Leaky-Wave Theory, Techniques, and Applications: From Microwaves to Visible Frequencies

The theory of electromagnetic leaky waves can explain a variety of electromagnetic phenomena and scenarios, from microwave to visible frequencies.

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ABSTRACT | Leaky waves have been among the most active areas of research in microwave engineering over the second half of the 20th century. They have been shown to dominate the near-field of several open wave-guiding structures, of great interest to tailor their radiation, guidance and filtering properties. The elegant theoretical analyses and deep physical insights in this area, developed in an era in which computational resources were limited, represent a fundamental scientific legacy that is still extremely relevant in today’s engineering society and beyond. In this regard, the relevance of leaky-wave concepts has been increasingly recognized in recent times over a broader scientific community, including optics and physics societies. In this paper, after revisiting the fundamental concepts of leaky-wave theory, we discuss and connect different relevant research activities in which leaky-wave concepts have been applied, with the goal of facilitating multidisciplinary interactions on these topics. In addition to the canonical microwave applications of leaky waves, particular attention is devoted to a few areas of interest in modern optics, such as directive optical antennas, extraordinary optical transmission, and embedded scattering eigenvalues, in which leaky waves play a fundamental role.

KEYWORDS | Antennas; leaky waves; metamaterials; optics; plasmonics

I. INTRODUCTION

In wave physics, the damping of harmonic oscillations in time and/or space is intuitively associated with losses in a dissipative system. However, it is well known that in open systems the oscillation energy can also be gradually lost in the form of radiation toward the remote boundaries of an open region, hence reducing the amplitude of oscillations even when the system is ideally nondissipative. Examples of this phenomenon are ubiquitous in several areas of physics and engineering, such as radioactive states in quantum mechanics, damped resonances in open acoustic or electromagnetic cavities, and leaky waves in open wave-guiding structures [1]. Although the concepts of “radiation loss” and “energy leakage” appear rather intuitive at first, the analysis of wave localization and guiding in open systems has proved to be very interesting and at the same time theoretically challenging, revealing surprising and counterintuitive features, as we discuss in the following. In this paper, we specifically focus on the fundamental and applied aspects of electromagnetic leaky waves, an exciting research topic that traces back its origins to the early stages of microwave engineering, and whose importance is now being increasingly recognized in many scientific communities. In the last decades, leaky-wave concepts have been successfully applied to design radiating systems (leaky-wave antennas) and to explain and interpret several phenomena throughout the electromagnetic spectrum, such as Cherenkov radiation, Wood’s anomalies, and extraordinary optical transmission. The goal of this paper is to review and connect these areas of research, spanning several disciplines, and provide the reader with the fundamental background information to understand and apply leaky-wave concepts in different electromagnetic scenarios, from microwave to visible frequencies. We hope that this effort...
may be beneficial to the microwaves, optics, engineering, and physics communities, so that the connections of leaky-wave theory and applications to recent advances in physics and optics may be appreciated in the context of the wide background of discoveries carried out by the microwave community in the past decades.

II. FUNDAMENTALS OF LEAKY-WAVE PHYSICS

In the early days of microwave field theory and engineering, during the first half of the 20th century, most of the attention was devoted to guided waves in closed systems, such as metallic waveguides [2], with many of these early research efforts brilliantly summarized in Marcuvitz’s handbook [3]. In closed lossless systems, any field distribution satisfying the boundary conditions can be represented as the superposition of source-free wave eigensolutions (i.e., self-sustained oscillations, or eigenmodes), which form an orthogonal and complete discrete spectrum of modes. Each of these modal, or proper, solutions is characterized by finite energy everywhere, i.e., it is absolute square integrable. In a waveguide—transversely closed in one or two directions—these eigenmodes correspond to pole singularities of an appropriate characteristic Green’s function in the transverse-wavenumber complex plane [1]. From the engineering standpoint, this characteristic Green’s function can be interpreted as the voltage (or current) in a transmission line along one of the transverse directions of the waveguide [4], [5]. Within this network formalism, the pole singularities correspond to resonances of the transverse network model, which can be conveniently calculated using analytical methods. For instance, as shown in Fig. 1(a) and (b), for a parallel-plate waveguide the transverse cross-section can be modeled as a transmission-line segment terminated by lumped impedances ($Z_{l1} = Z_{l2} = 0$ for a waveguide with perfectly conducting walls). The transverse wavenumbers of the source-free solutions are then found by solving the transverse resonance equation

$$\bar{Z} + \bar{Z}^* = 0$$

where $\bar{Z}$ and $\bar{Z}^*$ are the impedances looking at the two sides of an arbitrary reference plane at $x = x_0$. In closed, nondissipative wave-guiding structures, the eigenmodes are typically described by purely real or purely imaginary transverse wavenumbers, which represent, respectively, propagating waves with constant amplitude, or evanescent waves with constant phase along the waveguide axis [1]. When material losses are considered, these eigenmodes become complex, taking into account absorption and the corresponding modal decay. Conversely, open systems can commonly support complex eigenmodes even in the

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**Fig. 1.** Examples of closed and open wave-guiding structures. (a) A closed parallel-plate waveguide and (b) its transverse network model for the application of the transverse resonance method. If the walls of the waveguide are perfectly conducting (PEC), the load impedances are $Z_{l1} = Z_{l2} = 0$. $Z_d$ indicates the characteristic impedance of the transmission line, which depends on the material properties of the waveguide and the polarization under consideration. The position of the reference plane $x = x_0$ is arbitrary. (c) A grounded dielectric slab and (d) a grounded slab with a partially reflecting-surface (PRS) cover (e.g., a periodic arrangement of metal strips). The open structures in (c) and (d) can also be analyzed using the transverse resonance method, by applying the network model in (b) with $Z_{l2} = 0$ and $Z_{l1}$ consisting of a reactive part, modeling the discontinuity (assumed to be lossless), and a resistive part, representing the semi-infinite free space region where radiation may occur.
nondissipative scenario, due to radiation losses, as discussed in the following.\footnote{For the sake of completeness, we should also mention that also in a few closed geometries (e.g., dielectrically loaded or, more generally, inhomogeneous closed waveguides) complex modes can exist in the lossless limit. These modes appear in pairs, having opposite phase constants, and they correspond to leaky modes. Although they do not directly contribute to the proper spectral solution, and can therefore contribute to the local reactive power storage [200]–[202]. In addition, as discussed in details in [10], also complex modes in lossless open structures, i.e., leaky modes, always come in pairs, but only one of the waves in a pair contributes to the steepest-descent representation for the total field, leading to real power transport in the backward or forward direction (according to whether the leaky mode is proper or improper, respectively).}

Open waveguides as radiating systems were first investigated in the pioneering work of Hansen in the late 1930s [6], who proposed an antenna structure realized by opening a longitudinal slit in the side of a rectangular waveguide. The early development of these concepts, however, were fundamentally hindered by the limited understanding of the underlying physical mechanism of leaky waves. Soon, in fact, it was recognized that a leaking waveguide mode is characterized by a complex longitudinal wavenumber, with attenuation constant due to radiation losses. This fact is, in some instances, associated to seemingly unphysical results: a longitudinal attenuation may in fact correspond to a wave amplitude increase in the transverse plane toward infinity. Consider, for example, the cases depicted in Fig. 1(c), (d), and Fig. 2, with longitudinal direction \( z \) (parallel to the waveguide axis) and transverse direction \( x \), in which the top portion of the wave-guiding structures is now partially or fully open, such that a portion of the energy can leak out in the upper semi-infinite region. The leaking field on the aperture will have complex longitudinal (horizontal) wavenumber \( k_x = \beta - j\alpha \) (assuming an \( e^{j\alpha x} \) time-harmonic convention), where the real quantities \( \beta \) and \( \alpha \) are, respectively, the phase and attenuation constant. The field above the waveguide aperture is then characterized, using Helmholtz equation, by a transverse (vertical) wavenumber

\[
k_x = \pm \sqrt{k_0^2 - k_z^2} = \beta - j\alpha_x
\]

where \( k_0 \) is the free-space wavenumber. For a mode carrying power along the positive \( z \), \( \alpha > 0 \) due to passivity. A leaky mode, for which the phase also flows in the same positive \( z \) direction, i.e., \( \beta > 0 \), necessarily requires the branch cut choice \( \alpha_x < 0 \) in (2), implying that the eigenmode fields exponentially increase toward infinity in the transverse \( x \) direction [4], [7], [8], a result that puzzled the early pioneers in the field of leaky-wave theory, since it violates the conventional radiation condition for acceptable solutions of Helmholtz equation. In contrast, for a backward leaky mode, for which the phase flows backward with respect to the power flow, i.e., \( \beta < 0 \), as in some periodic leaky-wave structures, it follows that \( \alpha_x > 0 \) and the field properly decays towards infinity [8]. The seemingly unphysical behavior of forward leaky modes originally led to serious skepticism toward the very existence of leaky waves, despite the fact that energy leakage was experimentally measurable [9]. It was not until the late 1950s that this issue was solved, thanks to deeper theoretical understanding and physical insights provided by the fundamental works of Marcuvitz [1] and Oliner [4], [5].

While in closed systems the eigenmodal spectrum is purely discrete, in open systems a continuous eigenmodal spectrum arises, determined by the branch cuts of (2) that emanate from the branch points \( k_z = \pm k_0 \). For a wave-guiding structure open on one side, the sketch in Fig. 3 shows the complex plane of the longitudinal wavenumber \( k_z \), with the typical choice of branch cuts. A proper spectral solution of the source-excited electromagnetic problem implies an integration over the top Riemann sheet associated with waves decaying at infinity, namely, \( \text{Im}[k_z] < 0 \), or \( \alpha_z > 0 \), (radiation condition). However, as indicated in Fig. 3, pole singularities may also be present in the bottom Riemann sheet, and in this case they are denoted as nonmodal, or improper source-free solutions of the field equation. As shown by Marcuvitz [1], these complex poles can actually correspond to leaky modes. Although they do not directly contribute to the proper spectral solution, and can therefore

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Fig. 2. Sketch of leaky-wave radiation from a waveguide partially open \((z > 0)\). The contour plot on the \(xz\) plane depicts a time-snapshot of the field distribution of a guided fast wave, which becomes leaky when the wave-guiding structure is partially opened. The radiation is confined in the region \( x \times z \) tan \( \theta \), as discussed in Section 2. The plots at different cross-sections show the field amplitude along the transverse \( x \) direction (the case considered here is 2D; therefore, the fields are constant along \( y \)). The red curves highlight the exponential growth \((\alpha < 0)\) of the field amplitude along the transverse direction, and the exponential attenuation \((\alpha > 0)\) along the longitudinal direction. Roman numbers indicate the field amplitude on the aperture at three different values of \( z \).
be characterized by an unphysical growth toward infinity, they can nevertheless accurately describe the radiation field in limited spatial regions. In fact, leaky waves are defined only in a wedge-like region of space dependent on the location of the source, as shown in Fig. 2. The radiation angle is generally given by \( \theta = \tan^{-1}(\text{Re}[k_x]/\text{Re}[k_z]) \), which is usually approximated (for small values of \( \alpha \) and sufficiently long radiating structures) as \( [8] \)

\[
\theta = \cos^{-1}\left(\frac{\beta}{k_0}\right). \tag{3}
\]

\(^{2}\)It should be noted that, while for leaky-wave radiation it is common to define the radiation angle from the surface, as in Figs. 1 and 2, in different situations, e.g., for some of the scattering problems considered in the following sections, the angle is defined from the surface normal.

For a forward wave, the amplitude of the leaky mode indeed is expected to exponentially increase in the vertical (transverse) direction \( x \), as required by (2), but the mode is defined only in the spatial region \( x < z \tan \theta \), for a source placed at \( z = 0 \), implying that the leaky wave has always finite amplitude and, under suitable conditions, it dominates the near-field of the source and it becomes a valid representation of the radiation field [4]. From the mathematical standpoint, the direct relevance of these improper poles becomes evident when applying the steepest-descent method to solve for the integral contribution of the radiation continuum to the overall radiation pattern [8], [10], [11]. As the steepest-descent path passes close to the leaky poles, they can significantly contribute, or even dominate under certain conditions, the overall radiation pattern of an open waveguide in the region \( x < z \tan \theta \).

\[ \text{Fig. 3. Singularities on the two-sheeted complex plane of the longitudinal wavenumber} \ k_z = \beta - j\alpha \text{ for a wave-guiding structure open on one side, as in Fig. 1(c), (d), and Fig. 2 (if the waveguide were open on both sides, the complex plane would consist of four Riemann sheets). Also shown are the branch cuts (green lines) and the branch points} \ k_0, \ \pm jk_x, \text{expressed by (2). In particular, with this choice of branch cut, the lower half of the} \ k_z\text{-plane (which complies with the radiation condition, i.e., Im}[k_x] < 0 \text{ maps on the top Riemann sheet of the} \ k_z\text{-plane; conversely, the upper half of the} k_z\text{-plane (Im}[k_x] > 0 \text{ maps on the bottom Riemann sheet of the} k_z\text{-plane [10]. In the lossless case, a pole singularity [resonance of the network model in Fig. 1(b)] lying on the real axis (blue stars), with |Re}[k_x] > k_0 \text{ (slow wave region), corresponds to a bound surface mode, whereas a complex pole with |Re}[k_x] < k_0 \text{ (fast wave region, indicated by the dotted area) represents a leaky mode. Due to passivity, the upper half of the} k_z\text{-plane (light-red area) is forbidden for any mode carrying power along the positive} z \text{ axis. For forward phase propagation, Re}[k_x] > 0, \text{ the leaky pole (red dot) lies on the bottom Riemann sheet (Im}[k_x] < 0, \text{ and the leaky mode is said to be “improper” (it does not respect the radiation condition), whereas for backward phase propagation Re}[k_x] < 0, \text{ the leaky pole (orange dot) is “proper,” as it lies on the top Riemann sheet with Im}[k_x] > 0 \text{ (here, the “forward” and “backward” nature of a mode is defined according to whether its energy and phase velocities are in the same, or in the opposite, direction). In this plot, we show only one pole for each kind of complex modes; in reality, however, complex modes (proper, or improper) always come in pairs (having opposite phase constant} \beta, \text{ but only one soft return one contributes to real power transport [10])}. \]

\]
convenient. In such situations (e.g., the relevant case of open microstrip structures, or, more generally, strip-loaded dielectric slabs), full-wave numerical methods can be applied to calculate the complex propagation constants of leaky modes, including the mode-matching technique [12], the finite-difference time-domain (FDTD) method [13], different variations of the method of moments [14]–[16] often combined with the matrix-pencil method [17]–[19], among many others.

In this section, we have discussed the fundamental difference between closed and open electromagnetic systems, and the interpretation of leaky waves as source-free modal solutions in open waveguides. It is interesting to note that, while at microwave frequencies it is easy to define closed systems using impenetrable metallic walls (whose thickness can remain deeply subwavelength), in the optical range, most micro- and nanostructures are electromagnetically open, since materials change their conduction properties at high frequencies and ideal conductors are not available in optics, implying that field screening is more challenging at these scales. Therefore, leaky waves are expected to play a significant role at optical frequencies and several concepts and designs developed at microwave frequencies may be translated to the infrared and visible range to realize interesting coupling effects between guidance and radiation. In the next section we overview the general principles and recent developments in the field of leaky-wave antennas at microwave frequencies, which will allow us to explore the connections and relevance to optics in the following sections.

III. LEAKY-WAVE ANTENNAS: GENERAL PRINCIPLES AND RECENT TRENDS

Systematic research on leaky-wave antennas may be traced back to the theoretical foundations of complex guided modes laid down in the late 1950s, as briefly outlined in Section II. The field has been in continuous development since then, and leaky-wave antennas have become increasingly popular in the microwave range thanks to their appealing features, especially the possibility of realizing highly directive antennas without the need for complicated feeding networks typical of phased arrays. Since the body of literature on conventional leaky-wave antennas is remarkably large, and this review paper does not want to be a comprehensive survey of this classical field of antenna technology, we refer the reader to [7], [8] for several examples of microwave leaky-wave antennas and their design principles.

The advent of metamaterials [20], or artificial materials with unusual electromagnetic properties, has further boosted in the last fifteen years the interest in leaky-wave antennas, as the new concepts unveiled by these materials have offered novel inspiration for the design of leaky-wave structures, overcoming some of the limitations of conventional designs. In this section, we discuss recent advances in leaky-wave antennas, particularly at microwave frequencies, beneficial to introduce, in Section IV, the application of these concepts to the optical range. In order to fully appreciate the latest developments in this research area, we start by briefly reviewing the fundamental properties of leaky-wave antennas, highlighting the main challenges in their design and operation.

A. Basic Properties of Leaky-Wave Antennas

A leaky wave traveling along an open wave-guiding structure realizes an effective antenna aperture that radiates energy to the far field. As for any other antenna, the far-field radiation pattern may be obtained by performing the Fourier transform of the aperture field. The complex wavenumber of the leaky wave on the aperture directly determines the main features of its radiation pattern: direction of the main beam, beamwidth, and sidelobe level. We already discussed in the previous section how the angle of leakage of the main beam is determined by the leaky-wave phase constant \( \beta \), and how it can be approximately calculated using (3) if the phase and attenuation constants are invariant along the aperture. The attenuation constant \( \alpha \) controls the angular width of the main beam in the farfield, i.e., the antenna directivity. If the leaky waveguide is sufficiently long to avoid end reflections, the leakage rate directly determines the size of the effective antenna aperture: a large (small) \( \alpha \) implies a short (long) effective aperture, which corresponds—after a Fourier transformation—to a wide (narrow) beam in the farfield. For one-dimensional (1D) uniform leaky-wave antennas, the half-power beamwidth is linearly proportional to \( \alpha/k_0 \) through the approximate formula [8]

\[
\text{BW} = 2 \csc(\theta) \frac{\alpha}{k_0}.
\]  

Typical 1D leaky-wave antennas with a small attenuation constant produce narrow beamwidth in one plane (the \( xz \) plane in Fig. 2), but a much wider beam in the orthogonal plane (i.e., a \textit{fan beam}). However, if \( \alpha \) is small, the slow exponential decay of the aperture field implies poor sidelobe features [21]. This problem is usually tackled by tapering the antenna aperture, in such a way to maintain \( \beta \) constant, while slowly varying \( \alpha \) in order to achieve the desired performance.

Another fundamental property of leaky-wave antennas is their inherent ability to frequency scan the main lobe direction, due to the frequency dependence of the leaky-wave phase constant \( \beta \). For many conventional leaky-wave antennas, however, beam scanning close to the broadside (\( \theta = \pi/2 \)) and endfire (\( \theta = 0, \pi \)) directions has proven to be particularly challenging. Therefore, the possibility of achieving continuous frequency scanning from backward endfire to forward endfire, with constant beamwidth, has
been an important research direction in recent years, as discussed in the following. Different categories of leaky-wave antennas exhibit drastically different frequency-scanning responses, related to their individual geometry and operation principle. For many broadband antenna applications, frequency scanning of the main beam may actually be undesirable. In order to achieve constant beam direction over a wide bandwidth, the antenna structure should support a leaky mode with low dispersion of the phase constant \( \beta \). For example, almost nondispersive leaky-wave radiation can be supported by a long slot in a PEC sheet between different semiinfinite dielectrics, a behavior that can be exploited to design ultrawideband leaky-wave antennas [22]–[24].

Given the wide range of different designs proposed in the last sixty years, it is not an easy task to categorize leaky-wave antennas in different classes, and any possible classification will inevitably have elements of arbitrariness. In several papers, reviews and books on this topic, leaky-wave antennas are typically classified depending on whether the geometry of the guiding structure is uniform, quasi-uniform, or periodic. Another important distinction can be made between antennas with 1D, or two-dimensional (2D) guiding structures. In the present paper, we also adopt these classifications, with the caveat that the boundaries between different categories are not exactly defined, and there may be exceptions that do not fall in one specific class.

One-dimensional uniform leaky-wave antennas have a constant geometry along the length of the structure (although in some cases the antenna opening may be gradually tapered, as mentioned above). A typical example is the case of a rectangular waveguide with an open slit in the longitudinal direction. The fundamental mode of 1D uniform leaky-wave antennas is a fast wave (i.e., with phase constant \( 0 < \beta < k_0 \)), which can directly couple to a propagating plane wave if the guiding structure is open. When fed at one end, the main beam can be scanned in the forward quadrant, from broadside to endfire by increasing the frequency of operation (backward-radiating homogeneous leaky-wave antennas have also been proposed, based on biased ferrite materials [25]). In most designs, the performance typically deteriorates when reaching the extremes of this angular range. For example, for a leaky-wave antenna consisting of a slotted waveguide, broadside operation is difficult because it implies working at the waveguide cutoff frequency, while radiation exactly at endfire is forbidden due to the radiation null of an equivalent magnetic dipole at the waveguide slit, consistent with (4). A few solutions to overcome these problems have been discussed in the literature, e.g., [7] and [8].

In 1D periodic leaky-wave antennas, a periodic modulation of the guiding structure is introduced along the longitudinal direction of the antenna. In contrast to uniform structures, the fundamental mode is a slow wave \( (\beta > k_0) \), which therefore would not radiate even if the structure is electromagnetically open. However, due to the periodic corrugation, the guided wave consists of an infinite number of space harmonics (Floquet modes), with longitudinal wavenumber

\[
k_{z,n} = k_{z,0} + 2\pi n/p
\]

where \( k_{z,0} \) is the wavenumber of the fundamental mode (slightly different compared to the case without periodic perturbation), \( n \) is the harmonic order, and \( p \) is the period in the longitudinal direction. Although the fundamental mode is a slow wave, it is possible to design the periodic structure such that one of the space harmonics (typically the \( n = -1 \)) is fast and, thereby, it will radiate. In many realistic situations (e.g., periodically patterned microstrip lines [18], [19]), the periodic perturbation may also efficiently excite higher-order modes, and some of their space harmonics may significantly contribute to the overall antenna radiation.

One of the main advantages of 1D periodic leaky-wave antennas, compared to uniform structures, is the fact that the phase constant of the leaky mode can assume either positive or negative values at will, which allows scanning the main beam from the backward to the forward quadrant as the frequency is increased. However, the antenna performance generally degrades when we approach broadside, due to the presence of an open stopband of the periodic structure. From a physical viewpoint, at the open stopband frequency the traveling wave supported by the periodic leaky-wave antenna becomes a standing wave, behaving similar to an antenna array in which all the radiating elements are excited with same phase [8]. As a result, the attenuation constant drops to zero and all the reflections at the periodic discontinuities add in phase back to the input port, determining a purely reactive input impedance and large mismatch. The issue of poor broadside radiation in periodic leaky-wave antennas has been one of the main motivations to pursue novel metamaterial-inspired designs, as we discuss in the following.

Another important category of leaky-wave antennas is represented by quasi-uniform structures, which are also characterized by a periodic modulation of their geometry. In this case, however, the fundamental mode is a fast wave, as in uniform structures, and the period is chosen to be small enough such that radiation comes only from the

\[\text{References}\]

3The attenuation constant drops to zero at broadside only when the broadside point corresponds to an open stopband, i.e., a stopband of an open periodic structure where one of the space harmonics is radiating. This is different compared to the behavior in closed stopbands (occurring when the mode is bound), or below the guided-mode cutoff, in which cases there is no leaky-wave radiation, but the attenuation constant may be nonzero, since it corresponds to reactive evanescent decays of the fields. Consider, for example, an air-filled rectangular waveguide with a longitudinal slit, operating with the fundamental TE_{01} mode, which is leaky. As the frequency is lowered, the main beam tends to broadside, as the phase constant goes to zero at cutoff. At the same time, the attenuation constant grows, which corresponds to an increase of reactive attenuation, not of leakage [7].
fundamental mode, and is not coupled to other space harmonics [7]. In general, quasi-uniform structures with subwavelength period can be conveniently modeled with effective homogenous material parameters or surface impedance concepts. Although in quasi-uniform leaky-wave antennas the periodicity does not play a direct role in determining the radiation, since the fundamental mode is already a fast wave, the periodic modulation can be used to control the attenuation and phase constants of the leaky mode. A basic example of quasi-homogenous leaky-wave antenna is the so-called holey waveguide, first introduced in [26], in which a series of closely spaced holes is realized on the short side of a rectangular waveguide. The resulting radiating structure is quasi-uniform, as the leakage comes from the fundamental space harmonic of the periodic structure. Compared to a uniform slitted waveguide, this design has the advantage of providing smaller attenuation constants (and thereby a narrower beam), since the periodic holes do not completely disrupt the current lines on the waveguide wall, as a long slit would do [8]. A recent example of quasi-uniform radiating structures are the leaky-wave antennas based on transmission-line metamaterials, which allow controlling the complex propagation constant of the leaky modes to a large extent [25], as we discuss in the next subsection.

While historically 1D leaky-wave antennas have been the most explored geometries at microwave frequencies, 2D geometries have been attracting increasing attention in the past years, since several designs based on metamaterial concepts belong to this category. 2D leaky-wave antennas consist of a planar guiding structure, e.g., a parallel-plate waveguide with a partially reflective wall, supporting a cylindrical leaky wave radially propagating outward from the source (which may be a short dipole embedded in the open guiding structure). Notably, 2D leaky-wave antennas with homogenous or quasi-homogenous geometry can realize, at a given frequency, a directive pencil beam at broadside, with maximum broadside radiation when the phase and attenuation constants of the leaky mode are nearly equal [7], [8]. At other frequencies, the radiation will be in the form of a conical beam with axis parallel to the surface normal. Typical 2D leaky-wave structures are based on partially reflective metallic screens, or grounded dielectric and metamaterial slabs [7], [8]. A few examples involving metamaterials will be discussed in the following subsection.

Recent trends in leaky-wave antenna research include the planarization, miniaturization and tunability of the antenna structure, as well as the possibility to achieve continuous frequency scanning over the entire angular range, including broadside, which may be facilitated by exploiting metamaterial concepts, as discussed next. Another important trend is the investigation of leaky-wave antennas for frequencies above the millimeter-wave region, in particular the optical frequency range, which will be the main subject of Section IV.

B. Leaky-Wave Antennas Based on Artificial Surfaces and Transmission-Line Metamaterials

Planar leaky-wave antennas often involve periodically-modulated surfaces and artificial surfaces, e.g., patterned metallic screens, which offer further degrees of freedom in the design of their leakage properties. The investigation of such artificial surfaces is an important research direction, also in relation to the rising field of metasurfaces—the planarized version of metamaterials—particularly at optical frequencies [27]–[29].

Leaky waves on periodic surfaces were first investigated by Oliner and Hessel in their studies of guided waves on sinusoidally-modulated reactance surfaces [30]. Although this work was mainly motivated by improving the performance of endfire surface wave antennas, it has inspired many leaky-wave antenna designs (e.g., [31], [32]), in which the sinusoidal modulation allows an independent control of the phase and attenuation constants of the leaky mode. Interestingly, these ideas have also been recently applied to the THz frequency range, in the form of sinusoidally-modulated graphene leaky-wave antennas [33], [34]. In these designs, the complex conductivity of the graphene sheet can be modulated by applying DC bias voltages at different gating pads along the structure, as shown in Fig. 4, or launching an acoustic wave traveling along the surface, allowing a unique dynamic control of the leaky-wave radiation from the surface.

Another important related concept is the one of high-impedance surfaces, introduced by Sievenpiper, Yablonovitch, and co-workers [35]. While a perfect electric conducting surface allows propagation of transverse magnetic (TM) surface waves, but forbids transverse electric (TE) ones, a high impedance surface behaves as an artificial magnetic conductor, which provides the dual operation, forbidding TM surface waves, but supporting TE propagation in the form of leaky waves. High-impedance surfaces owe their interesting properties to periodic structures with a resonant unit cell, as shown in Fig. 5(a) and (b), which corresponds to a lumped inductor-capacitor (LC) resonator. The dispersion diagram for a typical artificial surface of this kind is depicted in Fig. 5(c), within the first Brillouin zone of the periodic structure. As seen from this diagram, the high impedance surface can be employed as a leaky-wave antenna by using the leaky portion of the TE mode above the "light line," namely, for \( k_\parallel < \omega/c \), where \( k_\parallel \) is the parallel wavenumber of the surface wave, \( \omega \) is the angular frequency and \( c \) the speed of light in vacuum. As usual, the main beam can be scanned with frequency, following the dispersion of the leaky mode; alternatively, it is also possible to steer the beam at a fixed frequency by changing the resonance frequency of the LC unit cell, which results in a modification of the modal dispersion as shown in Fig. 5(c). Based on these principles, several tunable and reconfigurable leaky-wave antennas have been proposed, which exploit a modification of the capacitance and/or inductance of the unit cell obtained with different mechanisms, e.g.,...
mechanically [36], or electronically [37]. Notably, artificial impedance surfaces have also been used to realize holographic surfaces [38]–[41], which have been interestingly connected to leaky-wave antennas [42]. Moreover, artificial surfaces with subwavelength resonant unit cells in a leaky-wave antenna configuration have been recently
exploited as “metamaterial apertures” for computational imaging [43], [44].

Other important advances for leaky-wave antennas have come from the field of transmission-line metamaterials, introduced independently by Caloz and Itoh [45], Eleftheriades, and co-workers [46]–[48], and Oliner [49], [50], in the early 2000s. The application of composite right/left handed (CRLH) transmission-line metamaterials to leaky-wave antennas has led to several important breakthroughs in this area of research and technology, the most important being the possibility of continuously scanning the main beam through broadside [51], [52].

As discussed above, conventional periodic leaky-wave antennas suffer from the open stopband problem, which leads to beam degradation when approaching the broadside direction. From a transmission-line point of view, it was realized that any periodic structure with only series or shunt radiating elements would always exhibit an open stopband at broadside [25]. A CRLH metamaterial is composed of a transmission-line structure (e.g., a microstrip line) altered by periodically loading it with so-called “left-handed elements,” namely capacitances in series and inductances in parallel, which are combined with the elements of a conventional transmission line, i.e., per-unit-length series inductances and shunt capacitances [25]. The unit cell of a CRLH metamaterial and an example of its practical implementation are shown in Fig. 6(a) and (b). When the unit cell periodicity is subwavelength, the structure is quasi-uniform, and radiation occurs from the fundamental $n = 0$ mode, which is a fast wave. The dispersion diagram in Fig. 6(c) (blue curve) shows that the fundamental mode has indeed a branch with negative phase velocity (backward radiation; antiparallel phase and group velocity) at lower frequencies, and a branch with positive phase velocity (forward radiation) at higher frequencies, separated by a gap at $\beta = 0$, which corresponds to the open stopband of the periodic structure. The edges of this bandgap are determined by the resonance frequency of the series and parallel branches of the unit cell, which are generally different [25]. These considerations reveal that the open stopband at the broadside point can be completely closed (Fig. 6(c), red curve) by designing a “balanced” structure with identical series and shunt resonance frequencies, corresponding to the following condition for the inductances and capacitances of the unit cell [53], [54]

$$L_R C_L = L_C R_C.$$  \hspace{1cm} (6)

If this condition is fulfilled, a 1D leaky-wave antenna based on CRLH metamaterials can scan the main beam through broadside without degradation, as the frequency is increased [Fig. 6(d)]. Interestingly, it has been noted that the series and shunt radiating elements must contribute equally in order to obtain efficient broadside radiation, a
situation achieved under the balanced condition (6); instead, broadside radiation would be poor if radiation came mostly from only series or shunt elements [55], [56]. Extensions of these concepts to 2D and 3D geometries have been discussed in [52]–[54]. It is also worth noting that the high-impedance surfaces discussed above (Fig. 5) may be interpreted as a 2D precursor of CRLH transmission-line metamaterials [54], in which the vertical vias provide the shunt inductances, while the gaps between plates introduce series capacitances, as seen in the unit cell in Fig. 5(a).

CRLH leaky-wave antennas have been an active area of research over the last few years. Recent advances include electronically tunable [57], active [25] and nonreciprocal designs [58]. Moreover, the dispersion of leaky modes in CRLH structures has also been interestingly exploited to realize microwave analog real-time spectrum analyzers [59]. Several other leaky-wave antenna designs based on transmission-line metamaterials have been extensively reviewed in [25], [60], [61].

C. Leaky-Wave Antennas Based on Plasma Layers and Plasmonic Metamaterials

2D leaky-wave antenna geometries typically consist of a partially reflective screen that covers a grounded dielectric slab. An interesting alternative is based on grounded plasma layers, which have been shown to support weakly attenuated leaky waves sustaining directive radiation [8]. In particular, guided waves in plasma slabs were originally investigated by Tamir and Oliner in the 1960s [62], [63], when the topic of electromagnetic radiation in plasmas was starting to be of strategic importance for military and space applications. They demonstrated that a plasma layer supports leaky modes only above its plasma frequency, where the permittivity is small and positive. At these frequencies, a 2D uniform leaky-wave antenna can therefore be realized by grounding the plasma slab on one side and embedding a source in it [62], [63]. Below the plasma frequency, instead, the slab is opaque and it supports TM surface waves at the plasma-air interface, corresponding to surface plasmon modes [64].

Interestingly, these concepts may be readily extended to optical frequencies by replacing the plasma layer with natural plasmonic materials, in particular noble metals, which have their plasma frequency in the visible/ultraviolet range [64]. At microwaves, instead, natural plasmas exist only in the form of ionized gases, which are not necessarily practical for antenna applications; however, an artificial plasma slab can be realized using engineered structures, such as dense arrays of conducting wires, which may be designed to exhibit a low permittivity at the frequency of interest [65]. Leaky-wave antennas based on grounded wire arrays have been first proposed in the pioneering work of I. Bahl and K. Gupta [65], [66]. With the advent of metamaterials, these ideas have been rediscovered and extended to more practical geometries, stimulating intensive research efforts on artificial plasmas by several research groups worldwide [67]–[74]. An example of low-permittivity leaky-wave antenna for microwave frequencies, based on an effective wired medium, is shown in Fig. 7(a).

Grounded metamaterial slabs with low positive permittivity (or permeability) have been shown to be particularly appealing to realize narrow beams at broadside, with increasing directivity as the permittivity (or permeability) is lowered, which can be intuitively interpreted as a “lensing effect” due to the low refractive index of the structure [75]–[77], as further discussed in Section VI. Interestingly, this phenomenon can also be explained as the result of the excitation of a polaritonic resonance in the low-permittivity planar slab, which determines a strong redirection of the power flow inside the structure, such that the emerging wavefront is almost planar [77],

4In the solid-state physics literature, polaritonic resonances, or material polaritons, indicate scattering resonances due to the coupling of impinging photons with collective excitations of the material, such as phonons, plasmons, excitons, etc., resulting in field distributions mainly concentrated in the material object, and strong redirection of the power flow.
may be directly related to the possibility of drastically enhancing the transmission through subwavelength holes in metal screens, and the realization of coherent thermal emitters, topics that have raised large interest at optical frequencies, treated extensively in Sections VI and VII.

IV. LEAKY WAVES IN OPTICAL ANTENNAS AND WAVEGUIDES

In the millimeter-wave region of the electromagnetic spectrum, above the GHz range, losses in metal structures become significant due to skin effects. In fact, the skin depth $\delta$ in conductors shrinks as the frequency increases, following the relation $\delta = \sqrt{2/\omega \mu \sigma}$, where $\mu$ is the magnetic permeability of the medium and $\sigma$ is its conductivity (the latter assumed to be constant and purely real at these frequencies). As a result, the resistance of any real metal increases at higher frequencies, implying that metallic waveguides exhibit nonnegligible Ohmic losses at millimeter wavelengths. To limit this issue, mm-wave leaky-wave antennas have been designed based on structures that are already open, such as dielectric waveguides, groove guides, microstrip lines, etc., avoiding as much as possible the presence of metal. These structures generally support a fundamental mode that is purely bound (slow wave), which is coupled to radiation modes by introducing proper perturbations in the geometry, e.g., asymmetries or periodic corrugations. Several designs have been proposed to realize mm-wave leaky-wave antennas with minimized losses and we refer the reader to [7], [81], and references therein, for an exhaustive treatment of this topic.

At even higher frequencies, in the infrared and optical ranges, the conduction properties of metals drastically change, as the real part of the conductivity decreases, and its imaginary part becomes dominant. This modification results in much larger field penetration in the metal, which becomes characterized by a dispersive permittivity with a finite negative real part. Therefore, the conventional design principles of waveguides and leaky-wave antennas, which exploit metals as impenetrable conductors to confine and guide electromagnetic fields, can no longer be directly applied above the millimeter-wave range.

Historically, the realization of guiding structures at optical frequencies has been mainly based on dielectric materials, while metals have been generally avoided due to their inherent losses. Given the open nature of dielectric structures, the concept of leaky waves plays an important role in the analysis and design of their guidance and radiation properties (see, e.g., [82]). Within a vast literature on this topic, notable examples include the design of planar dielectric strip waveguides [83] and the rigorous analysis of multilayered and periodic dielectric structures, e.g., dielectric gratings, in terms of surface and leaky modes [84], [85]. These research works have been mainly motivated by the increasing interest in integrated optical systems, in which a clear understanding of leakage and
radiation phenomena is important to design efficient optical couplers, or conversely, to avoid unwanted crosstalk among different parts of a photonic circuit. Leaky-wave concepts have also been applied to the important topic of optical fibers, particularly in the work of Snyder and co-workers [86]–[88]. By studying the complex solutions of the source-free field problem in dielectric cylindrical fibers, they demonstrated the existence of “leaky rays” that geometrical optics would predict to be trapped by total internal reflection. Families of leaky rays form weakly leaky modes, which are important in the accurate analysis of multimode optical fibers.

More recently, dielectric structures have also been used to realize directive leaky-wave radiation at optical frequencies, based, e.g., on photonic quasi-crystals [89], or silicon perturbations in a dielectric waveguides [90], [91]. As an example, the optical leaky-wave antenna proposed in [90] and shown in Fig. 8 is based on the excitation of the fundamental mode of a silicon nitride waveguide, a slow wave, coupled to radiation by periodic silicon perturbations, following similar design principles as for periodic leaky-wave antennas at microwave frequencies. Leaky-wave radiation is then obtained by exciting the antenna at one end of the dielectric waveguide, which produces a directive beam thanks to the low attenuation constant of the leaky mode in this structure.

A common problem of waveguides and leaky-wave antennas based on dielectric materials is the fact that their transverse dimension needs to be comparable to the wavelength in order for the field to efficiently interact with the periodic corrugations. More in general, this problem is fundamentally associated with the diffraction limit in optical structures, which implies that the electromagnetic energy guided in any open structure cannot be easily confined in a subwavelength volume, but it tends to spread over a region with transverse dimensions comparable to the wavelength [79]. At low frequencies, diffraction can be beaten by exploiting the high conductivity of metals, which can be used to shield and guide electromagnetic waves. As an example, a coaxial cable can confine power (carried by its TEM mode) in a region with cross-section significantly smaller than the signal wavelength. Also at optical frequencies the diffraction limit can be overcome using metals, but based on different principles, since metallic materials exhibit drastically different properties in the optical range, as aforementioned.

At sufficiently high frequencies, typically in the near-infrared range, the finite carrier density $n_e$ in metals causes the electrons to respond to the electromagnetic excitation with increasing time delay, which can no longer be neglected. Because of this noninstantaneous response, metals become characterized by a frequency-dispersive permittivity function, which is generally well approximated by a classical Drude model

$$
\varepsilon = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega(\omega + j\tau^{-1})}\right)
$$

(9)

with collision frequency $\tau^{-1}$ and plasma frequency $\omega_p = \sqrt{n_e e^2/m^* \varepsilon_0}$ (for a cold plasma), where $\varepsilon_0$ is the permittivity of free space, $e$ is the electron charge and $m^*$ is the effective mass of the electron (determined by the specific band diagram of the material) [92]. For noble metals, like gold and silver, the plasma frequency lies in the visible or near-ultraviolet range, implying that their permittivity has a small negative real part at infrared and visible frequencies, up to $\omega_p$. This property leads to the onset of plasmonic effects in metallic structures, associated with the excitation of collective oscillations of the electron gas, or
plasmons. When impinging photons couple to these electron oscillations, the resulting plasmon polaritons can sustain particularly strong light-matter interaction and field enhancement [64], [93]. In particular, the interface between a material with positive permittivity, e.g., air, and a material with negative real permittivity, e.g., a noble metal at optical frequencies, may support anomalous resonances (surface plasmon resonances), which determine light localization in deeply subwavelength regions, hence overcoming the diffraction limit. Such interface resonances may therefore be exploited to largely reduce the resonant dimensions of several electromagnetic systems. Based on these principles, many plasmonic optical nanoantennas have been recently proposed, which allow converting free-space propagating optical radiation into subwavelength localized, or guided, optical fields, and vice versa [94], [95]. Although most plasmonic nanoantenna geometries proposed to date belong to the category of standing-wave resonant antennas, such as nanodipoles or nanodimers, a few interesting traveling-wave and leaky-wave designs have also been put forward.

A first notable example of plasmonic leaky-wave nanoantenna is based on planar complementary bilayers, as discussed in Section III-C [Fig. 7(b)], in which one layer is chosen to be plasmonic, while the other is dielectric or insulating. As aforementioned, leaky-wave antennas of this kind can be very low-profile, while retaining high directivity, thanks to the subdiffusive interface resonance between complementary “oppositely signed” materials, which can readily be obtained at optical frequencies with plasmonic media. Besides, the directivity can be further enhanced exploiting the interesting properties of materials with low permittivity [79]. An interesting extension of these concepts to conformal structures has been proposed in [96], in which a cylindrical plasmonic shell with subwavelength cross section is shown to support a circularly symmetric resonant leaky wave. The resulting radiation pattern is omnidirectional in the azimuthal plane, but highly directional in the elevation angle, and can be scanned with frequency in the forward quadrant.

Another interesting category of subdiffusive plasmonic leaky-wave antennas at optical frequencies is based on linear arrays of plasmonic nanoparticles [Fig. 9(a)], which have been extensively studied in recent years (see, e.g., [97]–[100] and references therein). In particular, it has been shown that nanoparticle arrays with subwavelength transverse cross-section may support both slow and fast guided modes (eigenmodes with real and complex frequencies).
wavenumbers), which may be exploited, respectively, to realize subdiffractive nanotransmission lines [97], or optical leaky-wave antennas [98]. Interestingly, under the dipolar approximation for the nanoparticles forming the array (namely, the particles can be modeled as polarizable dipoles, neglecting higher-order multipolar contributions), the guidance and leakage properties of a linear array are fully determined by the array period $d$ and the real part of the particle electric polarizability $\Re[\alpha_{ee}]$, which depends on the geometrical and material parameters of the particles, while the imaginary part $\Im[\alpha_{ee}]$ is related to the background Green’s function and the local material loss for power conservation considerations.

The complete eigenmode spectrum of such linear arrays can be split into longitudinal and transverse polarized modes, as shown in Fig. 9(a), which determine different leaky-wave radiation properties, as exhaustively investigated in [98]. The longitudinal polarization has been found to be most suitable for leaky-wave operation, since it guarantees directive conical radiation, as seen in Fig. 9(b), as well as scanning capability in the forward direction. Instead, the transversal polarization supports backward-wave radiation, but it is intrinsically less efficient than the longitudinal mode. In general, periodic arrays of nanoparticles may offer more flexibility than thin plasmonic layers in the design of efficient subdiffractive leaky-wave antennas at optical frequencies.

A different example belonging to the broad category of linear nanoparticle arrays aimed at tailoring directive radiation is the optical Yagi-Uda antenna shown in Fig. 9(c) [102], [103]. Drawing inspiration from its radio-frequency counterpart, this optical antenna is composed of a finite array of nanoelements, only one of which is driven by a localized optical source, such as a quantum dot or a fluorescent molecule, while the other “parasitic” nanoelements direct and shape the radiated beam. Although the Yagi-Uda array is usually considered a slow wave antenna, supporting a surface wave that radiates towards endfire [101], modifications of the geometry or environment may transform it into a leaky-wave antenna. For example, in the experimental demonstration of an optical Yagi-Uda nanomaterial antenna reported in [103], the presence of the glass substrate causes the antenna to radiate not at endfire, but at an oblique angle, as seen in Fig. 9(d), because the traveling wave supported by the array continuously leaks into the substrate.

Broadband optical leaky-wave antennas can also be realized relying on nonresonant structures, such as plasmonic stripes, slot, or groove waveguides [104], [105], often inspired by wideband leaky-wave antennas at microwave frequencies [22]–[24]. For example, it has been shown in [104] that a long and narrow slot in a plasmonic sheet deposited on a silicon substrate supports a weakly dispersive leaky mode that radiates a directive beam into the substrate, similar to the behavior of its low-frequency counterpart [22]. Interestingly, the beam remains directive over a large fractional bandwidth (50% around the central wavelength of 1550 nm), and the nonresonant nature of this setup makes it more robust to fabrication tolerances.

The field of optical nanoantennas has attracted increasing attention from different scientific communities, as it holds the potential for unprecedented subdiffractive light-matter interactions at the nanoscale, efficient coupling between far-field radiation and localized nanosources, as well as the exciting possibility of realizing point-to-point wireless links in optical nanocircuits [94], [95]. For many of these applications, a directive beam is highly desirable, which however cannot be realized with single resonant optical antennas, such as nanodipoles. In this context, the optical leaky-wave antennas discussed in this section offer an ideal platform to achieve high directivity with a simple and compact structure (simpler than, for example, optical phased arrays [106]). For these reasons, we believe that the topic of optical leaky-wave antennas will gain increasing attention in the next years, as more ideas and techniques developed at microwave frequencies are translated and adapted to optical frequencies.

V. INTERPRETATION OF OTHER ESTABLISHED PHENOMENA IN TERMS OF LEAKY WAVES

Leaky waves represent a fundamental concept in wave physics and, although they are generally associated with antenna technology, it is easy to realize that evidence of these waves is ubiquitous in the physical world. In particular, it has been shown that leaky waves play a key role in several diverse phenomena. Some examples, such as Cherenkov radiation, Smith-Purcell effect, Wood’s anomalies and Goos-Hanchen effect, are briefly discussed in this section, while the more recent areas of extraordinary optical transmission (EOT), coherent thermal emission and embedded photonic eigenstates will be the subject of the following sections.

A. Cherenkov Radiation

A charged particle, or a beam of charged particles (usually electrons), traveling in, or near, a dielectric medium, emits radiation if the velocity of the particles exceeds the speed of light in the dielectric medium. This effect, known as Cherenkov radiation, was first observed by Cherenkov and Vavilov, and theoretically interpreted by Tamm and Frank in the 1930s (see, e.g., [107]). Cherenkov radiation occurs in the form of a radiation cone, around the particle beam, at an angle $\theta = \cos^{-1}(c/nv)$, where $c$ is the speed of light in vacuum, $n$ is the dielectric refractive index and $v$ is the particle velocity. Such radiation can be considered the electromagnetic analogous of bow waves in acoustics and fluid dynamics.

Interestingly, it has been shown by I. Palocz and A. A. Oliner that Cherenkov radiation, at least in some forms, can be conveniently interpreted in terms of leaky waves [108]. In particular, they rigorously studied the
problem of radiation from a realistic electron beam in the vicinity of a dielectric medium. Instead of starting from idealized current distributions for the moving electrons, as was usually done, Palocz and Oliner considered the more realistic scenario of a modulated electron beam of finite thickness and investigated its guided space-charge modes. Assuming that the electrons move along the longitudinal direction \( z \) and their velocity is weakly modulated [108], it is possible to prove that the total current density along \( z \) can be described by an effective displacement current density in a material with nonlocal relative permittivity

\[
\varepsilon_b = 1 - \frac{\beta_e^2}{(\beta_e - k_z)^2}
\]  

where \( k_z \) is the longitudinal wavenumber, \( \beta_e = \omega / v_0 \) is the electronic propagation wavenumber, \( v_0 \) is the average electron velocity and \( \beta_P \) is the plasma propagation wavenumber, as defined in [108]. Since there is no current in the transverse \( x \) and \( y \) directions, the electron beam region can then be substituted by an anisotropic (and spatially dispersive) material with tensor permittivity \( \varepsilon = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{bb} \). Therefore, in order to study the space-charge eigenmodes of the realistic electron beam, it is possible to apply a transverse resonance procedure based on this equivalent anisotropic slab. It follows that, when the electron beam is surrounded by free-space, all the space-charge waves are bound and decay exponentially in the surrounding region, as depicted in Fig. 10(a) (left panel). However, when a dielectric region is brought in close proximity to the beam, under certain conditions, a few space-charge modes may become leaky and radiate energy into the dielectric (red arrows in Fig. 10(a), right panel). As shown in [108], this energy leakage occurs, approximately, when the average electron velocity \( v_0 \) is larger than the phase velocity \( c/n \) in the dielectric medium, which indeed corresponds to the condition for Cherenkov radiation.

This elegant analysis based on leaky space-charge waves represents a self-consistent solution of the Cherenkov effect in the considered geometry, which fully takes into account the influence of radiation on the beam itself, as well as the realistic properties of the beam, such as its thickness. Furthermore, the leaky-wave solution provides new information and physical insights: it shows that the Cherenkov radiation is not determined by a single leaky space-charge wave, ideally emerging at one angle; instead, multiple leaky waves with similar, yet not identical, wavenumbers contribute to the radiation, determining a small angular spread (about 1 degree) [108]. The ability to predict this fine structure of the radiation confirms the power and elegance of the leaky-wave analysis applied to this problem.

B. Smith–Purcell Effect

In 1953, Smith and Purcell predicted and experimentally verified that a beam of electrons traveling close to a metal diffraction grating radiates electromagnetic waves (typically in the optical range) due to the periodic motion of charges induced on the metallic surface [109]. A crude model based on the Huygens principle predicts that the relation between the radiation angle \( \theta \) and the wavelength \( \lambda \) is given by \( \lambda = d(c/v - \cos \theta) \) (for the fundamental space harmonic), where \( d \) is the grating period and \( v \) is the electron velocity.

It is clear that the Smith–Purcell effect resembles the phenomenon of Cherenkov radiation discussed above, and indeed they both belong to the general category of “Cherenkovian effects” [110]. The fundamental connection of the Smith–Purcell effect with leaky waves was again elucidated by Palocz and Oliner [111]. Following a similar approach as described above, they studied the guided space-charge modes of a realistic electron beam in different environments, based on the transverse resonance method. In particular, when the electron beam passes between two parallel conducting plates [Fig. 10(b), left panel], all the space-charge eigenmodes are bound. However, if periodic openings are defined in the metal plates (Fig. 10(b), right panel), space harmonics arise, some of which may be leaky, hence radiating energy at specific directions and frequencies that depend on the grating. As shown in [111], this leaky-wave interpretation of the interaction of electron beams with periodic metallic
C. Enhanced Goos–Hänchen Effect, Optical Beam Couplers and Guided-Mode Resonances

When a light beam of finite size (e.g., a Gaussian beam) is totally internally reflected by a dielectric interface, the reflected beam may exhibit a lateral displacement, called Goos–Hänchen shift, due to the finite penetration and lateral power flow associated to the evanescent waves excited at the interface [112]. This effect, first experimentally observed by Goos and Hänchen in 1947 [113], is generally very weak. In fact, at the interface between two conventional dielectrics, the lateral displacement is typically a small fraction of the incident beam width.

A way to largely enhance the Goos–Hänchen shift has been proposed by Tamir and Bertoni [112], based on structures that support leaky waves, such as plane-stratified media, or periodic structures. In particular, for specific incident angles and frequencies that guarantee phase matching between the incident wave and a leaky mode of the structure, a large portion of the incident energy penetrates the structure and is guided laterally as a leaky wave. As the wave travels along the structure, it radiates back energy, which forms a reflected beam laterally shifted from the position predicted by geometrical optics. Such an effect, due to the excitation of leaky modes, may be much more pronounced than the conventional Goos–Hänchen shift, leading to lateral displacements in the order of the beam width.

The external excitation of guided leaky waves of the open structure corresponds to complex poles of the reflection coefficient on the transverse wavenumber plane. However, any propagating plane wave possesses a real wavenumber, and therefore it cannot directly excite the complex eigenmodes of the structure (consistent with the fact that the plane-wave reflection coefficient never goes to infinity, due to power conservation). Nevertheless, whenever the real wavenumber of the incident plane wave is close to the real part of the complex wavenumber of a leaky pole (corresponding to a phase matching condition), and its imaginary part is sufficiently small, the leaky wave can be strongly excited (as discussed in [114], this is essentially a forced resonance phenomenon). Moreover, in the analysis of the enhanced Goos–Hänchen effect, the width of the incident beam is assumed to be large (at least tens of wavelengths), such that, in a plane wave expansion (assuming an incidence angle $\theta$ from the surface normal), the principal contribution comes from the plane wave with transverse wavenumber $k_t = k_0 \sin \theta$, which can strongly couple with the leaky pole.

Another interesting feature of the enhanced Goos–Hänchen effect due to leaky waves is the fact that, while in stratified structures the displacement is generally in the forward direction, in periodic structures it may be either forward or backward [negative Goos–Hänchen effect, as depicted in Fig. 11(a)]. In the periodic case, if there are several radiating space harmonics, the incident beam may strongly couple to a leaky mode of the structure whenever...
it is phase matched to any of these space harmonics. Then, as shown in [112], all of the diffracted beams scattered by the periodic structure (including the specular reflected beam) undergo a forward or backward displacement, according to whether the incident beam is phase matched to a space harmonic of forward or backward type. Interestingly, similar considerations have also been applied to explain negative displacement of reflected beams in acoustics [115].

Besides periodic gratings, negative Goos–Hanchen shift can also be observed in homogeneous plasma slabs with negative permittivity in specific geometries, thanks to the excitation of backward-propagating leaky waves [112]. This effect has been predicted [116] and experimentally verified [117] at optical frequencies in metallic films coupled to a dielectric prism, which may support leaky surface-plasmon waves in the so-called Otto configuration (i.e., prism-air-metal) or Kretschmann configuration (i.e., prism-metal-air). Furthermore, in recent years several engineered structures, such as photonic crystals [118], negative-index metamaterials [119] and stratified hyperbolic media [120], have also been shown to support giant and negative Goos–Hanchen effect, based on different mechanisms.

The same physical mechanism underlying the enhanced Goos–Hanchen shift, namely, the coupling of an incident beam to a leaky mode of an open structure, plays a key role in the design of efficient beam-to-surface wave couplers (see, e.g., [112], [121]–[123]). For example, it has been shown in [112] that, in light of the large lateral displacement $D$ of an incident beam due to leaky waves, it may be possible to trap a large portion of the incident energy by avoiding the leakage after a distance $D/2$. This can be accomplished by modifying the structure such that the attenuation constant $\alpha$ is suppressed, while the wave impedance is almost unaffected, hence allowing the energy to continue propagating laterally in the form of a bound surface mode. These considerations are particularly relevant today in the analysis of the electromagnetic wave interaction with graded metasurfaces, particularly in reflection mode (e.g., [124]–[126]), which can often be interpreted in terms of well-established leaky-wave theory.

Finally, the interaction of an incident plane wave with the leaky modes of a structure has also been exploited to realize total absorption in lossy layered media [127] and anomalous filtering effects in periodic slab waveguides [128]–[131]. Notably, this latter category is based on so-called guided-mode resonances (or leaky-mode resonances), which occur when the incident plane wave excites a leaky waveguide mode due to phase matching [Fig. 11(b)], determining pronounced resonant peaks/dips in the reflection/transmission spectra, especially when the incidence is very close to the surface normal (or other special angles, such as the Brewster angle [129]). These resonant effects have been studied extensively in recent years to realize several functionalities, such as broadband and narrowband filtering and polarization control [130], [131].

D. Wood’s Anomalies and Fano Scattering Resonances

In 1902, Wood discovered anomalous sharp amplitude variations (i.e., narrow bright and dark bands) in the spectrum of an optical metallic reflection grating, under an illumination with almost constant spectral intensity. Since these features were not predicted by ordinary grating theory, they started attracting large attention in the scientific community. Lord Rayleigh found that the occurrence of the anomalies corresponds to the emergence of a new diffraction order at grazing angle, which determines a rearrangement in the amplitude of the other diffraction orders. However, this explanation accounted only for a specific class of anomalous spectral features (now known as Rayleigh anomalies), while it did not explain many experimental observations (for further details, see, e.g., [114] and [133]). The first theoretical breakthrough in the modern understanding of Wood’s anomalies came from the work of Fano in the 1930s [134], who recognized that some of these features arise from forced resonances associated with guided modes of the gratings. Interestingly, the asymmetric lineshape of Wood’s anomalies was one of the first observations of Fano resonances, later been shown to be ubiquitous in quantum and classical systems [135], [136], and now particularly popular in the optics literature [137]–[144]. These resonances are essentially interference phenomena occurring between a discrete oscillating state and a continuum, which, in the case of a grating, correspond, respectively, to a guided mode of the structure and the continuum of radiation modes of free space.

The explanation of Wood’s anomalies in terms of scattering resonances due to guided modes was made more quantitative in the work of Hessel and Oliner [114], [145], who elucidated the phenomenon in light of the modern concept of leaky waves. In particular, drawing inspiration from their previous work on guided waves on periodic structures [30], Hessel and Oliner rigorously studied the scattering from a periodic reactance surface representing the grating. By representing the fields in a Floquet-type expansion and solving the inhomogeneous system of equations associated with the boundary conditions, they calculated the amplitude spectra of all diffraction orders scattered by the periodic surface under a specific plane wave incidence. Interestingly, these amplitude spectra exhibit sharp asymmetric features, consistent with Wood’s anomalies in realistic gratings. Most importantly, the anomalous features always occur near a frequency value that guarantees phase matching between one of the diffraction orders of the incident plane wave and a leaky mode of the grating [114]. Therefore, in line with other phenomena discussed in Section V-C, also Wood’s anomalies can be explained as a form of forced scattering resonance arising from the external excitation of leaky waves supported by a periodic open structure.

The analysis of Wood’s anomalies by Hessel and Oliner did not consider realistic gratings, but rather an ideal periodic surface reactance; nevertheless, it correctly
predicted and explained, for the first time, important properties observed in optical experiments, such as the possibility of anomalies for both incident polarizations, and the peculiar reluctance of different anomalies to merge. It was later discovered that the leaky modes of metallic gratings at optical frequencies are associated to surface plasmon polaritons, which become leaky due to the periodicity of the grating. Although more accurate analyses of Wood’s anomalies have been recently developed, which take into account the realistic geometry and material properties of optical gratings [133], the elegant interpretation in terms of scattering resonances and leaky waves explains the fundamental physical mechanism and captures the main features of this phenomenon.

As an aside, we note that the recently studied anomalous Fano scattering resonances in metallic nanoparticles at optical frequencies [135]–[144] may be considered, in a sense, the equivalent of these leaky-wave anomalies for bounded 3D structures. These sharp scattering features, in fact, are also a form of forced scattering resonances involving a damped oscillatory state. As noted in [1], in fact, both leaky waves in open waveguides and radiation-damped oscillations in open cavities correspond to complex pole solutions of a source-free boundary-value problem. Besides, these Fano scattering resonances share some of the peculiar features of Wood’s anomalies, such as the asymmetric and narrow lineshape and the reluctance of different scattering resonances to merge [143], [144], features that can be exploited in several nano-optics scenarios.

VI. EXTRAORDINARY OPTICAL TRANSMISSION

In the field of optics, one of the most popular breakthroughs of the last couple of decades has been the discovery of extraordinary transmission of light through subwavelength apertures in metallic films [146], [147], which attracted large interest in the scientific and engineering communities. To better appreciate these findings, we should consider first the idealized case of a single hole in an infinitely thin perfectly conducting metal screen. For this configuration, Bethe showed long ago that the transmission through a hole of radius \( r \) is proportional to \( (r/\lambda)^4 \), where \( \lambda \) is the wavelength [148]; therefore, the transmission becomes rapidly very weak for subwavelength dimensions. In addition, if the thickness of the screen is taken into account, the transmission decreases even further, with exponential dependence on the hole depth if \( r < \lambda/4 \) (i.e., when no propagating modes are allowed in the hole).

Several years after Bethe’s seminal paper, Ebbesen and co-workers showed experimentally that, when several subwavelength holes are arranged in an array [146], or a single hole is surrounded by a periodic texture [147], power transmission can be dramatically enhanced, by several orders of magnitude compared to Bethe’s limit. In addition, it was observed that the transmitted energy can become much larger than the energy impinging on the holes, implying that also the light incident on the metal surface was “funneled” through the subwavelength apertures. Since this effect cannot be predicted by conventional diffraction theory and is somewhat counterintuitive, large attention was devoted by scientists and engineers to investigate its physical mechanism. It is interesting to note that the phenomenon of resonant transmission in arrays of apertures in metallic screens has been known for quite some time in the microwave engineering community, particularly in the context of frequency selective surfaces (FSSs) and filters [149]–[151]. However, resonant transmission in conventional FSSs typically involves aperture sizes comparable to half wavelength, whereas EOT effects have been observed for significantly smaller apertures [152]. It should be also noted that the narrow EOT peak always appears at frequencies close to the onset of the first grating lobe, whereas the broader resonant transmission peak associated with normal FSS operation is generally at lower frequency [153].

The narrowness of the EOT peak (especially for very thin metal screens) and the fact that it occurs in a region of small practical interest for FSS designers, due to the presence of undesired grating lobes, partially explain why this kind of transmission resonances was not observed (or noticed) in the microwave community before the work of Ebbesen and co-workers in 1998 [146], despite the fact that similar geometries were commonly investigated by FSS designers, as further discussed in [153].

Since the earliest attempts to interpret the EOT phenomena observed in optical experiments, there has been a general agreement among researchers that surface plasmons play a key role in the transmission enhancement (see, e.g., [146], [147], [154]–[159]); however, the details of the enhancement mechanism were initially not fully understood. A very important step in the understanding of the extraordinary optical transmission (EOT) effect was the recognition of the fundamental role played by leaky waves supported by the patterned metal screen. As discussed in Section IV, noble metals at optical frequencies behave like plasmas, characterized by a dispersive permittivity following a classical Drude model, as in (9). As mentioned above, a smooth planar interface between a metal with negative permittivity and air can support a TM surface wave, known as surface plasmon, propagating along the interface with longitudinal wavenumber [64]

\[
k_p = k_0 \sqrt{\frac{\varepsilon_r(\omega)}{1 + \varepsilon_r(\omega)}}
\]

where \( \varepsilon_r(\omega) \) is the relative permittivity of the metal with Drude dispersion. It is clear from (11) that, for any value \( \varepsilon_r(\omega) < -1 \), a surface plasmon on a smooth interface is a slow wave with \( k_p > k_0 \) and, thereby, it does not radiate. In
particular, for low frequencies \(\varepsilon_r(\omega)\) very large and negative, the dispersion curve of the surface plasmon follows the light line, i.e., \(k_p \approx k_0\), and the surface wave is only weakly bound to the interface. Instead, at frequencies approaching the value \(\omega = \omega_p/\sqrt{2} [\varepsilon_r(\omega) = -1\), according to (11)], the plasmon wavenumber \(k_p\) goes to infinity, implying that the surface wave is tightly bound to the interface, rapidly decaying on both sides, and corresponding to a so-called surface-plasmon resonance. As an aside, it is interesting to recognize the similarities and differences between surface plasmon waves at optical frequencies and the so-called Zenneck surface waves supported at the interface between free space and a medium with finite conductivity (for example, the poorly conducting surface of the Earth), which have been extensively studied since the pioneering days of wireless telegraphy. Indeed, like surface plasmons, Zenneck waves are TM surface waves decaying away from the conductor-vacuum interface, and their dispersion relation is consistent with (11) [160], with the relevant difference that, at the low frequencies usually considered in the analysis of Zenneck waves, the complex permittivity of the conductor can be assumed nondispersive. Another important difference is that, in realistic situations, Zenneck waves are significantly less localized on the interface than surface plasmons. For further details on the relation between Zenneck waves and surface plasmons, and the lingering controversy about the actual excitation of Zenneck waves, we refer the interested reader to [160]–[162].

As discussed in Section III in the context of leaky-wave antennas, a slow wave can be made to radiate by introducing a periodic modulation of the guiding structure (in this case the metallic interface), which determines the appearance of infinite space harmonics of the fundamental mode, with wavenumber given by (5). Therefore, in analogy with periodic leaky-wave antennas, a periodic modulation of the metallic surface can be designed such that a selected space harmonic of the plasmon mode is a fast wave and, as a result, the overall guided mode becomes a leaky surface plasmon that can efficiently couple energy to free-space radiation. D. R. Jackson, A. A. Oliner, and coworkers applied these considerations based on leaky-wave theory to provide the basis for a consistent explanation of the EOT effect [76], [163]–[166]. Consider, for example, the case of a single subwavelength aperture in a metallic screen surrounded by periodic corrugations, which has been shown to exhibit largely enhanced transmission compared to Bethe’s limit [147]. Fig. 12(a) depicts a 1D version of this geometry, with periodic corrugations on the exit face.
face. The subwavelength aperture essentially acts here as a magnetic line source, which radiates part of the power directly into free space [denoted as a space wave in Fig. 12(a)]; however, the associated radiation pattern is expected to be broad, due to the subwavelength size of the aperture. The source can in addition excite surface plasmons on the metallic surface, which, in the absence of corrugations, are bound and would radiate only at discontinuities (e.g., at the end of the metal screen). Instead, as discussed above, by applying a suitably tailored periodic perturbation, directive leaky radiation can be induced from one of the space harmonic of the fundamental plasmon mode, typically the \( n = -1 \), which becomes a leaky wave with complex wavenumber \( k_{p, -1} = \beta_{-1} - j\alpha \) (note that the attenuation constant does not have an index, since it is the same for all space harmonics). Therefore, the surface plasmons launched by the subwavelength hole can become leaky due to the periodic corrugations, and radiate two beams at angles \( \theta_{\pm 1} = \pm \sin^{-1}(\beta_{-1}/k_p) \), as depicted in Fig. 12(a). As the frequency varies, the beam direction is scanned, as in conventional leaky-wave antennas, until, at a certain frequency value, the two beams merge into a single beam around broadside. Although radiation at exactly broadside is forbidden due to the open stopband of the periodic structure, as discussed in Section III, it has been shown that optimum broadside radiation can be obtained for a phase constant \( \beta_{-1} \) slightly detuned from broadside, corresponding to the “merged beam” being on the verge of splitting [165]. Interestingly, this optimum point has been shown to correspond to the condition [76], [167]

\[
|\beta_{-1}| = \alpha, \tag{12}
\]

which guarantees maximum power radiated at broadside for the geometry in Fig. 12(a). Similar considerations can be extended to the 2D scenario [165].

Leaky surface plasmons can efficiently collect the power emerging from the subwavelength aperture and radiate it directly to free space, as they propagate laterally. The effective aperture from which the power is radiated is therefore much larger than the subwavelength hole, hence allowing a large directivity. If the corrugations are properly designed, as discussed above, at a certain frequency the energy will be radiated very efficiently within a narrow beam at broadside. As an example, Fig. 12(b) shows the numerically computed farfield radiation pattern for a magnetic line source (which models the aperture) on a silver film, with (blue line), or without (red line) optimized corrugations (taking into account the losses of the metallic material). The graph shows an impressive difference between the two cases, particularly at broadside, clearly demonstrating the directive beaming effect. To further confirm the leaky-wave explanation of this phenomenon, numerical simulations [166] reported in Fig. 12(c) show that, indeed, the field along the surface decays exponentially from the aperture (even in the lossless case), as expected for a traveling leaky mode.

The enhancement of power radiated at broadside, as described above, is exactly equivalent, by reciprocity, to the enhancement of power transmitted through the hole, when the periodic corrugations are placed on the entrance plane [165]. Therefore, the leaky-wave explanation of directive beaming also provides a consistent and accurate explanation of the EOT effect through a subwavelength aperture. Analogous considerations can also be applied to the case of hole arrays, in which the periodic corrugations for an individual hole are provided by the rest of the array. Interestingly, the fundamental role of leaky waves in the EOT effect was also recognized independently by other authors, although from different perspectives (see, e.g., [168]).

The leaky-wave theory of EOT provides practical design guidelines to optimize the apertures and periodic corrugations for enhanced transmission, based on well-established leaky-wave antenna principles. Moreover, this viewpoint reveals that the EOT phenomenon is actually just an example of the directive beaming effect based on leaky waves [76] and it is not restricted to optical frequencies or to plasmonic materials. For example, enhanced transmission effects have been demonstrated in subwavelength hole arrays at microwave and mm-wave frequencies [173]–[175], and beam collimation has been achieved in quantum cascade lasers at THz frequencies by patterning periodic corrugations on the laser facet [176]. Furthermore, given the universality of leaky waves, similar concepts can be applied to the acoustic and quantum realms. Notably, extraordinary acoustic transmission (EAT) and sound collimation have recently been demonstrated [169]–[172], which can indeed be explained in terms of excitation of leaky acoustic guided modes supported by the structure [172]. Moreover, in the domain of quantum physics, leaky matter waves have been shown to play a fundamental role in the phenomenon of enhanced transmission and directive beaming of atoms through apertures much smaller than the atomic de Broglie wavelength [177], [178], which may suggest intriguing applications of leaky-wave theory into the quantum regime.

Since leaky waves can also be supported by homogeneous structures, the presence of a periodic modulation is not a necessary requirement to achieve directive beaming and enhanced transmission. For example, we discussed in Section III-C that directive radiation at broadside can be obtained with planar leaky-wave antennas based on grounded homogeneous metamaterial slabs, where the source is a small dipole embedded in the slab, or a slot in the ground plane. Therefore, by reciprocity, these designs can be used to enhance the transmission from subwavelength apertures in a metallic screen. Notably, strong EOT and directive beaming effects have been obtained when a subwavelength hole in a perfectly conducting screen is covered, on both sides, by homogenous slabs with low
positive permittivity (and/or permeability) [77]. The principle of operation can be explained intuitively using ray-optics, as shown in Fig. 13. Consider, for example, the exit face: since the wavenumber in the metamaterial slab is much smaller than in free space, i.e., $|k| \ll k_0$, the phase accumulated by the different rays will be only slightly different on the exit plane and all the rays will refract in free space at almost the same angle, producing a narrow beam at broadside (in the ideal case of $k \to 0$, the radiation pattern tends to a delta function). Interestingly, this “lensing” effect, explained with ray-optics, has been shown to be fundamentally related to the weakly attenuated leaky waves supported by the slab [67], [77]. The interpretations in terms of ray-optics and leaky waves become equivalent for sufficiently thick slabs [179].

For the setup with metamaterial slabs on both sides, as in Fig. 13, reciprocity guarantees that the incident energy will be collected, transmitted through the subwavelength aperture, and radiated at broadside with very high efficiency. Compared to the other approaches to EOT discussed above, in this case no plasmonic effects and periodic corrugations are required, but the fundamental physical mechanism is still based on the excitation of leaky modes in an open guiding structure. Moreover, since no periodicity is involved, this solution avoids the open stopband problem discussed above. However, radiation at exactly broadside is still problematic, because it would require the permittivity (or permeability) to be identically zero, consistent with the ray-optics interpretation in Fig. 13 [77]. A practical disadvantage of the low-permittivity metamaterial design may be the large optimal thickness of the slabs [given by (7)], which may be reduced using metamaterial bilayers, as discussed in Section III-C.

VII. COHERENT THERMAL EMISSION

The ability of leaky-wave structures to realize highly directive radiation from low-directivity sources, which is at the basis of the EOT effect discussed above, also explains another intriguing optical effect, namely, the realization of coherent thermal emission with microstructured surfaces. Thermal sources, such as the filament of an incandescent bulb, typically emit light over a broad angular range, as well as a broad bandwidth determined by Planck’s law of thermal radiation. These are incoherent light sources, in which the emitted infrared light from different points of the structure does not interfere, in contrast with, for instance, the electromagnetic radiation from an antenna array.

Nevertheless, the technological need for low-cost light sources in the mid-infrared range has recently stimulated extensive research efforts to develop thermal sources that are spatially and temporally coherent, i.e., directional and frequency selective. In this context, it was shown that, by simply introducing periodic corrugations on the surface of a material, as shown in Fig. 14(a), the thermal emission can be made highly directional and narrowband [180], [181]. In particular, when such a grating is heated, the resulting thermal emission pattern exhibits two directive beams at opposite oblique directions, and strong frequency dependence [Fig. 14(b)]. It is evident that the underlying physical mechanism behind coherent thermal radiation is indeed related to the excitation of leaky modes on the corrugated structure supported within the Planck spectrum. To clarify this fact, consider the surface of a planar thermal source without corrugations. When the structure is heated, each volume element can be modelled as a random point source, which may excite a guided surface wave, such as a surface phonon, or a surface plasmon, in addition to free-space radiation. Guided waves do not directly contribute to thermal radiation, since they cannot couple to propagating plane waves. Thermal radiation is therefore determined by the low-directive radiation of random point sources, which contribute incoherently (i.e., without interference) to the overall emission. If the surface is periodically perturbed, as in Fig. 14(a), leaky-wave radiation can be induced from one of the space harmonics of the guided modes, consistent with (5). Such leaky waves, continuously radiating along the surface of the thermal source, determine a much wider effective aperture (in other words, a much longer “coherence length”), which results in directive radiation, as seen in Fig. 14(b). For a given temperature, the effect is particularly strong if the periodicity of the corrugations is selected such that leaky waves appear in the frequency range where Planck’s black body radiation is maximum for the given temperature.

Interestingly, leaky-wave theory connects the seemingly unrelated optical effects of extraordinary optical
transmission and coherent thermal emission, showing that they have a common underlying mechanism. This similarity is even clearer in a different implementation of a coherent thermal source, based on the “bull’s eye” geometry shown in the inset of Fig. 14(c) [182], [183]. In this case, concentric periodic corrugations on a metallic sheet are suitably designed to support directive leaky-wave radiation at broadside, very similar to the directive beaming effect described in Section VI. As seen in Fig. 14(c), at the design frequency the main beam can indeed be made very narrow near broadside, thanks to the large effective aperture of the thermal source. As the frequency is varied, the beam is expected to shift and change significantly, due to the frequency dispersion of the leaky plasmon modes. By fixing the observation angle and varying the wavelength [Fig. 14(d)], it is therefore possible to appreciate the narrow bandwidth of the emissivity peak, resembling the response of a coherent source, such as a laser, despite being based on a thermal process. Moreover, owing to Kirchhoff’s law of thermal radiation, which states that the emissivity of a body is equal to its absorptivity, a structure designed to work as a coherent thermal source can also be used as an absorber characterized by high selectivity in both angle and frequency.

As in the case of extraordinary optical transmission, leaky-wave theory reveals that plasmonic effects and periodicity are not necessary to obtain coherent thermal emission. In fact, any structure supporting leaky modes in the spectrum where Planck’s radiation is near its maximum can be exploited for coherent thermal emission engineering, including photonic crystal slabs [184], metamaterial wire medium slabs [185], leaky-wave frequency-selective surfaces [186], multilayered dielectric slabs on a metallic substrate [187], among many other examples.

VIII. EMBEDDED PHOTONIC EIGENVALUES

As we have discussed throughout this paper, a guided mode with phase constant $|\beta| < k_0$, supported by an open waveguiding structure, is a leaky wave that radiates energy into free space as it travels along the structure. In other words, the leaky mode can couple to the radiation modes of free space, namely, outgoing plane waves. Surprisingly,
however, recent theoretical and experimental findings have shown that, under specific conditions, a fast wave traveling along an open structure can avoid radiation and, instead, it can behave as a purely bound mode without coupling to propagating waves in the background [188]–[190]. This phenomenon has been experimentally observed at optical frequencies in a photonic crystal slab [189], shown in Fig. 15(a), formed by a dielectric slab waveguide with periodic corrugations (cylindrical holes). The dispersion diagram for this 2D periodic structure along the irreducible Brillouin zone is shown in Fig. 15(b).

As usual, the structure supports slow and fast waves, respectively, below and above the light lines. In particular, consider the first TM mode (indicated by the green line): when it lies within the light cone (namely, the region delimited by the light lines in different directions), the mode is expected to be leaky, since it can couple with the continuum of radiation modes of free space (in other words, phase matching can be fulfilled). However, at a specific point along the dispersion curve of the TM₁ mode [red circle in Fig. 15(b)], it has been found that the leakage rate actually vanishes, despite the presence of available radiation modes to which the energy can couple. At this specific condition, the guided mode becomes ideally bound and confined in the slab, as seen in the numerically computed field distribution in Fig. 15(c). In particular, the mode does not exhibit longitudinal attenuation, since there is no longer energy leakage (the attenuation constant of the leaky mode is suppressed), which corresponds to an oscillatory state with infinite quality factor (i.e., infinite lifetime), as seen in Fig. 15(c). Peculiar bound states of this kind, existing within the radiation continuum (the light cone), fall into the category of embedded eigenvalues [189], and can be considered the electromagnetic analogue of anomalous localized electron states in quantum mechanics [191], [192].

In leaky-wave theory, it is known that leaky modes may have anomalous responses at certain particular points of the dispersion diagram. For example, at the transition region in which a bound mode evolves into a leaky mode (across the light line), a so-called “spectral gap” may be present, in which the modal response may be quite complicated, and even become nonphysical [193], [194]. Moreover, in the dispersion diagram of a periodic structure, an open stopband may appear at the $\beta = 0$ point [denoted as $\Gamma$ in Fig. 15(b)], as we frequently mentioned in this paper, where leaky-wave radiation is forbidden, even though the mode lies within the radiation cone. Interestingly, this open stopband can be interpreted in terms of symmetry incompatibilities, which prevent the guided wave to couple...
to radiation modes [195]. As a result, a “symmetry-protected bound state” appears at the $F$ point, characterized by absence of radiation and an infinite quality factor, as seen in Fig. 15(c).

Both the spectral gap and the open stopband, however, occur at “exceptional” points of the dispersion diagram, where an anomalous leaky-wave response may be expected. Instead, as seen in Fig. 15(b), the embedded eigenvalues described earlier appear at seemingly unremarkable wavenumbers, not intuitively related to any specific physical mechanism. Experimentally, it has been shown that the reflectivity spectra exhibit asymmetric Fano resonances due to the external excitation of the leaky guided modes of the slab [189], consistent with our discussion in Section V-C and D. However, when the angle and frequency of the incident wave are scanned closer to the position of the embedded eigenvalue on the dispersion diagram, the resonance gets sharper and sharper, as its quality factor tends to infinity, and eventually vanishes, since the leaky mode becomes a purely bound state, decoupled from free-space radiation. From a physical viewpoint, the disappearance of leakage has been generally explained as the result of destructive interference among different “radiation channels” [189], or the coupling between different guided mode resonances, which can be studied with coupled-wave theory [190]. From the perspective of leaky waves, it appears that, as we approach the frequency of the embedded eigenvalue, the complex leaky pole on the wavenumber plane moves closer and closer to the real axis, until it becomes purely real in the ideal limit, which corresponds to a bound surface mode with no attenuation, despite being in the fast-wave region. The details of the physical mechanism in terms of leaky-wave theory, however, are still not fully unveiled, and represent an exciting open area of research.

Interestingly, analogous bound states in the radiation continuum have also been observed in crystal acoustics [196], [197]. In particular, surface acoustic modes that are normally leaky have been shown to become purely bound under specific circumstances. As in the electromagnetic case, the disappearance of leakage in these anomalous bound states, known as “secluded supersonic surface waves,” is not determined by symmetry incompatibilities.

Given the analogy between leaky waves in open guiding structures and radiation-damped oscillations in open cavities [1], recently the concept of embedded photonic eigenvalue has also been extended to three-dimensional open structures [198], [199]. It has been shown that composite plasmonic spheres of finite size, and possibly subwavelength dimensions, can be designed to support oscillatory states without radiation loss, even when surrounded by available and compatible radiation modes (outgoing spherical waves). Analogous to the leaky-wave scenario described above, the complex pole associated with radiation-damped oscillations moves closer and closer to the real axis of the complex frequency plane (instead of the wavenumber plane in the leaky-wave case). In the ideal limit, this three-dimensional embedded eigenvalue corresponds to an eigenmode (self-sustained oscillation) of the open cavity “embedded” along the real frequency axis, resulting in the disappearance of radiation loss and ideal light confinement, as demonstrated in [199].

IX. CONCLUSION

In this paper, we have presented and discussed the general theoretical principles and practical applications of electromagnetic leaky waves, with several relevant connections to microwave and optical physics and engineering. After over sixty years since its inception in the context of microwave and antenna engineering, we have discussed how this area of research is still very relevant today, and the importance of leaky-wave concepts is becoming increasingly more recognized in different scientific communities, beyond microwave engineering.

As we have discussed extensively along this paper, the theory of leaky waves has proved to be of fundamental significance in the understanding of different phenomena, including several anomalous optical effects that cannot be explained using conventional diffraction theory and geometrical optics. Moreover, since most micro- and nanostructures are electromagnetically open at optical frequencies (field screening is more challenging at these scales and wavelengths), leaky-wave phenomena are ubiquitous in nanooptics and play a key role in a variety of nano-optical components and systems. Of particular importance today is the application of leaky-wave concepts to design and optimize optical radiation with high directivity, as well as structures that exhibit enhanced optical transmission and embedded eigenvalues, which may lead to unprecedented manipulation of light at the subwavelength scale.

In conclusion, we believe that the alliance of well-established leaky-wave concepts with the new areas of metamaterials, plasmonics, and nanophotonics may open exciting new research directions, with particular emphasis to practical applications, extending the reach of leaky-wave theory and bringing to new relevance the pioneering studies of the scientists and engineers that have laid the foundations of this exciting research area.

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Monticone and Alù: Leaky-Wave Theory, Techniques, and Applications: From Microwaves to Visible Frequencies


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