HARK the SHARK: Realized Volatility Modelling with Measurement Errors and Nonlinear Effects

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joint work with

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Motivations: Despite its effectiveness, the HAR model suffers several type of misspecifications:

1. Measurement errors in RV $\rightarrow$ Bollerslev, Patton & Quaedvlieg (2016 BPQ)
2. Heterosckedasticity in residuals $\rightarrow$ Corsi, Mittnik & Pigorsch[2] (2008)
3. time-varying parameters $\rightarrow$ Chen, Härdle & Pigorsch (2010) and standard practice of estimating HAR on a moving window.

$$\Downarrow$$

We propose **new extensions of HAR model** addressing these effects separately to disentangle them and quantify their contributions to volatility forecasts

Structure of the paper

1. Combine the asymptotic theory of the realized volatility estimator with Kalman filter to account for measurement errors → HARK
2. Employ log transformations to remove nonlinearities (heteroskedasticity and autocorrelation of HAR residuals) → HARlog
3. Introduce score driven time-varying parameters → SHAR
4. **Combine the various approaches** in a single model able to account for all the above misspecifications → **SHARK**

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Theoretical framework

Assume log-price $X_s = \log(P_s)$ evolves according to a Brownian semimartingale:

$$dX_s = \mu_s ds + \sigma_s dW_s \tag{1}$$

The day t Integrated Volatility (IV) of X_s is defined as:

$$IV_t = \int_{t-1}^{t} \sigma^2(s) ds \tag{2}$$

The day t Realized Volatility (RV) is defined as:

$$RV_t = \sum_{i=1}^{M} r_{i,t}^2 \tag{3}$$

where the intraday returns $r_{i,t} = X_{t-1+i\Delta} - X_{t-1+(i-1)\Delta}$, $i = 1, \ldots, M$ are computed on M intraday time intervals of length $\Delta = 1/M$.
As the intraday period $\Delta \to 0$, the estimation error is mixed normal distributed:

$$RV_t = IV_t + \epsilon_t, \quad \epsilon_t \sim \text{MN}(0, 2\Delta IQ_t) \tag{4}$$

where $IQ_t = \int_{t-1}^{t} \sigma^4(s) ds$ the Integrated Quarticity (IQ) of the process.
IQ can be consistently estimated using the Realized Quarticity (RQ):
For later convenience, we also report the asymptotic distribution of $\log(RV_t)$.

$$\log(RV_t) = \log(IV_t) + \xi_t, \quad \xi_t \sim \text{MN}\left(0, 2\Delta \frac{IQ_t}{IV_t^2}\right) \tag{5}$$

HAR assumes $RV_t = IV_t$ and heterogeneous autoregressive structure:

$$RV_{t+1} = \beta_0 + \beta_1 RV_t + \beta_2 RV_{t-1|t-5} + \beta_3 RV_{t-6|t-22} + \eta_{t+1} \tag{6}$$

where $RV_{t_1|t_2}$ denotes the average of daily RV's from day t_1 to day t_2.

BPQ: measurement errors in RV induce attenuation bias of OLS β coefficients
For ex:

- if DGP is an AR(1) on IV: $IV_{t+1} = \phi_0 + \phi_1 IV_t + u_t$
- while we estimate an AR(1) on RV: $RV_{t+1} = \beta_0 + \beta_1 RV_t + u_t$
- With measurement errors $RV_t = IV_t + \epsilon_t$, the population parameter β_1 is

$$\beta_1 = \phi_1 \left(1 + \frac{2\Delta IQ}{Var(IV_t)}\right)^{-1} < \phi_1 \tag{7}$$

In presence of time-varying measurement errors BPQ propose the HARQ:

$$RV_{t+1} = \beta_0 + \underbrace{\left(\beta_1 + \beta_{1Q} RQ_t^{1/2}\right)}_{\beta_{1,t}} RV_t + \beta_2 RV_{t-1|t-5} + \beta_3 RV_{t-6|t-22} + \eta_{t+1} \tag{8}$$

State-space representation and the HARK model

A natural way of dealing with measurement errors is through the Kalman filter.

$$RV_t = IV_t + \epsilon_t \tag{9}$$

$$IV_{t+1} = \beta_0 + \beta_1 IV_t + \beta_2 IV_{t-1|t-5} + \beta_3 IV_{t-6|t-22} + \eta_{t+1} \tag{10}$$

which can be written in the standard state-space rapresentation

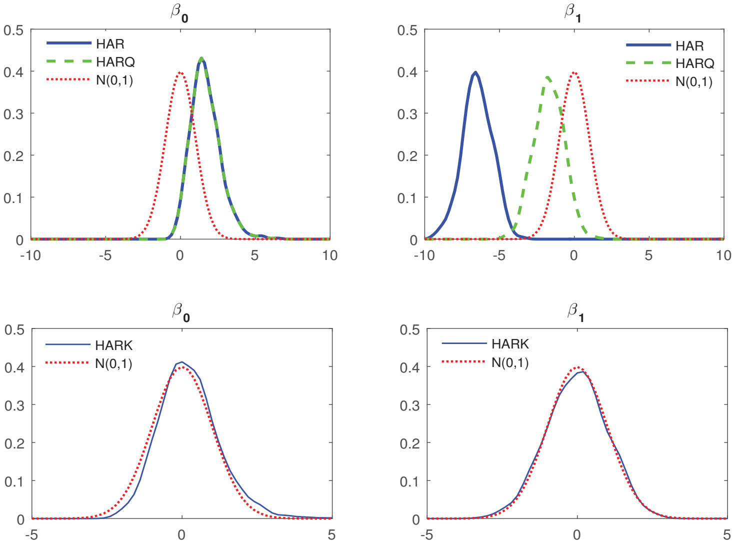
$$RV_t = Z\alpha_t + \epsilon_t, \quad \epsilon_t \sim \mathrm{NID}(0, H_t) \tag{11}$$

$$\alpha_{t+1} = c + T\alpha_t + \eta_t, \quad \eta_t \sim \mathrm{NID}(0, Q) \tag{12}$$

where $Z = (1, 0 \ldots, 0)$ is an n-dimensional row vector,

$$c = \begin{pmatrix} \beta_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \alpha_{t+1} = \begin{pmatrix} IV_{t+1} \\ IV_t \\ \vdots \\ IV_{t-n-2} \end{pmatrix}, T = \begin{pmatrix} \beta_1, & \frac{1}{n_w}\beta_2 & \overbrace{\cdots}^{n_w \text{ terms}} & \frac{1}{n_w}\beta_2, & \frac{1}{n_m}\beta_3 & \overbrace{\cdots}^{n_m \text{ terms}} & \frac{1}{n_m}\beta_3 \\ 1 & 0 & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & \ddots & & & & \vdots & \vdots \\ \vdots & & \ddots & & & \vdots & \vdots \\ \vdots & & & \ddots & & \vdots & \vdots \\ \vdots & & & & \ddots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 1 & 0 \end{pmatrix}$$

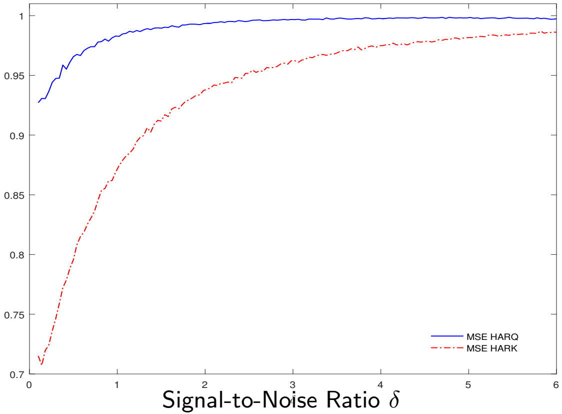
Simulations: DGP = HAR + Noise



Kernel densities of standardized estimation errors of β_0 and β_1

Simulations: DGP = HAR + Noise

Do better parameter estimates translate in better **out-of-sample forecasts**?



Relative MSE to HAR: $\frac{MSE\_HARQ}{MSE\_HAR}$, $\frac{MSE\_HARK}{MSE\_HAR}$

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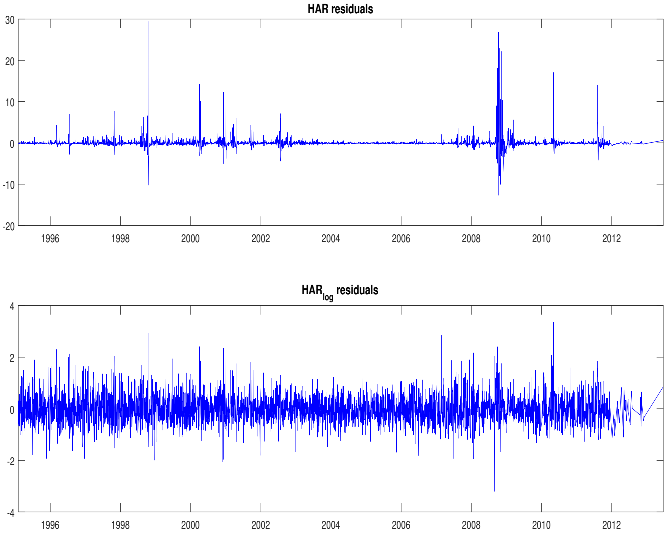
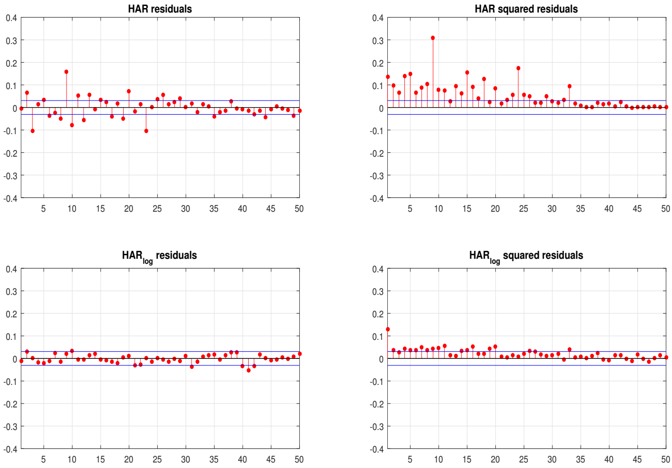
Heterosckedasticity in residuals: HAR vs HAR-Log (on S&P500 data)



Heteroskedasticity in residuals: HAR vs HAR-Log (on S&P500 data)



$\Rightarrow$ HARK will be estimated on $\log RV_t$

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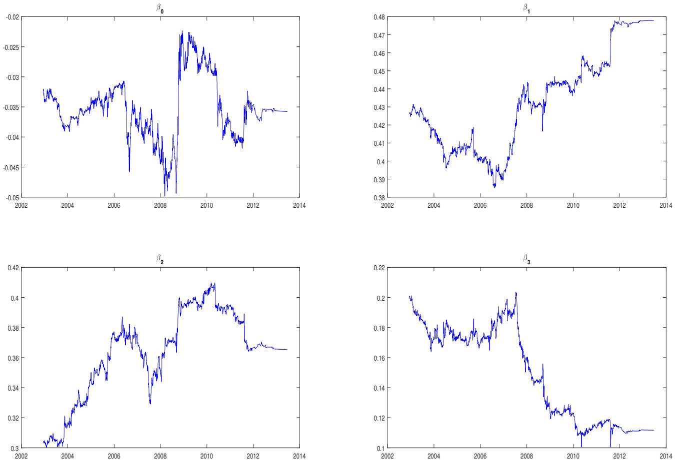
Time-varying parameters



HAR-Log parameters estimated on a moving window of 2000 observations

The SHAR model

Time variation in HAR parameters introduced by Score-driven HAR-Log (SHAR)

$$\log RV_{t+1} = \beta_{0,t+1} + \beta_{1,t+1} \log RV_t + \beta_{2,t+1} \log RV_{t-1|t-n_w} + \beta_{3,t+1} \log RV_{t-n_w-1|t-n_w-n_m} + \eta_{t+1} \quad (13)$$

with $\eta_{t+1} \sim \text{NID}(0, q_{t+1})$

The vector of time-varying parameters at time t is defined as:

$$f_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \log q_t)'$$

In score-driven models f_{t+1} is assumed to have the following updating rule:

$$f_{t+1} = \omega + A s_t + B f_t \quad (14)$$

where s_t is the scaled score vector:

$$s_t = (\mathcal{I}_{t|t-1})^{-1} \nabla_t, \quad \nabla_t = \left[\frac{\partial \log p(\log RV_t | f_t, \mathcal{Y}_{t-1}, \Theta)}{\partial f_t'} \right]', \quad \mathcal{I}_{t|t-1} = \text{E}[\nabla_t \nabla_t']$$

$\Theta = \{\omega, A, B\}$ is the set of all static parameters estimated by standard MLE

$$\log L(\log RV_1, \ldots, \log RV_T | \Theta) = \sum_{t=1}^{T} -\frac{1}{2} \left(\log q_t + \frac{(RV_t - \mu_{t|t-1})^2}{q_t} \right) \quad (15)$$

where $\mu_{t|t-1}$ is the conditional mean:

SHAR: residual diagnostic

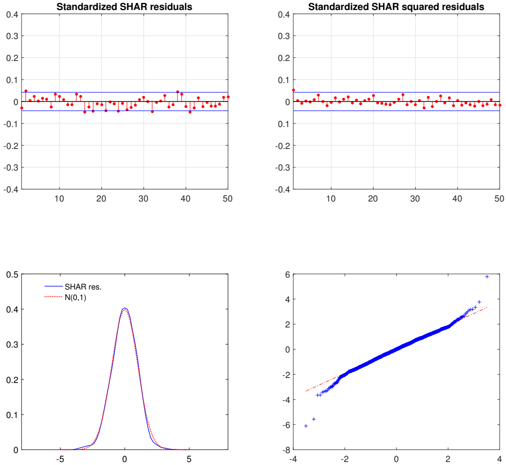


Figure: Sample autocorrelation of std. residuals and squared std. residuals of the SHAR model estimated on S&P500 future RV from 03-01-1995 to 21-06-2013. Kernel density estimates of SHAR std residuals and Q-Q plot.

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Score-driven HARK (SHARK): HARK with score-driven t.v.p. :

$\log RV_{t+1} = \log IV_{t+1} + \epsilon_t$
$\log IV_{t+1} = \beta_{0,t+1} + \beta_{1,t+1} \log IV_t + \beta_{2,t+1} \log IV_{t-1|t-n_w} + \beta_{3,t+1} \log IV_{t-n_w-1|t-n_w-n_m} + \eta_{t+1}$

with $\eta_{t+1} \sim \text{NID}(0, q_{t+1})$

which, again, can be written in state-space representation

$$RV_t = Z\alpha_t + \epsilon_t, \quad \epsilon_t \sim \text{NID}(0, H_t) \tag{16}$$

$$\alpha_{t+1} = c_t + T_t\alpha_t + \eta_t, \quad \eta_t \sim \text{NID}(0, Q_t) \tag{17}$$

As before, the vector of time-varying parameters at time t is:

$$f_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \log q_t)'$$

with the usual updating rule:

$$f_{t+1} = \zeta + Cs_t + Df_t \tag{18}$$

where $s_t = (\mathcal{I}_{t|t-1})^{-1}\nabla_t$.

The SHARK model: Estimation

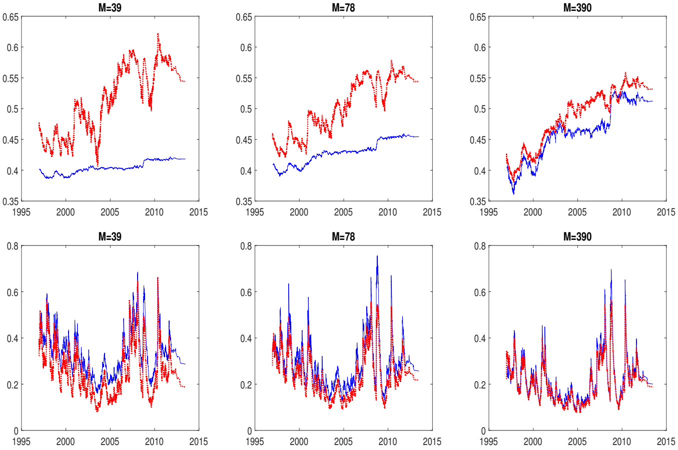
As shown by Delle Monache et al (2017), ∇_t and $\mathcal{I}_{t|t-1}$ can be computed as:

$$\nabla_t = -\frac{1}{2}\left[\dot{F}_t'(\mathbb{I}_{n_t} \otimes F_t^{-1})\text{vec}(\mathbb{I}_{n_t} - v_t v_t' F_t^{-1}) + 2\dot{v}_t' F_t^{-1} v_t\right] \quad (19)$$

$$\mathcal{I}_{t|t-1} = \frac{1}{2}\left[\dot{F}_t'(F_t^{-1} \otimes F_t^{-1})\dot{F}_t + 2\dot{v}_t' F_t^{-1}\dot{v}_t\right] \quad (20)$$

- v_t and F_t are the Kalman filter prediction error and its covariance matrix
- $\dot{v}_t = \partial v_t / \partial f_t'$ and $\dot{F}_t = \partial \text{vec}(F_t)/\partial f_t'$ are computed through a parallel filter that is obtained by deriving the Kalman filter recursions w.r.t. f_t
- Running the 2 recursions gives us α_t, f_t and the log-L in the prediction error form $\rightarrow$ static parameters $\{\zeta, C, D\}$ estimated with standard MLE.

SHAR vs SHARK parameter estimates at different frequencies



Dynamics of filtered parameters $\beta_{1,t}$ (top row) and variance q_t (bottom row) for the SHAR (blue) and SHARK (red) on S&P500 RVs computed at $10, 5, 1$-min

A nonlinear DGP

Two-factor stochastic volatility model of Huang and Tauchen (2005):

$$\begin{aligned}
dX_t &= \mu dt + \sigma_{ut}\nu_t(\rho_1 dW_{1t} + \rho_2 dW_{2t} + \sqrt{1-\rho_1^2-\rho_2^2})dW_{3t}) \\
\nu_t^2 &= s\text{-}\exp\{\beta_0 + \beta_1\nu_{1t}^2 + \beta_2\nu_{2t}^2\} \\
dv_{1t}^2 &= \alpha_1\nu_{1t}^2 dt + dW_{1t} \\
dv_{2t}^2 &= \alpha_2\nu_{2t}^2 dt + (1+\phi\nu_{2t}^2)dW_{2t} \\
\sigma_{ut} &= C + Ae^{-at} + Be^{-b(1-t)}
\end{aligned}$$

- Generate 1 sec. data assuming a trading day of 23400 sec. (09:30 - 16:00)
- Compute RV_t, RQ_t with $M = 39, 78, 390$ (corresponding to 10, 5, 1 min.)
- Generate $N = 1000$ daily RV (RQ) series of $T = 3000$ obs and compute out-of-sample forecasts of the last 1000 with a rolling window of 2000 obs.

A nonlinear DGP, cont'd

Average signal-to-noise ratio

	$M = 39$	$M = 78$	$M = 390$
HARK	7.54	13.62	61.17
SHARK	8.00	14.30	62.70

Much larger than what found on real data, but still interesting to see HARQ vs HAR_{log}

	HAR	HARQ	HAR_{log}	HARK	SHAR	SHARK
	Out-of-sample					
	$M = 39$					
MSE	1.0000	0.9892	0.9676	0.9769	**0.9605**	0.9869
MAE	1.0000	0.9869	0.9691	0.9575	0.9725	**0.9564**
QLIKE	1.0000	0.9705	0.9262	0.9415	**0.9227**	0.9484
	$M = 78$					
MSE	1.0000	0.9938	0.9734	0.9777	**0.9698**	0.9816
MAE	1.0000	0.9880	0.9715	0.9653	0.9753	**0.9644**
QLIKE	1.0000	0.9604	0.9248	0.9318	**0.9218**	0.9341
	$M = 390$					
MSE	1.0000	0.9985	0.9765	0.9917	**0.9740**	0.9768
MAE	1.0000	0.9887	**0.9731**	0.9760	0.9780	0.9737
QLIKE	1.0000	0.9560	0.9235	0.9497	**0.9207**	0.9236

Empirical application: Data

- Tick-by-tick data of S&P500 from 03-01-1995 to 21-06-2013 (4259 days)
- 18 of the most highly traded stocks on NYSE between 03-01-2006 and 31-12-2014 (2250 days)
- Use a rolling window of 2000 obs. to predict RV in the last $T - 2000$ days
- SHAR and SHARK estimated only once, while all other models are re-estimated each day
- Since one of the main purposes of this analysis is to assess the effect of measurement errors, we compute *RV* estimates at different frequencies:
  1. $M = 39 \Rightarrow$ 10-min returns
  2. $M = 78 \Rightarrow$ 5-min returns
  3. $M = 390 \Rightarrow$ 1-min returns

Out-of-sample - Index data

	HAR	HARQ	HAR_{log}	HARK	SHAR	SHARK
	S&P500					
	$M = 39$					
MSE	1.0000	0.9897	0.9927	0.9699*	0.9770	**0.9508***
	(0.033)	(0.033)	(0.033)	(0.233)	(0.033)	(1.000)
QLIKE	1.0000	0.9037	0.8863	0.8765	0.8643*	**0.8536***
	(0.015)	(0.022)	(0.022)	(0.057)	(0.200)	(1.000)
	$M = 78$					
MSE	1.0000	0.9714	0.9555	0.9284*	0.9452	**0.9163***
	(0.003)	(0.057)	(0.089)	(0.587)	(0.089)	(1.000)
QLIKE	1.0000	0.8689	0.7844	0.7668*	0.7733*	**0.7455***
	(0.000)	(0.000)	(0.013)	(0.180)	(0.180)	(1.000)
	$M = 390$					
MSE	1.0000*	0.9595*	0.9781*	0.9636*	**0.9524***	0.9538*
	(0.141)	(0.932)	(0.141)	(0.736)	(1.000)	(0.932)
QLIKE	1.0000	0.9244	0.9022	0.8911	0.8830*	**0.8759***
	(0.008)	(0.019)	(0.019)	(0.096)	(0.295)	(1.000)

Table: Relative out-of-sample losses computed on S&P500 data. We show in parenthesis the p-values of the MCS at 90% c.l. computed using the MSE and QLIKE. The presence of an asterisk indicates that the model is included in $\widehat{\mathcal{M}}_{90\%}$.

Out-of-sample - Individual stocks

	HAR	HARQ	HAR_{log}	HARK	SHAR	SHARK
			$M = 39$			
MSE	$1.0000^{(3)}$	$0.7651^{(5)}$	$0.5639^{(5)}$	$0.5433^{(14)}$	$0.5504^{(14)}$	$\mathbf{0.5377}^{(18)}$
QLIKE	$1.0000^{(3)}$	$0.9579^{(3)}$	$0.7366^{(10)}$	$0.7295^{(14)}$	$0.7242^{(16)}$	$\mathbf{0.7188}^{(17)}$
			$M = 78$			
MSE	$1.0000^{(2)}$	$0.9034^{(3)}$	$0.6612^{(7)}$	$0.6485^{(12)}$	$0.6525^{(15)}$	$\mathbf{0.6428}^{(18)}$
QLIKE	$1.0000^{(3)}$	$1.2628^{(3)}$	$0.7071^{(11)}$	$0.7010^{(13)}$	$0.6952^{(17)}$	$\mathbf{0.6890}^{(18)}$
			$M = 390$			
MSE	$1.0000^{(1)}$	$0.8492^{(1)}$	$0.5437^{(3)}$	$0.5691^{(9)}$	$0.5315^{(9)}$	$\mathbf{0.5216}^{(18)}$
QLIKE	$1.0000^{(2)}$	$2.3093^{(3)}$	$0.6002^{(10)}$	$0.5972^{(12)}$	$0.5944^{(17)}$	$\mathbf{0.5866}^{(17)}$

Table: Relative averages of out-of-sample losses computed on 18 individual stock data. We show in parenthesis the number of times the model is included in $\widehat{\mathcal{M}}_{90\%}$.

Longer forecast horizons (weekly)

	HAR	HARQ	HAR_{log}	HARK	SHAR	SHARK
			$M = 39$			
MSE	1.0000	0.9519	0.9302	$0.9047^{\star}$	0.9229	**0.8732***
	(0.000)	(0.001)	(0.008)	(0.276)	(0.091)	(1.000)
QLIKE	1.0000	0.9263	0.8253	$0.8051^{\star}$	$0.8021^{\star}$	**0.7729***
	(0.001)	(0.036)	(0.083)	(0.232)	(0.232)	(1.000)
			$M = 78$			
MSE	1.0000	0.9204	0.8853	$0.8652^{\star}$	$0.8607^{\star}$	**0.8460***
	(0.000)	(0.004)	(0.004)	(0.512)	(0.512)	(1.000)
QLIKE	1.0000	0.8917	0.7662	$0.7452^{\star}$	$0.7387^{\star}$	**0.7243***
	(0.000)	(0.010)	(0.023)	(0.544)	(0.544)	(1.000)
			$M = 390$			
MSE	1.0000	0.9422	0.9190	$0.9084^{\star}$	0.9140	**0.9033***
	(0.000)	(0.003)	(0.003)	(0.783)	(0.003)	(1.000)
QLIKE	1.0000	0.9387	0.8572	$0.8426^{\star}$	$0.8375^{\star}$	**0.8250***
	(0.000)	(0.003)	(0.003)	(0.568)	(0.568)	(1.000)

Table: Relative out-of-sample losses of HAR, HARQ, HAR_{log}, HARK, SHAR, SHARK models on S&P500 data for weakly ($j = 5$) variance forecasts.

Longer forecast horizons (monthly)

	HAR	HARQ	HAR_{log}	HARK	SHAR	SHARK
			$M = 39$			
MSE	1.0000	1.0081	0.8686	$0.8334^{\star}$	$0.8568^{\star}$	**0.7936***
	(0.000)	(0.000)	(0.008)	(0.2350)	(0.2350)	(1.000)
QLIKE	1.0000	0.9755	0.8023	0.7630	0.8045	**0.6935***
	(0.001)	(0.001)	(0.006)	(0.074)	(0.074)	(1.000)
			$M = 78$			
MSE	1.0000	0.9896	0.8319	$0.8039^{\star}$	$0.8050^{\star}$	**0.7847***
	(0.000)	(0.000)	(0.001)	(0.852)	(0.852)	(1.000)
QLIKE	1.0000	0.9747	0.7697	$0.7347^{\star}$	$0.7634^{\star}$	**0.6886***
	(0.000)	(0.000)	(0.000)	(0.218)	(0.218)	(1.000)
			$M = 390$			
MSE	1.0000	1.0187	0.9409	$0.9241^{\star}$	0.9400	**0.9014***
	(0.000)	(0.000)	(0.000)	(0.461)	(0.000)	(1.000)
QLIKE	1.0000	1.0150	0.8712	$0.8468^{\star}$	$0.8459^{\star}$	**0.7993***
	(0.002)	(0.002)	(0.002)	(0.157)	(0.157)	(1.000)

Table: Relative out-of-sample losses of HAR, HARQ, HAR_{log}, HARK, SHAR, SHARK models on S&P500 data for monthly ($j = 22$) variance forecasts.

Disentagling misspecifications

	HAR	HAR_{log}	HARK	SHAR	SHARK
HARQ			$M = 39$		
	0.9490	0.9129	0.9281	0.9627	0.9741
HARQ			$M = 78$		
	0.9321	0.9004	0.9235	0.9475	0.9737
HARQ			$M = 390$		
	0.9395	0.9405	0.9517	0.9779	0.9623

Table: R^2 resulting from regressing forecast errors of the HARQ against forecast errors of all other models.

Disentagling misspecifications

Current debate (rised by Cipollini Gallo Otranto): since $\rho(RV_t, RQ_t) \approx 0.93$, is the HARQ capturing measurement errrors or nonlinearity with a RV_t^2 term?

Thanks to the newly proposed models we can disentangle different misspecifications and quantify their contribution to volatility forecasts.

In fact, given a **Loss function** L we have:

1. $\varphi_{\text{het}} = \dfrac{\text{E}[L(\text{HAR}_{\log})]}{\text{E}[L(\text{HAR})]} \quad \rightarrow$ heteroskedasticity and nonlinear effects
2. $\varphi_{\text{err}}^{(1)} = \dfrac{\text{E}[L(\text{HARK})]}{\text{E}[L(\text{HAR}_{\log})]}, \quad \varphi_{\text{err}}^{(2)} = \dfrac{\text{E}[L(\text{SHARK})]}{\text{E}[L(\text{SHAR})]} \rightarrow$ measurement errors
3. $\varphi_{\text{tvp}}^{(1)} = \dfrac{\text{E}[L(\text{SHAR})]}{\text{E}[L(\text{HAR}_{\log})]}, \quad \varphi_{\text{tvp}}^{(2)} = \dfrac{\text{E}[L(\text{SHARK})]}{\text{E}[L(\text{HARK})]} \rightarrow$ time-varying param's

As Loss functions L we will consider: MSE and QLIKE

Assessing the relative impact of noise and non-linearity

	φ_{het}	$\varphi_{tvp}^{(1)}$	$\varphi_{tvp}^{(2)}$	$\varphi_{err}^{(1)}$	$\varphi_{err}^{(2)}$
			S&P500		
			$M = 39$		
MSE	0.9927	0.9842	0.9803	0.9771	0.9732
QLIKE	0.8863	0.9752	0.9739	0.9890	0.9876
			$M = 78$		
MSE	0.9555	0.9872	0.9922	0.9715	0.9764
QLIKE	0.7844	0.9746	0.9817	0.9775	0.9846
			$M = 390$		
MSE	0.9781	0.9737	0.9898	0.9852	1.0015
QLIKE	0.9022	0.9787	0.9829	0.9876	0.9919

Table: Forecast gains φ_{het}, $\varphi_{tvp}^{(1)}$, $\varphi_{tvp}^{(2)}$, $\varphi_{err}^{(1)}$, $\varphi_{err}^{(2)}$ using the MSE and QLIKE as loss measures.

Forecast gains measures at longer horizons, cont'd

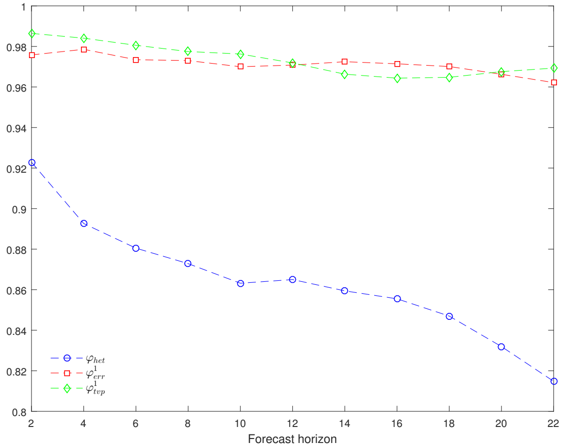


Figure: MSE-based forecast gains φ_{het}, φ^1_{err}, φ^1_{tvp} at different forecast horizons

	φ_{het}	φ_{err^1}	φ_{tvp^1}
	uSPA test		
t_{uSPA}	3.5273	2.8827	1.8295
p-value	0	0	0.0365

Table: MSE based uniform superior predictive ability test (uSPA) of Quaedvlieg (2017). We show the t_{uSPA} test statistics and related p-values based on a multi-horizon bootstrap with 5000 replications.

Robustness checks

$H_t = V_t \equiv 2\Delta \frac{RQ_t}{RV_t^2}$ only if the HARK (SHARK) is correctly specified.

Adopt more flexible specifications for H_t to test for model misspecification:

1. $H_t = \alpha V_t + (1-\alpha)V_{t-1}$
2. $H_t = \alpha V_t + (1-\alpha)\text{mean}(V_t)$
3. $H_t = \alpha V_t + (1-\alpha)\text{median}(V_t)$
4. $H_t = \beta V_t$

with $0 \leq \alpha \leq 1$, $\beta > 0$.

Results

- $\alpha \simeq 1$, with great significance $\rightarrow$ todays's V_t is the "best" proxy for H_t
- We obtain $\beta > 1$ for the HARK and $\beta \simeq 1$ for the SHARK

Conclusions

1. Identify the main sources of misspecifications of HAR model and propose **new HAR extensions** addressing these effects separately
    1. Measurement errors in RV $\rightarrow$ HARK
    2. Heterosckedasticity in residuals $\rightarrow$ HARlog
    3. Time-varying parameters $\rightarrow$ SHAR
    4. **All of them $\rightarrow$ SHARK**
2. **Quantify their contributions** to volatility forecasts by:
    1. L(HAR) / L(HARlog) $\rightarrow$ heteroskedasticity and nonlinear effects
    2. L(HARlog) / L(HARK) or L(SHAR) / L(SHARK) $\rightarrow$ measurement errors
    3. L(HARlog) / L(SHAR) or L(HARK) / L(SHARK) $\rightarrow$ time-varying param's
3. **Answer to current debate** on which is the most relevant effect: ... "it depends" ...on the frequency, i.e. when $M \uparrow$, measurement errors $\downarrow$
4. Show typical **performance ranking** of different models: L(HAR) > L(HARQ) > L(HARlog) > L(SHAR) > L(HARK) > **L(SHARK)**

So ... Thank You and ...