

938II – Electronics and communication technologies (2026/27)

Communication technologies: Signals & systems

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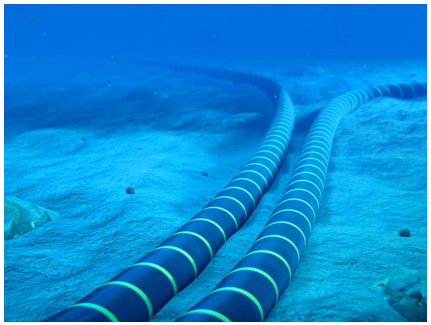
Introduction to the course

Categories of communication systems (1/3)

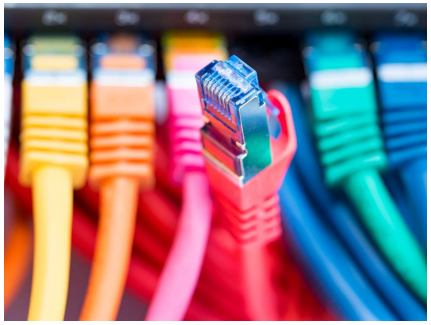
- **Access vs. transport**



vs.



- **Wired vs. wireless**



vs.



Categories of communication systems (2/3)

- **Broadcast (point-to-multipoint) vs. unicast (point-to-point)**



vs.



- **Terrestrial vs. satellite**



vs.





Categories of communication systems (3/3)

- **Broadband vs. narrowband**

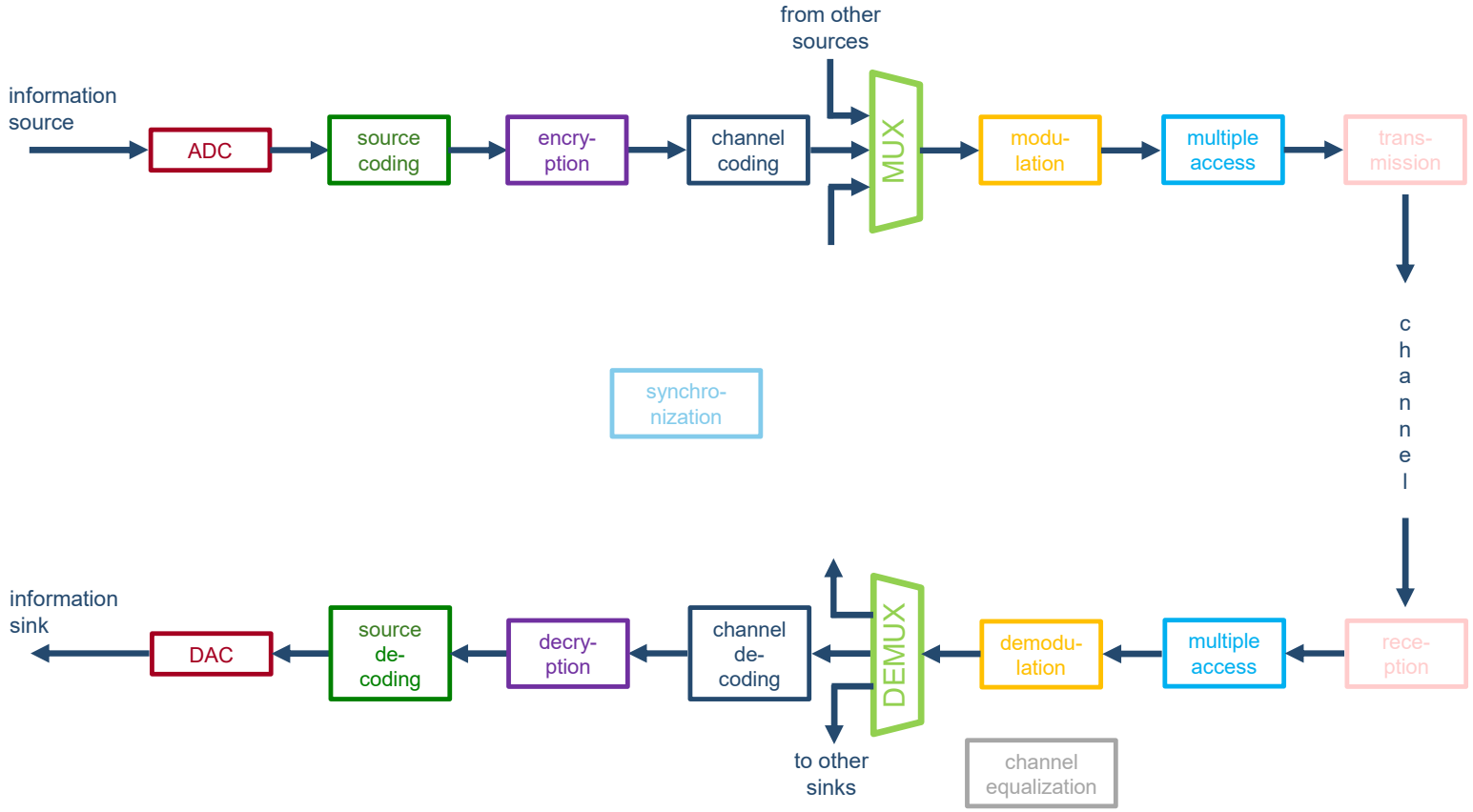


vs.



… and many more

Elements of a digital communication system





What is a signal? (1/7)

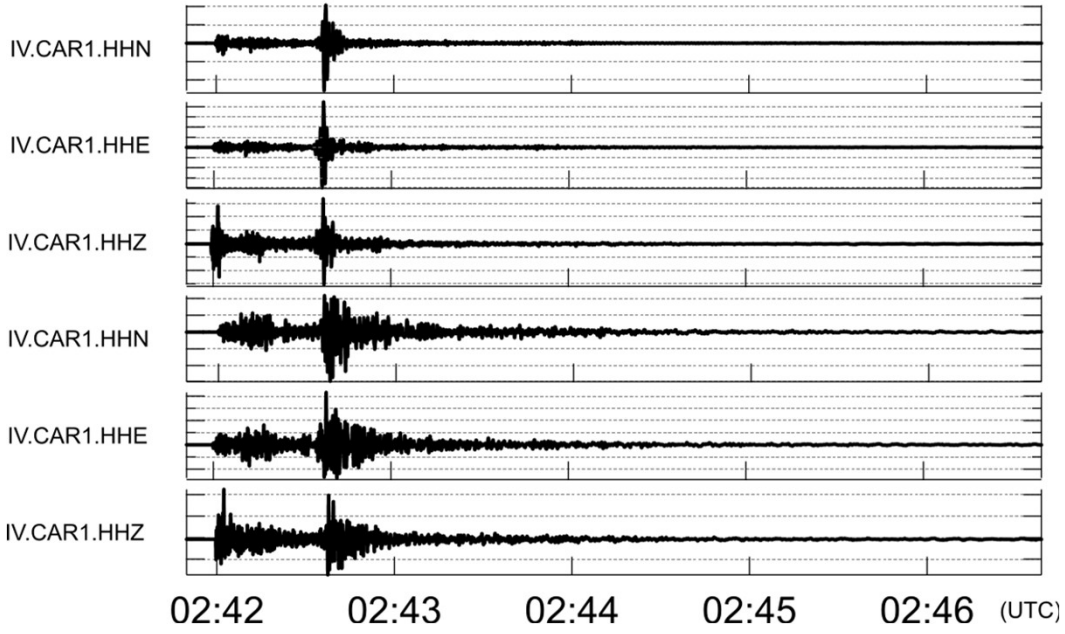
A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"



$$x(t) : \Re \rightarrow \Re$$

What is a signal? (2/7)

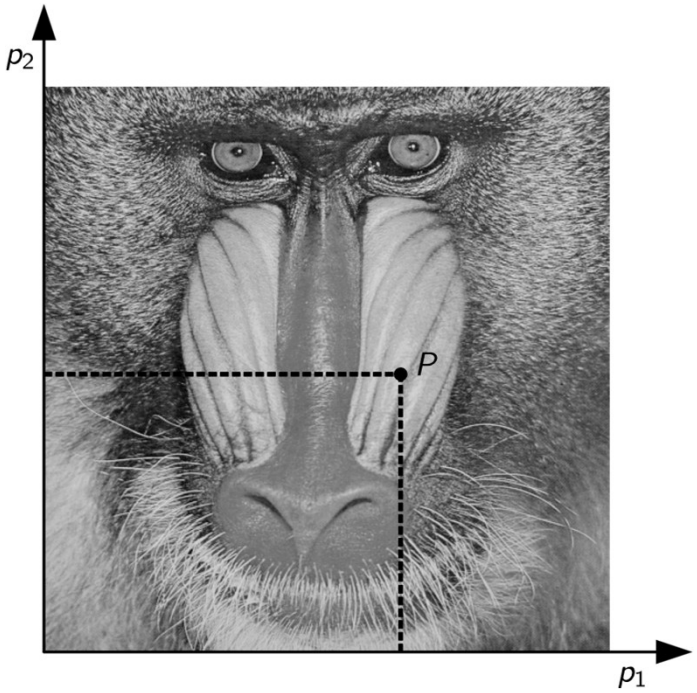
A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"



$$\mathbf{x}(t) : \Re \rightarrow \Re^n$$

What is a signal? (3/7)

A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"



$$z(p_1, p_2) : \Re^2 \rightarrow \Re$$

What is a signal? (4/7)

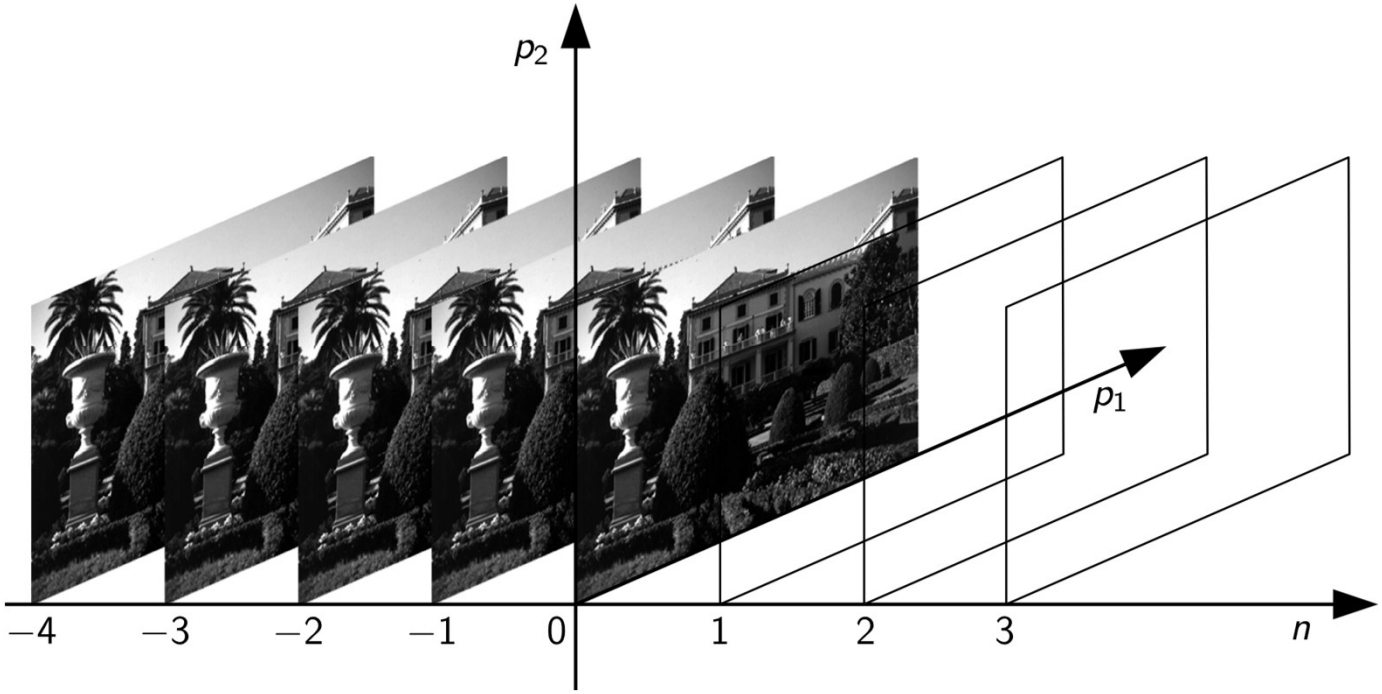
A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"



$$z_{\mathrm{R}}(P) : \Re^2 \rightarrow \Re$$
$$z_{\mathrm{G}}(P) : \Re^2 \rightarrow \Re$$
$$z_{\mathrm{B}}(P) : \Re^2 \rightarrow \Re$$
$$\mathbf{z}(P) = [z_{\mathrm{R}}(P), z_{\mathrm{G}}(P), z_{\mathrm{B}}(P)]$$
$$\mathbf{z}(P) : \Re^2 \rightarrow \Re^3$$

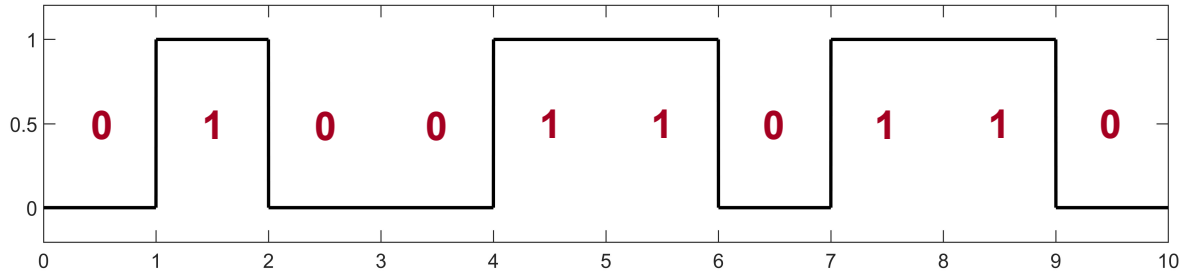
What is a signal? (5/7)

A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"

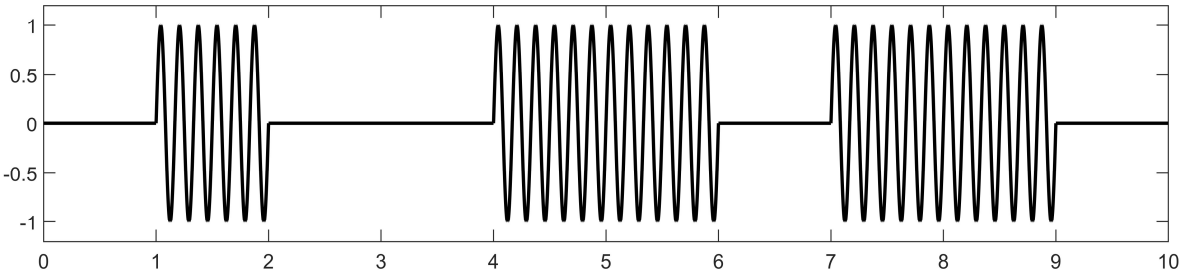


What is a signal? (6/7)

A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"



physical signal conveying a sequence of bits

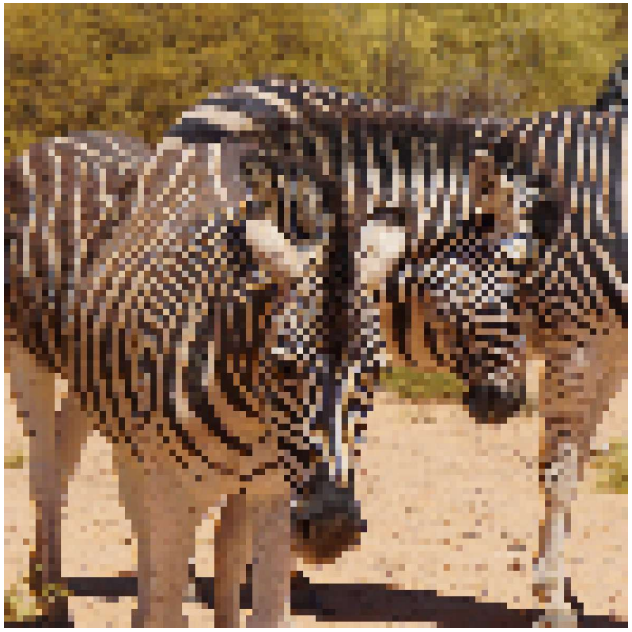


OOK signal @ 194 THz



What is a signal? (7/7)

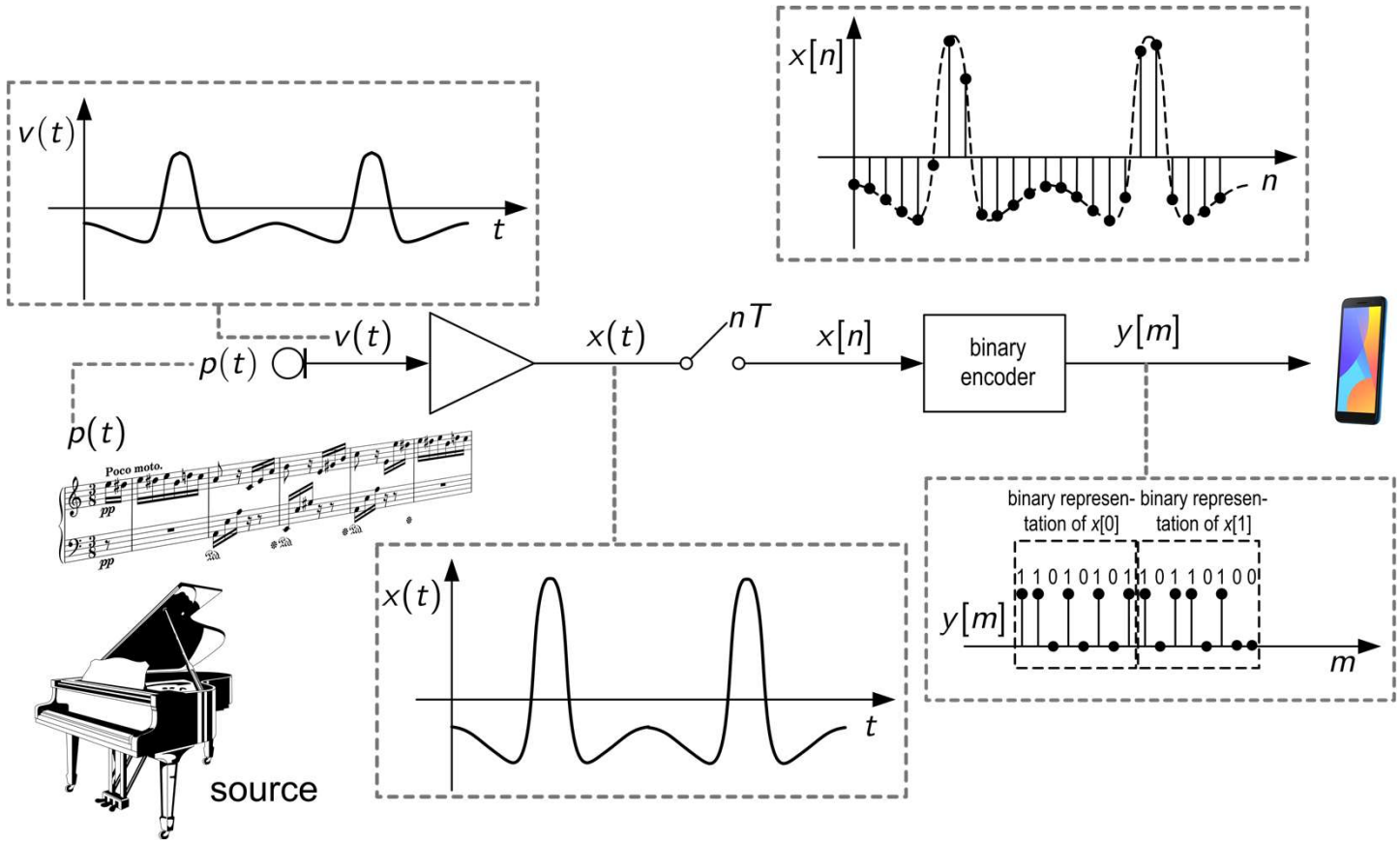
A signal is any variable physical quantity that carries a certain amount of information, that makes the signal itself "interesting"



$$\mathbf{z}(P) = [z_{\mathrm{R}}(P), z_{\mathrm{G}}(P), z_{\mathrm{B}}(P)]$$
$$\mathbf{z}(P) = \mathbf{z}(p_1, p_2) : \Re^2 \rightarrow \Re^3$$

$\mathbf{z}[n_1, n_2]$, with each RGB component expressed over 8 bits

Example: digital recording of an audio signal

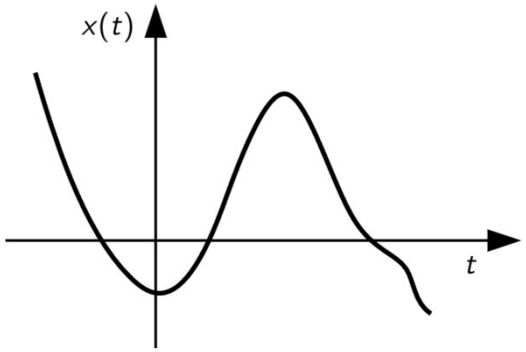


Classes of signals

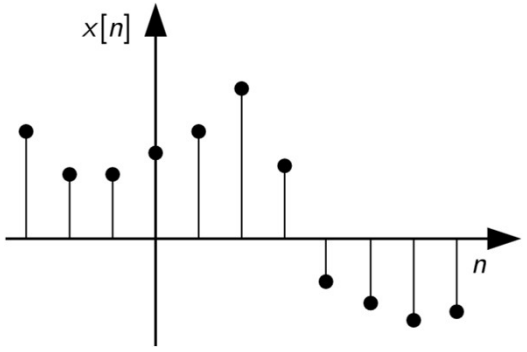
continuous time (x-axis)

discrete time (x-axis)

continuous amplitude (y-axis)

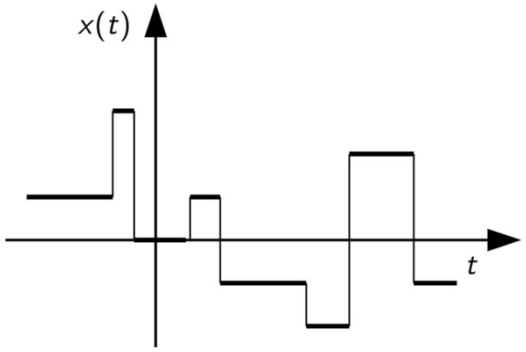


analog signals

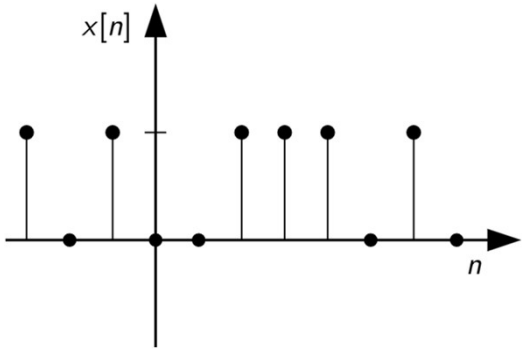


discrete-time signals (sequences)

discrete amplitude (y-axis)



quantized signals

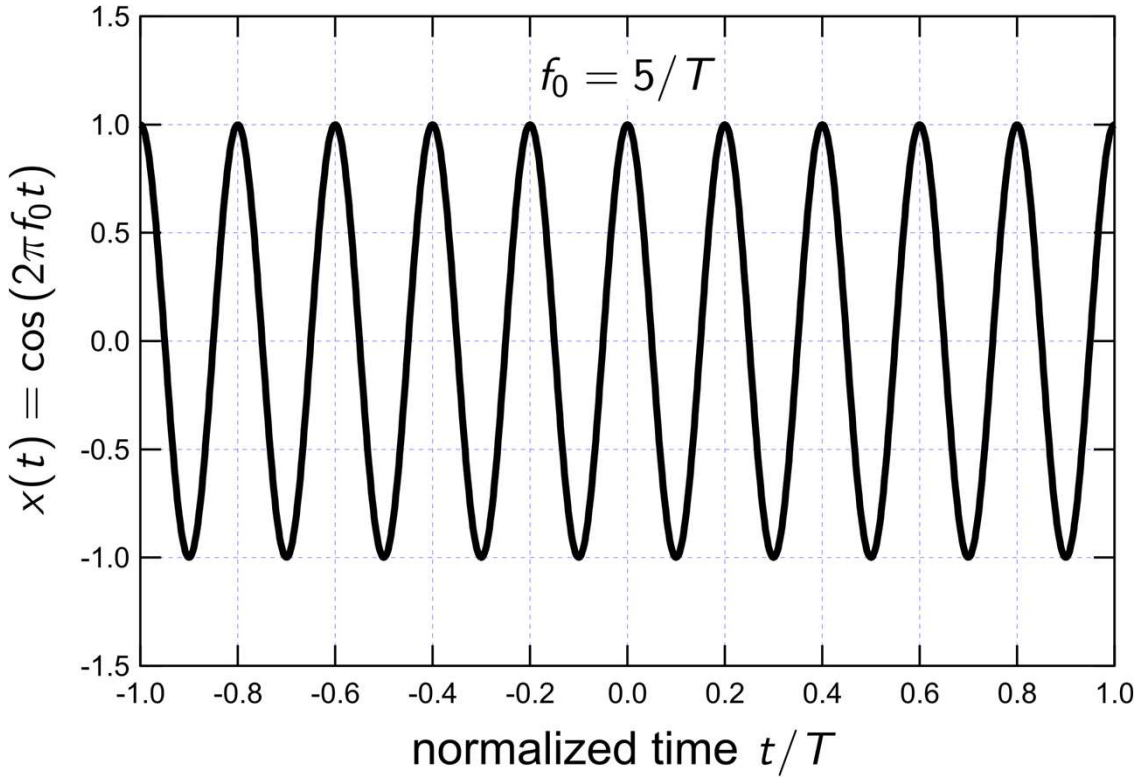


digital signals



Basic of Fourier analysis and synthesis

One (simple) sinusoidal signal

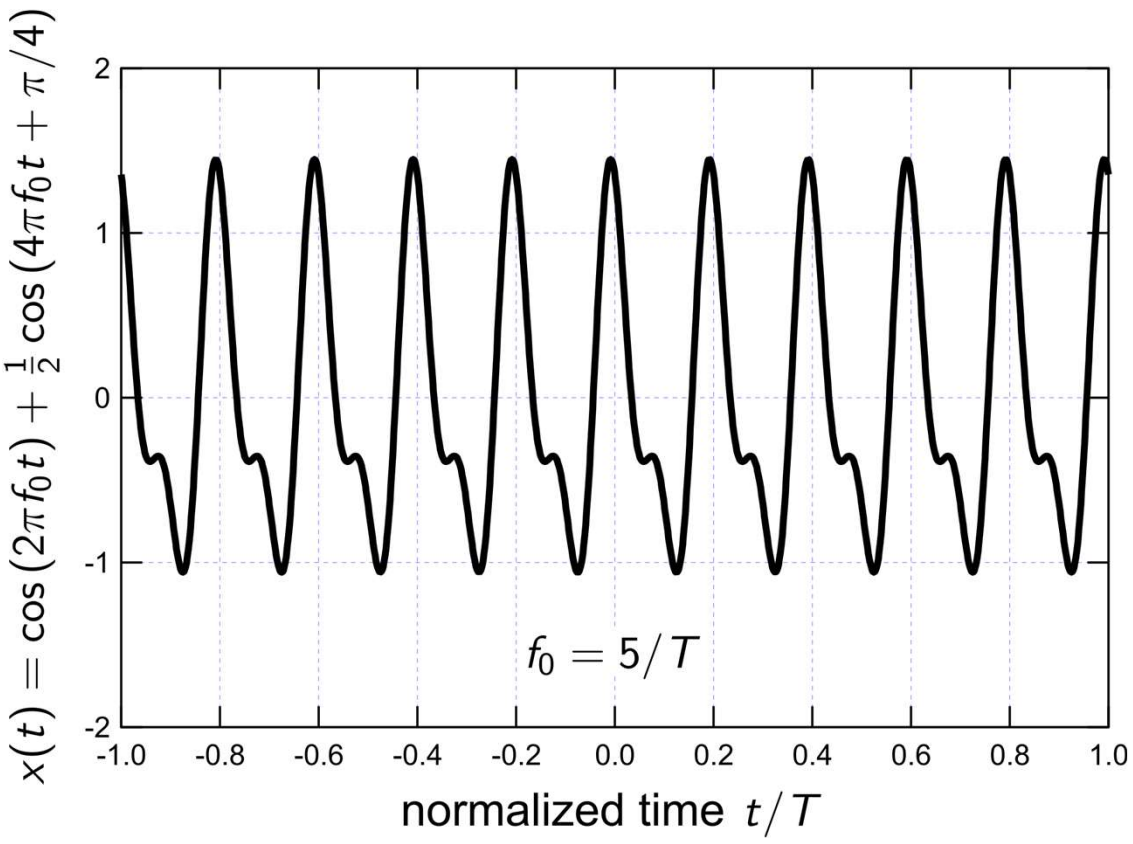


$$x(t) = \cos(2\pi 5t/T)$$

In general,

$$x(t) = A\cos(2\pi f_0 t + \varphi)$$

Two (simple) sinusoidal signals

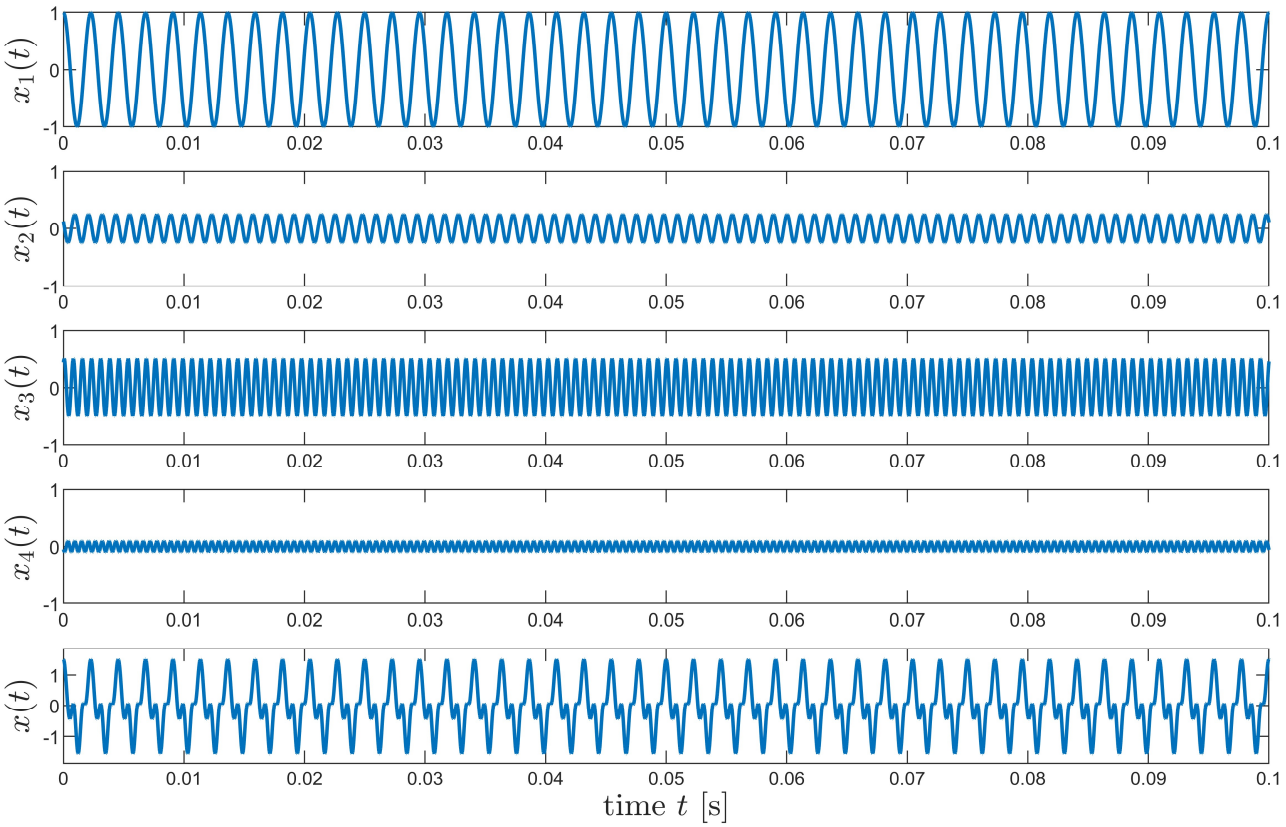


$$x(t) = \cos(2\pi 5t/T) + \frac{1}{2}\cos\left(2\pi 10t/T + \frac{\pi}{4}\right)$$

In general,

$$x(t) = \sum_{k=0}^{+\infty} A_k \cos(2\pi k f_0 t + \varphi_k)$$

Many (simple) sinusoidal signals



$$x(t) = \sum_{k=1}^{4} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$f_0 = 440\,\text{Hz}$$

$$\mathbf{A} = \left[1, \frac{1}{4}, \frac{1}{2}, \frac{1}{10}\right]$$

$$\boldsymbol{\varphi} = \left[0, \frac{\pi}{3}, -\frac{\pi}{6}, \frac{2\pi}{3}\right]$$



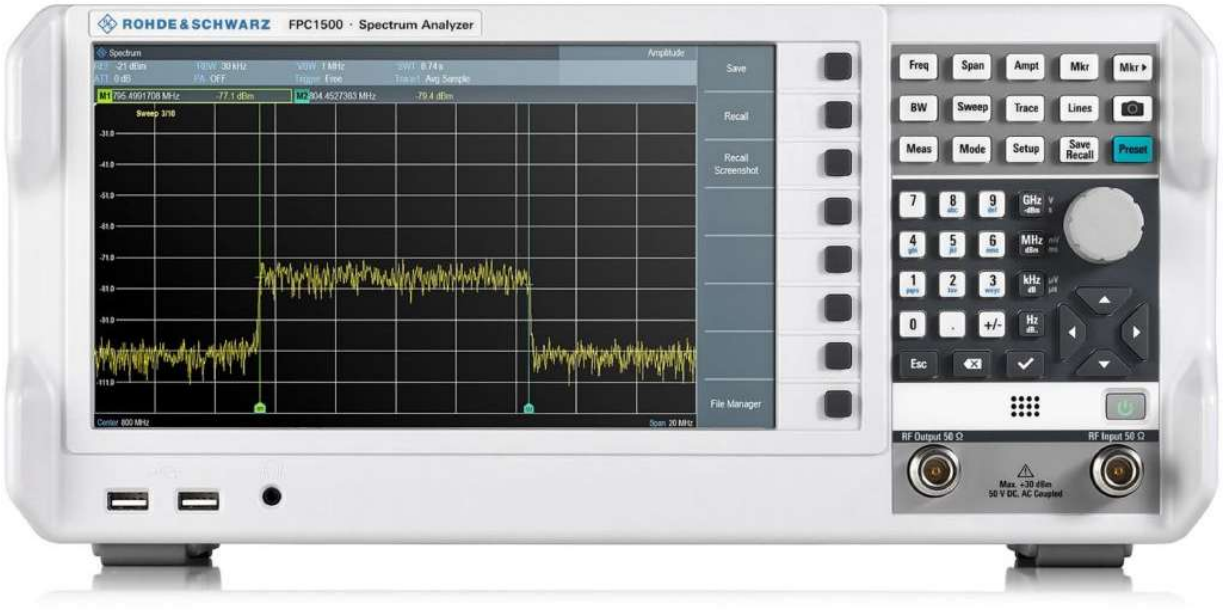
General Fourier series (1/2)

synthesis: $$x(t) = \sum_{k=0}^{+\infty} A_k \cos(2\pi k f_0 t + \varphi_k)$$



General Fourier series (2/2)

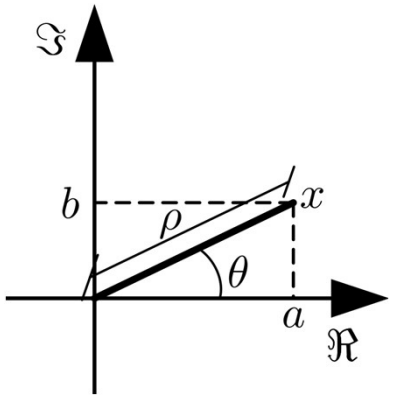
analysis: $$A_k e^{j\varphi_k} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\left[\cos(2\pi k f_0 t) - j\sin(2\pi k f_0 t)\right] dt$$



Cheat-sheet of complex numbers

$$x = \underbrace{a + jb}_{\text{rectangular representation}} = \underbrace{\rho e^{j\theta}}_{\text{polar representation}} \in \mathbb{C}, \qquad j \triangleq \sqrt{-1}$$

$$x^{\mathsf{H}} = a - jb = \rho e^{-j\theta}$$ Hermitian conjugate



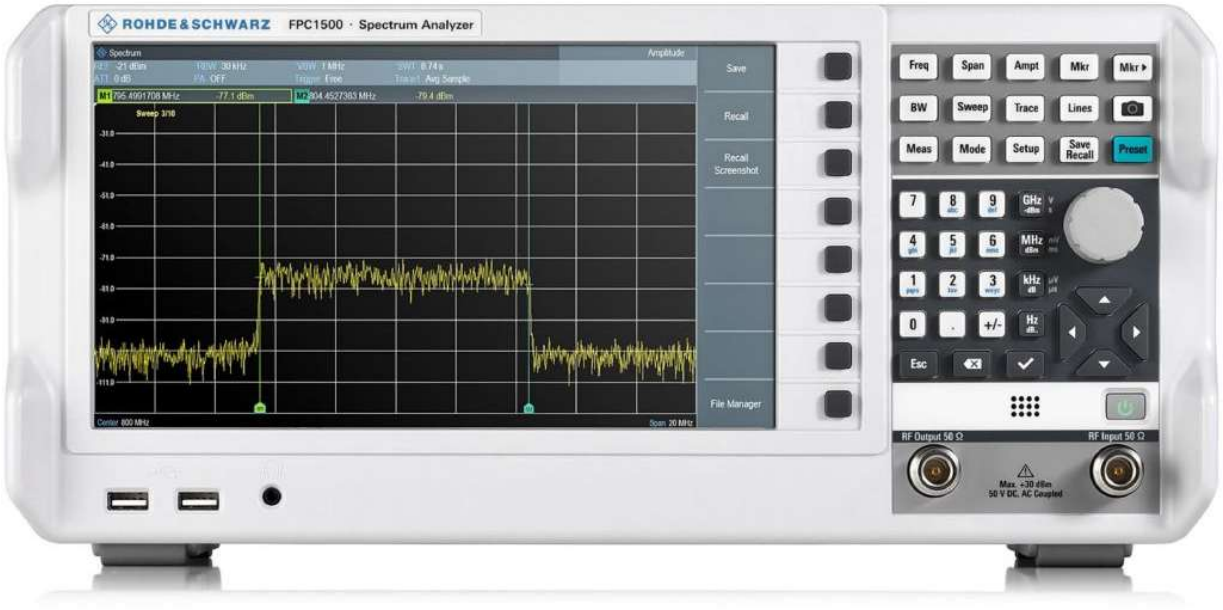
$$\rho \triangleq \|x\| = \sqrt{x \cdot x^{\mathsf{H}}} = \sqrt{a^2 + b^2}, \qquad \theta \triangleq \angle x = \tan^{-1}\left(\frac{b}{a}\right)$$

$$a \triangleq \Re\{x\} = \rho \cos\theta, \qquad b \triangleq \Im\{x\} = \rho \sin\theta$$

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}, \qquad \sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$ Euler's formulas

Fourier transform: A quick recap (1/2)

analysis: $$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt$$

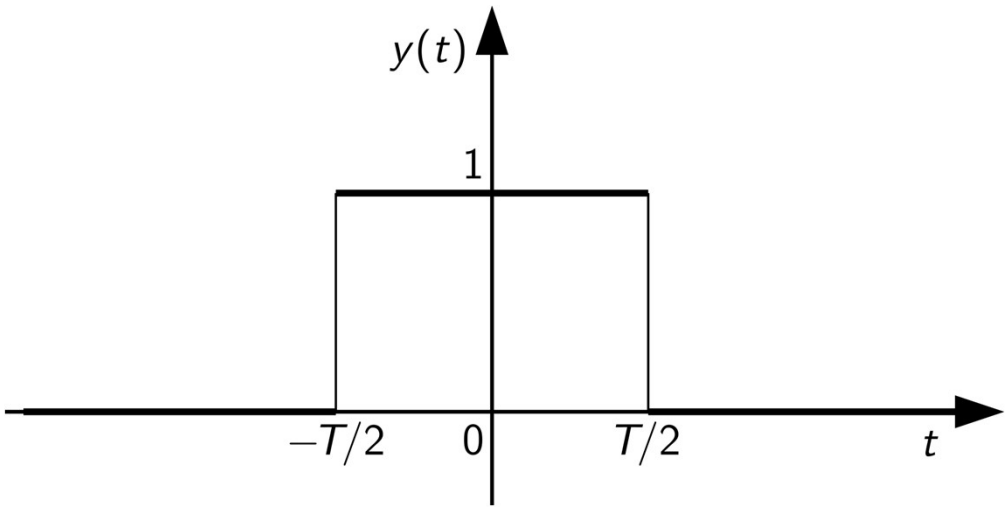


Fourier transform: A quick recap (2/2)

synthesis: $$x(t) = \int_{-\infty}^{+\infty} X(f) e^{-j2\pi f t} df$$



The (aperiodic) rectangular pulse (1/4)

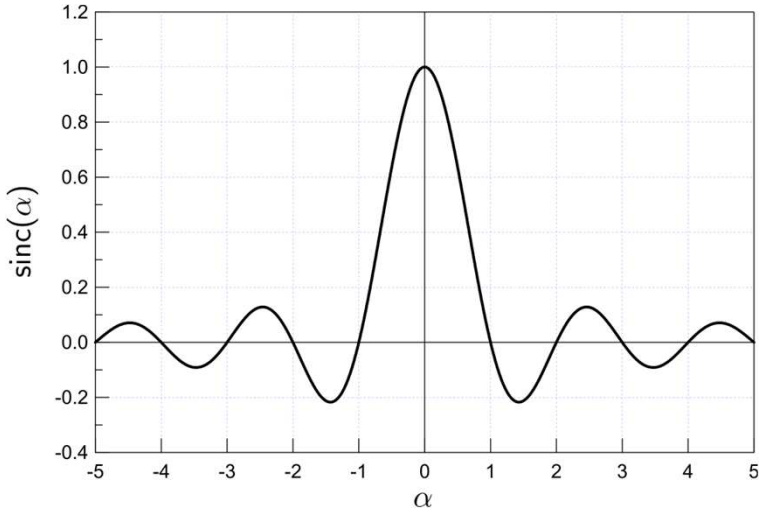


$$y(t) = \text{rect}\left(\frac{t}{T}\right), \qquad \text{rect}(\alpha) = \begin{cases} 1, |\alpha| \leq \frac{1}{2} \\ 0, |\alpha| > \frac{1}{2} \end{cases}$$

The (aperiodic) rectangular pulse (2/4)

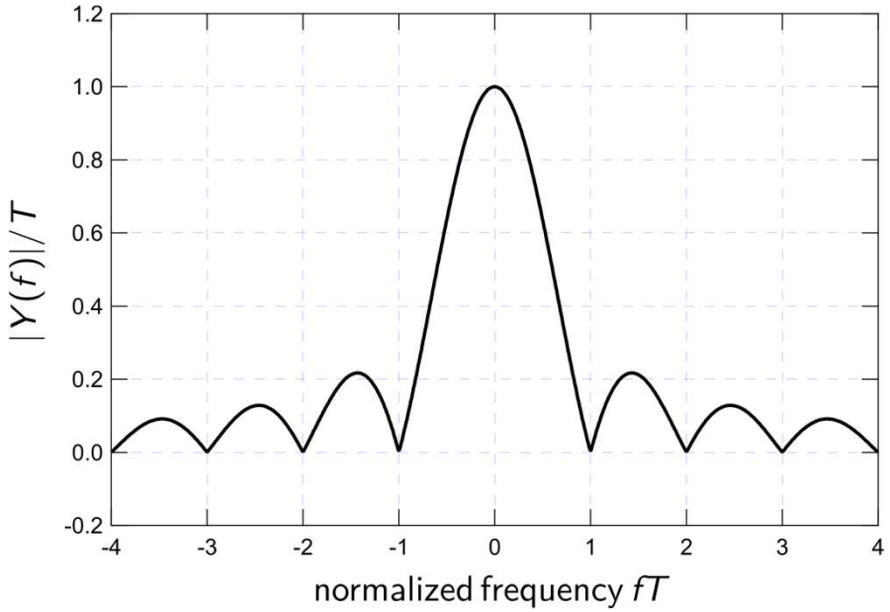
$$
\begin{aligned}
Y(f) &= \int_{-\infty}^{+\infty} x(t) \cdot e^{-j2\pi k f_0 t} dt \\
&= \int_{-T/2}^{T/2} 1 \cdot e^{-j2\pi f t} dt \\
&= \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{-T/2}^{T/2} \\
&= T \cdot \frac{\sin(\pi f T)}{\pi f T} \\
&= T \cdot \mathrm{sinc}(fT)
\end{aligned}
$$

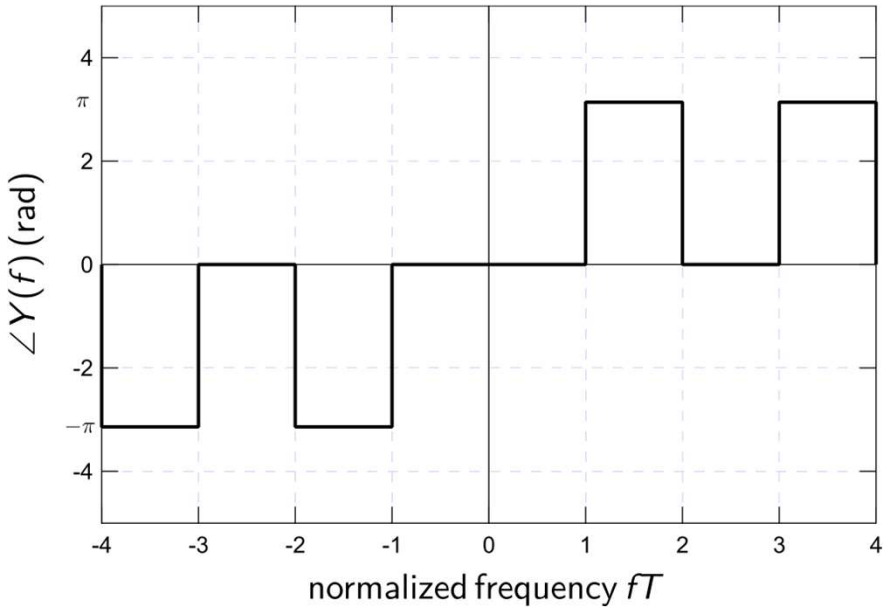
$$\mathrm{sinc}(\alpha) = \frac{\sin(\pi\alpha)}{\pi\alpha}$$



The (aperiodic) rectangular pulse (3/4)

$$Y(f) = \int_{-\infty}^{+\infty} y(t) \cdot e^{-j2\pi ft} dt = T\,\mathrm{sinc}\,(fT)$$

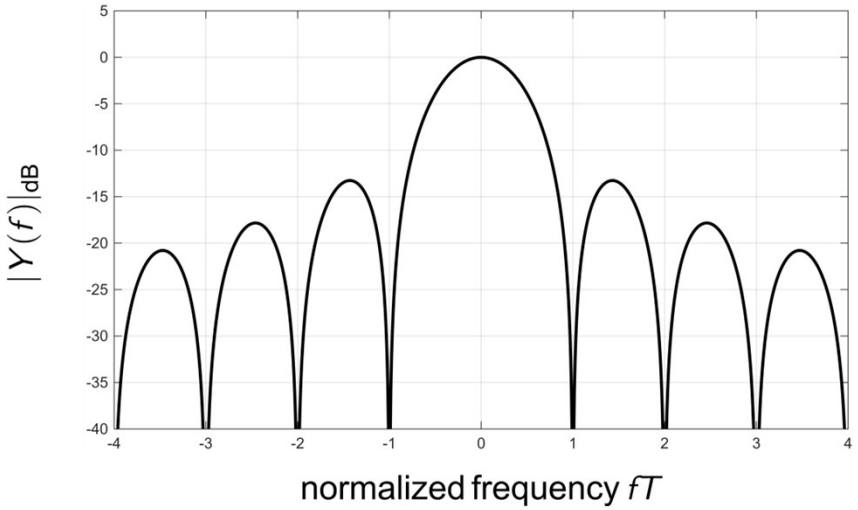




The (aperiodic) rectangular pulse (4/4)

In many cases, it is useful to evaluate the amplitude response via an equivalent, but often more convenient, representation, using a logarithmic scale:

$$|Y(f)|_{\mathrm{dB}} = 10 \log_{10} \frac{|Y(f)|^2}{|Y(f_0)|^2} \qquad [\mathrm{dB}]$$

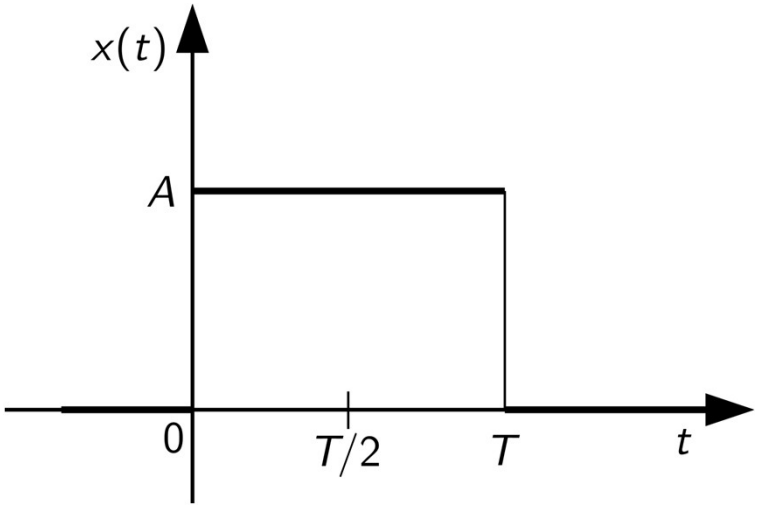




Useful properties of the Fourier transform

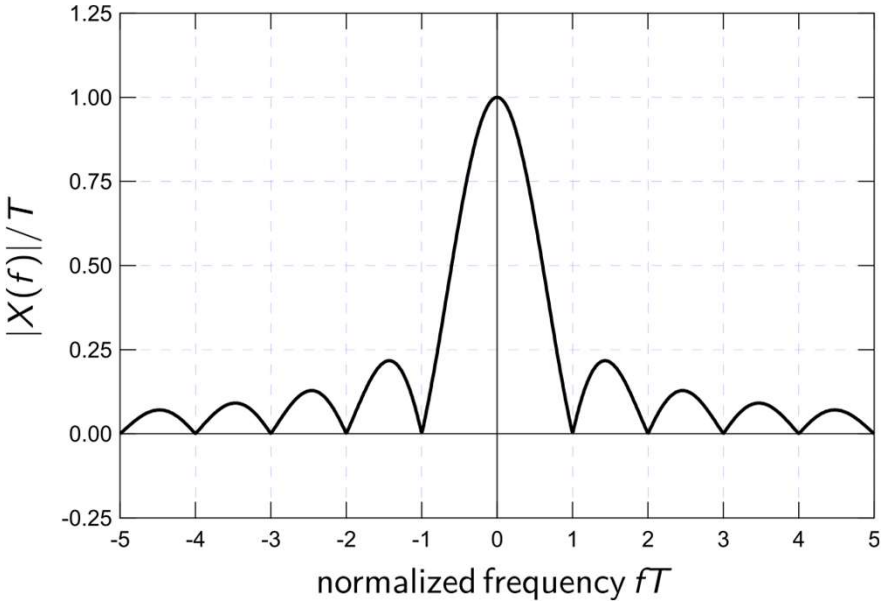
Delay theorem (1/2)

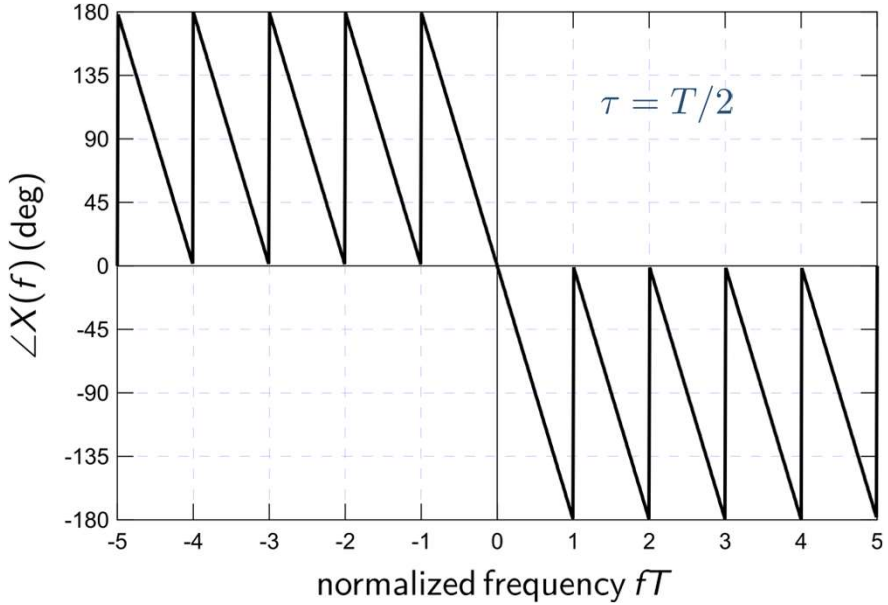
$$x(t-\tau) \Leftrightarrow X(f)e^{-j2\pi f\tau}$$



Delay theorem (2/2)

$$x(t-\tau) \Leftrightarrow X(f)e^{-j2\pi f\tau}$$





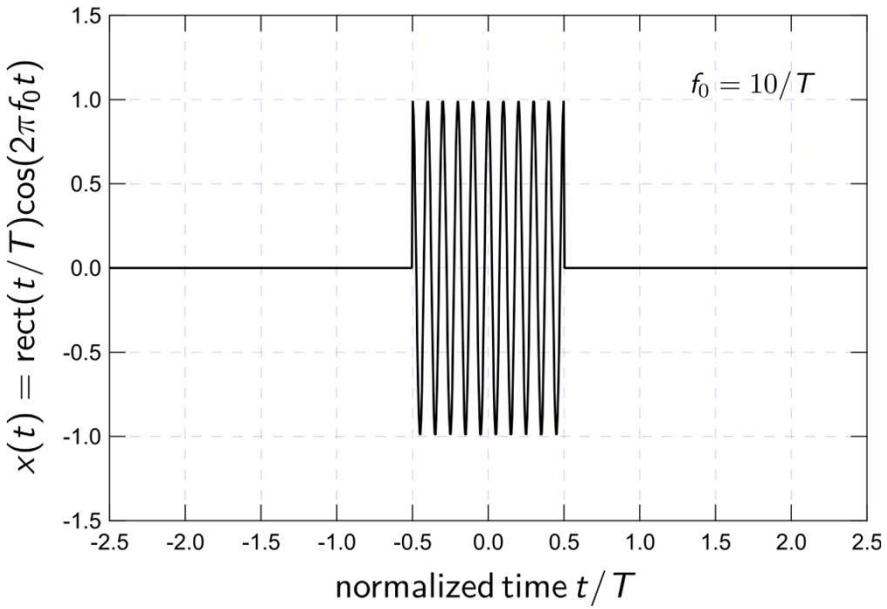


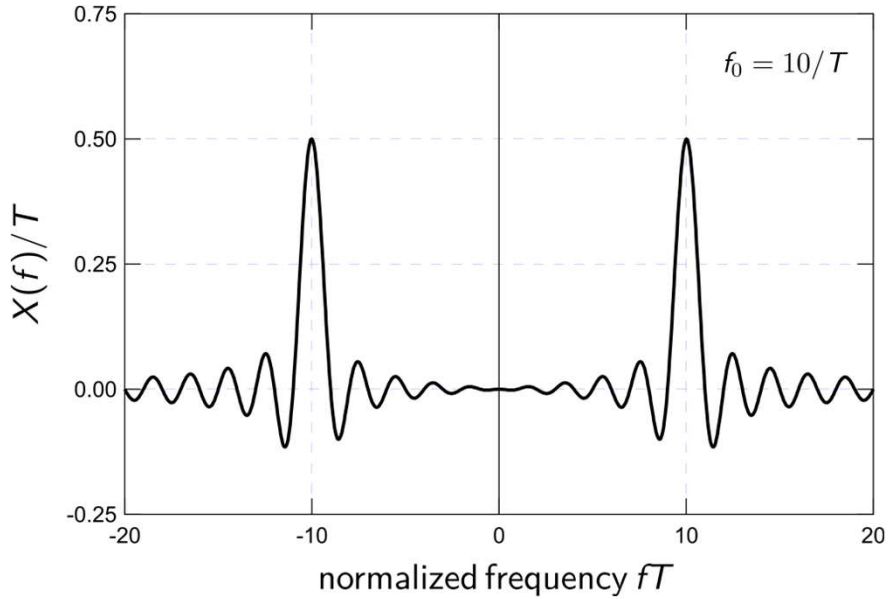
Linearity theorem

$$x(t) = ax_1(t) + bx_2(t) \Leftrightarrow X(f) = aX_1(f) + bX_2(f)$$

Modulation theorem

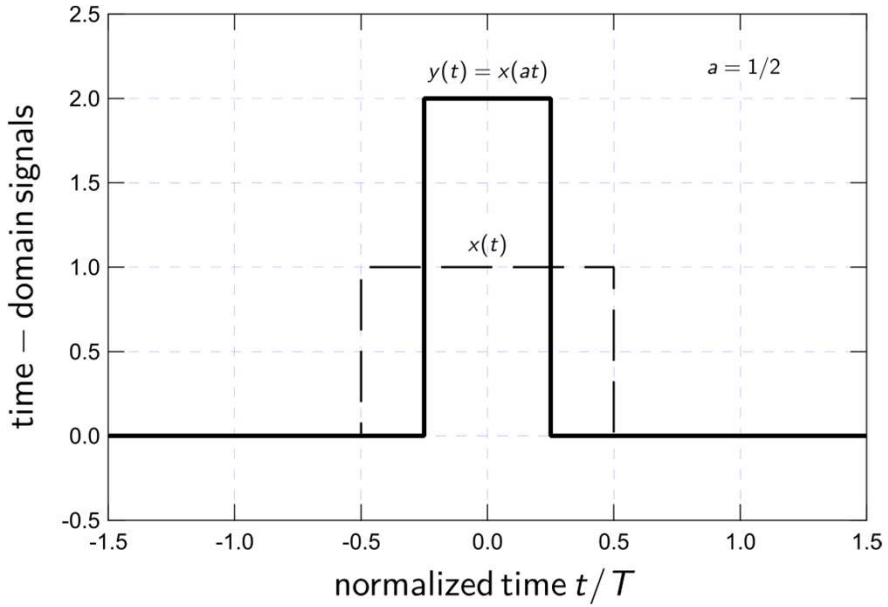
$$x(t)\cos(2\pi f_0 t) \Leftrightarrow \frac{X(f-f_0)+X(f+f_0)}{2}$$

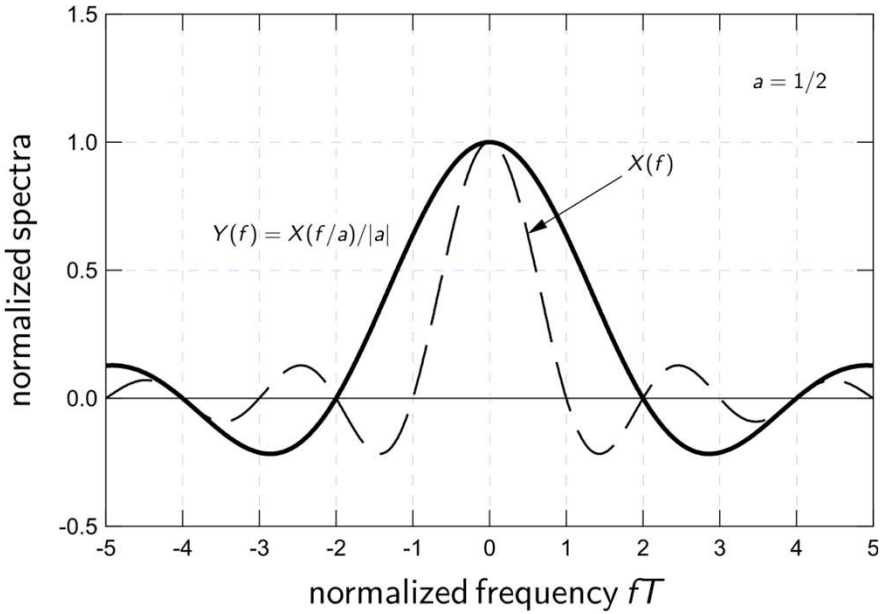




Scale change theorem

$$x(at) \Leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

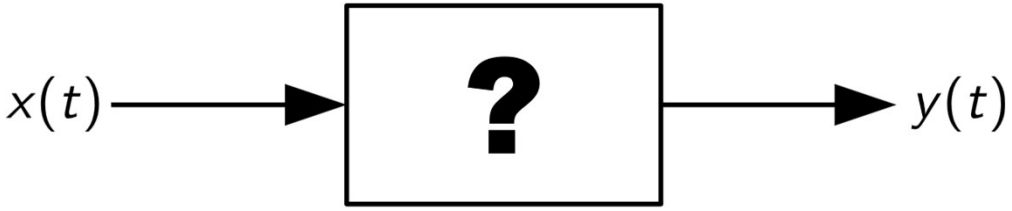




Linear time-invariant (LTI) systems



What is a system?

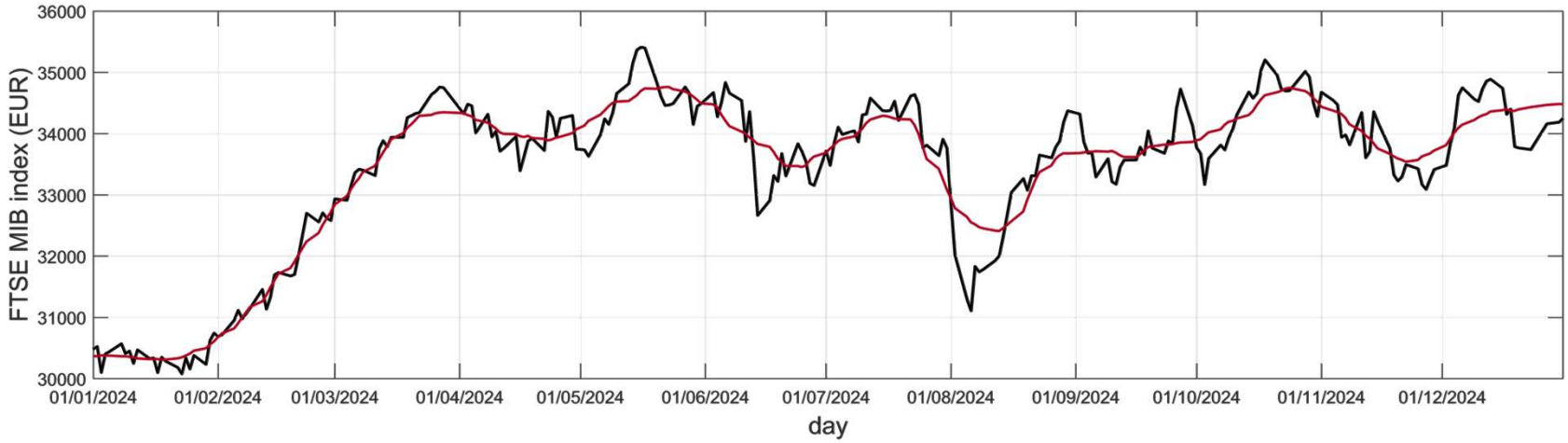


The “black box” approach means that the output value of *y* at a certain time instant *t* depends on the whole shape of the input signal *x* from its beginning up to *t*

$$y(t) = \mathcal{T}\left[x(\alpha); t\right]$$

An example: the moving-average filter

We want to smooth the yearly behavior of the FTSE MIB index:





Frequency response of an LTI system (1/2)

A notable class of systems (considered throughout the course) is represented by the linear time-invariant (LTI) systems

- Linearity means that, if the input signal is a linear combination of two separate inputs, the output is a linear combination of the separate outputs, using the same coefficients:

$$x(t) = \alpha x_1(t) + \beta x_2(t) \Rightarrow y(t) = \mathcal{T}[x(t)] = \alpha \mathcal{T}[x_1(t)] + \beta \mathcal{T}[x_2(t)]$$

- Time invariance (a.k.a. stationarity) means that the system features do not change with time:

$$\mathcal{T}[x(t - t_0)] = y(t - t_0)$$



Frequency response of an LTI system (2/2)

An LTI system can be fully characterized by its frequency response $H(f)$

$H(f)$ can be computed, for each frequency f, by measuring the amplitude and phase variations observed at the output, when the input signal is a sinusoidal wave oscillating at frequency f:

$$H(f) = \left. \frac{y(t)}{x(t)} \right|_{x(t)=e^{j2\pi ft}}$$

The frequency response is useful to relate input and output spectra:

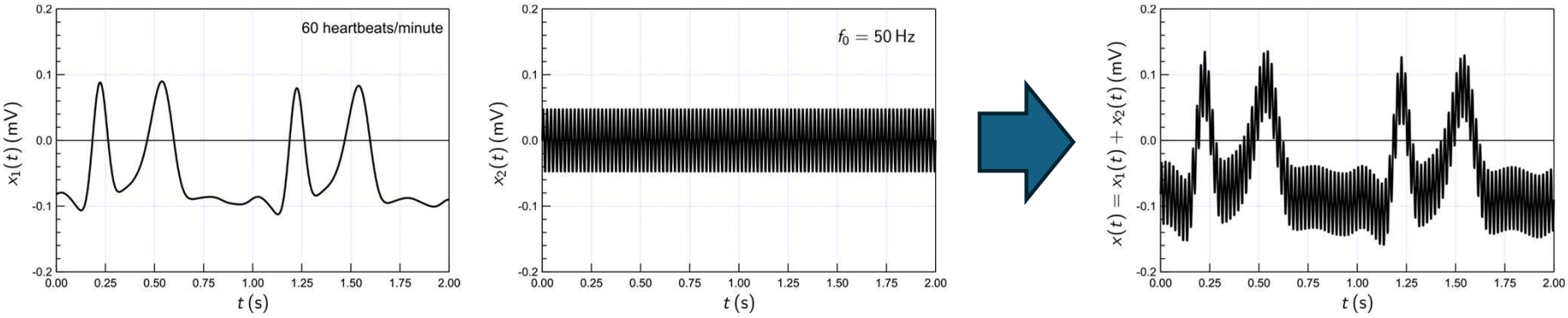
$$Y(f) = H(f) \cdot X(f)$$

A special class of LTI systems: filters (1/5)

Among the LTI systems, filters play a fundamental role in digital communications

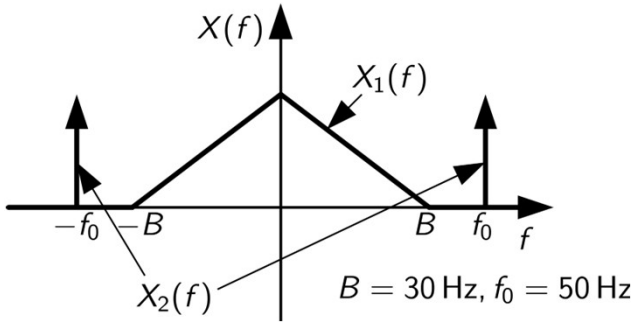
Filters possess a selective behavior in the frequency domain, which is able to maintain some frequency components while rejecting some others

Example: we want to remove the undesired interference generated by AC power in an ECG signal

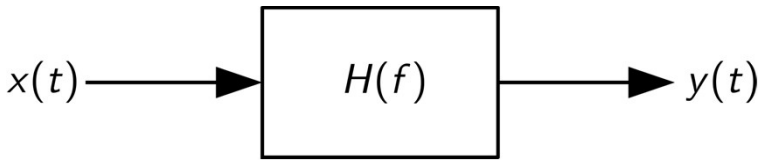


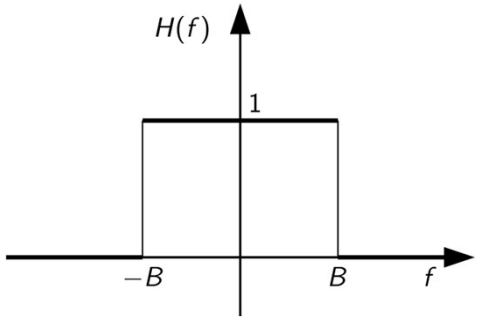
A special class of LTI systems: filters (2/5)

We can face the problem in the frequency domain:



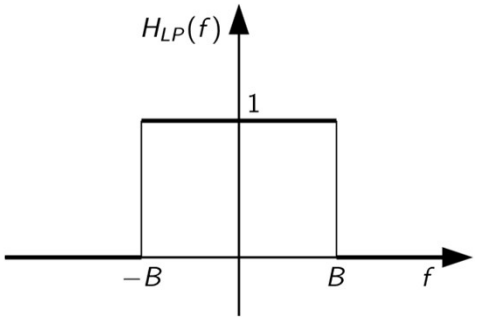
We can apply a low-pass filter which reject the components larger than B:



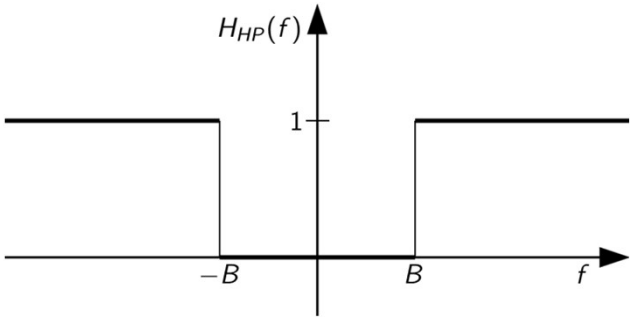


A special class of LTI systems: filters (3/5)

Low-pass (LP) filters: preserve low frequencies while rejecting high frequencies

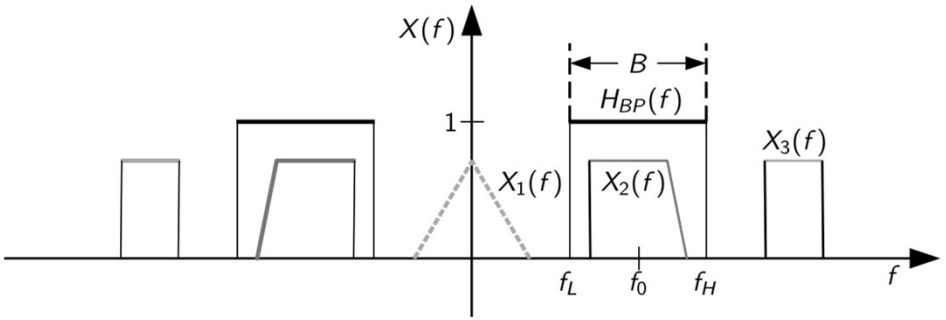


High-pass (HP) filters: preserve high frequencies while rejecting low frequencies

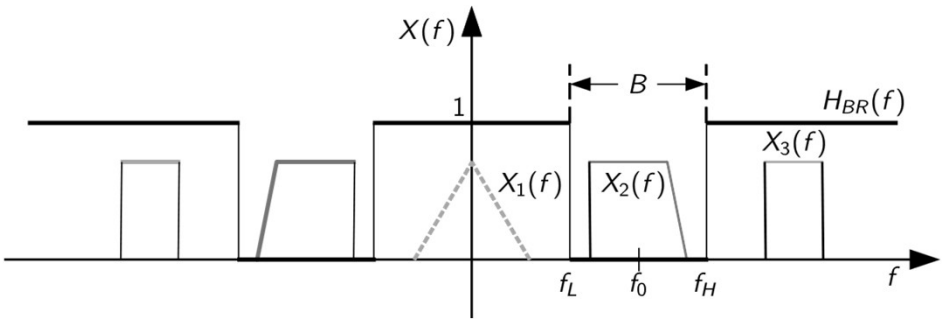


A special class of LTI systems: filters (4/5)

Band-pass (BP) filters: preserve a specific band while rejecting all other bands

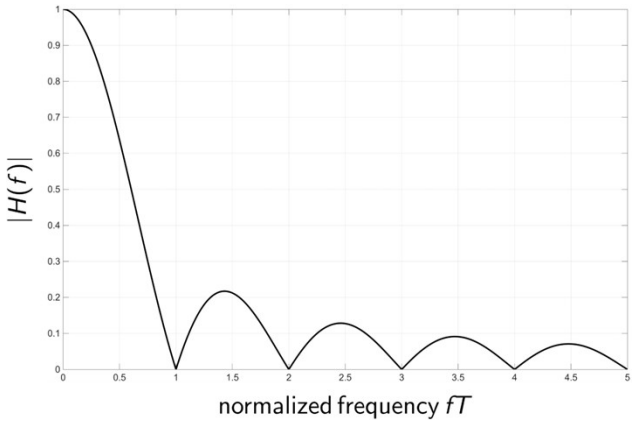


Band-reject (BR) filters: reject a specific band while preserving all other bands



A special class of LTI systems: filters (5/5)

In practice, ideal filters cannot be implemented. Feasible filters show some transition bands to relax the design constraints



To better evaluate the properties of the filter, it is worth plotting the amplitude response on a logarithmic scale, using the decibel (dB):

$$|H(f)|_{\text{dB}} = 20\log_{10}|H(f)|$$

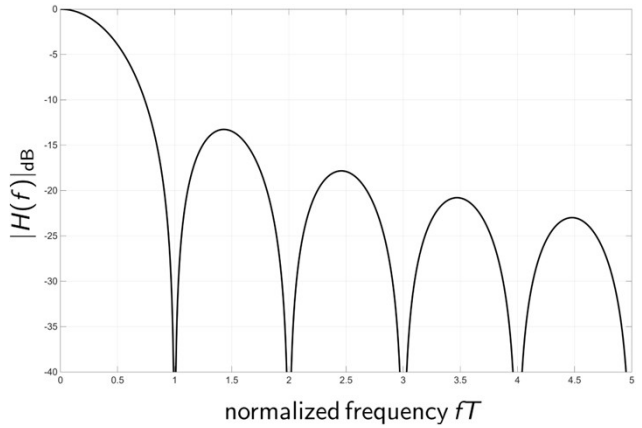
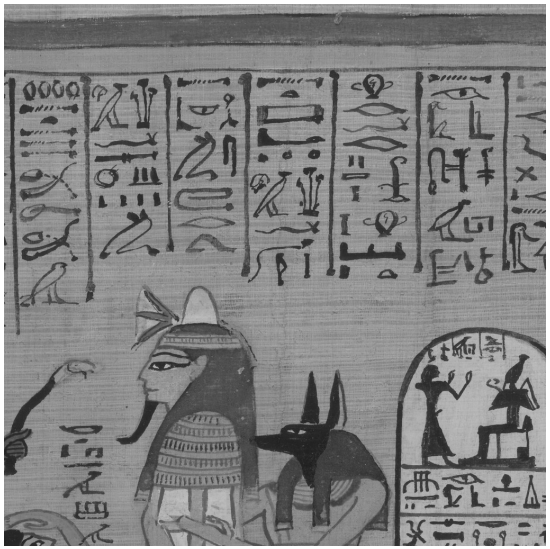


Image filtering (1/3)

Filtering can also be done for images. For instance, LP filtering corresponds to blurring the image:



LPF





Image filtering (2/3)

HP filtering can be used to enhance the details or for edge detection:

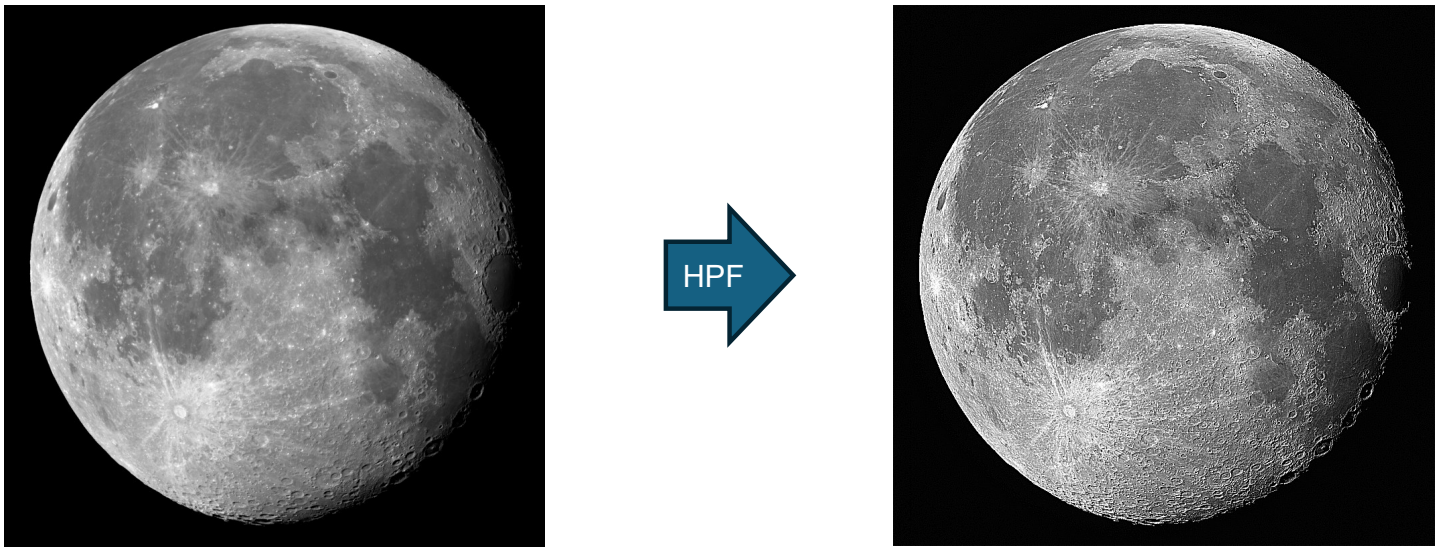
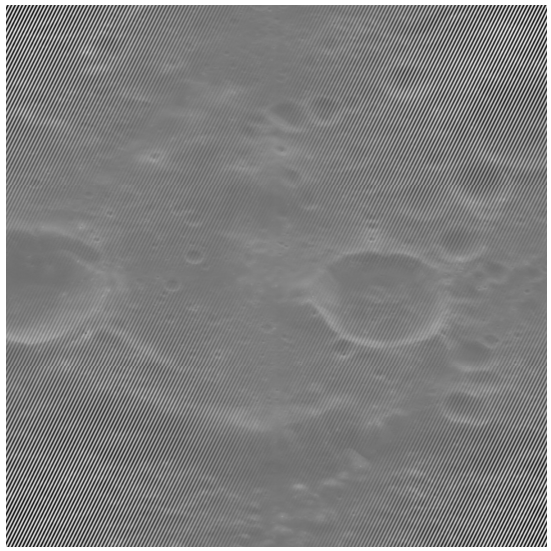


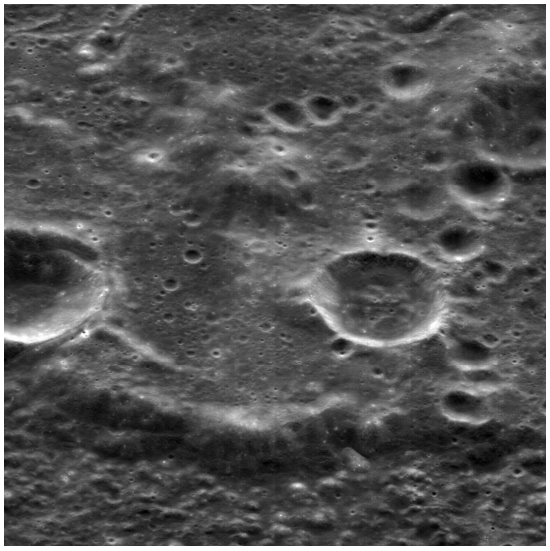


Image filtering (3/3)

BR filtering can be used to remove periodic interference from an image:



BRF





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