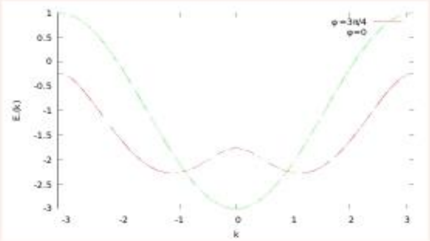
What: Meissner-to-Vortex phase transitions
For What: Spin & Density Structure and Transport
Where: 1D coupled chains
How: Using artificial magnetic fields

Concept

Non-interacting spin-1/2 bosons with s. o. c.

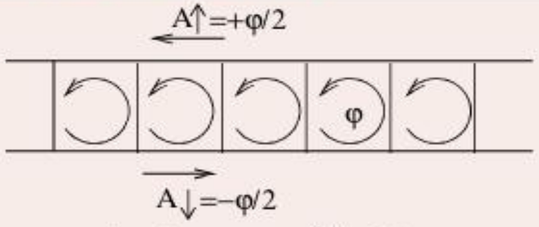
Hamiltonian and dispersion

$$H = -J_{\parallel} \sum_{j,\sigma} (b^{\dagger}_{j,\sigma} e^{i\sigma\varphi/2} b_{j+1,\sigma} + \text{H.c.}) - J_{\perp} \sum_{j} (b^{\dagger}_{j,\uparrow} b_{j,\downarrow} + \text{H.c.})$$



Equivalence with a two-leg ladder in a flux

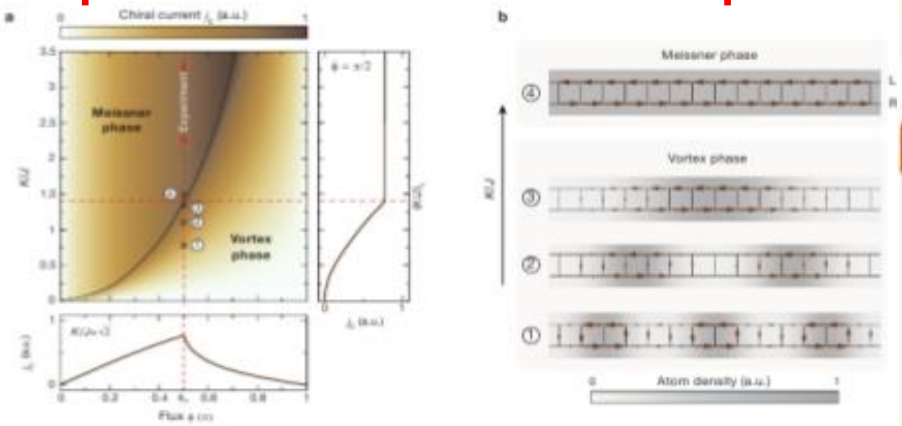
Spinless bosons on a ladder with flux φ



Landau gauge: $\vec{A} \parallel$ chains

Spin&Densityfluctuations compete and cooperate to manage magnetic flux

Meissner persistent current or Vortex phases



From Atala et al. Nat. Phys. (2014)

Meissner phase: single minimum in dispersion

$$j_{\parallel}(j) = J_{\parallel} \sin(\varphi/2)\ ;\ j_{\perp}(j) = 0.$$

Vortex phase: two minimas at $\pm k_c$ in dispersion

$$j_{\parallel}(j) = \frac{J_{\perp}^2 \cos(\varphi/2)}{2\sin^2(\varphi/2)\sqrt{J_{\perp}^2 + 4J_{\parallel}^2 \sin^2(\varphi/2)}} + C_1(\varphi)\cos(2k_c j)$$

$$j_{\perp}(j) = C_2(\varphi)\sin(2k_c j)$$

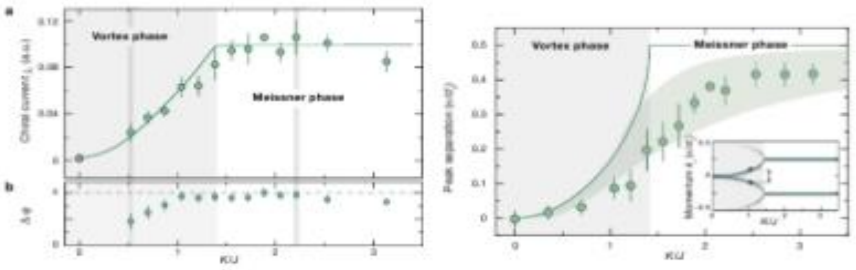
Definition of the currents in the ladder

$$
\begin{aligned}
j_{\parallel} &= -iJ_{\parallel}\sum_{j,\sigma}\sigma(b^{\dagger}_{j,\sigma}e^{i\sigma\varphi/2}b_{j+1,\sigma} - \text{H.c.})\\
j_{\perp} &= -iJ_{\perp}\sum_{j,\sigma}\sigma b^{\dagger}_{j,\sigma}b_{j,-\sigma}
\end{aligned}
$$

Mapping to spin-1/2 bosons with s. o. c.

- $j_{\parallel} \rightarrow$ spin current
- $j_{\perp} \rightarrow$ local magnetization along $\hat{\mathbf{y}}$

Experimental measurements



From Atala et al. Nat. Phys. (2014)

Experiment

Theoretical analysis via Bosonization

What is the effect of interaction ?

Model Hamiltonian

$$
\begin{aligned}
H &= -J_{\parallel}\sum_{j,\sigma}(b^{\dagger}_{j,\sigma}e^{i\sigma\varphi/2}b_{j+1,\sigma} + \text{H.c.}) - J_{\perp}\sum_{j}(b^{\dagger}_{j,\uparrow}b_{j,\downarrow} + \text{H.c.})\\
&\quad + U\sum_{j,\sigma}n_{j,\sigma}(n_{j,\sigma} - 1)
\end{aligned}
$$

Limit $J_{\perp} \ll J_{\parallel}, U$

Bosonization approach [EO, T. Giamarchi PRB **64**, 144515 (2001)]

Spin Charge separation

Change of variables

$$
\begin{aligned}
\Pi_c &= \frac{1}{\sqrt{2}}(\Pi_{\uparrow} + \Pi_{\downarrow})\ ;\ \Pi_s = \frac{1}{\sqrt{2}}(\Pi_{\uparrow} - \Pi_{\downarrow})\\
\phi_c &= \frac{1}{\sqrt{2}}(\phi_{\uparrow} + \phi_{\downarrow})\ ;\ \phi_s = \frac{1}{\sqrt{2}}(\phi_{\uparrow} - \phi_{\downarrow})\\
H &= H_c + H_s\\
H_c &= \int \frac{dx}{2\pi}\left[u_c K_c(\pi\Pi_c)^2 + \frac{u_c}{K_c}(\partial_x\phi_c)^2\right]\\
H_s &= \int \frac{dx}{2\pi}\left[u_s K_s(\pi\Pi_s - \varphi/(\sqrt{8}a))^2 + \frac{u_s}{K_s}(\partial_x\phi_s)^2\right]\\
&\quad -2J_{\perp}A_0^2\int dx \cos\sqrt{2}\theta_s
\end{aligned}
$$

In the Meissner phase

Observables

$$
\begin{aligned}
\langle j_{\parallel}\rangle &= \frac{uK}{4\pi}\varphi\\
\langle S^z_j S^z_0\rangle &= O(e^{-|j|/\xi})\\
\langle j_{\perp}(ja)j_{\perp}(0)\rangle &= O(e^{-|j|/\xi})\\
\langle b_{j,\sigma}b^{\dagger}_{0,\sigma'}\rangle &\sim \frac{1}{j^{1/(2K_c)}}
\end{aligned}
$$

Mermin-Wagner $\Rightarrow$ no long-range order

In the vortex phase

Observables

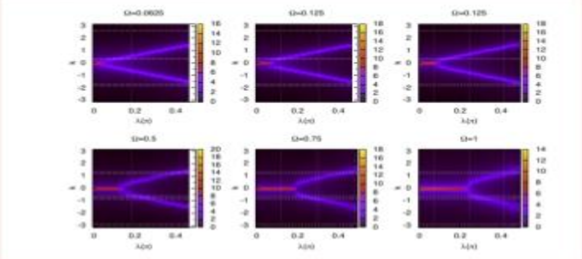
$$
\begin{aligned}
\langle j_{\parallel}\rangle &= \frac{uK}{4\pi}(\varphi - \sqrt{\varphi^2 - \varphi_c^2})\\
\langle S^z_j S^z_0\rangle &= -\frac{K_s^*}{4\pi^2 j^2} + B_0^2\frac{\cos(2\pi\rho_0 j)}{|j|^{K_s^*/2+K_c/2}}\\
Q &\propto \sqrt{\varphi^2 - \varphi_c^2}\\
\langle j_{\perp}(ja)j_{\perp}(0)\rangle &\sim \frac{\cos(2Qj)}{|j|^{1/K_s^*}}\\
\langle b_{j,\sigma}b^{\dagger}_{j,\sigma'}\rangle &\sim \delta_{\sigma\sigma'}\frac{\cos(Qj/2)}{|j|^{1/(2K_c)+2/(2K_s^*)}}
\end{aligned}
$$

Numerical exp.: DMRG Results

Density Matrix Renormalization Group approach

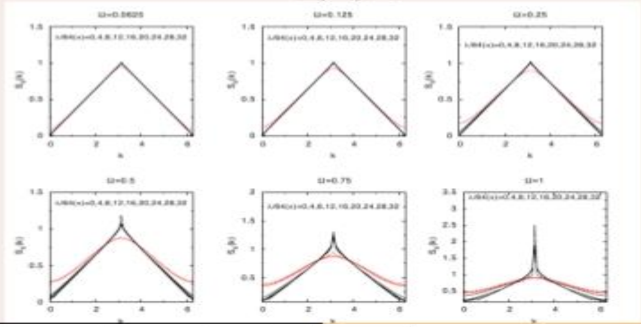
Momentum distribution [M. Di Dio *et. al.*, EO unpub. (2015)]

$n(k) = \sum_{k,\sigma} e^{-ikj} \langle b_{j\sigma} b^{\dagger}_{0,\sigma} \rangle \;\; \Omega \leftrightarrow J_{\perp} \, , \, \lambda \leftrightarrow \varphi \, , \, U = +\infty.$

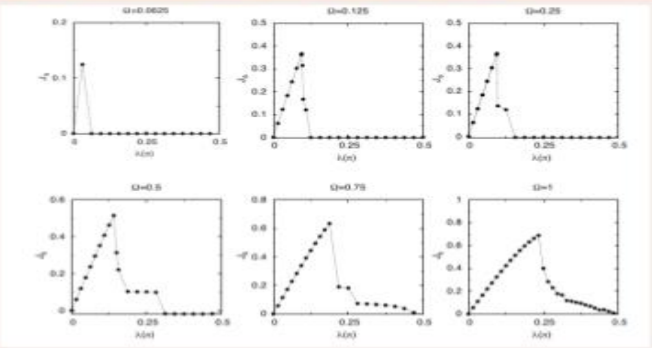


density wave correlations [M. Di Dio *et. al.*, EO unpub. (2015)]

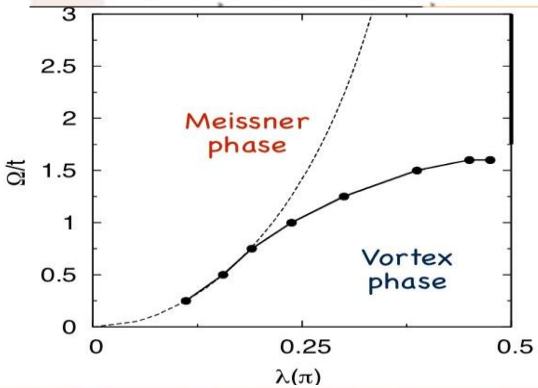
$S_s(k) = \sum_j \langle S^z_j S^z_0 \rangle e^{ikj}$



Leg current [M. Di Dio *et. al.*, EO unpub. (2015)]

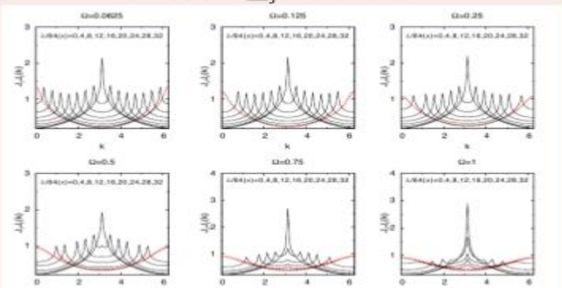


Phase Diagram

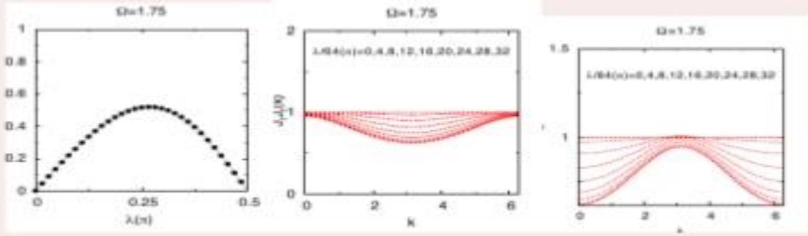


Rung current [M. Di Dio *et. al.*, EO unpub. (2015)]

$\langle J_r J_r \rangle (k) = \sum_j \langle j_{\perp}(ja) j_{\perp}(0) \rangle e^{ikj}$



Loss of commensurate-incommensurate transition



$\Rightarrow$ Breakdown of field theory for large Ω.

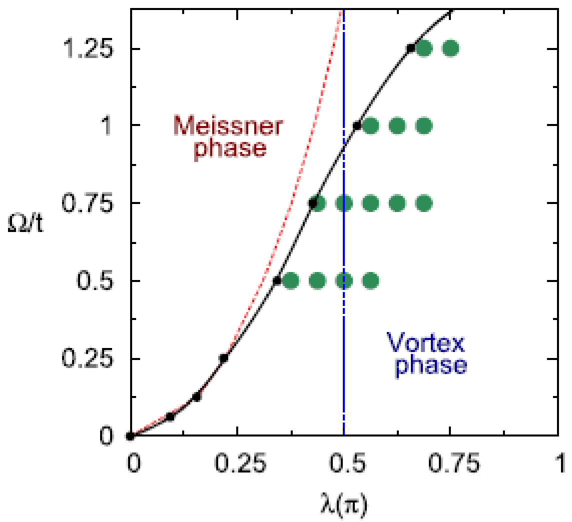


Figure 8. DMRG simulation results at $L = 64$ in PBC. Phase diagram for a hard-core bosonic system on ladder as a function of flux per plaquette λ and Ω/t, at the filling value $\rho = 0.5$. The occurrence of the two incommensurations is evidenced as follows. The black solid line represents the phase boundary between the Meissner and the first incommensuration, a standard Vortex phase. The dark–green solid dots are the points where the second incommensuration appears. The dashed blue line marks the critical $\lambda = 0.5\pi$ at which the second incommensuration is expected. For comparison, the phase boundary between the Meissner and Vortex phase for a non-interacting system is represented as well, by the red-dashed line. Notice the enhanced size of the Meissner region in the hard-core repulsive with respect to the non-interacting case.

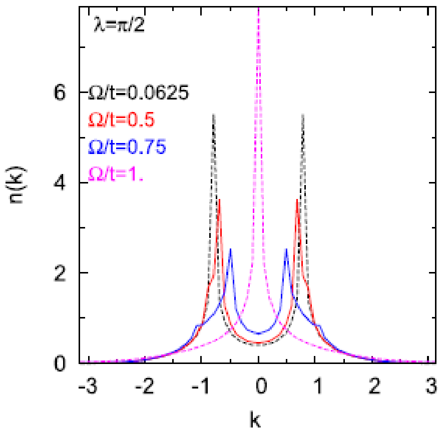


Figure 6. Second incommensuration. DMRG simulation results at $L = 64$ in PBC. Momentum distribution $n(k)$ at $\rho = 0.5$ and fixed applied flux $\lambda = \pi\rho$, for different values of Ω/t as in the legend. Dashed black line: $\Omega/t = 0.0625$, where the system is in the standard vortex phase (first incommensuration). Dashed magenta line: $\Omega/t = 1$, where the system is in the Meissner phase. Red and blue solid lines: $\Omega/t = 0.5$ and $\Omega/t = 0.75$, respectively, where the occurrence of the second incommensuration is signaled by the appearance of the secondary peaks.

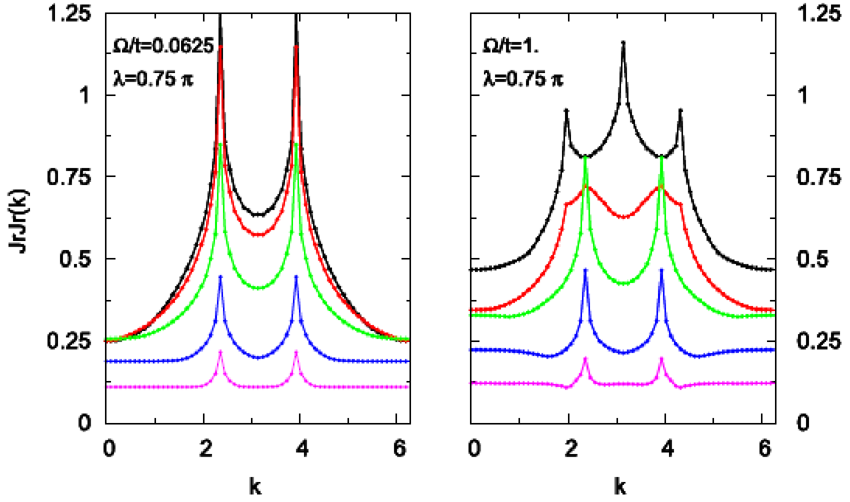


Figure 7. First and second incommensuration. DMRG simulation results at $L = 64$ in PBC. FT of the rung-current correlation function $C(k)$ at fixed applied flux $\lambda = 0.75\pi$ for different fillings as in the legend: $\rho = 1.0, 0.75, 0.5, 0.25$ and 0.125 are represented by black, red, green, blue and magenta solid lines, respectively. Left panel: case with $\Omega/t = 0.0625$. Right panel: case with $\Omega/t = 0.75$. Data at $\Omega/t = 0.75$ and $\rho = 1$ has been shifted to make more evident the second incommensurations peaks.

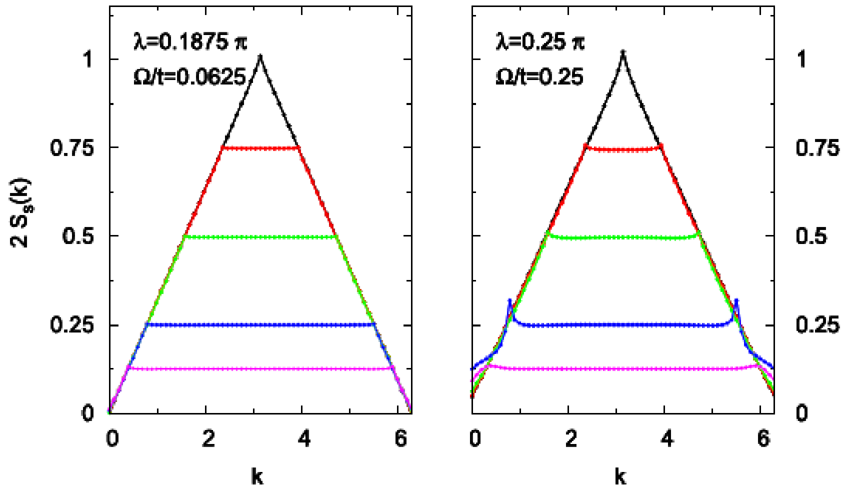


Figure 9. DMRG simulation results at $L = 64$ in PBC. Spin static structure factor $S_s(k)$ for different fillings as in the legend: $\rho = 1.0$, 0.75, 0.5, 0.25, and 0.125 are represented by the black, red, green, blue, and magenta solid lines, respectively. Left panel: $\lambda = 0.1875\,\pi$ and the small value of interchain hopping $\Omega/t = 0.0625$, where the system is always in the standard Vortex phase at all fillings. Right panel: $\lambda = 0.25\,\pi$ and $\Omega/t = 0.25$. Notice that the system at $\rho = 0.125$ is in the Meissner phase.