The Ricardian System and the Industrial Revolution

Davide Fiaschi, Rodolfo Signorino

1. Introduction

Famously, right in the opening lines of the Preface to his *On the Principles of Political Economy and Taxation* (1817 1st ed., 1821 3rd ed.) David Ricardo claimed that «To determine the laws which regulate [the distribution of the produce of the earth — all that is derived from its surface by the united application of labour, machinery, and capital, … among three classes of the community; namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated], is the principal problem in Political Economy» (*Works* I, 5).

Employing contemporary language, Ricardo identified the determination of factor shares as the main theoretical problem economists are called to analyse. According to Ricardo, what makes this specific problem hard to tackle lies also in the fact that factor shares significantly change in the course of time. He highlighted «the actual fertility of the soil, the accumulation of capital and population, and the skill, ingenuity, and instruments employed in agriculture» (*ibid.*) as the main drivers of change in income distribution.

The goal of the paper is to study the determinants of the dynamics of the labour share (the ratio between total labour income and output) in a Ricardian economy, where factors' compensations do not strictly follow the rule of marginal returns and the main driver of changes is technological progress and the accumulation of circulating *and* fixed capital. In particular, we intend to apply our Ricardian model to the explanation of the dynamics of factor shares in UK in the period between 1770 and 1860, *i.e.* during the Industrial Revolution. Our analysis centers on the labour share given its deep impact on several topics, particularly those related to the distinctive features of a capitalist economy concerning the division of gains from economic growth among the various social classes (see

Atkinson 2009). Moreover, we focus on the labour share because we intend to contribute to the debate concerning the “Engels’ pause”, *i.e.* the prolonged period of constant wages in the midst of rising output per worker triggered by the Industrial Revolution (Allen 2009).

In the interwar period the debate on the determinants and dynamics of the labour share was enlivened by Hicks (1932) and Keynes (1939). In more recent times Kaldor (1961), Solow (1958), Marglin (1984), Blanchard et al. (1997), Bentolila & Saint-Paul (2003), Karabarbounis & Neiman (2014) and Grossman & Oberfield (2022) are among the main contributors to the literature. Bentolila and Saint-Paul (2003) present a valuable introduction to the theory of labour share from a neoclassical point of view (see also Section III in Elsby et al., 2013, Section 2 in Schneider, 2011 and Grossman & Oberfield, 2022). Karabarbounis (2024) provides a comprehensive review. Section 2.4 in Bentolila and Saint-Paul (2003) and Section 6 in Grossman & Oberfield (2022) discuss the case for a theory of wage determination different from the marginal productivity theory, as it is the case in a Ricardian economy. Piketty & Zucman (2014), Karabarbounis & Neiman (2014), Allen (2009) provide different (though sometime discordant) long time series on factor shares (from the eighteenth century onwards).

In this Chapter our primary sources of inspiration are Chapter XXXI, “On Machinery”, described by Sraffa (1951) as «the most revolutionary change in the third edition» of Ricardo’s *magnum opus*, and Chapter IX, “The Industrial Revolution”, of Hicks’ *A Theory of Economic History* (1969). Actually, our analysis takes its clue from two observations by Hicks.

The first is that what characterises the Industrial Revolution is not so much the increase in the rate of capital investment *tout court* as the increase in the «*range and variety* of the fixed capital goods in which investment was embodied» (Hicks 1969, 143, Hicks’ emphasis). In short, what sets traditional manufactures apart from the new manufactures that emerge in the course of the Industrial Revolution is that the former relied almost exclusively on circulating capital, whereas the latter significantly utilized fixed capital. The second is that the initial generation of machines was costly to construct and run due to heavy reliance on scarce specialized human labor (and waterpower). By contrast, later generations of machines have progressively become less expensive and more effective in their operation, as they were constructed and repaired largely by means of other machines — essentially, they were «machines made by machines» (Hicks 1969, 143).

What about Ricardo? As acknowledged by Hicks (see in particular Hicks 1973, 98-99), Ricardo was the first Classical economist to systematically examine the economic implications of the Industrial Revolution, which he described not merely as a process of refinement of existing machinery, but rather as a process of "introduction of machinery" that is, the generalized substitution of techniques characterized by a high fixed to circulating capital ratio to ones where fixed capital was negligible and capital was basically circulating capital. Moreover, as concerns the impact of industrialization on the labour share «Ricardo had candour and courage; he followed his reasoning where it led him, not just where he (or his friends) wanted it to go» (Hicks 1969, 151). (McCulloch was one of Ricardo's friends who were upset by his "recantation" on the machinery question. In a letter to Ricardo, dated June 5, 1821, now in *Works* VIII, p. 382, McCulloch wrote: «Excess of candour has in this instance occasioned your doing a very serious injury to your favourite science».)

Nevertheless, Ricardo's analysis has a weak point, at least from an empirical point of view. According to Ricardo, the replacement of human labor with machines and the resulting rise in labour unemployment is contingent on the relative cost of machines to human labor. Indeed, Ricardo believed that it is the increase of wages that motivates entrepreneurs to opt for more fixed capital-intensive methods. Without technological progress in agriculture and free importation of cheap foreign corn, population growth in a densely populated country such as England compels the cultivation of progressively less fertile plots of land. Hence, the market prices of subsistence goods and, as a result, wages are inevitably driven upwards. The relevant passage is the following[1]:

> With every increase of capital and population, food will generally rise, on account of its being more difficult to produce. The consequence of a rise of food will be a rise of wages, and every rise of wages will have a tendency to determine the saved capital in a greater proportion than before to the employment of machinery. Machinery and labour are in constant competition, and the former can frequently not be employed until labour rises. In America and many other countries, where the food of man is easily provided, there is not nearly such great temptation to employ machinery as in England, where food is high, and costs much labour for its production.

[1] In this instance, Ricardo is aligning with Barton's 1817 analysis, the pamphlet that prompted Ricardo to reconsider his stance on the machinery question. According to Barton, it is the rise (fall) of real wages that compels entrepreneurs to accumulate fixed capital (wage funds): see Barton (1817, 17 - 18).

The weak point within Ricardo's analysis is that empirical data show that British real wages did not increase significantly during the first decades of the Industrial Revolution. As Hicks (1969, 148) pointed out, there is no doubt that *in the long run* real wages increased enormously in all industrialised countries and that such an increase would not have occurred without the productivity gains generated by the industrialisation process. The puzzle is that such a rise was "so long delayed" (*ibid.*). As recalled *supra*, such a puzzle has recently been christened by Allen (2009) as the "Engel's pause". The point is that, granted constant real wages, the Ricardian incentive to substitute machines for workers loses much of his strength. It is here that Hicks' second observation mentioned above comes to the aid of Ricardo's analysis. While real wages have remained constant, the cost of production of machines has progressively fallen since the labour coefficient in the production of newer machines went down. To put it briefly, it turns out that Ricardo was right for a wrong reason: he was right in pointing out that the choice of technique depends on the relative price of human labour/machines; he was wrong in thinking that rising wages would have triggered a substitution of machines to human labour in "old" countries such as England. Fortunately for Ricardo, the reduction of the price of machines yielded a similar outcome.

The structure of the paper is as follows. In Section 2 we discuss the empirics of the labour share in the long run and we question Keynes' and Kaldor's view concerning the stability of the labour share. Moreover, we briefly present Allen's 2009 results on the path followed by labour, land and capital shares in UK in the period 1770 – 1910. We take these results as the benchmark against which to assess the validity of our Ricardian model. In Section 3 we first show that a simple Ricardian model without fixed capital and in which circulating capital coincides with the wages fund (corn) is unable to match Allen's 2009 results. Then, we show that a Ricardian model where capital consists of i) the wages fund (corn), agricultural tools (say, spades) *and* iii) fixed capital (say, tractor) is able to achieve a better match with Allen's 2009 results. Section 4 concludes.

2. The dynamics of the labour share over time

As sarcastically noted by Solow (1958), «Ever since the investigations of Bowley and Douglas it has been widely believed that the share of the national income accruing to labor is one of the great constants of nature, like the velocity of light or the incest taboo» (Solow 1958: 618). To mention just two well-known examples of such a "belief", Keynes (1939) claimed that labour wage stability is «one of the most surprising, yet best-established, facts in the whole range of economic statistics, both for Great Britain and for the United States» (Keynes 1939, p. 48). (Keynes based his analysis on data over the period 1911 - 1935 for UK and 1919 - 1934 for US.) In the same vein, Kaldor (1961) proposed elevating to the rank of "stylized fact" a steady share of profits and wages[2] (for a methodological-oriented assessment of Kaldor's facts see Arroyo Abad and Khalifa 2015).

Pace Keynes and Kaldor, Solow's skepticism is all but unfounded: looking at Figure 7, p. 256, in Brown and Weber (1953), reported for the sake of convenience in Figure 1, the stability of the profit share is far from evident.

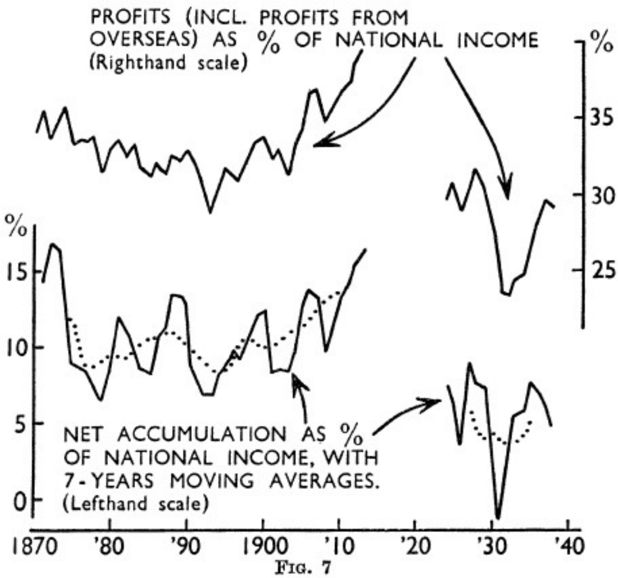


Figure 1: Figure 7, p. 256, in Brown and Weber (1953).

Figure 2 shows how the share of the labour income displays fluctuations and trend in the long run.

[2] Solow himself drew inspiration from Kaldor's stylized facts in formulating his growth theory and analyzing the sources of growth in his seminal 1956 and 1957 papers. We ignore whether Solow attempted to reconcile his 1956-7 position with his skeptical 1958 stance.

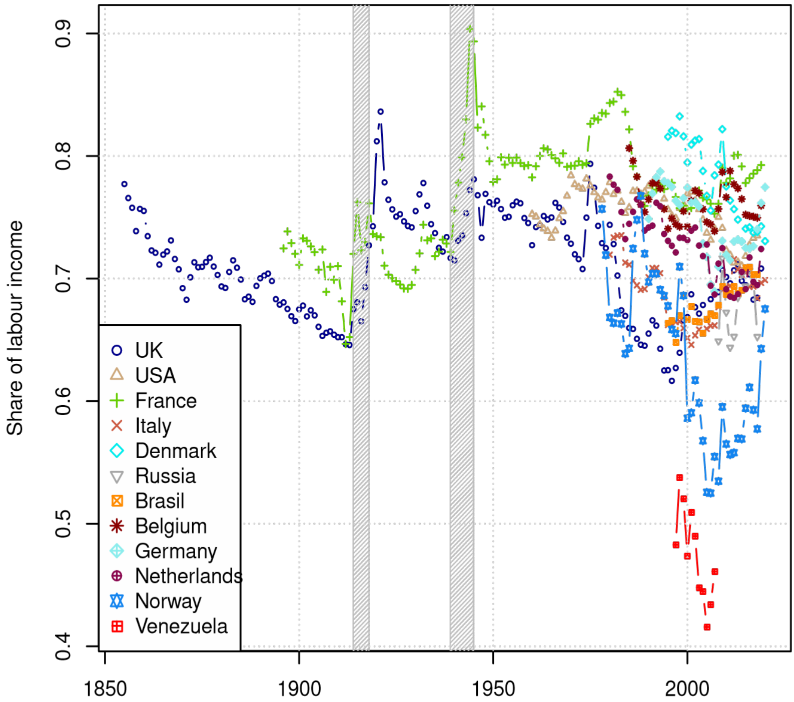


Figure 2: The share of labour income 1850-2020 for selected countries.
Sources: World Inequality Database Dataset (2023).

Taking a long-run perspective on the labour share, Figure 2 shows the large fluctuations over time for UK (the country for which we have the longest time series) and the wide heterogeneity across countries (excluding Venezuela, from 0.5 to 0.8). Before the First World War the trend was declining, while both World Wars have had a positive impact on the labour share. As concerns more recent experience, Karabarbounis and Neiman (2014) document a global decline of the labour share after 1975, a fact also evident for most of the countries reported in Figure 2.

Figure 3 below reproduces the values of the three income shares and the rate of profits for the period 1770-1910 for Great Britain taken by Allen (2009). As concerns the time period we are interested in this paper (1770 – 1860), the rent share is constantly declining; the wage share is roughly constant till 1800, then declines till 1860 and start raising only afterwards; while the profit share and the rate of profits are roughly constant till 1800 and then they both raises afterwards.

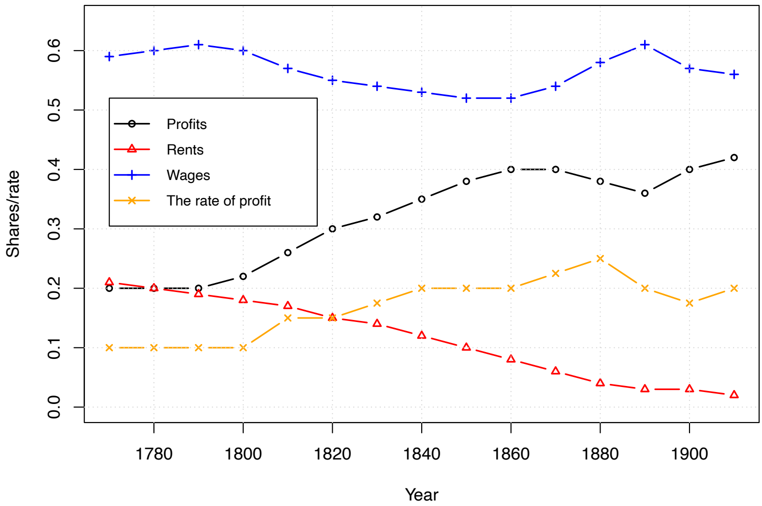


Figure 3: the dynamics of the share of profits, rents and wages on total GDO, and the rate of profit, for Great Britain in the period 1770–1910. Sources: Allen (2009).

To sum up, in order to take data seriously, a Ricardian model must be able to replicate the above path concerning the dynamics of factors' share. In what follows we propose a formulation of the Ricardian model suitable to perform this task.

3. The labour share in the Ricardian model

In this section, we first discuss how the standard one-commodity Ricardian model in which there is no fixed capital and circulating capital coincides with the wages fund is unable to reproduce the dynamics of factor shares and the rate of profits as reported by Allen (2009). Then, we show how a Ricardian model with fixed capital is better suited to the task of matching Allen's findings.

3.1. The simplest Ricardian model

The simplest Ricardian model is a one-commodity model, such as the famous corn model, in which corn is produced by means of corn and labor, there is no fixed capital and circulating capital consists solely of an anticipated wages fund. (As concerns the timing of factors' payment, rents are paid *ex post*, after the corn harvest. Farmers' capital consists of the wages fund which must be paid to workers *ex ante*, before the corn harvest. Hence, farmers earn profits as a residuum: profits are what remains of the corn harvest after rent payment.) This model was originally proposed by Kaldor (1955-6) and later formalized by Pasinetti (1960) (see also Pasinetti 1977, pp. 8-17)[3]. Salvadori (2021) provides an up-to-date formal restatement of Ricardo's theory of growth and distribution. In an economy with only wages fund as capital the labour share s_l is given by:

$$s_l = \frac{wN}{Y}, \tag{1}$$

where N is the total employment, w the real wage, and Y the total output. The share of profits s_c is instead given by:

$$s_c = \frac{rK_C}{Y} = \frac{rwN}{Y}, \tag{2}$$

where r is the rate of profit and $K_C = wN$ the stock of circulating capital coinciding with the wages fund; substituting for the latter into Eq. (1) leads to[3]:

$$s_c = rs_l. \tag{3}$$

Any attempt to reproduce by means of Eq. (3) the dynamics of factor shares since the start of the Industrial Revolution is doomed to fail. In this perspective, consider Figure 3 again: for a rate of profit of 0.1 and a labour share of 0.6 in 1770, according to Eq. (3) the capital share should have been 0.06 whereas in fact it was 0.2. For the rest of period, the possible match is still worse[4]. The same applies to the share of rents s_r, i.e.:

[3] Eq. (3) resembles the first one of Piketty's laws on the determinants of inequality, i.e. $s_c = r\ (K/Y)$, where K is the stock of physical capital, Y the total output, but adapted to a Ricardian world with only circulating capital.

[4] The sources are "Chapter6TablesFigures.xlsx" (Table TS6.1) from Piketty and Zucman (2014) and Allen (2009).

$$s_r = 1 - s_c - s_l = 1 - \ rs_l - s_l = 1 - (1+r)s_l,$$

which for a rate of profits of 0.1 and a share of wages of 0.6 in 1770 (see Figure 3) predicts a rent share of 0.34 against an observed of 0.2 and still worse if we consider 1860, where for a rate of profits of 0.2 and a share of wages of 0.5, the predict rent share should be 0.4 instead of the observed 0.1.

As frankly acknowledged by Pasinetti (1960, 92), «Ricardo's model is built on very crude assumptions». Yet, it turns out "to be logically consistent and determinate in all its macro-economic features and even in its sectoral details, except for some particular sectoral variables in which Ricardo was not interested" (*ibid.*). Hence, the absence of fixed capital appears to be an acceptable first approximation assumption in the analysis of the conflicting forces driving the dynamics of aggregate income. Nevertheless, the above discussion shows that the model with only wages fund as capital, whatever the chosen specification of the production function and other mathematical details, is unable to match the empirical evidence emerging from the Industrial Revolution in UK. We claim that the absence of fixed capital, e.g. tractors to be used in agriculture, and, to a lesser extent, of a kind of circulating capital different from the wage fund, e.g. spades with a full yearly depreciation, undermine the capacity of Ricardian model to mimics the dynamics of the factor shares and the rate of profits. The next section substantiates our claim.

3.2. A Ricardian model with fixed capital

As pointed out by Pasinetti (1960, 90 ff.), the analysis of an economy that produces more than one commodity, say *n* wage or subsistence goods and *m* luxury goods, does not pose significant analytical challenges, granted Say's Law of Markets and very specific assumptions about the commodity composition of final demand. Although such assumptions may appear *ad hoc* to contemporary readers, they are justifiable since Ricardo lacked a microeconomic theory of consumers' demand. As concerns the spending behaviour of workers and capitalists, the fundamental assumption is that workers spend their wages to consume a fixed basket of wage goods, whereas capitalists use their profits to buy the same kind of commodities. Landowners, on the other hand, are assumed to spend all of their rents on luxury commodities. Granted such assumptions, the production conditions of wage goods determine all the key variables in the Ricardian model, given the quantities produced of luxury goods. By the same token, Pasinetti considered the absence of fixed capital within the standard Ricardian model as an unduly restrictive assumption. Nonetheless, he noted that analyzing fixed capital requires a further assumption, *i.e.* that the ratio of fixed to circulating capital is uniform across sectors.

From this perspective, Pasinetti observes that Ricardo's inclusion of a new Chapter specifically dedicated to the machinery question in the third and final edition of his *Principles* underscores Ricardo's awareness of the importance of this issue within his broader analysis of the determination of labor employment. In fact, if the process of mechanization is not uniform among sectors, the various sectors end up being characterized by different ratios of fixed to circulating capital. In such a circumstance, the overall level of labour employment ceases to be determined solely by the total amount of capital and the given natural wage. In order to determine the overall level of labour employment and to show the possibility that mechanization may prove «injurious to the labouring class, as some of their number will be thrown out of employment, and population will become redundant, compared with the funds which are to employ it» (*Works* I), it is necessary to consider not only the total amount of capital but also its intersectoral allocation and thus the composition of final demand. This, in a nutshell, is one of the messages that, for Pasinetti, emerge from Ricardo's chapter on machinery.

In the elaboration of a Ricardian model with fixed capital, we start from the contributions by Casarosa (1978) and Kurz (2010). In particular, we assume that workforce dynamics is driven by the difference between the *market wage* w and the *natural wage* w_N, i.e.

$$\dot{P} = Pa(w - w_N), \tag{4}$$

where $a > 0$ is a known parameter. We also assume that natural wage changes through time. Natural wages dynamics depends on the gap between the market and the natural wage, i.e.:

$$\dot{w}_N = w_N b(w - w_N), \tag{5}$$

where $b > 0$ is a known parameter.

Differently from Casarosa (1978) and Kurz (2010), in the production of the agricultural (or final consumption) good, we consider also fixed capital together with land and labour, i.e.:

$$Y = F(L, N, K),$$

where Y is the total output, L the quantity of land, N the labour used in the production and K the stock of fixed capital. We assume that at any point in time there are fixed proportions in the use of factors; in particular, one unit of land requires β units of labour and $\beta\gamma$ units of fixed capital, i.e.

$$Y = Amin\left(L, \frac{N}{\beta}, \frac{K}{\beta\gamma}\right) = Af(L), \tag{6}$$

where $f(L)$, which summarizes the level of three inputs by considering only the amount of land, is assumed to have positive first derivative and negative second derivative in agreement with the standard assumption of declining marginal productivity of land. The ratio between land and labour is given by β, while the ratio between labour and capital is given by γ. Fixed capital is produced with a technology employing only labour, in particular one unit of fixed capital is produced with c_K units of labour[5]. In a competitive economy, each unit of fixed capital is therefore sold at a price equal to $c_K w$.

The technology is represented by A, β, γ and c_K, and technological progress is therefore represented by how is modelled their dynamics over time. An increase in A corresponds to the case of *landsaving innovation*, while a decrease in β and/or γ to the case of *capital-and-labour saving innovation* (see Blaug 1997, 106). Following the suggestion by Hicks (1969, 153), we also consider the possibility of a continuous decrease in the cost of producing fixed capital c_K, as the outcome of technological progress in the sector of production of fixed capital. Intuitively, a decline in c_K can lead to an increase in γ, i.e. a continuous substitution of land and labour with fixed capital in the agricultural sector.

The rate of profits r on the marginal land is given by (normalizing the price of output (corn) to one):

$$r = \frac{f' - \beta w - \beta\gamma c_K w}{\beta w + \beta\gamma c_K w}, \tag{7}$$

where f' is the production of cultivating an additional unit of land (i.e. the marginal productivity of land) and $\beta w + \beta\gamma c_K w$ is the cost of employing one unit

[5] The exclusion of fixed capital and land from the inputs in the production of new fixed capital is in line with the Solow tradition, where the production of new physical capital goods is made by a one-to-one technology transforming final goods into investment (Solow, 1956). The same choice of labour as the only input in the production of intermediate goods is present in several contributions of the literature on endogenous growth theory (see, e.g., Aghion and Howitt, 1992).

of land, i.e. β workers whose cost is βw, and γ units of fixed capital, whose cost is $c_K \beta \gamma w$.

The relationship between wage w and rate of profit r is therefore:

$$w = \frac{f'}{\beta(1 + \gamma c_K)(1 + r)}, \tag{8}$$

which is the same of the standard Ricardian model when $\beta = 1$ and $\gamma = 0$ (i.e. $K = 0$) (see, e.g., Casarosa, 1978). In equilibrium, the rate of profits on fixed capital K_F and circulating capital K_C is identical. Hence the rate of profits in this economy is given by:

$$s_c = \frac{r(K_C + K_F)}{Y} = \frac{r(\beta w N + \gamma c_K w N)}{Y} = \frac{r(\beta + \gamma c_K) w N}{Y}, \tag{9}$$

where circulating capital is given by:

$$K_C = \beta w N, \tag{10}$$

while the value of the stock of fixed capital as:

$$K_F = \beta \gamma c_K w N. \tag{11}$$

When $\beta > 1$, the value of circulating capital, K_C, may encompass more than just anticipated wage costs. It can also include expenditures on rapidly depreciating capital goods, such as small tools used in agriculture say, spades, which are produced by means of a one-to-one technology with labor.

The value of the fixed capital stock is derived by the equality $K = \gamma N$ and by evaluating the stock of fixed capital to its replacing cost $c_K w$ per unit. The labour share is instead given by:

$$s_l = \frac{wN}{Y} = \frac{f' N}{\beta(1 + c_K \gamma)(1 + r) Y} \tag{12}$$

and therefore:

$$s_c = r(\beta + \gamma c_K) s_l = \frac{r(\beta + \gamma c_K) f' N}{\beta(1 + c_K \gamma)(1 + r) Y}. \tag{13}$$

and

$$s_r = 1 - s_c - s_l = 1 - s_l[1 + r(\beta + \gamma c_K)] = 1 - \left[\frac{f'N}{\beta(1 + c_K\gamma)(1 + r)Y}\right][1 + r(\beta + \gamma c_K)]. \quad (14)$$

The comparison of Eqq. (3) and (11) highlights the distinction between models with and without fixed capital, particularly when the focus is on factor shares. Considering Allen's (2009) data for 1770, as depicted in Figure 3, the relationship holds for $\gamma = 2.3, \beta = 1, c_K = 1$. For the other years, γ is oscillating with a slightly increasing trend, peaking at 2.9 in 1900, signalling an increasing intensity in the use of fixed capital with respect to labour as expected. The intensity in the use of fixed capital would be greater if c_K had shown a decreasing trend as discussed in Hicks (1969).

The rest of the model needs to specify the dynamics of circulating and fixed capital. Given the assumption of fixed proportion in the use of factors, the ratio K_C/K_F is equal to $1/(c_K\gamma)$; therefore, it is necessary only to specify the dynamics of K_F. In particular, we follow Casarosa (1978) and Kurz (2010) assuming that the rate of profits is the main driver, but considering the possibility of depreciation, i.e.:

$$\dot{K_F} = K_F[\mathrm{c}(\mathrm{R} - R_{min}) - \delta], \quad (15)$$

where $R = (1 + r)$ is the rate of profit augumented by 1, R_{min} the minimum rate of profit (plus one) requested to make additional investment, δ is the depreciation rate of fixed capital and c is a known positive parameter.

Finally, we consider the possibility that technological progress has a positive trend, i.e.

$$\dot{A} = g_A A, \quad (16)$$

where g_A is the rate of technological progress, and the same for γ, β, and c_K, i.e.:

$$\dot{\gamma} = g_\gamma \gamma; \quad (17)$$

$$\dot{\beta} = g_\beta \beta; \text{ and} \quad (18)$$

$$\dot{c_K} = g_{c_K} c_K. \quad (19)$$

Initial conditions on w, w_N, A,γ, β, c_K, and N, i.e.:

$$w(0) = w_0, w_N(0) = w_{N0}, A(0) = A_0, \gamma(0) = \gamma_0, \beta(0) = \beta_0, c_K(0) = c_{K0},$$
$$P(0) = P_0, \textit{ and } K_F(0) = K_{F0} = \gamma_0 c_{K0} w_0 N_0 \quad (20)$$

conclude the description of the model, together with the dynamics of wages:

$$\frac{\dot{w}}{w} = \frac{\dot{K}_F}{K_F} - \frac{\dot{N}}{N} - \frac{\dot{\gamma}}{\gamma} - \frac{\dot{c_K}}{c_K}, \tag{21}$$

which is directly derived by Eq. (11), and with the constraint for the allocation of workforce between the production in the primary sector N and in producing fixed capital N_F, i.e.

$$P = N + N_F.$$

We observe that a strong dynamic of accumulation of fixed capital is compatible with stagnant wage if labour used in the primary sector increases, but also if γ and/or c_K increase, i.e. γc_K raises.

3.3. The augmented Ricardian model in action

In this section we investigate the ability of the Ricardian model to explain the dynamics of labour, capital and rents shares in UK taking the evidence reported in Allen (2009) as our benchmark.

For the sake of simplicity, we consider the following production function:

$$Y = AL^{\alpha}; \tag{22}$$

this implies that (remember that $L = N/\beta$):

$$s_l = \frac{\alpha}{(1 + c_K\gamma)R}; \tag{23}$$

$$s_c = \frac{(R-1)(\beta + \gamma c_K)\alpha}{(1 + c_K\gamma)R}; \quad \text{and} \tag{24}$$

$$s_r = 1 - \left[\frac{\alpha}{(1 + c_K\gamma)R}\right][1 + (R-1)(\beta + \gamma c_K)], \tag{25}$$

from which s_l decreases with r (R), s_c increases with r, and s_r decreases with r if $\beta + \gamma c_K > 1$.

Consequently, the empirical evidence presented in Figure 3 aligns with our model under the specified condition. This condition is not satisfied in the standard Ricardian model, where the rate of profit r is the main driver of factor share dynamics. In particular, the period of an increasing r from 1800 until 1880 is associated with a decline in s_l and s_r and a surge in s_c; while the period of decreasing r after 1880 is associated with a decline in s_c and a surge in s_l. The continuos decline in s_r, instead, can be explained with a decline in γc_K, which shifts the composition of factors in favour of fixed capital.

3.4. The dynamics of the rate of profits

In Section 3.2 we have shown that the dynamics of the factor shares crucially depends on the dynamics of r (R). The latter evolves according to:

$$\frac{\dot{R}}{R} = \frac{\dot{A}}{A} + (\alpha - 1)\frac{\dot{N}}{N} - \frac{\dot{w}}{w} - \alpha\frac{\dot{\beta}}{\beta} - \frac{\dot{c_K\gamma}}{1 + c_K\gamma}; \tag{26}$$

Along the period of interest 1770-1850, while real wages were almost constant, A increased (land became more fertile) and β decreased (the number of workers per unit of land dropped) thanks to technological progress in agriculture. These two facts likely counterbalanced the negative effect of an increase in N during the same period. By contrast, the contribute of $c_K\gamma$ is ambigous since γ increased as the effect of increasing fixed capital per worker, but the cost of producing new units of fixed capital c_K was decreasing over time.

3.5. The sectoral allocation of labour

To conclude our analysis, we briefly study the allocation of workforce in the production of fixed capital, i.e.

$$N_F = \frac{\dot{K_F} + \delta K_F}{c_K W} = N\beta\gamma c(R - R_{min}), \tag{27}$$

which implies that the ratio N_F/N increases with the rate of profits and with $\beta\gamma$, i.e. the ratio between fixed capital/land. By the same token we can express the employment in the agricultural sector as a function of total workforce P, i.e.:

$$N = \frac{P}{1 + \beta\gamma c(R - R_{min})}. \tag{28}$$

We therefore conclude that, in the period 1770-1850, the positive trend in $\beta\gamma$ and an increasing rate of profits have likely induced a reallocation of the employment of available workforce from the agricultural sector to the sector devoted to the production of fixed capital.

4. Final remarks

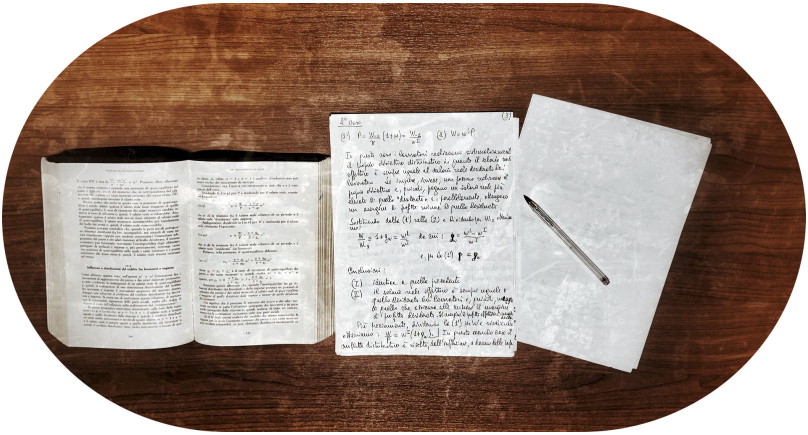
This Chapter has developed a Ricardian model augmented with fixed capital (machinery) in the spirit of Chapter XXXI, “On Machinery”, of David Ricardo’s *Principles of Political Economy and Taxation* and Chapter IX, “The Industrial Revolution”, of John Hicks’ *A Theory of Economic History*. Through this model we have analyzed the dynamics of factor shares during the British Industrial Revolution in the time span from 1770 to 1860. We have highlighted the pivotal role played by the specification of technological progress in both the agricultural sector and the sector producing fixed capital goods. By means of an appropriate set of parameters, an increasing rate of profits is compatible with a downward trend in the labor and rent shares and an upward trend in the profit share. Furthermore, an increasing rate of profits can also align with a sustained accumulation of fixed capital provided that technological progress lowers the land/fixed capital and labor/fixed capital ratios in the agricultural sector. A reduction in the production costs of fixed capital triggers such dynamics.

We wish to emphasize the distinctive features of the Ricardian model, which make it especially useful for studying the determination of factor shares. Once augmented with fixed capital, it offers a compelling alternative explanation for the distribution of output among factors and is able to integrate various aspects of technological progress that were prevalent during the Industrial Revolution, though they still remain somewhat marginal in the debate about the impact of technological progress on wages and employment (see, e.g., Caballero et al. 2017).

Even though the model here presented is fully specified, a detailed study of its dynamics is postponed to a future work. To paraphrase Ricardo, in this Chapter we have been desirous “only to elucidate the principle” concerning the role of fixed capital in the explanation of income distribution. Hence our object has been “to simplify the subject” (see *Works* I, 121 – 122, quoted in Kurz 2015).

5. Acknowledgments

We sincerely thank without in the least implicating Carlo Casarosa for his time and patience in discussing various aspects of this paper. Davide Fiaschi has been particularly influenced by Casarosa's critical approach, particularly when Carlo was assessing his own work. The photo witnesses Casarosa's teaching methods: he systematically challenged his own ideas, never taking for granted their correctness and persuasiveness.



References

Aghion P., Howitt P., *A Model of Growth Through Creative Destruction*, in «Econometrica», 60(2), 1992, pp. 323-351.

Allen R.C., *Engels' pause: Technical change, capital accumulation, and inequality in the British industrial revolution*, in «Explorations in Economic History», 46(4), 2009), pp. 418-435.

Arroyo A.L., Khalifa K., *What are stylized facts?* In «Journal of Economic Methodology», 22(2), 2015, pp. 143-156.

Atkinson A.B., *Factor shares: the principal problem of political economy?* «Oxford Review of Economic Policy», 25(1), 2009, pp. 3-16.

Barton J., *Observations on the Circumstances which Influence the Condition of the Labouring Classes of Society*, printed for John and Arthur Arch, Cornhill, by W. Mason, Chichester, 1817, available on-line at b21947041 (wellcomecollection.org).

Bellino E., Pasinetti L.L., *On Ricardo*, in H.D. Kurz and N. Salvadori (eds). *The Elgar Companion to David Ricardo*, Cheltenham, UK and Northampton, MA, USA, Edward Elgar, 2015, pp. 396-404.

Bentolila S., Saint-Paul G., *Explaining movements in the labour share*, in «Contributions in Macroeconomics», 3(1), 2003.

Blaug M., *Economic Theory in Retrospect*, 4th ed. Cambridge, Cambridge University Press, 1985.

Brown E.H., Weber B., *Accumulation, productivity and distribution in the British economy, 1870-1938*, in «The Economic Journal», 63(250), 1953, pp. 263-288.

Caballero, R. J., Farhi E., Gourinchas P.O., *Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares*, in «American Economic Review», 107(5), 2017, pp. 614-620.

Casarosa C., *A new formulation of the Ricardian system*, in «Oxford Economic Papers», 30(1), 1978, pp. 38-63.

Ciccone R., Trabucchi P., *Corn Model*, in H.D. Kurz and N. Salvadori (eds), *The Elgar Companion to David Ricardo*, Cheltenham, UK and Northampton, MA, USA, Edward Elgar, 2015, pp. 92-104.

Elsby M.W., Hobijn B., Şahin A. *The decline of the US labor share*, in «Brookings Papers on Economic Activity», 2, 2013, pp. 1-63.

Grossman G.M., Oberfield E., *The elusive explanation for the declining labor share*, in «Annual Review of Economics», 14, 202, pp. 93-124.

Hicks J.R., *The Theory of Wages*, London, Macmillan, 1932.

Hicks J.R., *A Theory of Economic History*, Oxford, Clarendon Press, 1969.

Hicks J.R., *Capital and Time. A Neo-Austrian Theory*, Oxford, Clarendon Press, 1973.

Kaldor N., *Alternative Theories of Distribution*, in «The Review of Economic Studies», 23(2), 1955-1956, pp. 83-100.

Kaldor N., *Capital accumulation and economic growth*, in *The Theory of Capital: proceedings of a conference held by the International Economic Association*, London, Palgrave Macmillan, 1961, pp. 177-222.

Karabarbounis L., *Perspectives on the labor share*, in «Journal of Economic Perspectives», 38(2), 2024, pp. 107-136.

Karabarbounis L., Neiman B., *The global decline of the labor share*, in «The Quarterly Journal of economics», 129(1), 2014, pp. 61-103.

Keynes J.M., *Relative movements of real wages and output*, in «The Economic Journal», 49(193), 1939, pp. 34-51.

Kurz H.D., *Technical progress, capital accumulation and income distribution in Classical economics: Adam Smith, David Ricardo and Karl Marx*, in «The European Journal of the History of Economic Thought», 17(5), 2010, pp. 1183-1222.

Kurz, H.D., *David Ricardo: on the art of "elucidating economic principles" in the face of a "labyrinth of difficulties"*, in «The European Journal of the History of Economic Thought», 22(5), 2015, pp. 818-851.

Marglin S.A., *Growth, Distribution, and Prices* (No. 155), Cambridge, Mass. (US), Harvard University Press, 1984.

Pasinetti L., *A Mathematical Formulation of the Ricardian System*, in «The Review of Economic Studies», 27(2), 1960, pp. 78-98.

Pasinetti L., *Lectures on the Theory of Production*, New York, Columbia University Press, 1977.

Piketty T., Zucman G. *Capital is back: Wealth-income ratios in rich countries 1700–2010* in «The Quarterly Journal of Economics», 129(3), 2014, pp. 1255-1310.

Ricardo D., *The Works and Correspondence of David Ricardo*, edited by Piero Sraffa with the collaboration of Maurice H. Dobb, 11 vols, Cambridge, Cambridge University Press, 1951-73.

Salvadori N., *Ricardo's Theory of Growth and Accumulation: A Modern View*, London and New York, Routledge, 2021.

Solow R.M., *A Contribution to the Theory of Economic Growth*, in «The Quarterly Journal of Economics», 70(1), 1956, pp. 65-94.

Solow R.M., *A skeptical note on the constancy of relative shares*, in «The American Economic Review», 48(4), 1958, pp. 618-631.

Schneider D., *The labor share: A review of theory and evidence*, World Inequality Dataset 2011, (updated October 2023).

Sraffa P., *Introduction to The Works and Correspondence of David Ricardo*, Cambridge, Cambridge University Press, 1951, pp. vii-lxii.