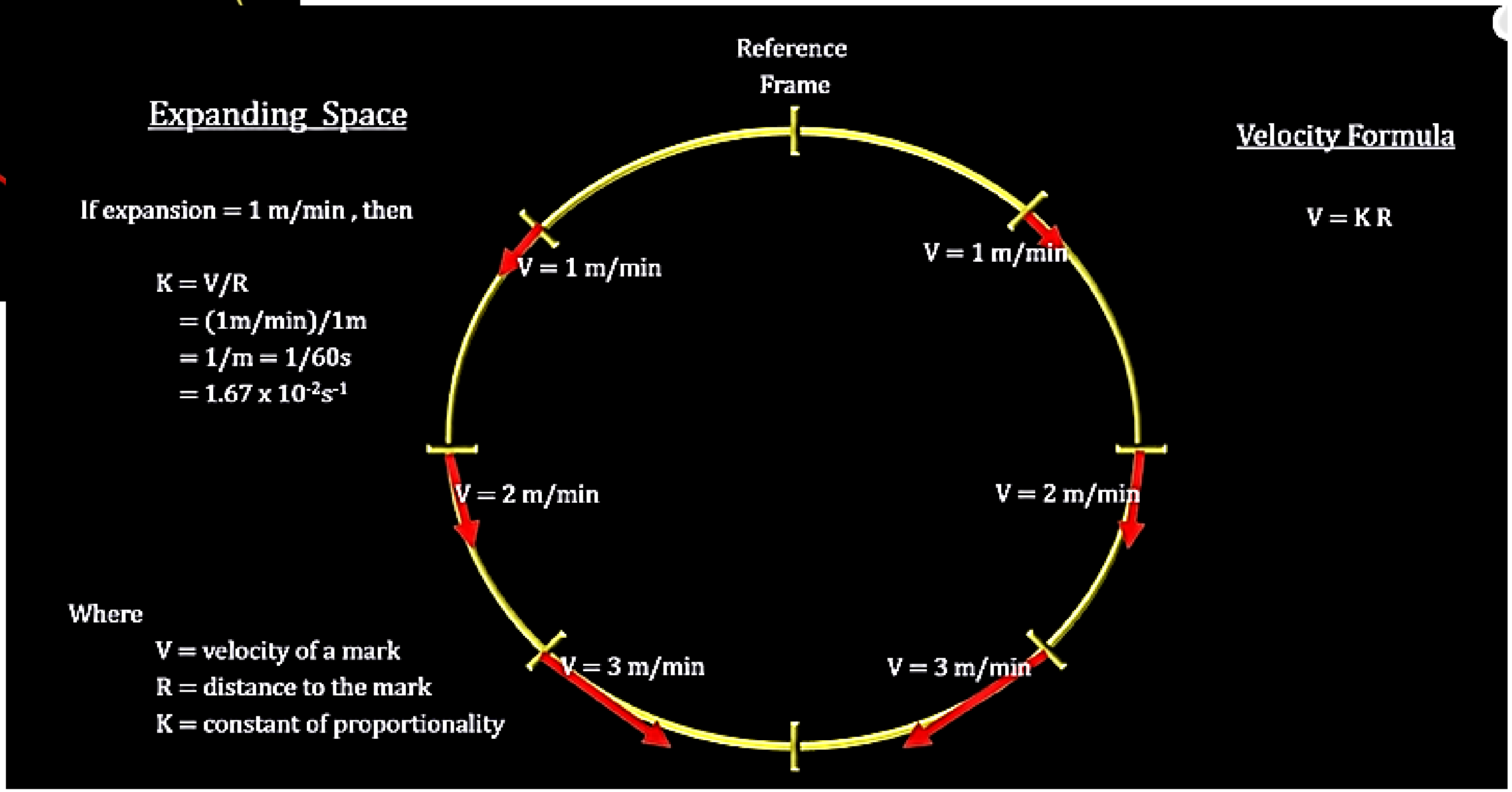


Extra Material

- Consider a fixed 1D space of 8 meters, for the TOP reference all segments move away, but from the BOTTOM reference this is not the case, the system is not homogeneous

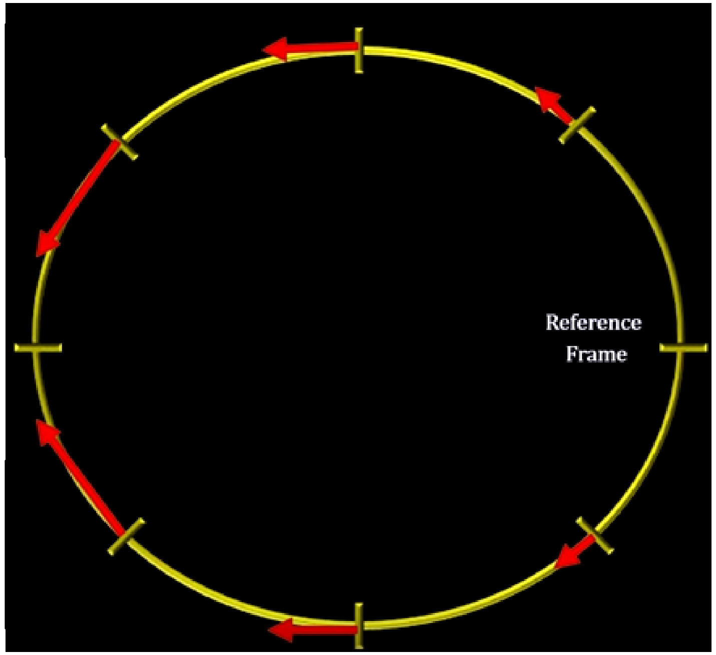


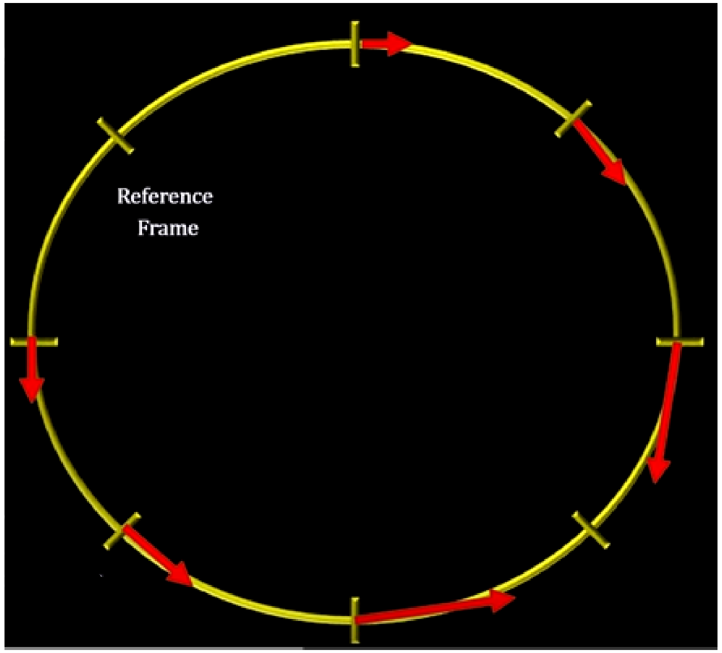
- Assume that each segment expands at a rate of 1 meter every minute



Extra Material

- This holds true for every reference point!



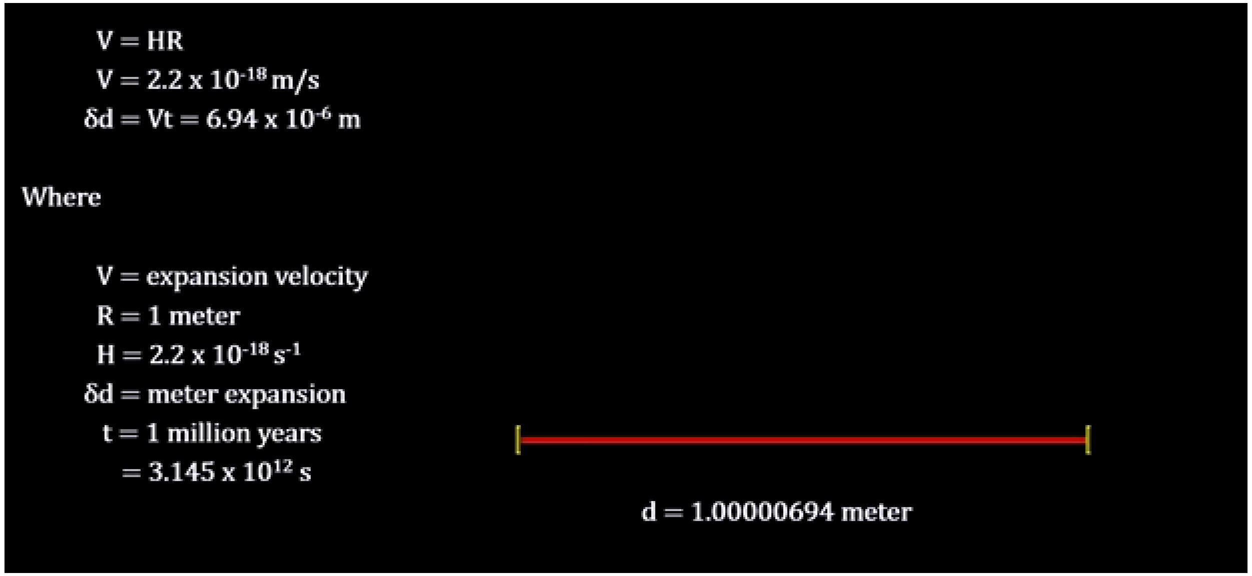


- Note: in this example the expansion rate is much faster than Hubble's constant H

$$\frac{K = 1.67 \times 10^{-2} s^{-1}}{H = 2.27 \times 10^{-18} s^{-1}} = 0.74 \times 10^{20}$$

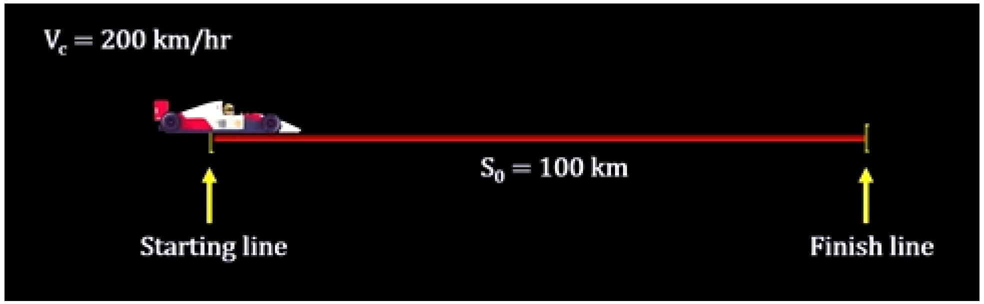
$$K = 74000000000000000000H$$

- In reality, universe expansion is MUCH slower, It would take 1 Million years to expand 1 meter of 0.000694 cm



- Matter in space is NOT expanded since it is held by internal forces (which do not change)

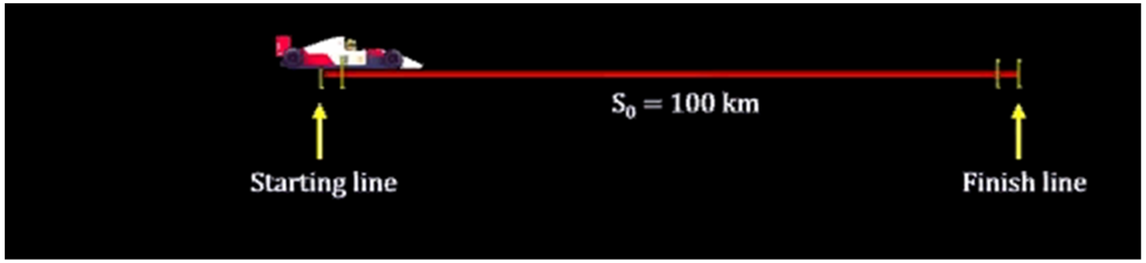
- Consider a not expanding 1D universe



- The Odometer reads 100 Km after 0.5 hours

Extra Material

- Consider an expansion of the track of 50 Km/hr



1. How long would it take to reach the finish line?
2. How far would the car have travelled?

$$S_0 = 100km$$
$$V_C = 200km / h$$
$$V_X = 50km / h$$

Distance travelled by the car $S = S_0 + S_x/2$

$V_C t = S_0 + V_x t \rightarrow t = S_0/(V_C - V_X/2) \Rightarrow$ **$t = 0.571$ hr**

Space expansion $S_X = V_x t = 50 \times 0.571 = 28.57$ km

Total Distance $S = 100 + 28.57/2 \Rightarrow$ **$S = 114.286$ km**

Ending track distance $S_E = S_0 + S_X \Rightarrow S_E = 128.57$ km