Price Discovery in High Resolution

by

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Discussion

by

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Quick summary

General Goal

Study the incorporation of information into prices across markets and timescales

Framework

Hasbrouck's (1995) pioneering work who introduces:

- VECM for prices of the same asset formed in different markets
- Information Share (IS) as measure of price discovery of different markets

Main issue still open in the literature: allocation of correlation terms $\rightarrow$ IS bounds

Approach of the paper

- Shrinking IS bounds by increasing resolution of obs up to microseconds,
- Problem: Huge n$^\circ$ of lags in VECM ($\sim 10^6$)
- Solution: Retain parsimony with "extreme" versions of HAR structure

Main results

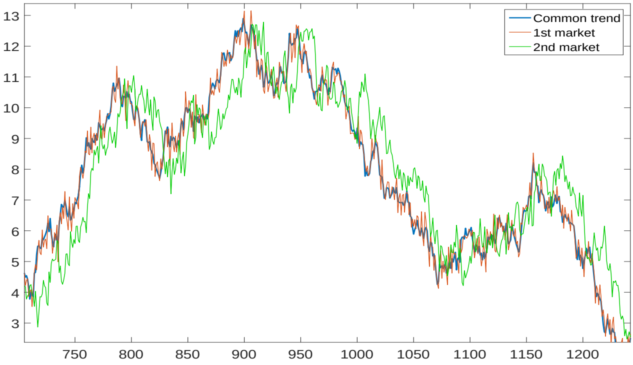
- Successfully collapsing the IS bounds as resolution increases
- Strong information dominance of direct subscribers over consolidated tape
- Slight information dominance of listing exchanges over other exchanges
- Strong information dominance of quotes over trades and lit over dark trades

Ad Hoc Simulation I: VECM with one Lag

- We simulate a VECM for two market prices with 1 lag:

$$\Delta p_t = \gamma B p_{t-1} + \phi_1 \Delta p_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \Omega), \qquad \Omega \text{ is diagonal}$$

- With a chioce of ϕ_1 that induces a sizable lag for the 2^{nd} market



IS 1^{st} market = [0.486, 0.483],

IS 2^{nd} market = [0.513, 0.516],

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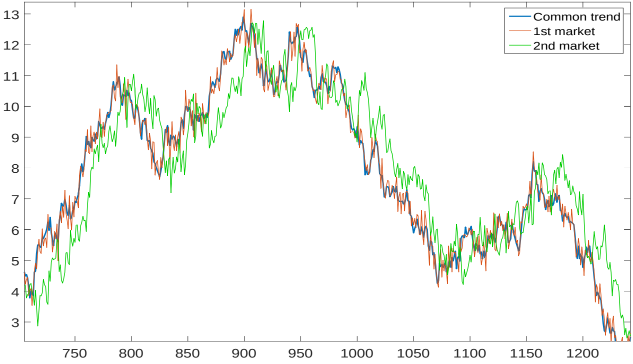

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IS and Autoregressive Lags ϕ_i

- Using the common trends representation

$$p_t = \theta(1)_* \left(\sum_{s=1}^{t} \epsilon_s \right) \iota + \theta^*(L)\epsilon_t \equiv m_t \iota + s_t$$

with $\theta(1)_*$ a generic row of $\theta(1)$ and ι a vector of ones.

- The Information Share (IS) is defined as

$$IS_i = \frac{\theta(1)^2_{*,i} \;\; \Omega_{i,i}}{\theta(1)_* \;\; \Omega \;\; \theta(1)'_*}$$

with $\theta(1)_{*,i}$ the *i-th* element of $\theta(1)_*$

- But, from the Granger's representation theorem we can analytically compute

$$\theta(1) = B_\perp \underbrace{\left[\gamma'_\perp \left(I_N - \sum_{i=1}^{k} \phi_i \right) B_\perp \right]^{-1}}_{\substack{(N-r)\times(N-r) \\ if \;\; r = N-1 \;\Rightarrow\; \text{scalar}}} \gamma'_\perp$$

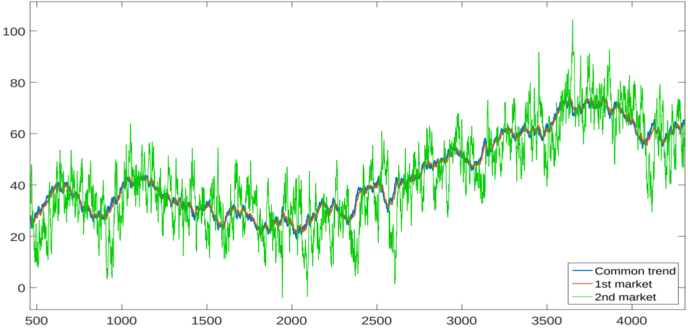
- Hence, the term containing the autoregressive lags ϕ_i cancel out in the IS ratio
$\Rightarrow$ IS depends on γ and Ω, but not DIRECTLY on ϕ's.

Ad Hoc Simulation II: VECM with Different Variances

- We simulate a VECM for two markets (with no lags):

$$\Delta p_t = \gamma B p_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \Omega), \qquad \Omega \text{ is diagonal}$$

  - 1^{st} market is fast adapting (γ_1 small) and with small innovation variance
  - 2^{nd} market is slower (γ_2 larger) and with substantial innovation variance



IS 1^{st} market = [0.380, 0.389],

IS 2^{nd} market = [0.619, 0.610].

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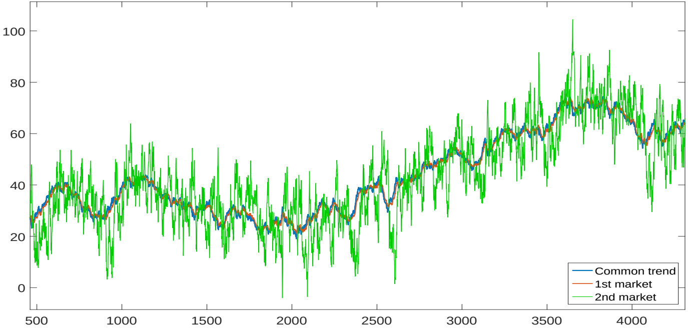

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IS and Market Variances $\Omega_{i,i}$

- In the simulations with a reduced form VECM, the IS of a market depends:
  - Positively on the speed of adjustment of the market:

    $$\text{small } \gamma_i \Rightarrow \text{high } IS_i$$

  - Positively on the noise variance of the market:

    $$\text{large } \Omega_{i,i} \Rightarrow \text{high } IS_i$$

- $\Rightarrow$ IS of a mkt with sufficiently large $\Omega_{i,i}$ can be larger than a mkt with a small γ_j (Update: see also the presentation here at SoFiE pre-conf of Christian Nguenang)
- Not clear if the reduced form model VECM contains MM noise or not (see Dias, Fernandes, Scherrer 2018)
- Difficulties of interpretation might stem from the reduced form nature of VECM
- Look for a structural model (as in De Jong & Schotman 2010) but with a lagged adjustment mechanism
- We consider a multivariate generalization of the partial price adjustment model of Hasbrouck & Ho (1987)

Structural Model with Lagged Adjustments

- In the proposed model the efficient price m_t is the scalar process

$$m_t = m_{t-1} + u_t, \qquad u_t \sim \text{NID}(0, \Sigma)$$

- The latent prices with lagged adjustment X_t is assumed to be

$$X_t = X_{t-1} + \Psi(m_t \,\iota - X_{t-1})$$

  the matrix Ψ is the speed of adjustment matrix.

  If $\Psi = \mathbb{I}$, then $X_{i,t} = m_t$, i.e. instantaneous price adjustment in every market

- Finally, the vector of observed log-prices is:

$$P_t = X_t + \varepsilon_t, \qquad \varepsilon_t \sim \text{NID}(0, H)$$

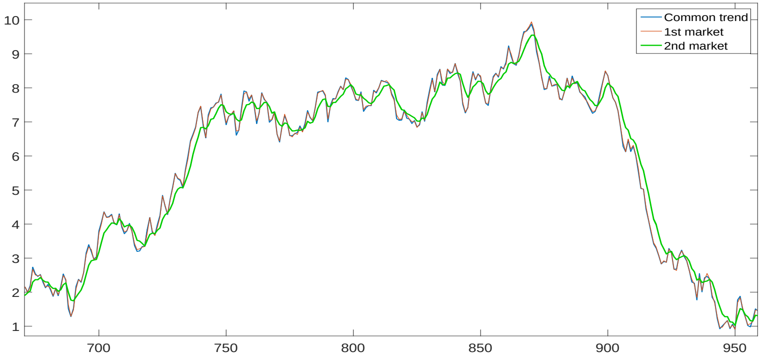
- This model can be conveniently estimated with the standard Kalman filter (treating zeros as missing values) assuming (identification assumption) $u_t \perp \varepsilon_t$

Structural Model with Partial Price Adjustment

- We simulate the structural model for two markets:

$$P_t = X_{t-1} + \Psi(m_t \iota - X_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Omega), \qquad \Omega \text{ is diagonal}$$

  - 1^{st} market is fast adapting ($\Psi_{1,1} = 0.9$)
  - 2^{nd} market is slower ($\Psi_{2,2} = 0.3$)



IS 1^{st} market = [0.997, 0.087]

IS 2^{nd} market = [0.912, 0.002]

Concluding Remarks

Some additional questions:

- What are the effects of large number of zero returns on IS?
- What is the effect of the strong one-second periodicity observed in UHF data?

What I liked about the paper:

- Clean and effective solution to the problem of non-uniqueness of IS
- Heterogeneous/Multifrequency view of the market
- Unified approach to modeling price dynamics across a full range of frequencies
- Consistency of results across resolutions
- Emergence of clearer pictures as resolution increases.