Low-energy Excitation Spectrum of 1D Dipolar Quantum Gases

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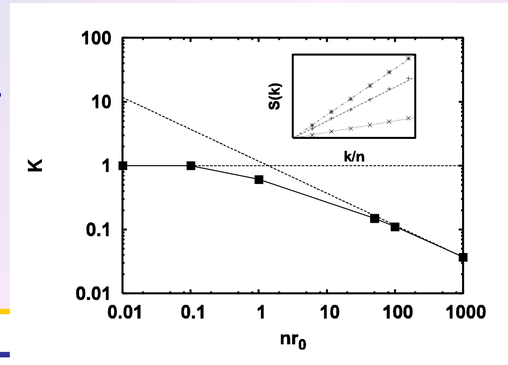


Motivations

- Dipolar quantum gases are emerging as competitive realizations of quantum devices and as a laboratory to investigate novel quantum phases in strongly correlated regimes taking advantage of enhanced quantum fluctuations
- Enhanced quantum fluctuations may come from various manipulations such as lowering the temperature, tuning the interactions in both the short-range part and the long-range tail, and reducing dimensionality
- In bosonic 1D dipolar gases all of these manipulations can be realized in current experiments

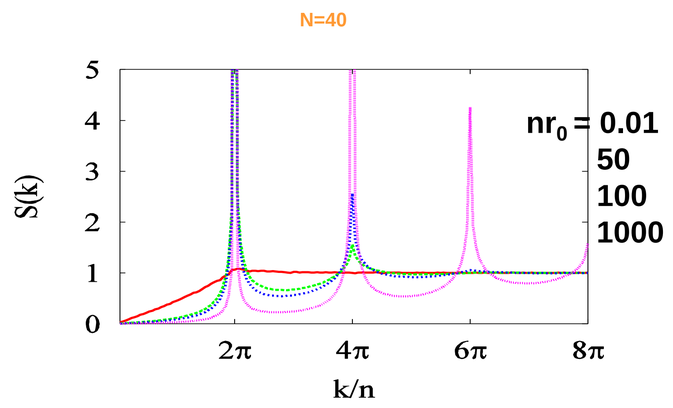
- Bosonic 1D dipolar quantum gases are in a super-strongly correlated regime well beyond Tonks-Girardeau (TG). Their behavior crosses over from TG at very low density to a dipolar density wave (DDW) condition at very high density
- Back-to-back comparison with Reptation Quantum Monte Carlo data shows that the 1D dipolar gas is a Luttinger liquid with K<1 in the whole crossover, continously decreasing from K=1 in the TG limit to K->0 in the DDW limit
- A clear-cut demonstration of Luttinger liquid behavior requires understanding of the excitations in the homogeneous gas

Open questions

- Do roton-like excitations show up at finite wavevectors?
- Do quantum fluctuations prevent the existence of long-range order in the DDW phase?

If the quantum gas were in a Luttinger state, answer to both questions would be NO

What we are going to demonstrate from now is that this is the indeed the case



The Model

N atoms with mass M on a line with density n having permanent dipolar moments with Hamiltonian

$$H = -\frac{1}{r_s^2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x^2} + \frac{1}{r_s^3}\sum_{i<j}^{N}\frac{1}{|x_i - x_j|^3}$$

$r_s = 1/(n\, r_0)$

In effective Rydbergs

$$Ry^* = \frac{\hbar^2}{2Mr_0^2}$$

$$r_0 = MC_{dd}/(2\pi\hbar^2)$$

Magnetic dipoles μ_d

$$C_{dd} = \mu_0\mu_d^2$$

Electric dipoles d

$$C_{dd} = \frac{d^2}{\varepsilon_0}$$

The Method

- Determine ground-state properties and the low-energy excitation spectrum by Reptation Quantum Monte Carlo (RQMC) simulations with N=40, 60, 80, 100
- RQMC in essence a path-integral technique at T=0 where ground-state distribution is sampled in the internal part of the path
- Advantages: computation of fluid structure and of imaginary-time correlation functions is conceptually straightforward and practically easy (no biases from mixed averages)
- Compute $F(q,\tau)=\langle\rho_q(\tau)\,\rho_q^+(0)\rangle/N$ creating a density fluct. with q
- $F(q,\tau)$ is related to dynamical structure factor $S(q,\omega)$ by FT
- Assume $S(q,\omega)=\Sigma_n|\langle n|\rho_q|0\rangle|^2\,\delta(\omega-\omega_n)$. Thus $F(q,\tau)$ is a sum of exponentials
- Fit $F(q,\tau)$ to find energy dispersion ω_n, including as many modes as it is needed to make the chi-square minimum (see FIG. 1 below)

Use a trial wavefunction with 2-body Jastrow factors

$$\psi_{trial}(R) = \prod_{i<j}\exp[u(|x_i - x_j|)]$$

Use of different u(x): Luttinger $u(x)=C\sin^{1/K}(\pi x/L)$ (L box size) or Gaussians centered at lattice sites yields negligible differences

Results

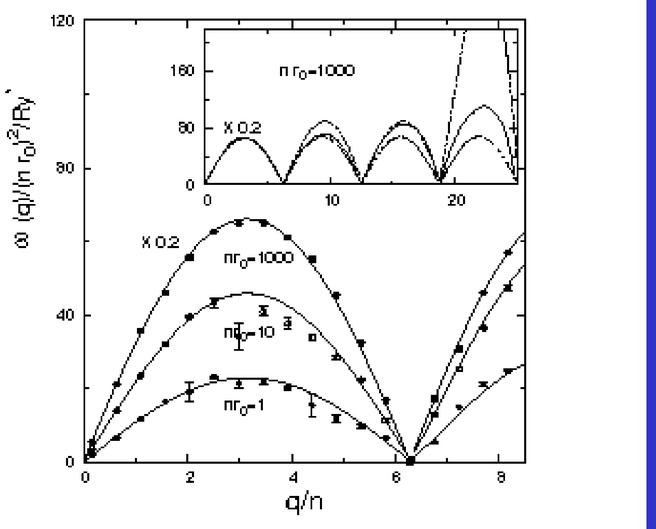


FIG. 2: Lowest excitation energies $\omega(q)$ in Ry^* units and scaled by $(nr_0)^2$, for a dipolar gas with $N = 40$ and different values of $nr_0 = 1, 10$ and 1000 as in the legend. Symbols with error bars are energies extracted using (3), the solid line is a guide to the eye. The curve at $nr_0 = 1000$ is depressed by a factor of 5 for graphical reasons. Inset: zoom on the $\omega(q)$ at $nr_0 = 1000$ up to $q/n = 8\pi$ for different $F(q,\tau)$ models: multimode model (3) (solid) and Feynman (dashed) approximation. Dotted line: periodic replica of the first bump.

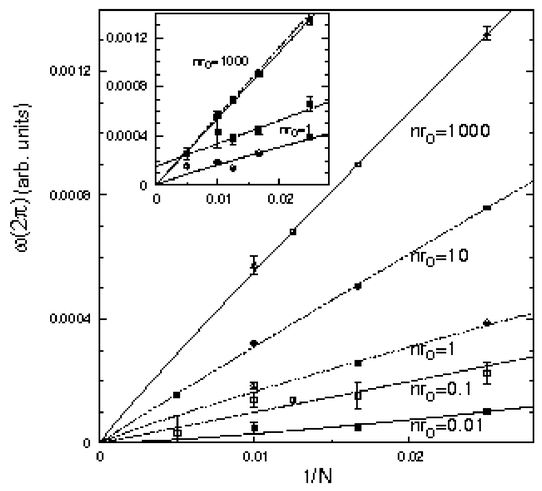


FIG. 3: $1/N$ scaling of $\omega(q = 2\pi)$ in arbitrary units at $nr_0 = 0.01$, 0.1, 1, 10 and 1000 as in the legend. Symbols with error bars: RQMC data. Solid lines: fit to the data. Inset: zoom of the RQMC data and related fits for the cases with $nr_0 = 1$ and 1000. Filled symbols and dashed lines: Feynman relation (single-mode approximation). Open symbols and solid lines: multimode analysis yielding best χ^2, with $\alpha = 1$ for the highest density case, $\alpha = 3$ or 4 for the other cases.

No roton gap and thus no superfluidity as from Landau criterion

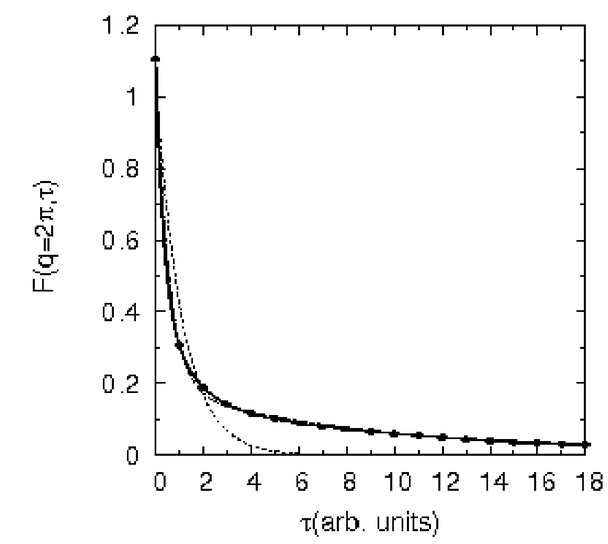


FIG. 1: $F(q,\tau)$ for $nr_0 = 0.1$ and $N = 100$. Symbols: RQMC data. (the error bars are not visible on this scale). Curves: fits to the RQMC data after including additional modes, that is $\alpha = 1$ (dotted line), 2 (dashed) and 3 (solid line) modes. Adding a fourth mode did not improve the quality of the fit. The reduced χ^2 changed from 1000 for $\alpha = 1$ to 0.2 for $\alpha = 3$.

- Using these results, we can also derive a strict upper bound for the order parameter of the solid $\rho_q = \langle\Sigma_m \exp(iGr_m)|\rangle$ with G reciprocal lattice vector, closely following the reasoning of Pitaevskii and Stringari and applying the uncertainty-principle inequality to operators ρ_{q+G} and $d\,\rho_q/dt$
- Obtain that in the long-wavelength limit, the order parameter of the solid vanishes as $\rho_G{}^2 < q^{\min(2K,1)}$ → No long-range order

References

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