Automation and Robotics Laboratory

# Robotica Aerospaziale – 712II
# Laurea Magistrale in Ingegneria Robotica e dell’Automazione
# Università di Pisa

INGEGNERIA
ROBOTICA E DELL’AUTOMAZIONE

Alias: Modellistica e Controllo di Sistemi Aerospaziali

**Anno Accademico 2022 - 2023**

**6 CFU – Primo Periodo**

**DOCENTI**

Mario INNOCENTI, Email: mario.innocenti@unipi.it
Giordana BUCCHIONI, Email: giordana.gucchioni@ing.unipi.it
Lorenzo POLLINI, Email: lorenzo.pollini@unipi.it
Dipartimento di Ingegneria dell'Informazione
RICEVIMENTO: Mercoledì 15:30-18:30

**ORARIO ~ 60 ore (26/09/2022-17/12/2022)**

Martedì: 09:30 – 11:30 (SI7)
Giovedì: 14:00 – 17:00 (B34)

**PREREQUISITI / Corequisiti**

- Concetti di base di Fisica I (Meccanica)
- Concetti di base di Controlli Automatici
- Interesse sugli argomenti del corso
- MATLAB

**PARTECIPAZIONE COMPETENZE E VERIFICA**

- Lezioni in presenza
- Esame orale
- Progetto numerico da portare all'esame (da decidere)

# Course Information

- **Objectives:**
  - Concise review of mechanical motion
  - General principles of orbital mechanics in the two-body model
  - Introduction to non Keplerian dynamics and proximity operations
  - Forces and moments on an aircraft (rigid body)
  - Aircraft dynamics and control
  - Aerial drones' dynamics and control (maybe)

- **Tools:**
  - Class notes and reference books (see next page)

- **Evaluation:**
  - Oral exam
  - Course project

# Course Information

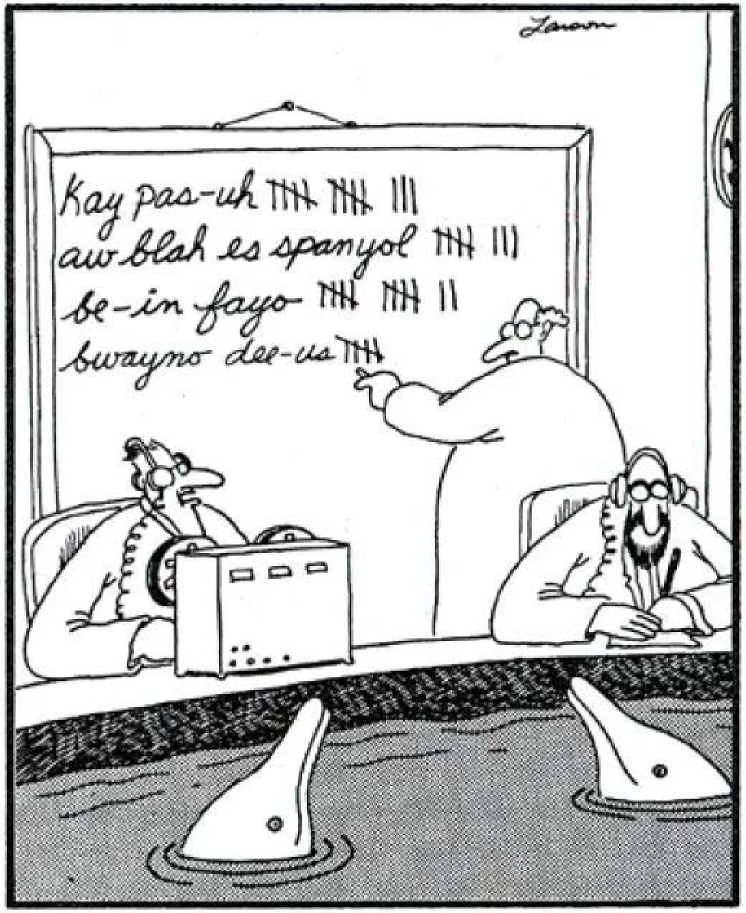
- Class notes were written with the help of material taken from the following sources:
  - MIT Opencourseware website (appropriate courses)
  - Stanford Engineering everywhere website
  - Prof. R. Stengel Princeton University
  - University of New Mexico, Prof. Chris Hall
  - Profs. Wie (Iowa State), Mengali (UNIPI), Peets (Arizona State)
  - Prof. Napolitano, University of West Virginia

**The author wishes to acknowledge their contribution.**

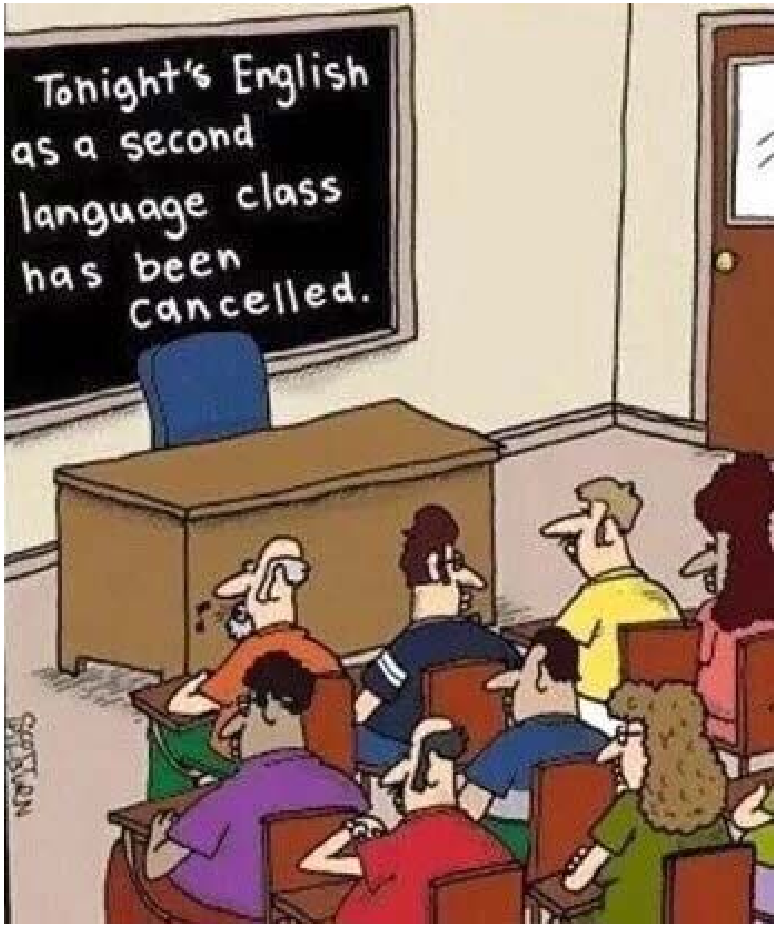
**References:**


- **Nelson R., Flight Stability and Automatic Control, McGraw Hill, 1997**
- **Stengel R., Lecture Notes, MAE 331, Princeton University**
- Blakelock, J., Automatic Control of Aircraft and Missiles, Wiley, 1991
- Mengali G., Elementi di Dinamica del Volo, Edizioni Plus, 2001
- Wie B., Space Vehicle Dynamics and Control, AIAA Education Series, 1998.
- Battin R., An Introduction to the Mathematics and Methods in Astrodynamics, AIAA Education Series, 1987.
- **Curtis H., Orbital Mechanics for Engineering Students, Elsevier 2014**.
- Mengali G., Meccanica del Volo Spaziale, Edizioni Plus, 2013.


- **The course material is an overview of the topics covered in 24 CFU of the MS degree in Aerospace Engineering**

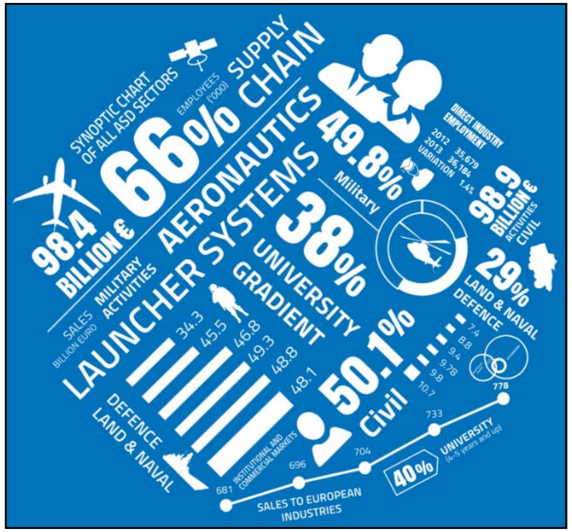


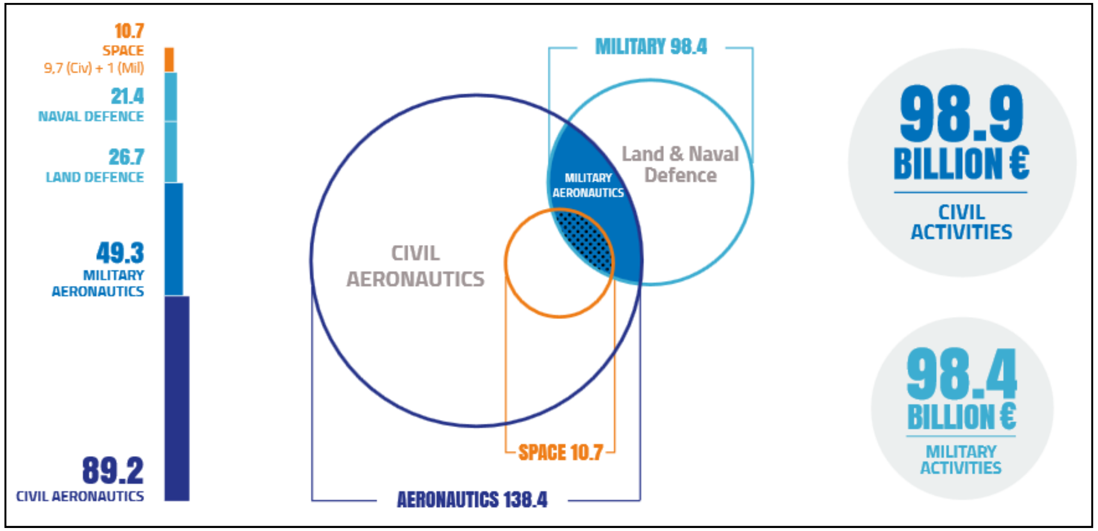

"Matthews ... we're getting another one of those strange 'aw blah es span yol' sounds."

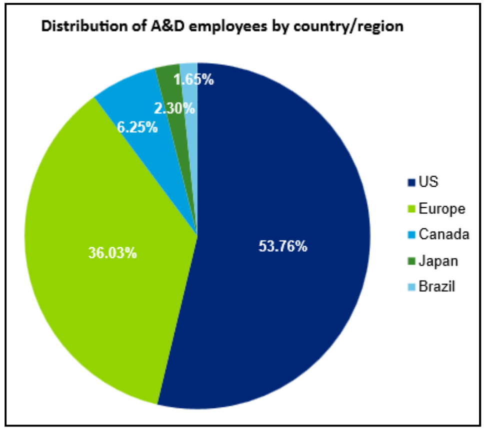


# Introduction

## Industrial Impact of Aerospace:




| NAICS Codes | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Search, detection, navigation and guidance systems manufacturing (334511) | $84,046 | $86,980 | $89,740 | $91,147 | $93,946 | $96,339 |
| Software services (541511) | $83,693 | $87,986 | $90,868 | $92,508 | $91,997 | $95,972 |
| Designing computer systems (541512) | $79,922 | $83,615 | $88,036 | $88,018 | $88,772 | $92,401 |
| Radio and television broadcast and wireless communication equipment manufacturing (334220) | $77,051 | $80,868 | $82,857 | $82,693 | $86,276 | $90,005 |
| Research and experimental development services (541710) | $85,868 | $89,130 | $89,349 | $89,295 | $89,001 | $89,103 |
| Aerospace products and parts manufacturing (33641) | $72,879 | $78,073 | $78,766 | $79,684 | $81,570 | $83,985 |
| Computer related services (541519) | $70,409 | $72,927 | $76,252 | $78,667 | $79,134 | $83,801 |
| Engineering services (541330) | $68,411 | $72,425 | $76,108 | $79,183 | $81,473 | $83,359 |
| Management consulting services (541618) | $72,518 | $77,103 | $78,366 | $83,146 | $79,533 | $83,109 |
| Data processing services (518210) | $64,047 | $67,574 | $73,088 | $73,923 | $76,281 | $79,315 |

# Introduction

## Industrial Players:

The Boeing Company has more than 143,000 employees

The Airbus Company has more than 133,671 employees

The Lockheed Martin Company has more than 110,000 employees

The Thales Company has more than 80,000 employees

The BAE Systems has more than 87,000 employees

The Leonardo Company has more than 33,000 employees

The SpaceX Company has more than 7,000 employees

The Avio SpA Company has more than 900 employees

The Arianespace Company has more than 4,000 employees

## Research Players:

NASA has more than 17,000 employees

CNES has more than 2,800 employees

ESA has more than 2,200 employees

JAXA has more than 1,550 employees

ASI has more than 200 employees

China's CNSA and CASC ….

| Agency (country/region) | Budget (in millions of US $) |
|---|---|
| NASA (United States) | 22,629 |
| CNSA (China) | 11,000 |
| ESA (Europe) | 7,430 |
| DLR (Germany) | 4,233 |
| CNES (France) | 3,384 |
| Roscosmos (Russia) | 2,530 |
| ISRO (India) | 2,000 |
| ASI (Italy) | 1,800 |
| JAXA (Japan) | 1,699 |
| KARI (South Korea) | 583 |
| UKSA (United Kingdom) | 500 |
| ASA (Algeria) | 360 |

Amazon Company has more than 790,000 employees

# Introduction

**Euro 110 Mil.**

**Euro 220 Mil.**

**Euro 420 Mil.**

67P/Churyumov–Gerasimenko

**€1.4bn**

Rosetta mission **2004-2014**
Land on a comet for the
first time in human history

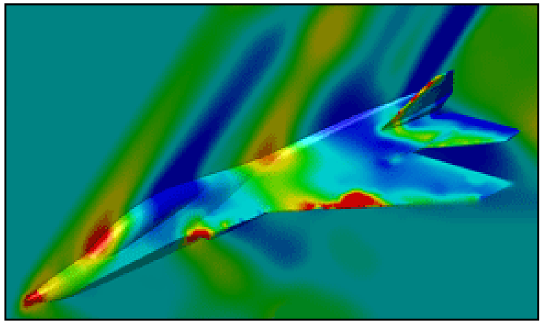
**€3.50**
**cost per European citizen**
(from 1996–2015, so
**€0.20**/person/year)

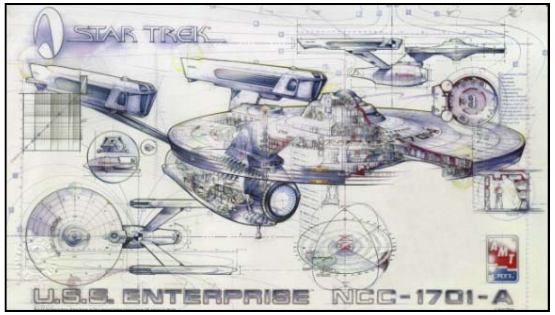
Cost of Rosetta per person **€3.50**

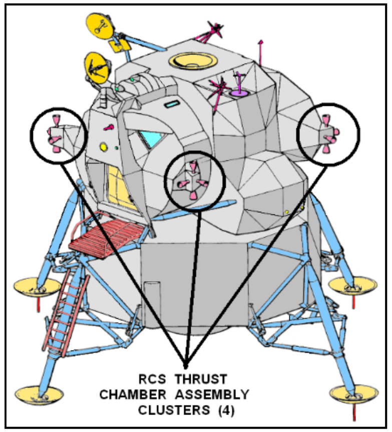
Cost of a cinema ticket to see *Interstellar* **€8.50**

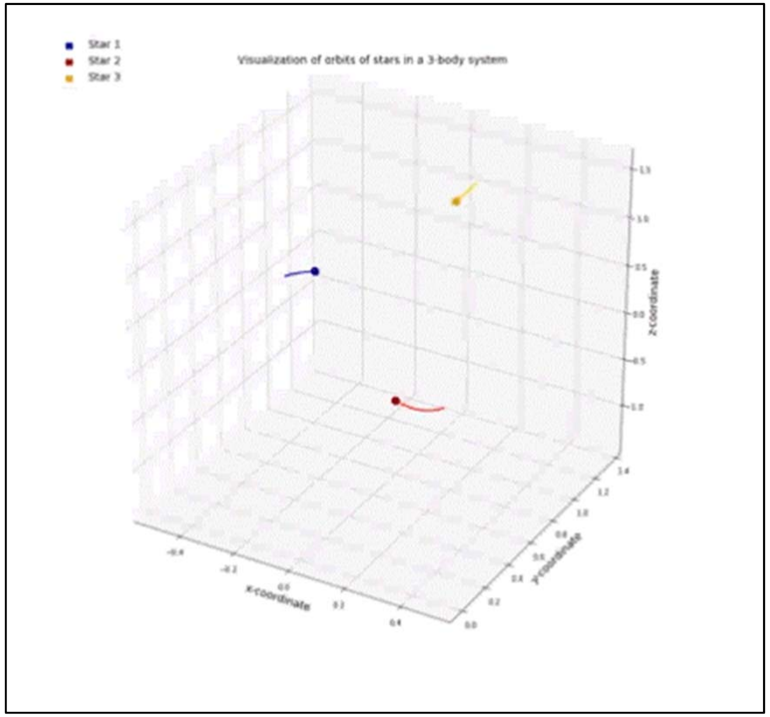
# Modeling

$$
\begin{aligned}
&\nabla \cdot \boldsymbol{E} = \rho / \varepsilon_0 \\
&\nabla \cdot \boldsymbol{B} = 0 \\
&\nabla \times \boldsymbol{E} = -\frac{d\boldsymbol{B}}{dt} = -\left\{ \frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{v} \cdot \nabla(\boldsymbol{B}) \right\} \\
&\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{d\boldsymbol{E}}{dt} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\left\{ \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{v} \cdot \nabla(\boldsymbol{E}) \right\} \\
&\nabla \cdot \boldsymbol{J} = -\frac{d\rho}{dt} = -\left\{ \frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla(\rho) \right\}
\end{aligned}
$$

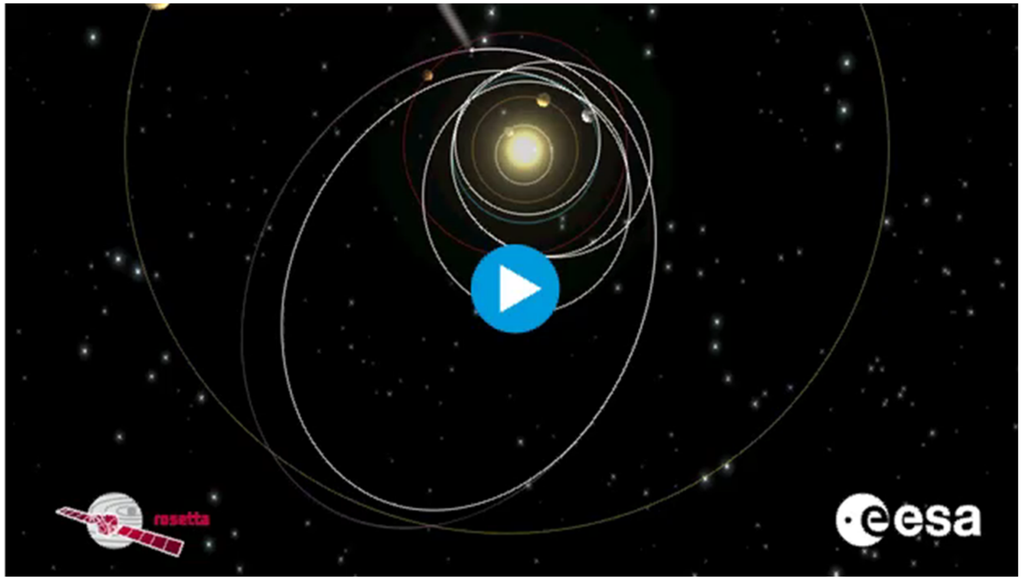


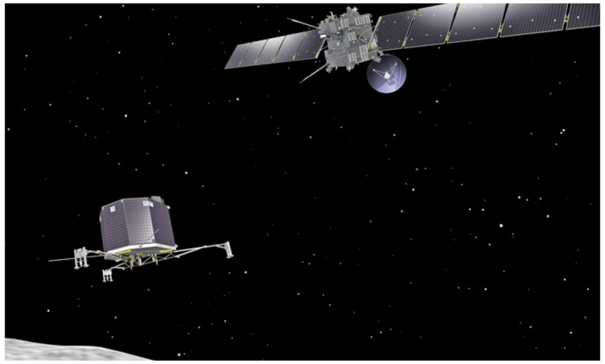


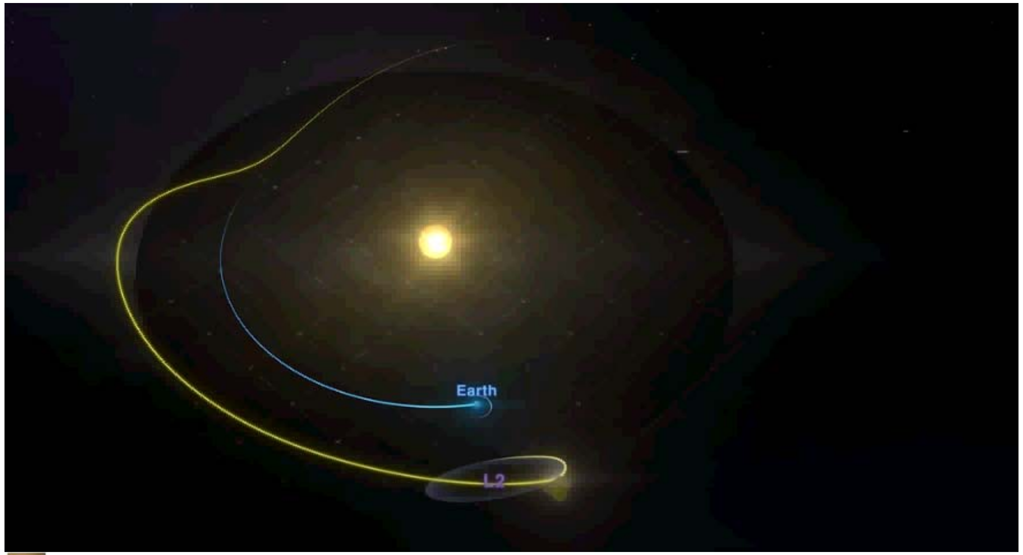



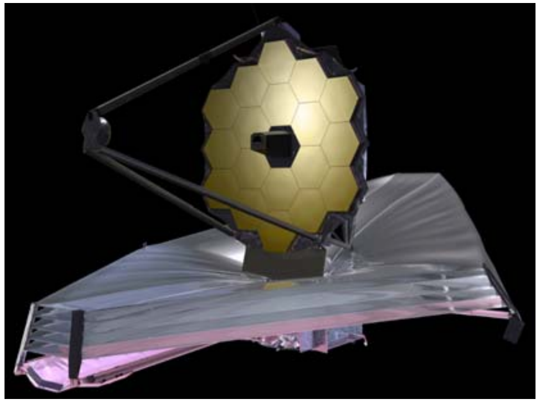



# Modeling





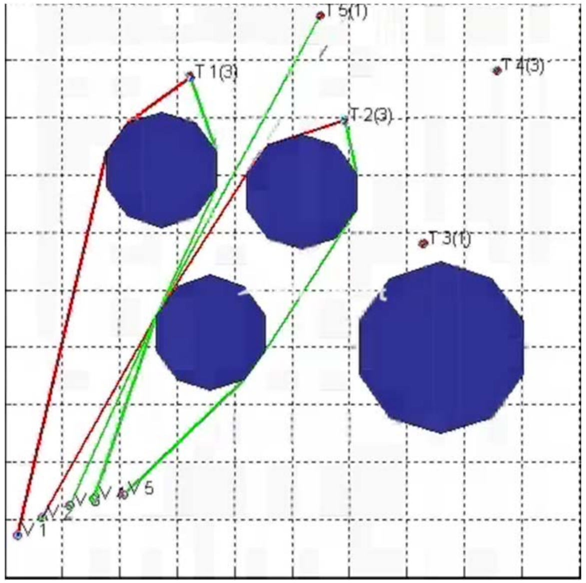
# Modeling

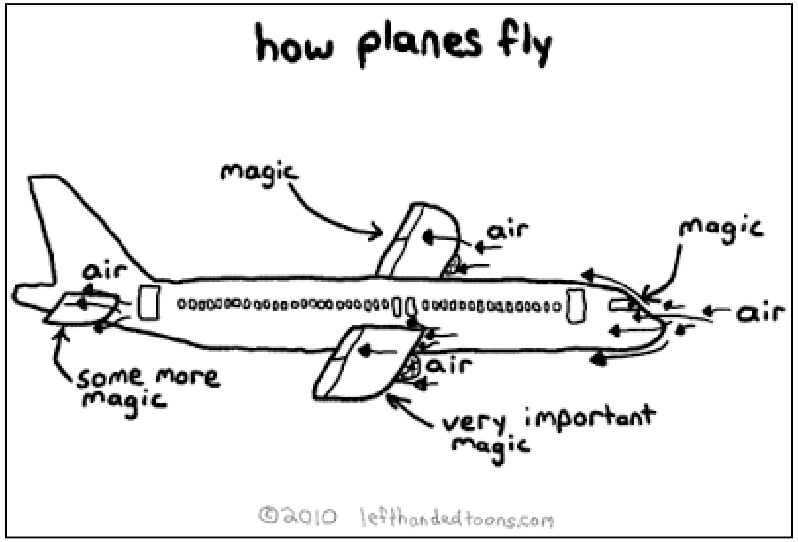
The **Mars Ingenuity**, a small, autonomous rotorcraft, is used to demonstrate the viability and potential of heavier-than-air vehicles on the Red Planet.

- Aerodynamics
- Wind
- Temperature (20 - -153 Celsius)
- Pressure (1% of Earth)
- Gravity (40% lower)
- Time delay (4 – 24 minutes)
- Flying on the surface of Mars is equivalent to flying 100,000 feet above sea level

https://www.youtube.com/watch?v=S6qs6L0shTg

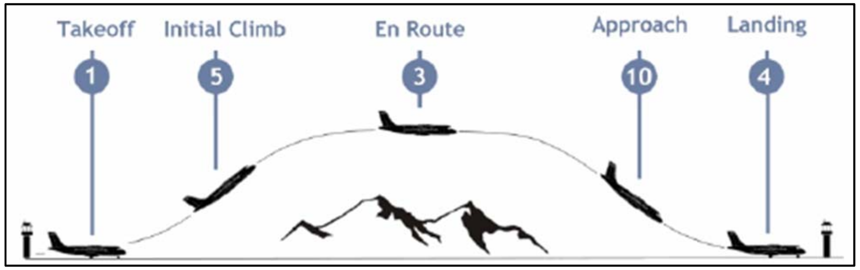
https://www.youtube.com/watch?v=CoZSbN9lm44



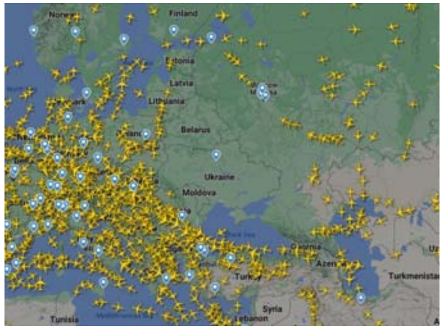



# Modeling

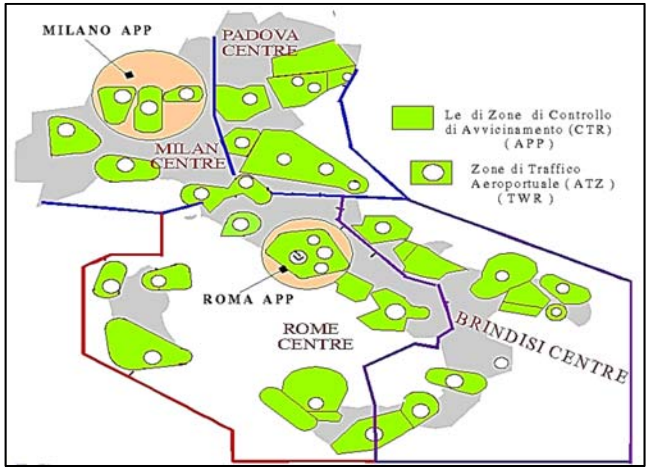
- Standard subdivision of phases of flight




- Flight Plan:
  - Airline Reqs.
  - Weather
  - Airport Capacity
  - Payload
  - Forbidden Areas
  - …

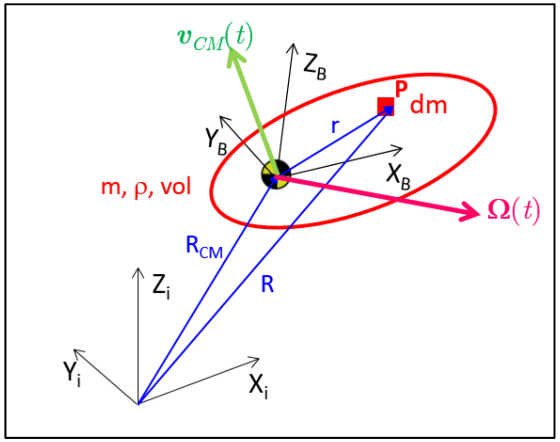


- Tower and Ground Controller (TWR)
- Approach Controller (APP)
- Area Controller (ACC)



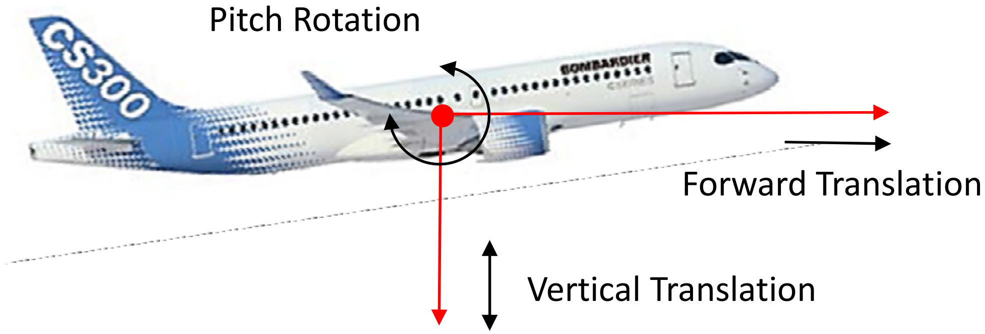


# Modeling – Nomenclature

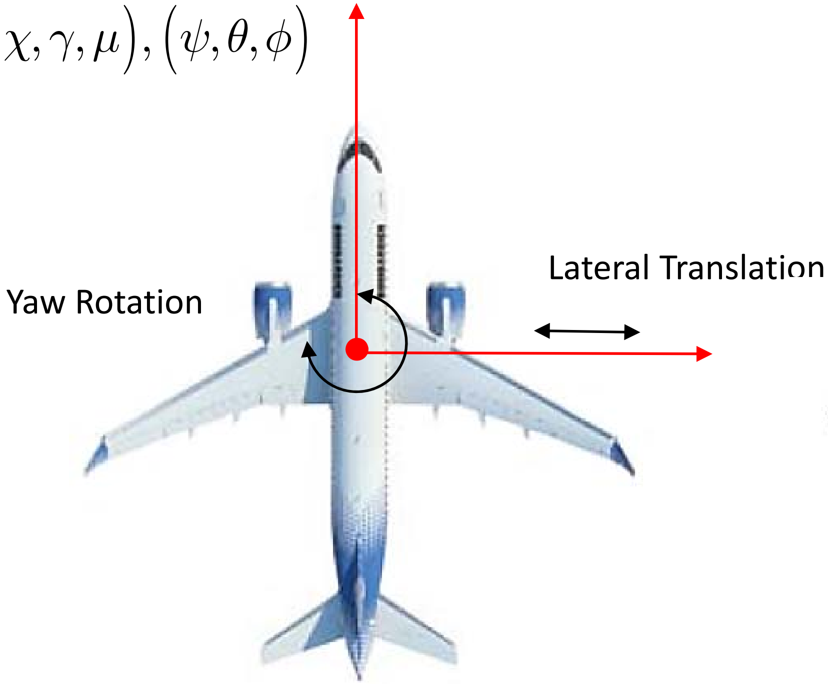


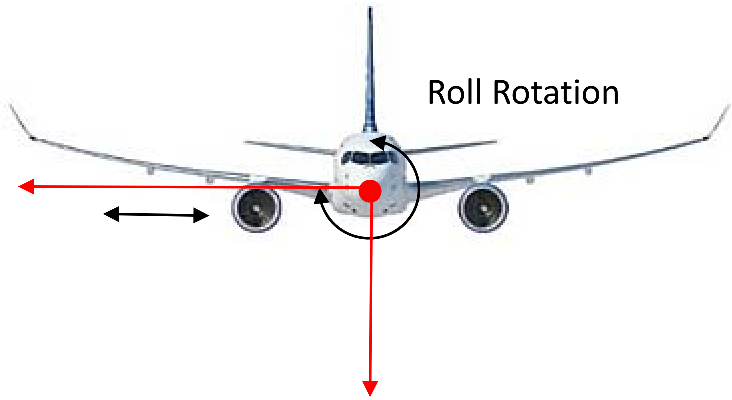

$$\left(\chi,\gamma,\mu\right),\left(\psi,\theta,\phi\right)$$

- 3 DOF Longitudinal Motion




- 3 DOF Lateral - Directional Motion

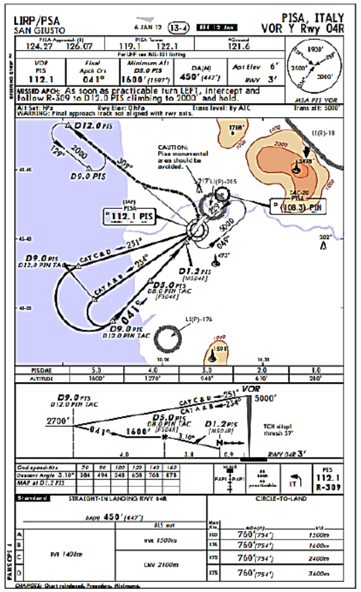


# Modeling – Autopilot Example

ILS categories for precision instrument approach and landing

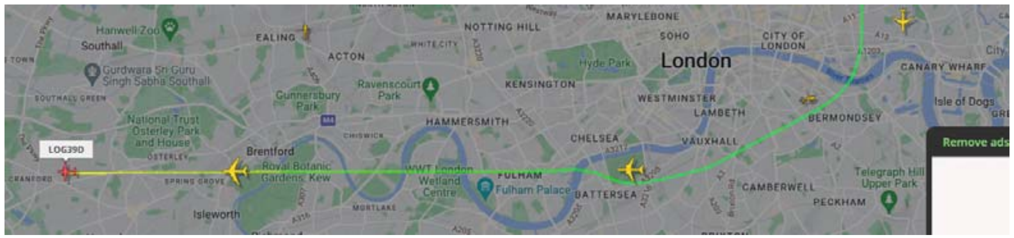
| Approach category | Decision height | Runway visual range (RVR) | Visibility minimum | Notes |
|---|---|---|---|---|
| I | 200 ft (61 m) or more[6] | 550 m (1,800 ft);[6] at some airports, 370 m (1,210 ft) is approved.[9] For single crew operations, increased to 800 m (2,600 ft). | 800 m (2,600 ft). | Either visibility not less than 800 m (2,600 ft) or a runway visual range (RVR) not less than 550 meters (1,800 ft) on runway with touchdown zone and centerline lighting. FAA Order 8400.13D allows for special authorization of CAT I ILS approaches to a decision height of 150 feet (46 m) with RVR ≥ 1,400 feet (430 m).[10] The aircraft and crew must be approved for CAT II operations and a heads-up display in CAT II or III mode must be used to the decision height. CAT II/III missed approach criteria apply.[10] |
| II | less than 200 ft and more than 100 ft (30 m)[6] | 1,000 feet (300 m)[8] | N/A | ICAO and FAA: 350 meters (1,150 ft) or EASA: 300 meters (980 ft).[7] |
| IIIa | less than 100 ft and more than 50 ft (15 m)[8] | 600 feet (180 m)[8] | N/A | |
| IIIb | less than 50 ft (15 m)[6] | 150 feet (46 m)[8] | N/A | |
| IIIc | No limitations[8] | None[8] | N/A | As of 2012, this category is not yet in operation anywhere in the world, as it requires guidance to taxi in zero visibility as well.[citation needed] Category IIIc is not mentioned in EU-OPS.[citation needed] |




**NOTE: 747 cockpit at 7.5 m!**

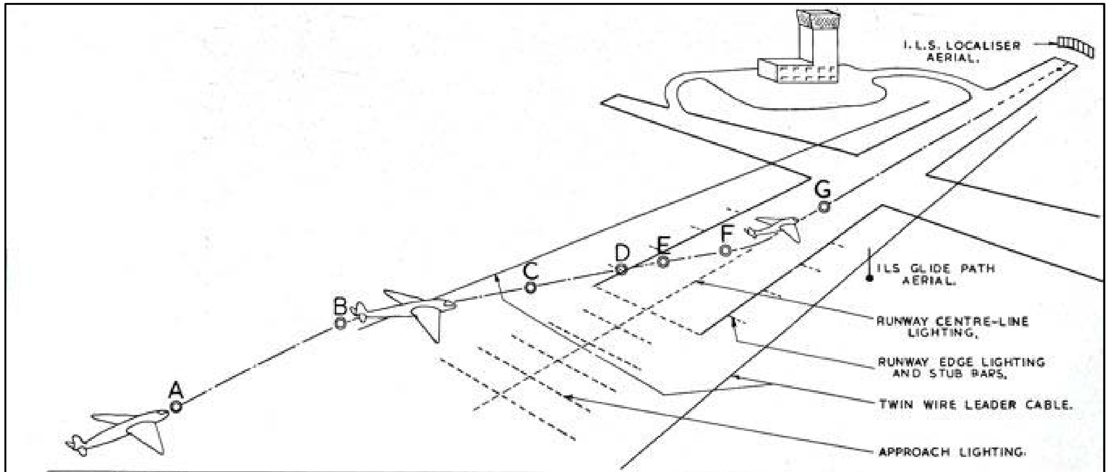
## ❑ Automatic Landing System (Autoland):

- Landing is a complex maneuver that involves controlled motion over both motion planes. The primary variables to be controlled are: Heading, Speed, Attitude (Pitch Angle), Vertical Speed and Flight Path. The maneuver sequence can be fully mechanized with appropriate control systems, leading to the so called automatic landing system (autoland).



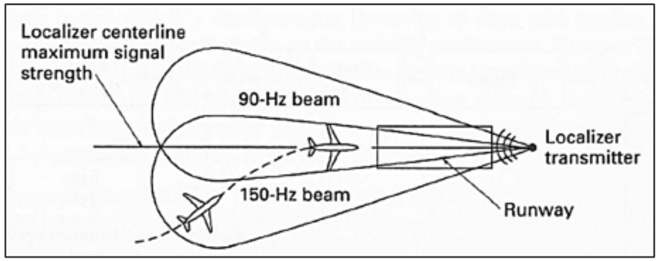
# Modeling – Autopilot Example

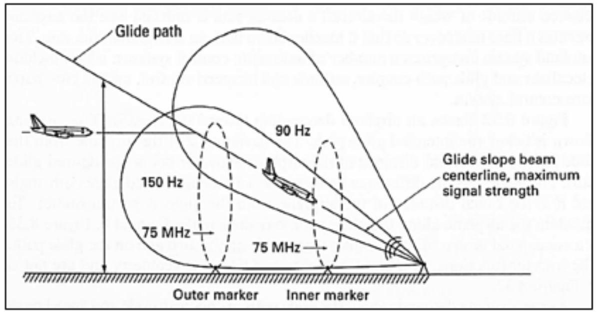
- **3 Main components: The primary aids are VOR (localizer), ILS and GPS**

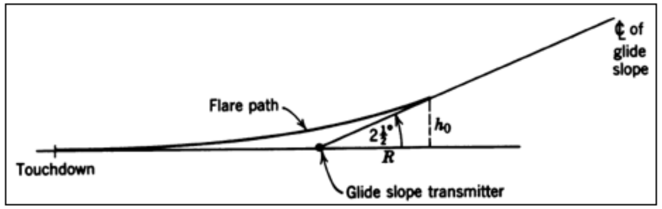



| POSITION | NOMINAL HEIGHT, FT. | PHASE JUST COMMENCED | GUIDANCE: AZIMUTH | GUIDANCE: ELEVATION |
|---|---|---|---|---|
| A | CIRCUIT | AUTOMATIC APPROACH (TRACK). | I.L.S. LOCALISER, COMPASS. | BAROMETRIC HEIGHT. |
| B | CIRCUIT | AUTOMATIC APPROACH (GLIDE). | I.L.S. LOCALISER. | I.L.S. GLIDE PATH. |
| C | 300 | AUTOMATIC LAND (LEADER CABLE). | LEADER CABLE. | I.L.S. GLIDE PATH. |
| D | 150 | AUTOMATIC LAND (ATTITUDE). | LEADER CABLE. | FIXED ATTITUDE. |
| E | 60 | AUTOMATIC LAND (FLARE). | LEADER CABLE. | RADIO ALTIMETER. |
| F | 20 | AUTOMATIC LAND (KICKING OFF DRIFT). | COMPASS. | RADIO ALTIMETER. |
| G | 0 | MANUAL GROUND RUN. | LEADER CABLE, COMPASS. | ——— |

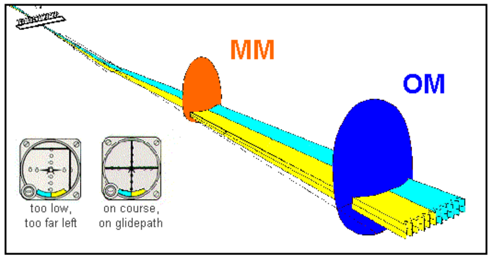
AUTOMATIC LANDING SYSTEM.



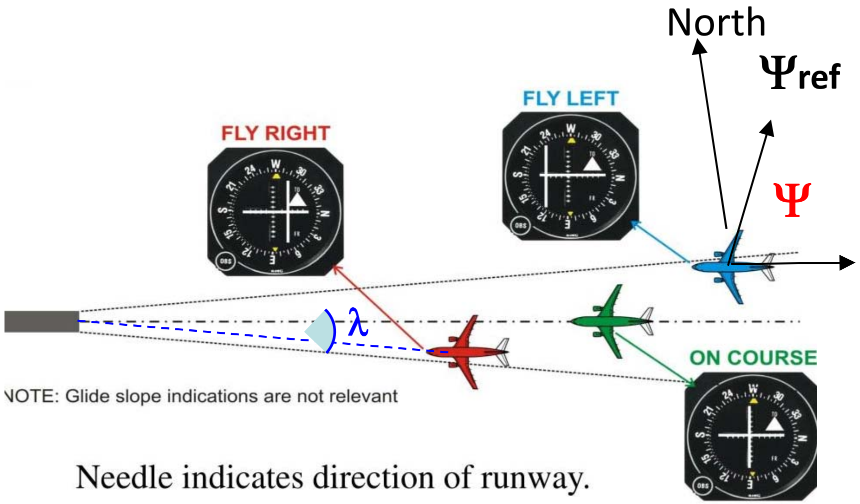





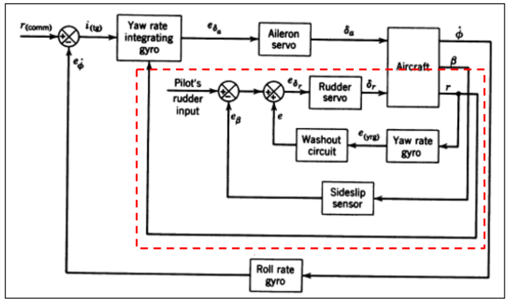



- **Autopilot Example:** Localizer Autopilot during landing (part of Autoland system)

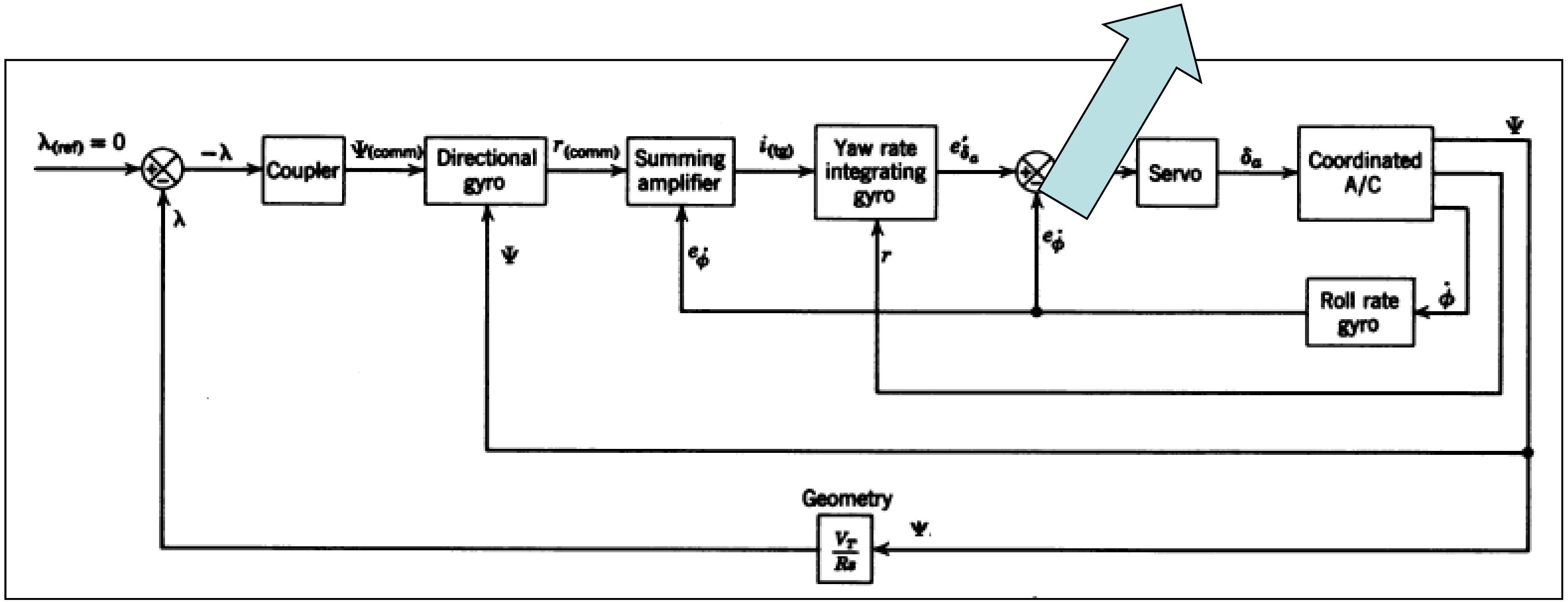



Needle indicates direction of runway.

- The angle λ must be kept to zero

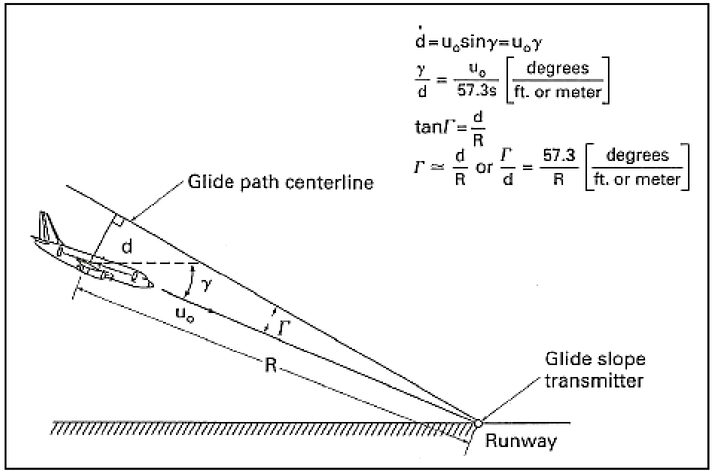


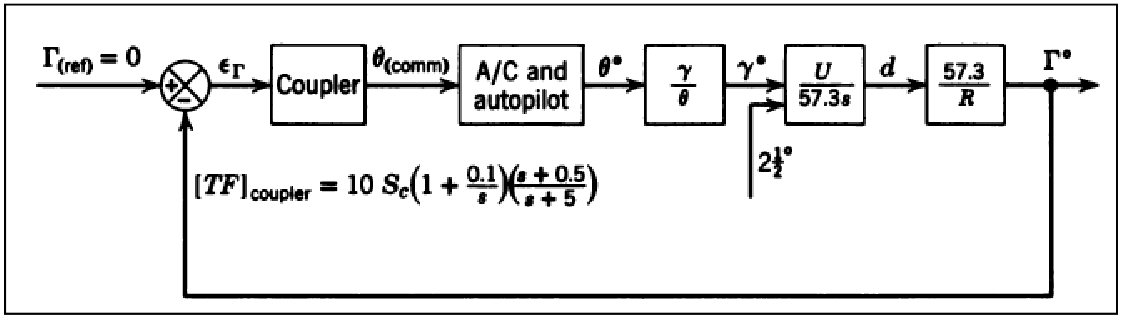

SAS Yaw damper

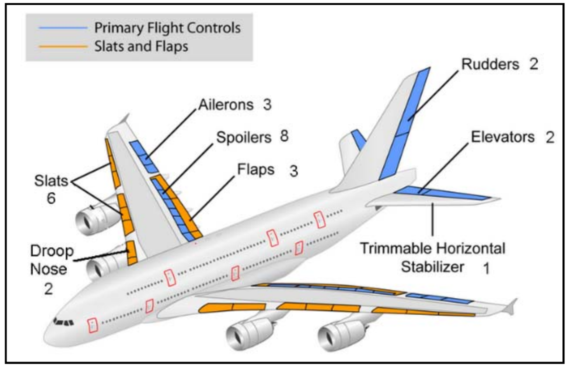


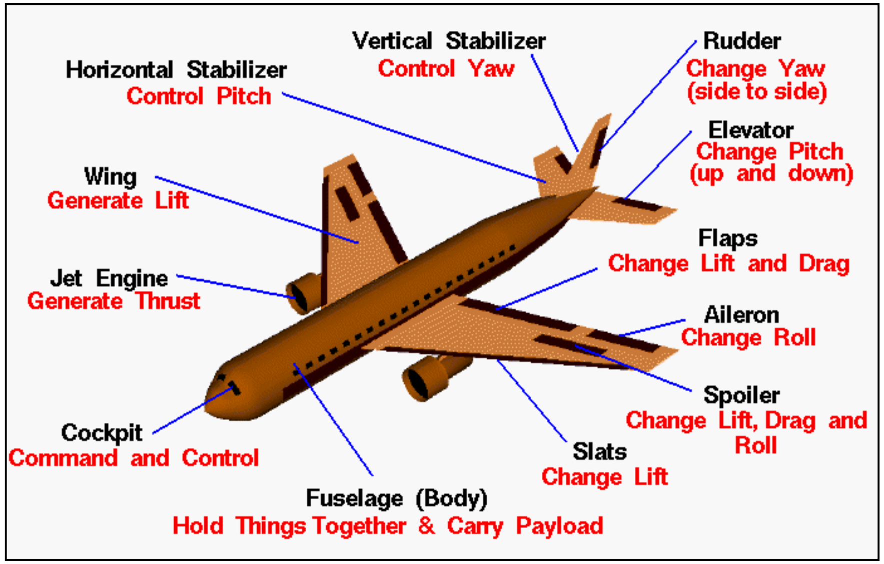
# Modeling – Autopilot Example

- **ILS – Glide Path Control:** The glide path (glide slope) provides an aircraft with information about whether it is flying above or below a preferred descent trajectory (nominally -2.5°/-3°) to the airport (Markers provide information about the actual distance to the runway).

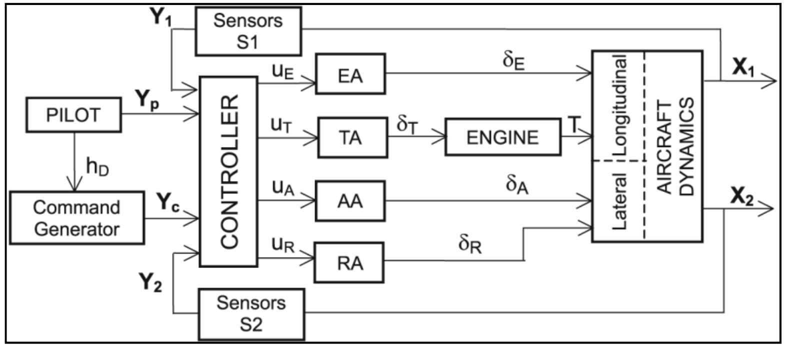


# Modeling




https://www.youtube.com/watch?v=vJzDRsEKDa0&t=14s

https://www.youtube.com/watch?v=mlw43gpH57U

https://www.youtube.com/watch?v=UV_vWtAJIow

Boston Landing

https://www.youtube.com/watch?v=jAKGu2Ki0Wc

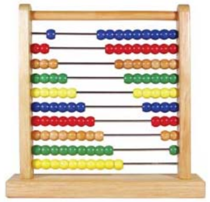
# Introduction

The avionics system in the F-22 Raptor, the current U.S. Air Force frontline jet fighter, consists of about 1.7 million lines of software code. The F-35 Joint Strike Fighter, scheduled to become operational in 2010, will require about 5.7 million lines of code to operate its onboard systems. And Boeing's new 787 Dreamliner, scheduled to be delivered to customers in 2010, requires about 6.5 million lines of software code to operate its avionics and onboard support systems.


These are impressive amounts of software, yet if you bought a premium-class automobile recently, "it probably contains close to 100 million lines of software code," says Manfred Broy, a professor of informatics at Technical University, Munich, and a leading expert on software in cars. All that software executes on 70 to 100 microprocessor-based electronic control units (ECUs) networked throughout the body of your car.

Alfred Katzenbach, the director of information technology management at Daimler, has reportedly said that the radio and navigation system in the current S-class Mercedes-Benz requires over 20 million lines of code alone and that the car contains nearly as many ECUs as the new Airbus A380 (excluding the plane's in-flight entertainment system). Software in cars is only going to grow in both amount and complexity. Late last year, the business research firm Frost & Sullivan estimated that cars will require 200 million to 300 million lines of software code in the near future.

Even low-end cars now have 30 to 50 ECUs embedded in the body, doors, dash, roof, trunk, seats, and just about anywhere else the car's designers can think to put them. That means that most new cars are executing tens of million of lines of software code, controlling everything from your brakes to the volume of your radio [see table, "How and Where Is Software Used in Cars? "].



```
START: JUMP   LOOP          # jump past sensor and constant locations
RSV:   0000                 # read right sensor value here
LSV:   0000                 # read left sensor value here
RMP:   0000                 # write right motor power level here
LMP:   0000                 # write left motor power level here
OFF:   0000                 # store motor-off constant here
ON:    0100                 # store motor-on constant here
LOOP:  LOAD   1      RSV    # load right sensor value into register 1
       LOAD   2      LSV    # load left sensor value into register 2
       SUB    1   2  3      # subtract 1 from 2 and store result in 3
       LOAD   1      OFF    # load motor-off constant into register 1
       LOAD   2      ON     # load motor-on constant into register 2
       BRANCH 3      RGT    # if the left sensor is greater than the
LFT:   STORE  2      RMP    # right then turn the right motor on
       STORE  1      LMP    # and turn the left motor off
       JUMP   LOOP          # and then jump to beginning of the loop
RGT:   STORE  2      LMP    # else turn the left motor on
       STORE  1      RMP    # and turn the right motor off
       JUMP   LOOP          # and then jump to beginning of the loop
```

## Certification Process

# Program

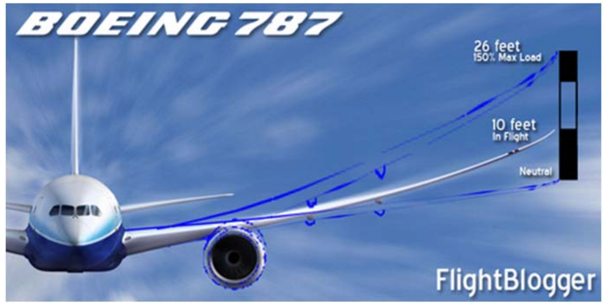
- **Review of Mechanics:**
  - Main assumptions/definitions
  - Classical / Relativistic
  - Newton
  - Reference Systems
- **Orbital Mechanics 1:**
  - Law of Gravitation, 2-Body Problem
  - Keplerian Orbits, Kepler Equation
  - Orbit Transfer, Lambert Theorem
  - Patched Conics
  - Interplanetary Missions
- **Orbital Mechanics 2:**
  - 3-Body Problem, Motivation
  - CR3BP and Lagrangian Points
  - Non-Keplerian Orbits
  - Invariant Manifolds and CR3BP Pathways
  - Proximity Operations: relative motion, rendezvous
- **Atmospheric Flight Dynamics:**
  - Rigid Body Motion
  - A/C Nonlinear Equations of Motion
  - Applied forces and moments
  - Geometry, Flight Envelope, Equilibrium Conditions
  - Linearization, Natural Modes, Stability Derivatives
- **A/C Control**
  - Sensors, Actuators, Primary Control Loops
  - Stability Augmentation, Autopilots
  - Pilot in the Control Loop

# Program

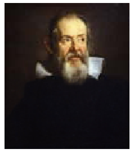
- **Objective**: Study of the motion (uncontrolled and controlled) of a body moving inside a medium
  - **Medium:**
    - Air
    - Space
  - **Body:**
    - Aircraft, drone, …
    - Spacecraft, planet

- The body has different mathematical characterizations (models) such as:
  - Point mass
  - Rigid body
  - Flexible body
  - …






- The motion is defined by the appropriate physical description, which determines at each instant, position and velocity of the body. In the case of a rigid body, this translates into 6 DOF or 12 state variables. These variables change according to the relevant differential equations of motion

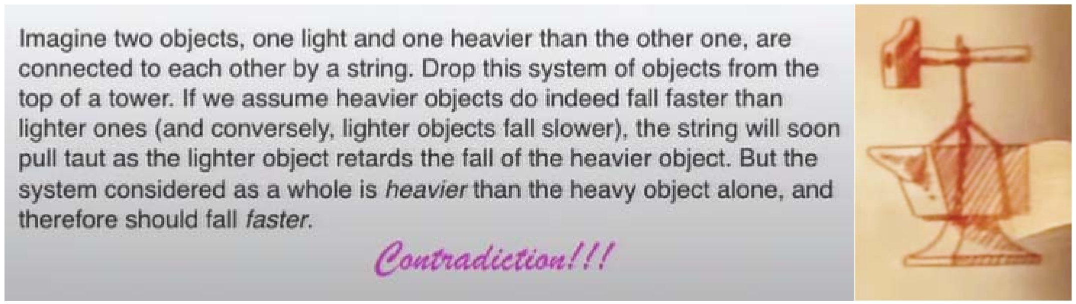


## ❑ Galileo Galilei (1564 – 1642)

Aristarchus of Samos (310 – 230 BC), Copernicus (1473 – 1543)

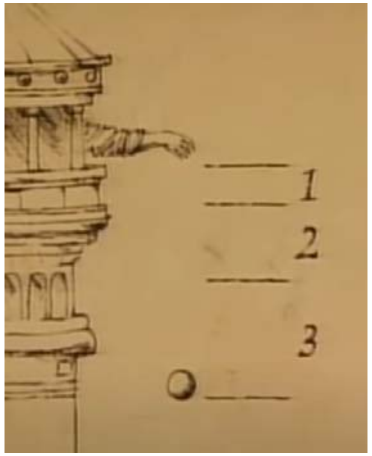
- <u>Invention of telescope</u> (9x, 30x) : Discovery of moons of Jupiter (Io, Europa, Ganymede, Callisto), which contributed to his financial well being but caused the condemnation of the Church

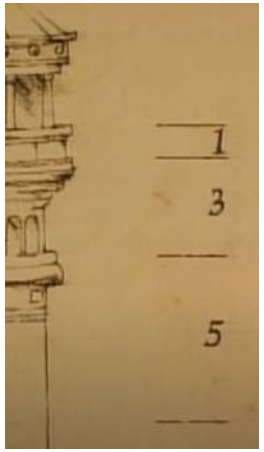
- <u>The Law of falling bodies</u>: the law states that the distances traveled are proportional to the squares of the elapsed times, or, **in vacuum**, **all bodies** fall with the **same, constant acceleration**
  - NO differential calculus (distance, velocity, acceleration)
  - Use of inclined plane

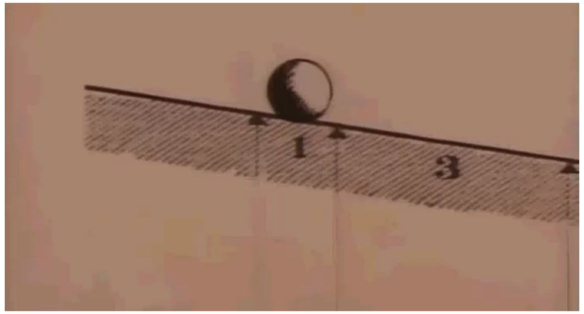



<u>Apollo 15 Experiment</u>

Leonardo: 1 unit distance in 1 unit time




- **Galileo: distance follows odd numbers of unit time**
- **Total distance as function of time: 1, 4, 9, 16, 25, …**

$s(t) = kt^2$ • Measure the distance a body falls after 1 second: ≈ 4.9 m (3.28 ft), then a body falls k = 4.9

$s(2) = 4k = 19.6m$ • The body speed increases with time

- Average speed = average distance/average time $\boxed{v_{avg} = \dfrac{19.6m}{2\sec} = 9.81m/\sec}$

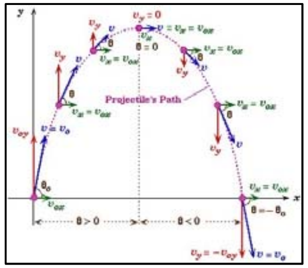
- Problem with instantaneous speed (speed at any given instant) we would get 0/0 !

$$v_{avg} = \frac{s(t+h)-s(t)}{h} = \frac{k(t+h)^2 - kt^2}{h} = 2kt + \cancel{kh} = 2kt = v_{ins}$$

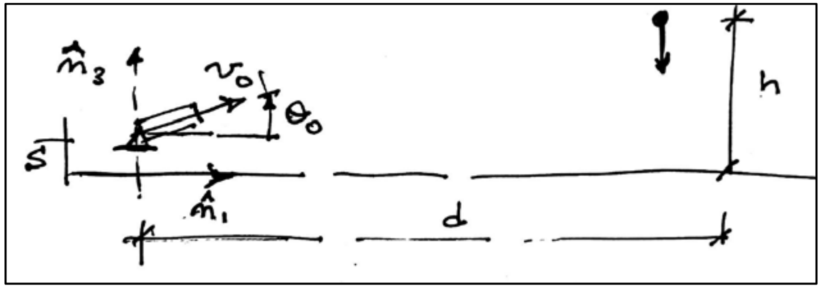
$$a_{avg} = \frac{v(t+h)-v(t)}{h} = \frac{2k(t+h)-2kt}{h} = 2k = a_{ins}$$

$$\boxed{a(t) = 2k = 32.2\frac{ft}{\sec^2} = 9.81\frac{m}{\sec^2} = g}$$

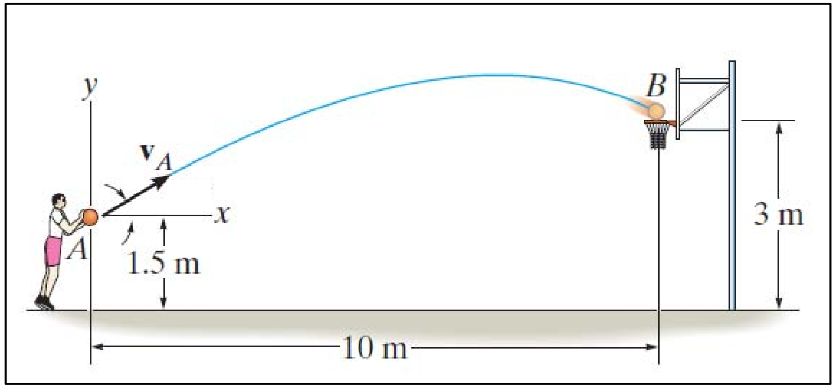
- Example: Motion of a body under gravity




$$x = x_0 + (v_0)_x t$$
$$y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$




$$T_f = \sqrt{\frac{2h}{g}}; \tan\theta_0 = \frac{h-s}{d}; v_0 = \frac{d}{T\cos\theta_0}, T < T_f$$



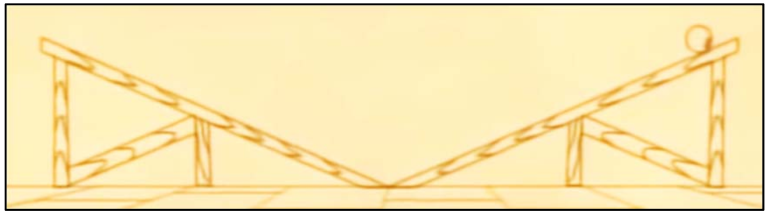
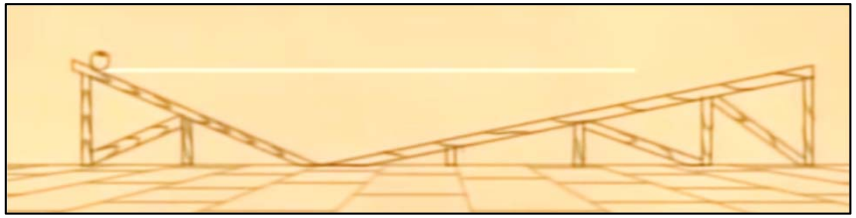

- Find the initial velocity and the angle so that the ball hits the basket after 1 second

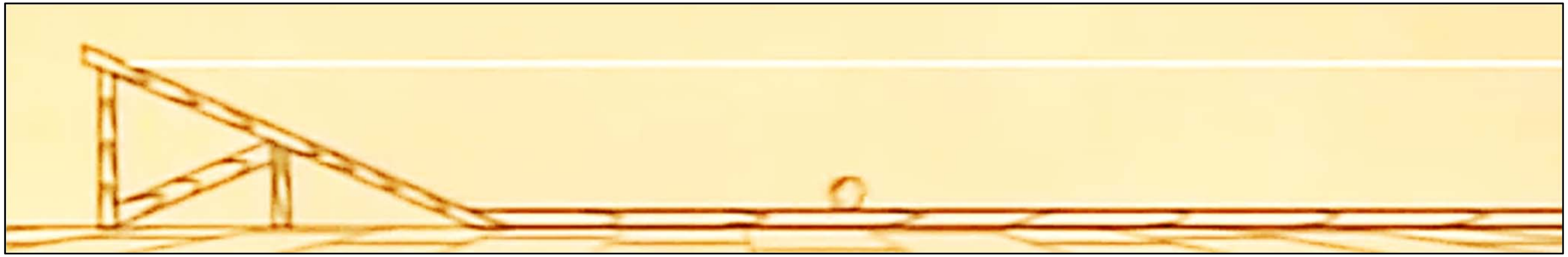
$$x = x_0 + v_{Ax} t \Rightarrow 10 = v_{Ax} t$$
$$y = y_0 + v_{Ay} t - \frac{1}{2} g t^2 \Rightarrow 3 = 1.5 + v_{Ay} t - 4.905 t^2$$

$$10 = v_{Ax}$$
$$6.405 = v_{Ay}$$

$$v_A = \sqrt{v_{Ax}^2 + v_{Ay}^2} = 11.87 \frac{m}{\text{sec}}$$
$$\theta = \tan^{-1}\left(\frac{v_{Ay}}{v_{Ax}}\right) = 32.64 \deg$$

- <u>Law of inertia (vacuum)</u>
  - **Inertia**: is property of a body by virtue of which it opposes any agency that attempts to put it in motion or, if it is moving, to change the magnitude or direction of its velocity.



  - <u>all external impediments removed</u>, a heavy body on a spherical surface concentric with the earth will maintain itself in that state in which it has been; if placed in movement towards the west (for example), it <u>will maintain itself in that movement</u>

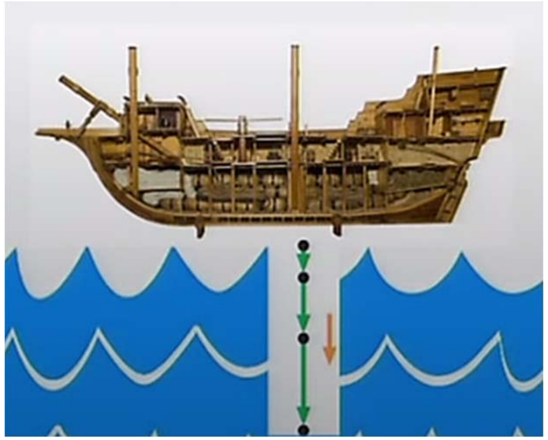
- Galilean transformations (inkling about light)


*"All the motion is relative. Whether a body is at rest or in motion it depends exclusively on the observer"*

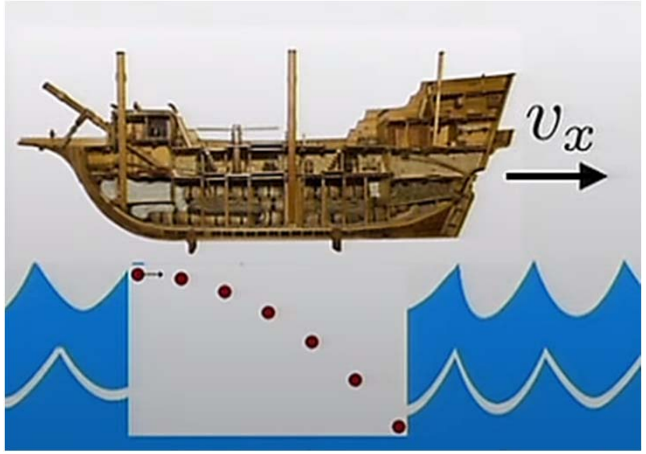
# Classical Mechanics

"Rinserratevi con qualche amico nella maggior stanza che sia sotto coperta di alcun gran navilio, e quivi fate di aver mosche, farfalle e simili animaletti volanti; siavi anche un gran vaso d' acqua, e dentrovi dei pescetti, sospendasi anco in alto qualche secchiello, che a goccia a goccia vadia versando dell' acqua in un altro vaso di angusta bocca, che sia posto in basso: e stando ferma la nave, osservate diligentemente come quelli animaletti volanti con pari velocità vanno verso tutte le parti della stanza, e i pesci si vedranno andar notando indifferentemente per tutti i versi; le stille cadenti entreranno tutte nel vaso sottoposto; e voi, gettando all' amico alcun cosa, non più gagliardamente la dovrete gettare verso quella parte che verso questa, quando le lontananze sieno uguali, e saltando voi, come si dice, a piè giunti, egual spazii passerete verso tutte le parti. Osservate che avrete diligentemente tutte queste cose, benchè niuno dubbio vi sia che mentre il vassello sta fermo non debbano succedere così; **fate muovere la nave con quanta si voglia velocità: ché (pur che il moto sia uniforme e non fluttuante in qua e in là) voi non riconoscerete una minima mutazione in tutti li nominati effetti, né da alcuno di quelli potrete comprendere se la nave cammina oppur sta ferma**..."
(Galileo Galilei, Dialogo sopra i massimi sistemi del mondo, p.196)

**Reference system fixed with the ship**
**Moving at constant speed**



**Reference system fixed with the port (at rest)**



- Conclusions:
  - the motion is relative, and all observers who are in uniform motion are equivalent.
  - Any observer (reference frame) moving at constant speed (or at rest) defines an inertial frame.

  - The laws of Physics are the same in all inertial frames.
  - Time is absolute in all inertial frames
  - Lengths of objects (space) are invariant as they move
  - Mass of an object is invariant

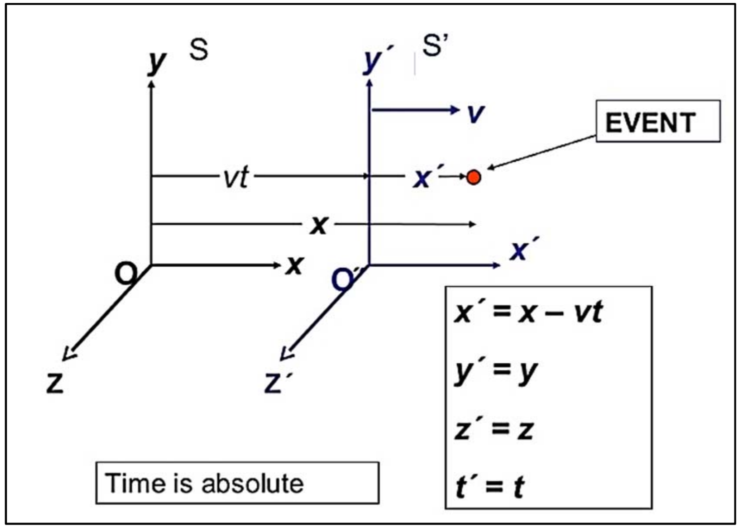
  - Velocities are (of course) different in different inertial frames (Galileo transformations)
  - Positions of objects are different in different inertial systems (coordinate transformation: Galileo, Descartes)
  - Acceleration is the same in different inertial frames

- These conclusions were against Aristotle's postulate (the ball would follow the same path from the mast whether the observer is on the boat or on the port)

- Galilean Transformation and Relativity

1. Consider two inertial reference frames S (at rest) and S’ moving at constant speed v in the x direction. The clocks are synchronized for t = t’ = 0 when the two reference frames are coincident

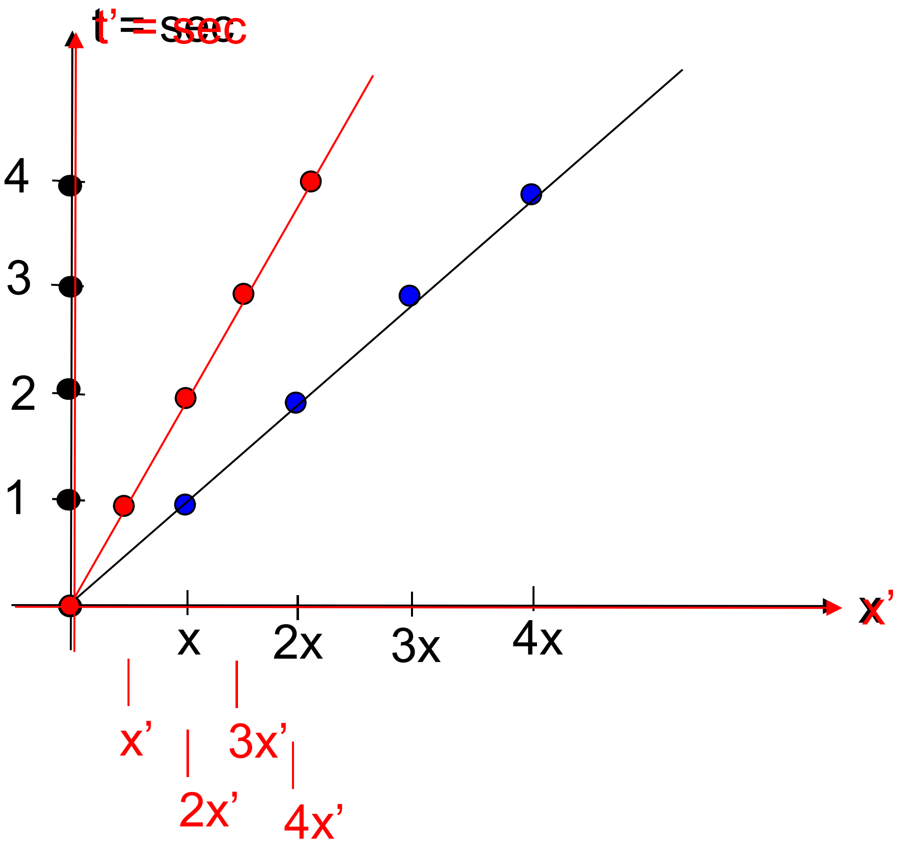
   - an event is the instantaneous physical situation or occurrence associated with a point in spacetime
   - How is the event perceived by the two reference systems?




$$\begin{cases} x' = x - vt \\ \dot{x}' = \dot{x} - v \\ \ddot{x}' = \ddot{x} \\ t' = t \end{cases}$$

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & -v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}$$

2. Consider a Galilean transformation from a different point of view, using the spacetime diagram (one dimensional motion).




1. Reference frame S is inertial at rest, with observer at the origin at time t = 0.
2. As time goes by, the position of observer is given by ●
3. Assume at time t = 0 an event E occurs, which moves at a speed $v_E = x/t = 1$. The position of the event as seen by S is given by ●
4. Reference frame S' is inertial and moves at constant speed $v' = 0.5v_E$. At time t = t' = 0 both frames are coincident ●
5. What is the position of event E now E' as seen from S' ?

- The lines indicating positions and time are called **world lines**.

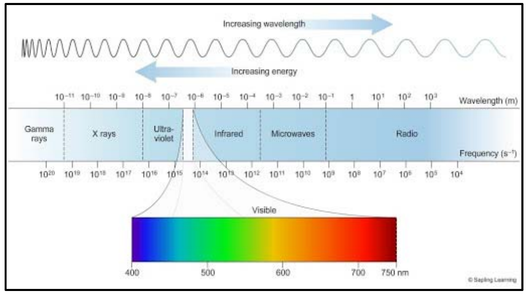
- Galilean transformation/relativity was adopted by Newton and considered the basis in Mechanics until the problem with **LIGHT** appeared

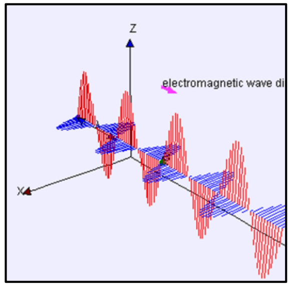
# Relativity Interlude

- Ancient astronomers thought the speed of light to be infinite
- Galileo imagined light to have a VERY HIGH speed
- Roemer (1676) measured the speed of light checking Io eclipse delay in different points of the Earth orbit (200,000 km/s) -> Huygens, Fizeau, Foucault, Bradley (298,000 km\s)

$$\boxed{c = 299{,}792.458 \text{ Km/sec or } 186{,}282 \text{ miles/sec} \approx 3\text{x}10^8 \text{ m/sec in vacuum}}$$

- **James Clerk Maxwell (1831 – 1879):** Theory of Electromagnetism (1865): unification of electricity, magnetism into a single field entity propagating in vacuum at a specific speed (the speed of light)



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \text{ (Faraday's law)}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \boxed{\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}} \text{ (Ampère-Maxwell law)}$$

The divergence of the fields is described by the differential equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \text{ (Gauss' law - electrostatics)}$$

$$\nabla \cdot \mathbf{B} = 0 \text{ (Gauss' law - magnetostatics)}$$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad f = \begin{cases} less\ than\ 1 Hz \\ \approx 10^{25}\ Hz \end{cases}$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

- In a wave equation, the wave propagation speed is given by:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{(8.854 \times 10^{-12})(1.257 \times 10^{-6})}}$$

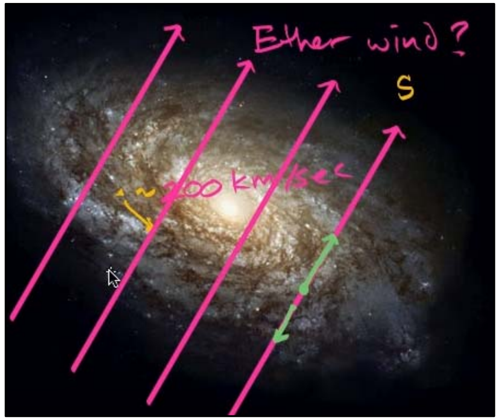
$$\boxed{= 2.998 \times 10^8 \approx 3 \times 10^8\ m/s}$$

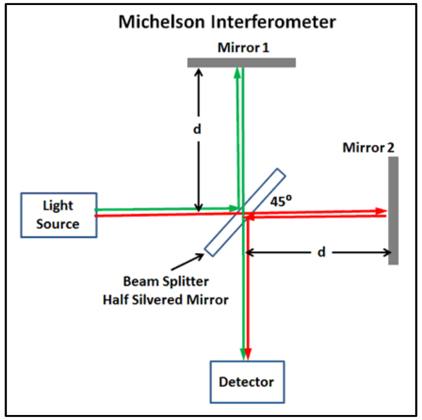
$$\begin{cases} \varepsilon_0 = & 8.8542 \times 10^{-12} & \dfrac{C^2}{N \cdot m}, \\ \mu_0 = & 4\pi \times 10^{-7} & \dfrac{W}{A \cdot m} \end{cases} \Rightarrow V = c = 299{,}792\ km / \sec$$

# Relativity Interlude

- <u>Luminiferous Aether:</u> A transparent fluid that is very stiff like granite to allow for the light's high speed and at the same time inviscid to maintain the stability of the planets motion (The concept was already known in ancient times: see Aristotle), The Aether is at rest and operates as an inertial reference.

- <u>Michelson – Morley Experiment </u>(1887 Cleveland OH)




The experiment compared the speed of light in perpendicular directions in an attempt to detect the relative motion of matter through the stationary luminiferous aether ("aether wind"). The result was negative, in that Michelson and Morley found no significant difference between the speed of light in the direction of movement through the presumed aether, and the speed at right angles.

- **Recall some of the main points up to Special Relativity (as known at the time)**:
  - Laws of physics are the same in all inertial references
  - There is no way of knowing if an inertial reference is at rest or moving at constant speed
  - Mass opposes motion (inertial mass)
  - Space and time are absolute
  - **Since EM theory is general, it should be valid in every inertial reference system**

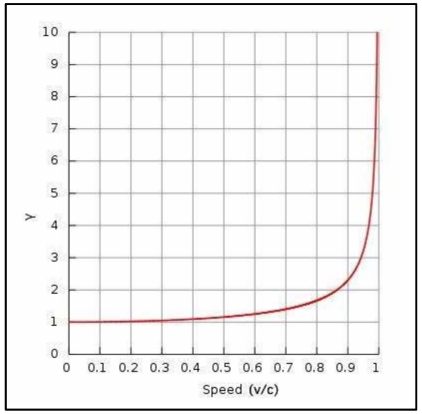
- **Maxwell's equations do not (*):**
  - Gauss' magnetic law satisfies Galilean relativity
  - Faraday's law satisfies Galilean relativity
  - Gauss' electrostatic law does not satisfy Galilean relativity
  - Ampère's law does not satisfy Galilean relativity

**(*) European Journal of Physics 30 (2009) 381-391**

- **Hendrik Lorentz (1853 – 1928)**
  - Lorentz derived a coordinate transformation, in order to show that Maxwell's equations were valid in every inertial reference, starting from Michelson – Morley experiment.
  - The result is the concept of relativity applied to time and space and their interconnection, leading to the new reference frame called **spacetime**.
  - Consider two reference frames F and F' moving with relative constant speed $v$. Assume for simplicity the direction of motion be along the x axis and the frames be coincident at **t = t' = 0**

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \gamma \begin{bmatrix} 1 & -v \\ -\frac{v}{c^2} & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}; \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & -v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}$$



# Relativity Interlude

## ❑ Albert Einstein (1879 – 1955), (On the Electrodynamics of Moving Bodies, June 1905)

- **The Laws of Physics are the same in any Inertial Frame of Reference**. (Such frames move at constant velocities with respect to each other). These Laws include in particular Maxwell's Equations describing electric and magnetic fields, which predict that light always travels at a particular speed $c$.
- It follows that any measurement of the speed of any flash of light by any observer in any inertial frame will give the same answer $c$ (in vacuum). In other words, **all inertial observers measure the same speed of light**, no matter the velocity and direction of their reference system.

1. The concepts of space and time must be defined precisely and inertial observers may measure different distances and times.
2. **There must be a different relativity transformation to yield all laws of physics valid for any inertial observer (Lorentz transformation)**
3. The speed of light is the theoretical maximum allowed in any inertial reference, so that principle of causality is maintained.

**If one of the two events represents the sending of a signal from one location and the second event represents the reception of the same signal at another location, then as long as the signal is moving at the speed of light or slower, the mathematics of simultaneity ensures that all reference frames agree that the transmission-event happened before the reception-event (Causality is satisfied). However, in the case of a hypothetical signal moving faster than light, there would always be some frames in which the signal was received before it was sent so that the signal could be said to have moved backward in time. Because one of the two fundamental postulates of special relativity says that the laws of physics should work the same way in every inertial frame, if it is possible for signals to move backward in time in any one frame, it must be possible in all frames. This means that if observer A sends a signal to observer B which moves faster than light in A's frame but backwards in time in B's frame, and then B sends a reply which moves faster than light in B's frame but backwards in time in A's frame, it could work out that A receives the reply before sending the original signal, challenging causality in every frame and opening the door to severe logical paradoxes (see tachyons paradox).**

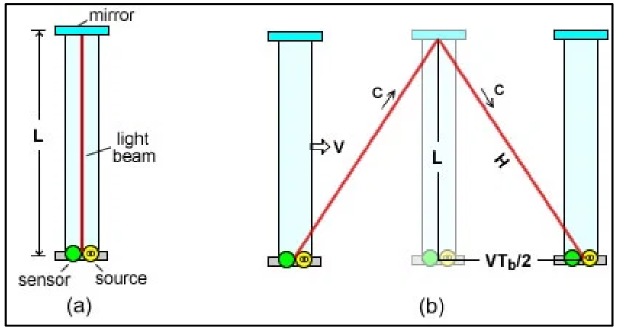
# Relativity Interlude

- **Main consequences of special relativity**
  - Time dilation $\boxed{\Delta\tilde{t} = \Delta t \gamma}$
    - Moving clocks run slower
  - Length contraction $\boxed{\Delta\tilde{l} = \Delta l \frac{1}{\gamma}}$
    - Moving objects appear shorter in the direction of motion
  - New definition of simultaneity

    Lorentz Transformation: The mechanical universe Episode 42

    Muon Example

    Causality




- **Main consequences of general relativity**
  - Motion is warped in spacetime
  - Mass is intrinsically connected to energy and its momentum is not only dependent on speed

# Classical Mechanics

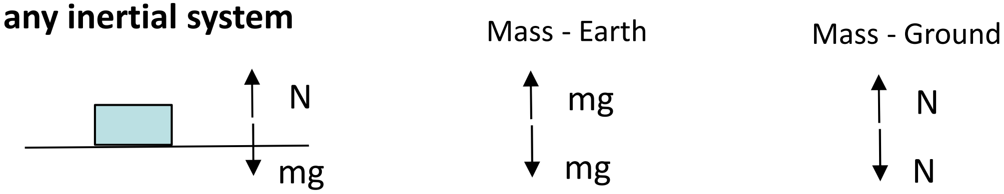
## ❑ Isaac Newton (1643 – 1727)

- Formalization of **classical mechanics** as we know it
- Invention of differential calculus together with Leibniz
- Aristotle stated that an object will reach the rest position unless acted upon by a force

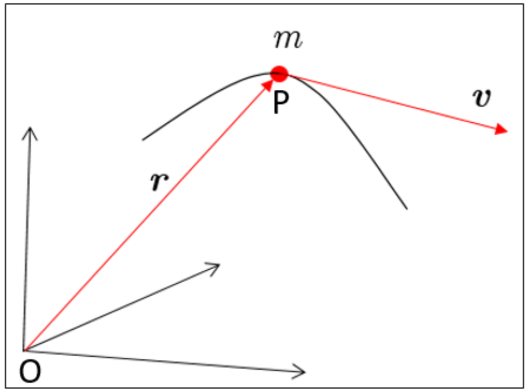
$$x(t) \Rightarrow \dot{x}(t) \Rightarrow \frac{dx}{dt}$$

- **First Law of Motion or Law of inertia**: An object will not change its motion unless acted upon by an unbalanced force. If at rest, it will remain at rest. If in motion, it will remain in motion following a **straight line** with the same velocity
  - Inertia: resistance of any object to changes in its motion
  - First law derived by Galileo's findings
  - It implies Galilean relativity (inertial system and fixed time)
  - **The law is valid in any inertial system**

- **Third Law of Motion:** When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.
  - For every action, there is an equal and opposite reaction
  - Component of conservation of energy
  - Component of Universal Law of Gravitation
  - The **law is valid in any inertial system**



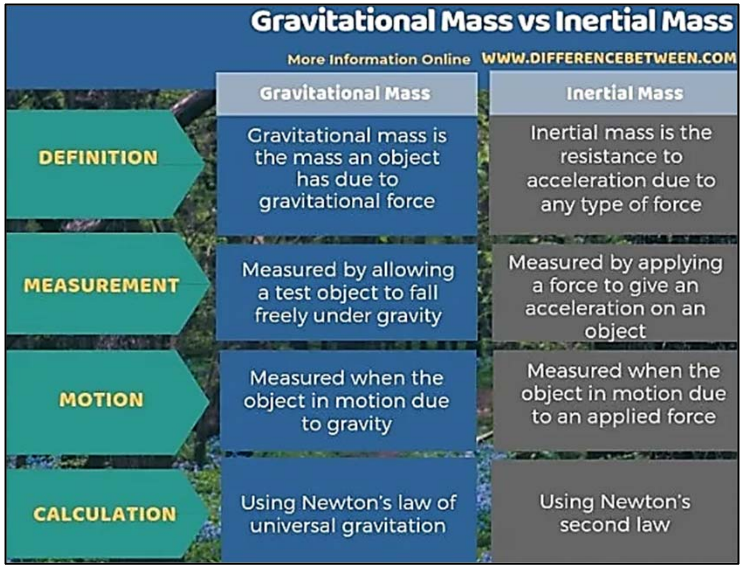
- **Second Law of motion**: In an inertial frame of reference, the vector sum of the forces $\boldsymbol{F}$ on an object is equal to the mass $m_{IN}$ of that object multiplied by the acceleration $\boldsymbol{a}$ of the object: $\boldsymbol{F} = m_{IN}\boldsymbol{a}$.




$$F(t) = m_{IN} \left. \frac{d^2\left[\boldsymbol{r}(t)\right]}{dt^2} \right|_{IN} ; \boldsymbol{r}(t) = P - O$$

- **Mass: amount of "stuff" of which a body is made**

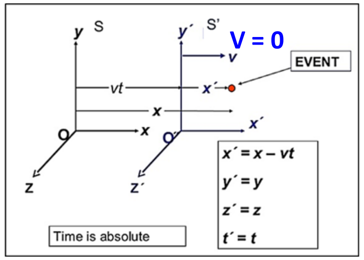
- **Difference between inertial mass and gravitational mass**



# Classical Mechanics

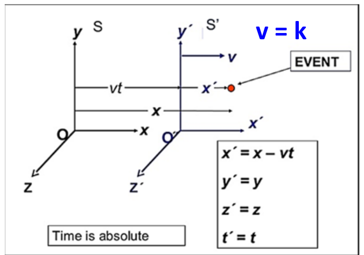
- Newton's Laws are the same in any inertial frame, that is a reference frame moving at constant speed. A non inertial frame is an accelerating or rotating frame

1. Consider two reference frames with different origin




$$x' = x + k$$

$$F = m\frac{d^2(x')}{dt^2} = m\frac{d^2(x+k)}{dt^2} = m\frac{d^2x}{dt^2} + m\cancel{\frac{d^2k}{dt^2}} = m\frac{d^2x}{dt^2}$$ ✓




2. Consider two reference frames in relative motion with constant speed v

$$x' = x + vt$$

$$F = m\frac{d^2(x')}{dt^2} = m\frac{d^2(x+vt)}{dt^2} = m\frac{d^2x}{dt^2} + m\cancel{\frac{d^2vt}{dt^2}} = m\frac{d^2x}{dt^2}$$ ✓

# Classical Mechanics

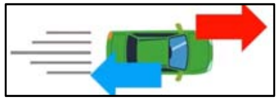
3. Consider an accelerating reference frame

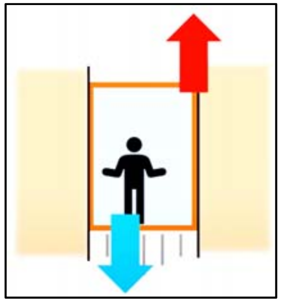
$$x' = x + \frac{1}{2}at^2$$

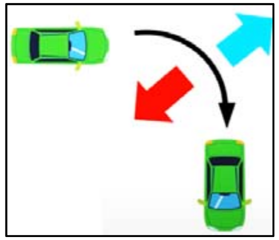
$$F = m\frac{d^2(x')}{dt^2} = m\frac{d^2(x + \frac{1}{2}at^2)}{dt^2} = m\frac{d^2x}{dt^2} + m\frac{d^2(\frac{1}{2}at^2)}{dt^2} = m\frac{d^2x}{dt^2} + ma \neq m\frac{d^2x}{dt^2}$$ ✗

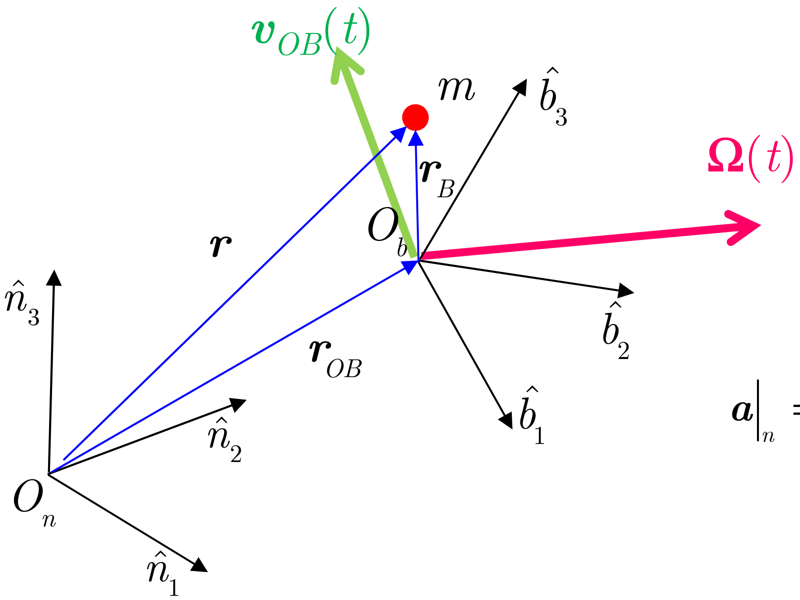
- For the observer in a non inertial reference frame, the laws of motion are not the same

$$\boxed{\vec{F} = m\vec{a} + \vec{F}_{ficticious}}$$









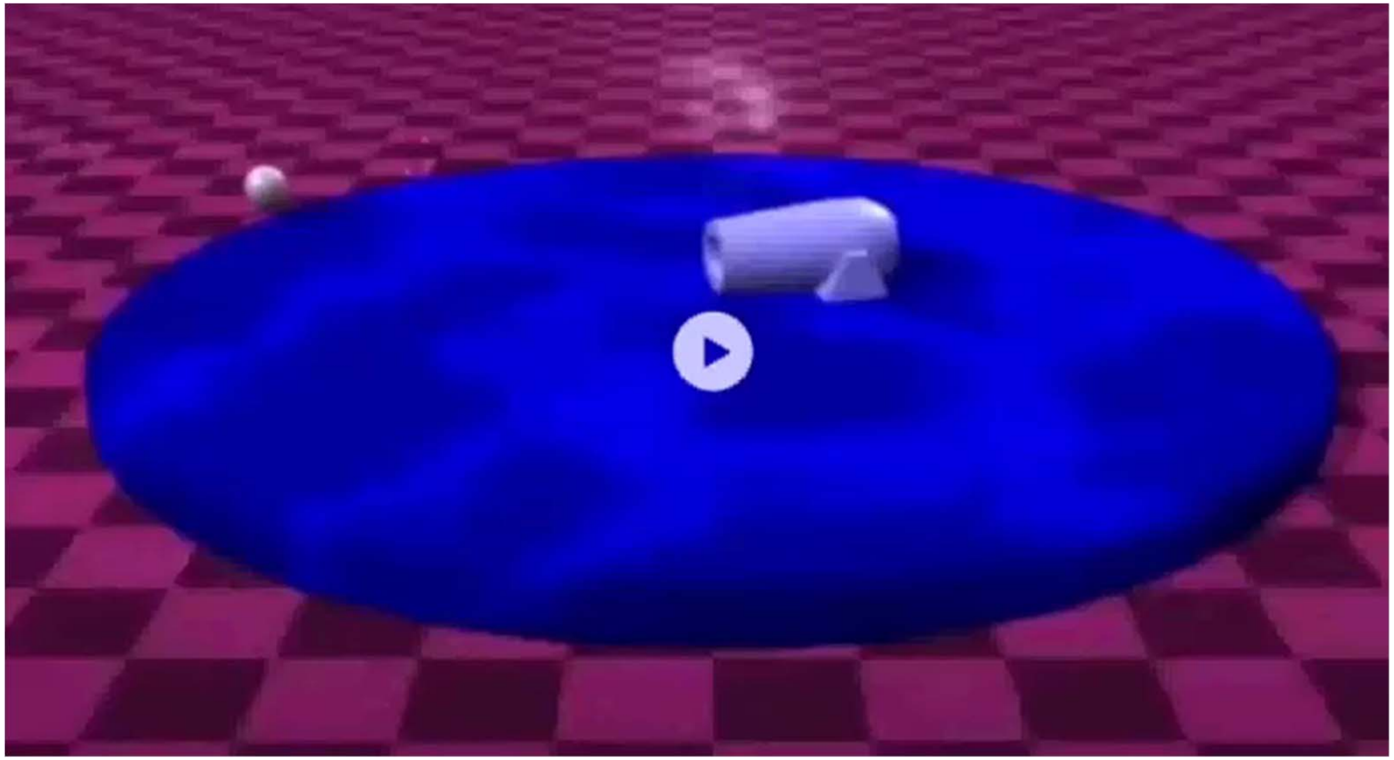

From Kinematics

$$\boldsymbol{v}\big|_n = \boldsymbol{v}_{OB} + \boldsymbol{v}_B + \boldsymbol{\Omega} \times \boldsymbol{r}_B$$

$$\boldsymbol{a}\big|_n = \boldsymbol{a}_{OB} + \boldsymbol{a}_B + \dot{\boldsymbol{\Omega}} \times \boldsymbol{r}_B + 2\boldsymbol{\Omega} \times \boldsymbol{v}_B + \boldsymbol{\Omega} \times \left[\boldsymbol{\Omega} \times \boldsymbol{r}_B\right]$$

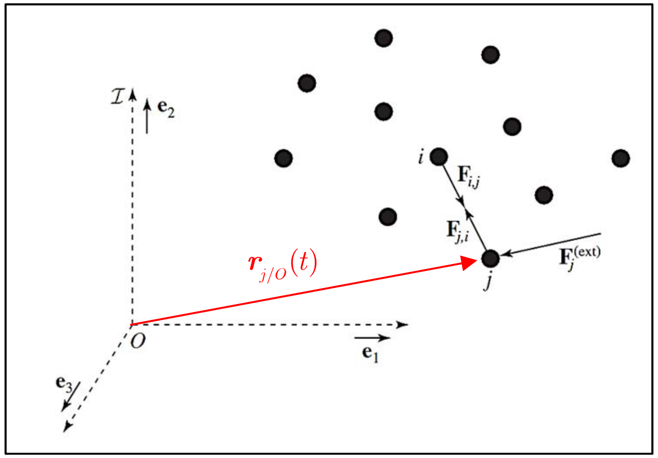
- Coriolis acceleration
- Centripetal acceleration

- **Minimal Review**



# Classical Mechanics

- Extension to a system of particles

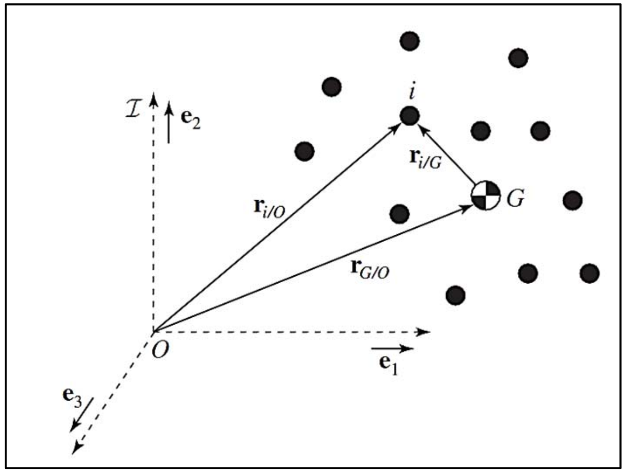



$\boldsymbol{F}_j^{(ext)}$ = external force on particle j

$\boldsymbol{F}_{j,i}$ = internal force on particle j due to particle i ( a spring, etc.)

$$F_j^{(ext)} + \sum_{\substack{i=1 \\ i=j}}^{N} F_{j,i} = m_j \left. \frac{d^2 \left[ \boldsymbol{r}_{j/O} \right]}{dt^2} \right|_{IN}$$

- Recall Newton's third law: $\boldsymbol{F}_{j,i} = -\boldsymbol{F}_{i,j}$

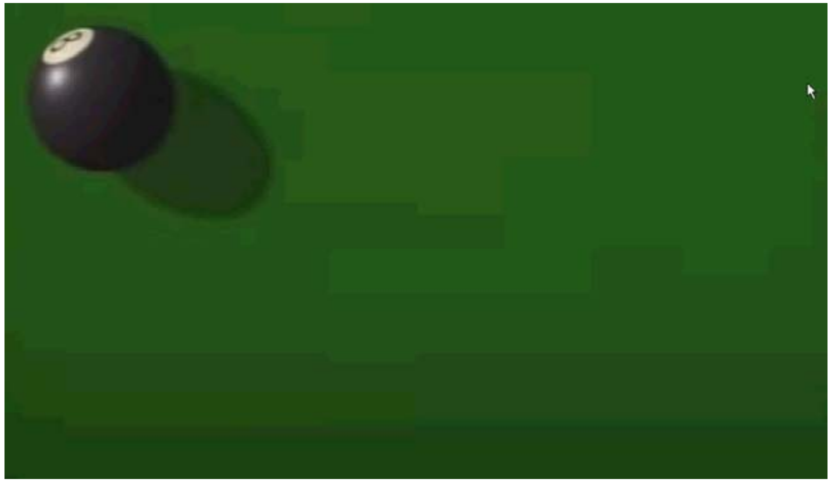


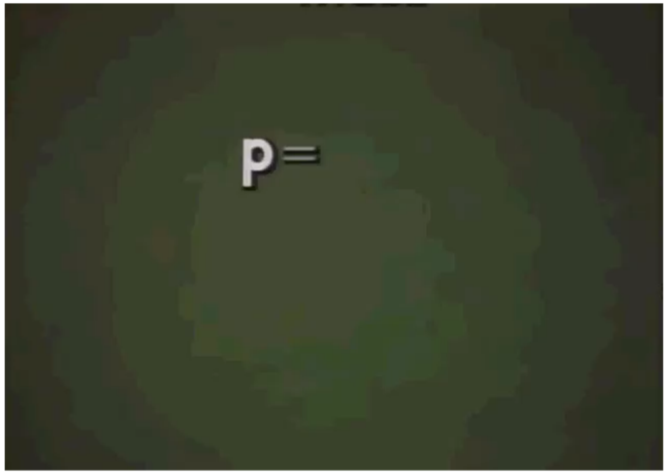

- **Center of Mass:** The center of mass (CoM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero. In analogy to statistics, the center of mass is the mean location of a distribution of mass in space. Center of mass and center of gravity (CoG) are different in general. First introduced by Archimedes

$$\begin{cases} M = \sum_i m_i \\ \boldsymbol{r}_{G/O} = \dfrac{1}{M} \sum_i m_i \boldsymbol{r}_{i/O} \end{cases}$$

- Linear momentum of a system of particles

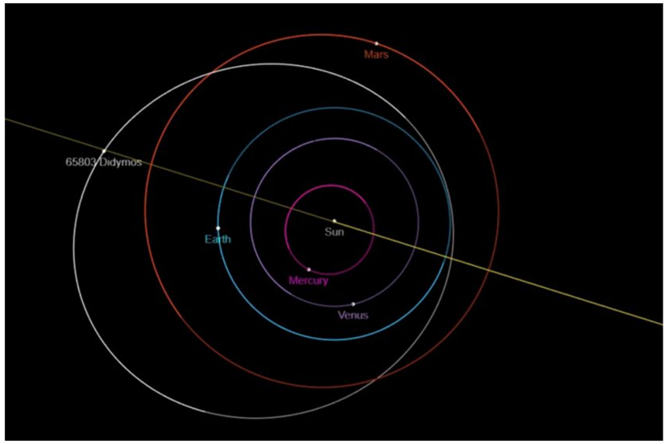
  - For a generic particle i: $$\boldsymbol{p}_i = m_i \left.\frac{d\left[\boldsymbol{r}_{i/O}\right]}{dt}\right|_{IN} = m_i \left.\boldsymbol{v}_{i/O}\right|_{IN}$$






https://www.youtube.com/watch?v=C0XNyTp5brM

- DART Mission


Didymos orbits the Sun at a distance of 1.0–2.3 AU once every 770 days (2 years and 1 month). Its orbit has an eccentricity of 0.38 and an inclination of 3° with respect to the ecliptic. Its approach to Earth in November 2003 was especially close (in relative terms), with a distance of 7.18 million km; it will not come that near until November 2123, with a distance of 5.9 million km. Didymos also occasionally passes very close to Mars: it will fly by Mars at a distance of 4.69 million km in 2144.[1]

https://www.youtube.com/watch?v=iPKvnmaE94s

https://www.youtube.com/watch?v=KK69hej1A6w

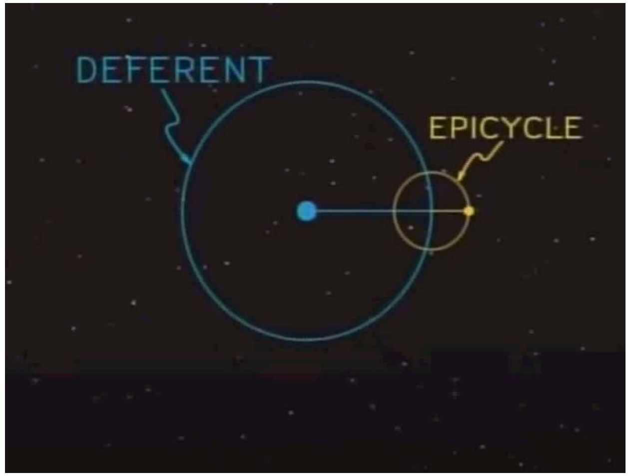
# Classical Mechanics

Nicolaus Copernicus → Tycho Brahe → **Johannes Kepler** → Isaac Newton

Nicolaus Copernicus → Galileo Galilei → Isaac Newton

Copernicus

1. The planetary orbit is a circle with epicycles.
2. The Sun is approximately at the center of the orbit.
3. The speed of the planet in the main orbit is constant.

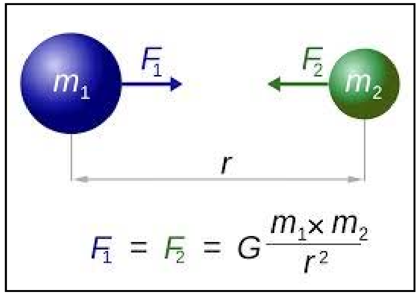



Kepler

1. The planetary orbit is not a circle with epicycles, but an <u>ellipse</u>.
2. The Sun is not near the center but at a <u>focal point</u> of the elliptical orbit.
3. Neither the linear speed nor the angular speed of the planet in the orbit is constant, but the area speed (closely linked historically with the concept of <u>angular momentum</u>) is constant.

- **Newton's Law of Gravitation:** Two particles (bodies) of mass m1 and m2, separated by a distance r, attract each other with a force of magnitude

$$\begin{cases} F = \dfrac{Gm_1m_2}{r^2} \\ G = 6.67259 \times 10^{-11} Nm^2 \,/\, Kg^2 \end{cases}$$



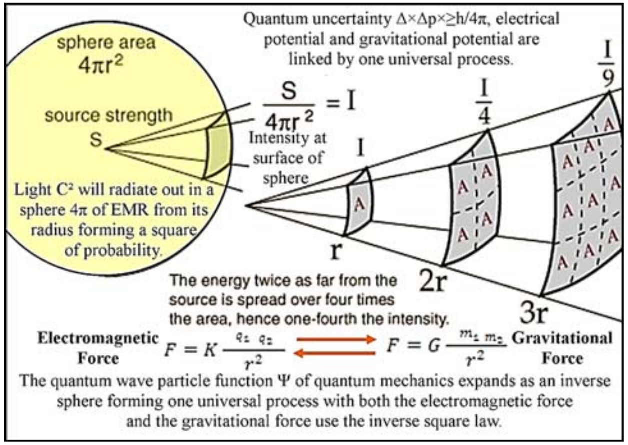

- In vector form, the gravity force acting on mass $m_2$ by $m_1$, with $\hat{\boldsymbol{r}} = \dfrac{\boldsymbol{r}}{|\boldsymbol{r}|}$ the unit position vector of $m_2$ relative to $m_1$ is given by:

$$\boldsymbol{F} = -\frac{Gm_1m_2}{r^2}\boldsymbol{r} = -\frac{Gm_1m_2}{r^3}\hat{\boldsymbol{r}}; \left|\boldsymbol{F}\right| = \frac{Gm_1m_2}{r^2}$$

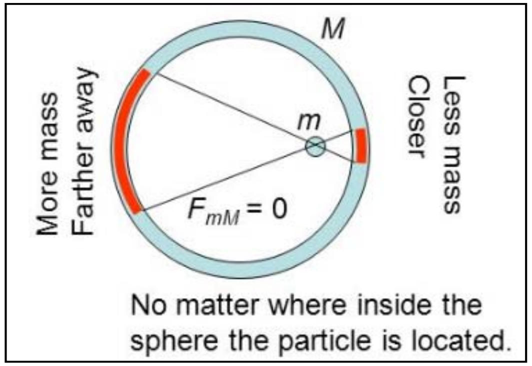
1. During the '600 it was generally accepted that forces generated by **point masses** in electrostatics and magnetism followed the inverse squared law
2. Work by Hooke implied that "gravity" would be a force following the same law
3. Newton unified the theory using his three laws of motion simultaneously proving Kepler's laws and Galileo's falling body results.
4. Newton used his shell theorem (guscio sferico) and calculus, proving that inertial and gravitational mass are the same.

# Classical Mechanics

- In physics, an **inverse-square law** is any physical law stating that a specified physical quantity or intensity is inversely proportional to the square of the distance from the source of that physical quantity. The fundamental cause for this can be understood as geometric dilution corresponding to point-source radiation into a uniform three-dimensional space. Being strictly geometric in its origin, the inverse square law applies to diverse phenomena. Point sources of gravitational force, electric field, light, sound or radiation obey the inverse square law.




- **Newton's Shell Theorem**: the theorem is part 2 of the law of gravitation.
  - A spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point at its center.
  - If the body is a spherically symmetric shell, no net gravitational force is exerted by the shell on any object inside, regardless of the object's location within the shell.



# Classical Mechanics

$$\begin{cases} F = \dfrac{G m_1 m_2}{r^2} \\ G = 6.67259 \times 10^{-11} Nm^2 \,/\, Kg^2 \end{cases}$$

- Newton's Law of Gravitation, attractive
- G = universal constant of gravity

$$\begin{cases} F = \dfrac{K_e q_1 q_2}{r^2} \\ K_e = 9 \times 10^9 Nm^2 \,/\, C^2 (As)^2 \end{cases}$$

- Coulomb's Law of electrostatics, attractive or repulsive
- $K_e$ = Coulomb's constant (permittivity in vacuum)

$$\begin{cases} F = \dfrac{K_m q_{m1} q_{m2}}{4\pi r^2} \\ K_m = 4\pi \times 10^{-7} Tm \,/\, A \end{cases}$$

- Force between two <u>pointsize</u> magnetic poles
- $K_m$ = Permeability constant in vacuum
- Tesla measures the density of magnetic flux

$$1T = 1\frac{N}{Am} = 1\frac{Kg}{As^2}$$

- **The Apple and the Moon:** If the apple falls to the ground, why the moon does not?

$$\begin{cases} F = \dfrac{G m_g M_E}{R_E^2} \\ F = m_{in} a \end{cases}$$

- <u>Consequences:</u>

1. All bodies fall to Earth with a constant acceleration that does not depend on the mass
2. Inertial and gravitational mass are the same

$$a = g = \frac{GM_E}{R_E^2} = 9.8m/s^2 = 32.2ft/s^2$$

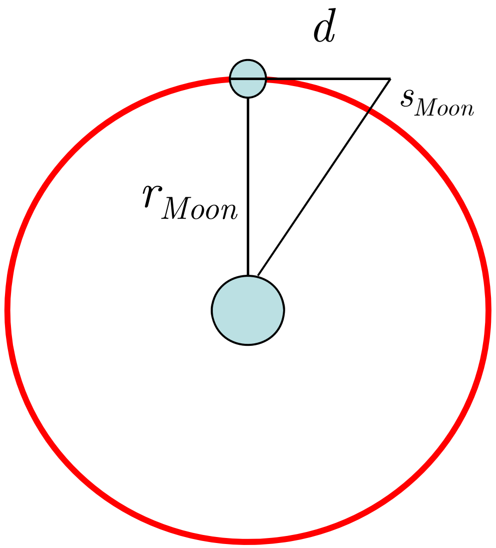
$$x = x_0 + (v_0)_x t$$
$$y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$

1. A body dropped from a height of 16.1 ft (4.9 m) reaches the ground in 1 second (Galileo)
2. A body with initial velocity of 32.2 ft/sec (9.8 m/sec) reaches the ground traveling 32.2 ft (9.8 m) after 1 second (Galileo)
3. If the initial velocity were large enough (11.19 km/s) then the body would never fall! (escape velocity, orbit)

$$\frac{a_{Moon}}{g} = \left(\frac{R_E}{R_{Moon} = 60R_E}\right)^2 = \left(\frac{1}{60}\right)^2$$

- So in 1 second the moon falls 16x12/3600 inches, compared to an apple (16x12 inches), that is 1/20 of an inch or 1.27 mm

# Classical Mechanics




$$\frac{2\pi r_{Moon}}{1\,month} = speed$$

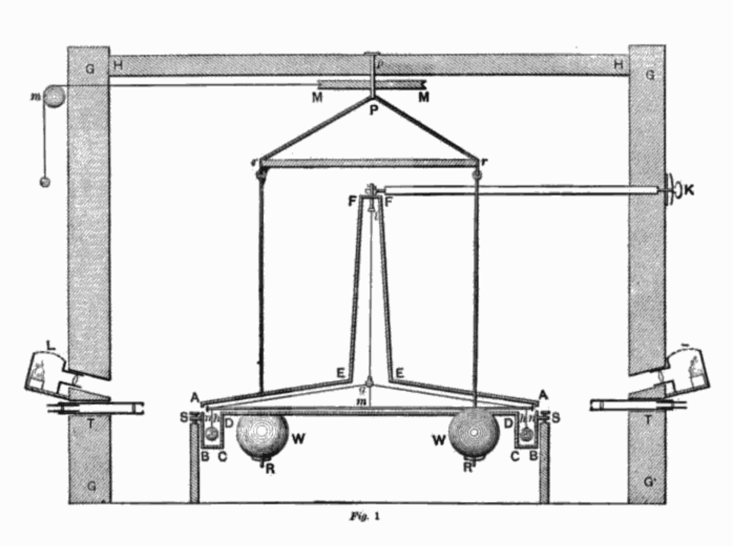
- From the law of inertia, the Moon "wants" to move in a straight line with the above speed
- In 1 second the straight distance covered is d = 40,281 inches (1,023.14 m)

$$(s_M + r_M)^2 = r_M^2 + d^2$$

$$s_M^2 + 2s_M r_M + r_M^2 = r_M^2 + d^2 \Rightarrow d^2 \approx 2s_M r_M$$

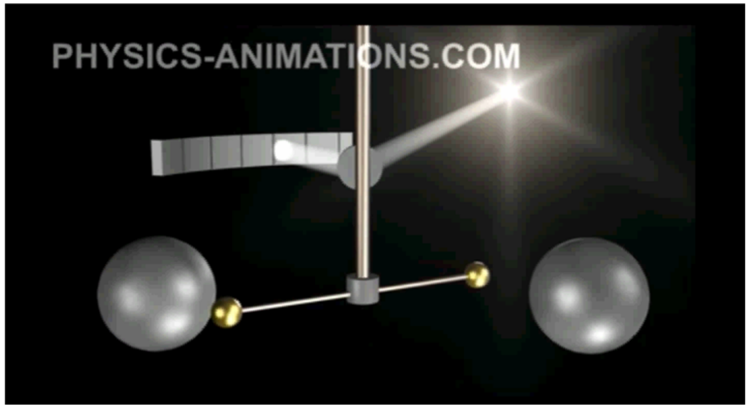
- The Moon, in its orbit around Earth "falls" of an amount **$s_M$ = 1/20 of an inch or 1.27 mm**
- The apple dropped from a height of 16.1 ft (4.9 m or 16x12 inches) reaches the ground in 1 second

- **Cavendish Experiment**: (circa 1797 – 1798) $G = 6.67259 \times 10^{-11} Nm^2 / Kg^2$




The apparatus constructed by Cavendish was a torsion balance made of a six-foot (1.8 m) wooden rod suspended from a wire, with a 2-inch (51 mm) diameter 1.61-pound (0.73 kg) lead sphere attached to each end. Two 12-inch (300 mm) 348-pound (158 kg) lead balls were located near the smaller balls, about 9 inches (230 mm) away, and held in place with a separate suspension system. The experiment measured the faint gravitational attraction between the small balls and the larger ones.

- Under the assumption that the physics of type 1A supernovae are universal, analysis of observations of 580 type Ia supernovae has shown that the gravitational constant has varied by less than one part in ten billion per year over the last nine billion years




- Once the gravitational constant was measured, it was possible to determine the mass of the Earth since $g$ and $R_E$ were known

$$M_{\oplus} = (5.9722 \pm 0.0006) \times 10^{24} \text{ kg}.$$

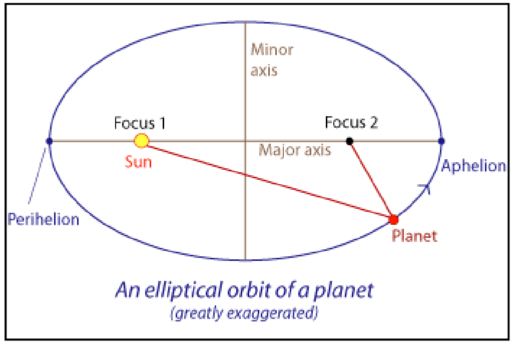
## Comments:

- Formal development of the inertia laws of motion based on infinitesimal calculus (Leibniz)
- Proof of Galileo’s falling body proposal and motion of planets, thus explaining Kepler’s laws
- Definition of inertial reference as basis of mechanics of bodies, whereby the laws of physics are the same with respect to any inertial frame
- Use of Galilean relativity hypothesis (the motion is relative, all inertial frames are the same)
- Time is absolute, space is absolute (in an inertial frame)

- Did not explain the instantaneous nature of gravity and the generation of gravitational force at a distance
- Law of gravitation did not explain Mercury’s orbit anomaly (perihelium precession)
- It required a medium in order to verify the third law in space (third law operates by contact)
- Light is a wave that moves at infinite speed (based on current knowledge of the times)

# Keplerian Mechanics

- **Basic Comment:** unless otherwise stated, we will study the motion of celestial bodies and artificial satellites under mutual gravitational attraction limited to TWO bodies, according to Newtonian Mechanics



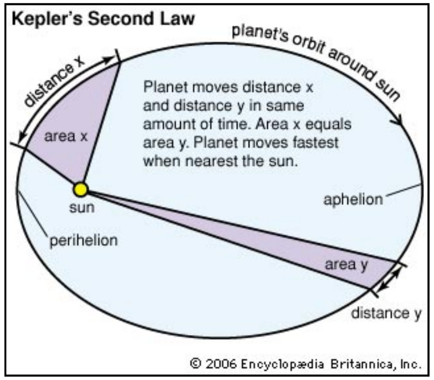

An elliptical orbit of a planet
(greatly exaggerated)

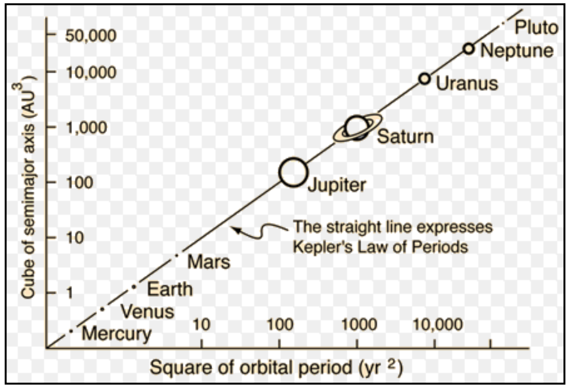
- **Kelper's Observations (for celestial bodies):**

**Law 1:** Planets move in elliptic orbits with one focus at the planet they orbit.

**Law 2:** Planets sweep out equal areas of the ellipse in equal time.

**Law 3:** The square of the period of the orbit is proportional to the cube of the semi-major axis.



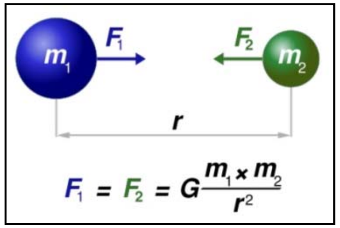

$$\Pi^2\,(\boldsymbol{years}) = a^3\,(\boldsymbol{astr.\,units})$$

**1 AU ~ 1.5x10$^6$ Km mean distance Earth - Sun**

[FEYNMAN LOST LECTURE]

- **Newton's Law of universal Gravitation**

N-Body problem




**Cavendish Experiment 1797**

"Big G" $G = 6.67 \times 10^{-11} N\ m^2 / kg^2$

- **Statement:** a particle attracts every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- **Gravitational two-body problem:** is the motion of two mass point particles that interact only with each other, due to gravity. This means that influences from any third body are neglected.
- **Central-force problem:** to determine the motion of a particle under the influence of a single central force. A central force is a force that points from the particle directly towards (or directly away from) a fixed point in space, the center, and whose magnitude only depends on the distance of the object to the center. In many important cases, the problem can be solved analytically, i.e., in terms of well-studied functions such as trigonometric functions. Examples include gravity and electromagnetism as described by Newton's law of universal gravitation and Coulomb's law, respectively.

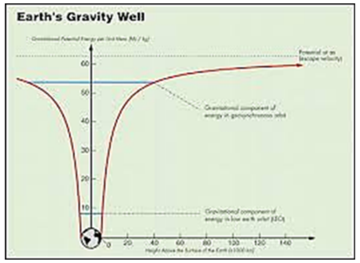
https://www.youtube.com/watch?v=et7XvBenEo8 NBODY

# Newton Mechanics

- A conservative system is a system in which work done by a force is
  - 1. Independent of path.
  - 2. Equal to the difference between the final and initial values of an energy function.
  - 3. Completely reversible.

- The two most notable conservative systems are gravitational and electric fields. With gravity for example, the gravitational potential energy acquired or lost by a mass depends only on the difference between heights (or between distances from the origin of the force), and not on the path taken to get from one state to the other.

## ❑ Main Properties of a Gravitational Field




$$\boldsymbol{F} = -\frac{Gm_1 m_2}{r^3}\boldsymbol{r};\ \left|\boldsymbol{F}\right| = \frac{Gm_1 m_2}{r^2}$$

- The total mechanical (specific) energy is constant:

$$\begin{cases} m_1 \gg m_2 \\ E_{SPEC} = \left(\dfrac{v^2}{2} - \dfrac{\mu}{r}\right), \mu = Gm_1 \Rightarrow \dfrac{dE_{SPEC}}{dt} = 0 \end{cases}$$

- The total (specific) angular momentum is constant:

$$\begin{cases} \boldsymbol{h}_{SPEC} = \boldsymbol{r} \times \dot{\boldsymbol{r}} \\ \dfrac{d\boldsymbol{h}_{SPEC}}{dt} = 0 \end{cases}$$

- The total linear momentum is constant:

$$(m_1 + m_2)\dot{\boldsymbol{r}}_{cm} = \mathbb{C}$$

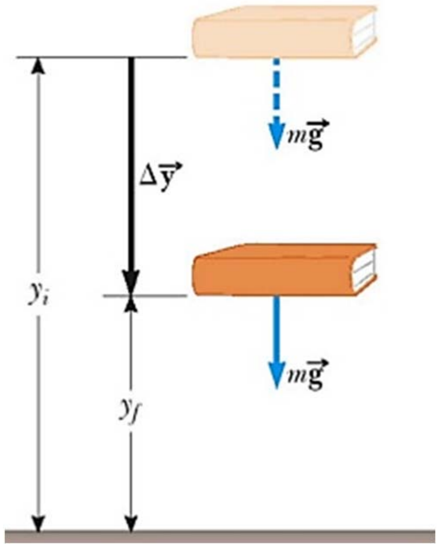
# Newton Mechanics

- The gravitational potential $\boldsymbol{U}$, generates a potential energy $\boldsymbol{V}$

$$V = -U = -\frac{Gm_1 m_2}{r}$$

$$U = \int \boldsymbol{F}(\boldsymbol{r}) d\boldsymbol{r}$$

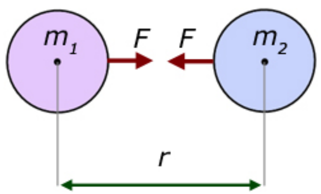
- Potential energy is a state of the system, a way of storing energy as of virtue of its configuration or motion, while work done in most cases is a way of chaining this energy from one body to another. When a body is dropped from a height, its gravitational energy a virtue of its configuration with respect to the Earth is converted into kinetic energy a virtue of its motion, due to the work done by gravity in bringing it down.




- Variation of potential energy due to height for a mass m (relative measure w.r.t. Earth).

$$\Delta V = mg\Delta y$$

- Seek a more general expression using Newton's law of gravitation




$$F = G\frac{mM}{r^2}$$

# Newton Mechanics

- The gravitational potential energy of mass $m$ in the gravitational field of $M$ is therefore

$$V = mG\frac{M}{r^2}r = G\frac{mM}{r}$$

- **Note:** there is a paradox in that, according to the above, the potential energy decreases the higher the mass $m$ is (!). So the convention is a sign change that makes the potential energy always negative, but with a maximum value of zero at the surface.

$$V = -G\frac{mM}{r}$$

❑ **Example:** How much potential energy is needed to lift a spacecraft of mass m = 9000 Kg, to an altitude of 400 Km?

$$\begin{cases} M_E = 5.96 \cdot 10^{24}\,Kg \\ R_E = 6.37 \cdot 10^3\,m \\ G = 6.674 \cdot 10^{-11}\,m^3 Kg^{-1} s^{-2} \end{cases}$$

$$\Delta V = V_{400} - V_0 = -G\frac{mM_E}{R_E + 400} - \left[-G\frac{mM_E}{R_E}\right] = 3.32 \cdot 10^{10}\,J$$

- Avg. Italian family consumption: 225 Kwh/month = $0.81 \cdot 10^9$ J = 41 months

- **Example:** Compute <u>escape velocity</u> (velocità di fuga) from the surface of $M$. The escape velocity $\boldsymbol{v_{ESC}}$ is the minimum velocity necessary to escape the gravity of a mass, in general it refers to planets and stars.

$$\begin{cases} t = 0 & v_1 = v_{ESC} \\ t \to \infty & v_2 = 0 \end{cases}$$

- Apply conservation of mechanical energy

$$\frac{1}{2} m v_{ESC}^2 - G \frac{mM}{r_1} = 0 - G \frac{mM}{\infty} = 0$$

1. Escape velocity from Earth: $v_{ESC}$ = 11.29 Km/sec = 40,000 Km/h

2. Escape velocity from the Sun: $v_{ESC}$ = 42 Km/sec = $1.68 \cdot 10^6$ Km/h

   - the velocity of rotation of the Earth at the equator is 1.7 Km/sec. The velocity of revolution of the Earth about the Sun is 30 Km/sec

## Equations of motion for variable mass

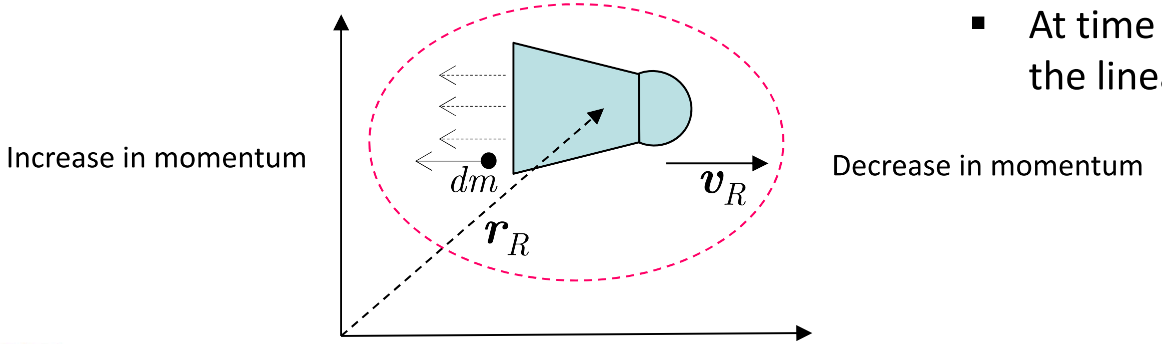
$$\boldsymbol{F} = \left.\frac{d\boldsymbol{p}}{dt}\right|_{In} = \left.\frac{d(m\boldsymbol{v})}{dt}\right|_{In}$$

When the mass changes with time (decrease or increase), application of Newton's law requires recalling the principle of conservation of linear momentum.

$$\boldsymbol{F} = \left.\frac{d\boldsymbol{p}}{dt}\right|_{In} = \left.\frac{d(m\boldsymbol{v})}{dt}\right|_{In} = \frac{dm}{dt}\boldsymbol{v} + m\frac{d\boldsymbol{v}}{dt} = \dot{m}\boldsymbol{v} + m\boldsymbol{a}$$

**Paradox:** when the external force is zero, if the mass is at rest it remains at rest, however, if the mass is moving with constant velocity $\boldsymbol{v}$, then it also accelerates due to a "force" equal to $-\boldsymbol{v}dm/dt$ !

- Consider the case of mass **Loss (and no external forces)**

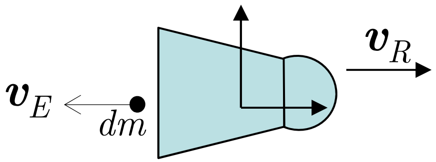



- At time t, the particles expelled change the linear momentum of the **system**

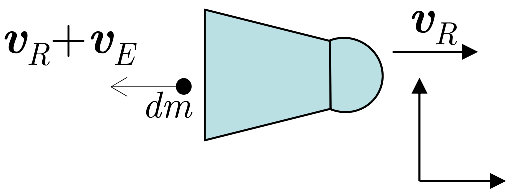
$$\boldsymbol{r}_R, \boldsymbol{v}_R, m_R(t)$$

# Newtonian Mechanics

Since the mass (fuel) is expelled, we can define an exhaust velocity (velocità di scarico): $\boldsymbol{v}_E$ of a particle $dm$




From a non inertial observer




From an inertial observer

- At time t: $\boldsymbol{r}_R, \boldsymbol{v}_R, m_R(t)$
- What is the mass of the rocket at time t+dt? $$m_{R_{t+dt}} = m_{R_t} - dm$$
- Change in mass in the interval dt: $$\frac{dm_R}{dt} = -\frac{dm}{dt}$$

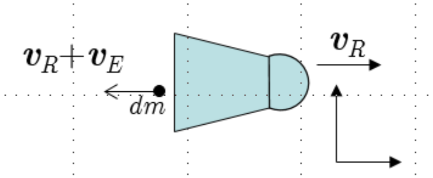
# Newtonian Mechanics

- How does the linear momentum of the system change? (from an inertial reference)

  - At time t: $\boldsymbol{p}_{SYS_t} = m_R \boldsymbol{v}_R$
  - At time t+dt: $\boldsymbol{p}_{SYS_{t+dt}} = \boldsymbol{p}_{SYS_t} - dm(\boldsymbol{v}_E + \boldsymbol{v}_R)$

$$\boxed{\frac{d\boldsymbol{p}_{SYS}}{dt} = -\frac{dm}{dt}(\boldsymbol{v}_E + \boldsymbol{v}_R)}$$

- **dm/dt is called mass flow rate (portata di massa)**




- **Rewrite the equation using the state variables** $\boldsymbol{v}_R, m_R(t)$

$$\frac{d\boldsymbol{p}_{SYS}}{dt} = \frac{d(m_R \boldsymbol{v}_R)}{dt} = \frac{dm_R}{dt}\boldsymbol{v}_R + m_R \frac{d\boldsymbol{v}_R}{dt} = \frac{dm_R}{dt}(\boldsymbol{v}_E + \boldsymbol{v}_R)$$

$$m_R \frac{d\boldsymbol{v}_R}{dt} = \frac{dm_R}{dt}\boldsymbol{v}_E$$

- Some comments:

$$m_R \frac{d\boldsymbol{v}_R}{dt} = \frac{dm_R}{dt}\boldsymbol{v}_E \qquad \frac{Kg}{\sec}\frac{m}{\sec} = Kg\frac{m}{\sec^2} = N$$

$$+ \quad + \qquad - \quad -$$

1. Check the signs of the terms:

Acceleration to the right

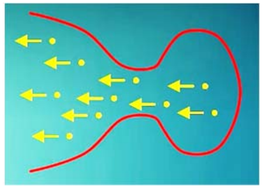
2. How does the vehicle acceleration change?

$$m_R \uparrow \Rightarrow \frac{d\boldsymbol{v}_R}{dt} \downarrow$$

$$\frac{dm_R}{dt} \uparrow \text{ and / or } \boldsymbol{v}_E \uparrow \Rightarrow \frac{d\boldsymbol{v}_R}{dt} \uparrow$$

# Newtonian Mechanics

- Tsiolkovski Equation (Rocket Equation)



$$m_R \frac{d\boldsymbol{v}_R}{dt} = \frac{dm_R}{dt}\boldsymbol{v}_E \qquad m_R d\boldsymbol{v}_R = dm_R \boldsymbol{v}_E$$

$$\int_{m_R^I}^{m_R^F} \frac{dm_R}{m_R} = \boldsymbol{v}_E \int_{\boldsymbol{v}_R^I}^{\boldsymbol{v}_R^F} d\boldsymbol{v}_R \quad \Rightarrow \quad \boxed{\Delta\boldsymbol{v} = \boldsymbol{v}_R^F - \boldsymbol{v}_R^I = \boldsymbol{v}_E \ln\frac{m_R^I}{m_R^F}}$$

- Other forms

$$\boxed{\Delta\boldsymbol{v} = I_{SP} g_0 \ln\frac{m_R^I}{m_R^F}} \qquad \boxed{F_{th} = g_0 I_{SP}\dot{m}}$$

- **Specific Impulse $I_{SP}$ (sec):** Efficiency of the propulsion system. How many seconds the propellant can accelerate its own initial mass at the gravity level of $g_0$ = 1g. The higher the specific impulse the more acceleration can be obtained (for the same mass).
- How many seconds 1 Kg of fuel produces 1 N of thrust

# Newtonian Mechanics

- Some examples of $I_{SP}$ :
  - $H_2$ - $O_2$ = 380 sec
  - Hydrazine = 240 sec
  - Ion Prop. = 1500 – 4000 sec

- Some examples of $\boldsymbol{v}_E$ :
  - $H_2$ - $O_2$ = 4.4 Km/sec
  - Solid rocket = 2.7 Km/sec

- Velocity needed:
  - To go to orbit = 8 Km/sec
  - To go to the Moon = 14 Km/sec
  - To go to Mars = 16 Km/sec

- Propellant needed to go to orbit:
  - $H_2$ - $O_2$ = 83% propellant mass
  - Solid rocket = 96% propellant mass

- Comparison of vehicles:
  - % Propellant
  - Train = 7%
  - Aircraft = 40%
  - Rocket = 85%
  - Molotov = 54%

  - % Payload
  - Train = 30% - 60%
  - Aircraft = 20% - 40%
  - Rocket = 2% - 4%
  - Molotov = 0

## Comments and Assumptions :

1. The equations of motion refer to a rigid body
2. The motion description needs modification to account for origin of the body – fixed reference not at the center of mass
3. The equations of motion contain additional terms if the mass and/or inertia are time varying
4. Mass and/or Inertia have specific geometries and density distribution depending on a particular rigid body
5. Different inertial systems and non inertial systems may be used depending on the vehicle, and/or application
6. Applied forces and moments may be different and of different relative importance depending on the vehicle and/or application
7. Singularities in the motion description may be present due to the Euler angles definition (alternative use of Quaternions)
8. Depending on the operational envelope of the vehicle, linearization is possible
9. No flexibility of the body is considered

| | |
|---|---|
| Linear Momentum | Quantità di moto |
| Point Mass | Punto materiale |
| Angular Momentum | Momento della quantità di moto |
| Center of Mass >(Gravity) | Centro di massa (di gravità) |
| Rigid Body | Corpo rigido |
| Speed, Velocity | Modulo della velocità, Velocità |
| Elevation Angle | Angolo di elevazione |
| Bank Angle | Angolo di inclinazione |
| Yaw, Pitch, Roll | Imbardata, Beccheggio, Rollio |
| Dyadic, Inertia Tensor | Diadica, tensore d'inerzia |
| Quaternion | Quaternione |
| Universal Gravitational Constant | Costante di gravitazione universale |

305 m

[Have you ever seen the Moon?]

[FAST Radio Telescope Short]

[FAST Radio Telescope Long]