Task 6 - Safety Review and Licensing
On the Job Training on Stress Analysis

F.M. with Finite Element analysis - Different calculation techniques + Numerical examples (ANSYS Workbench) 1/2

Davide Mazzini – **Ciro Santus**

BEYOND THE RESEARCH

UNIVERSITÀ DI PISA

Pisa (Italy)
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Content

- Different FE techniques
- ANSYS Workbench
  - Crack geometry element mesh preparation
  - $K_{I(II,III)}$ and J calculation
  - Examples
- ANSYS Apdl
  - The quarter point technique
  - Examples

FM parameters computed with FE

Stress Intensity Factor(s) SIFs: $K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}$ ← Stress parameters

Energy release rate: G

Contour integral: J

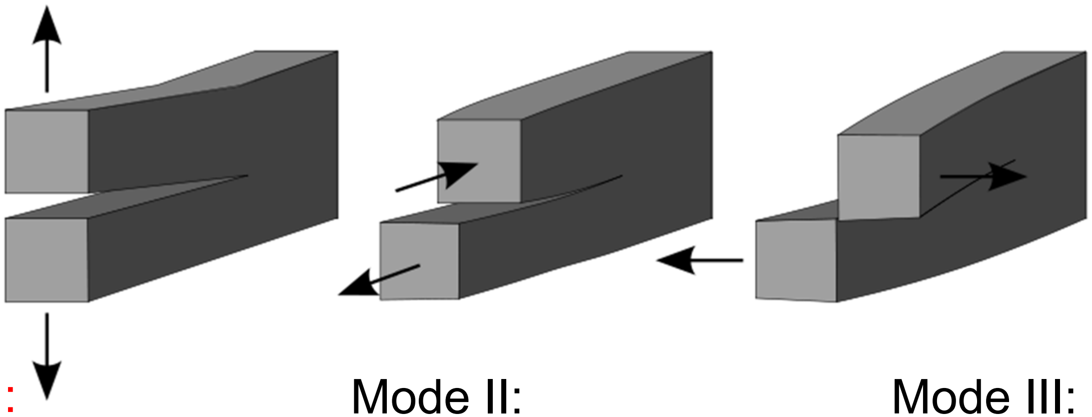
(Energy parameters: G, J)

Crack Mouth Opening Displacement CMOD

Crack Tip Opening Displacment CTOD

(Deformation parameters: CMOD, CTOD)

Fracture Mechanic parameters for the three modes



Mode I:
Opening

Mode II:
In-plane shear

Mode III:
Out-of-plane shear

$$J = \frac{K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2}{E'} + \frac{K_{\mathrm{III}}^2}{2G}$$

ANSYS help:

$$J = \int_{\Gamma} \left(w\,\mathrm{d}y - T_i \frac{\partial u_i}{\partial x} \mathrm{d}s \right)$$

11.3. Numerical Evaluation of Fracture Mechanics Parameters

Several tools are available for evaluating fracture mechanics parameters:

- The J-Integral calculation is based on the domain integral approach and is performed during the solution phase of the analysis (**CINT**).
- Direct energy-release rate calculation, based on the virtual crack closure technique (VCCT), is performed at solution (**CINT**).
- Stress-intensity factors calculation with the interaction integral approach during solution (**CINT**)
- Stress-intensity factors calculation with extrapolation during postprocessing (**KCALC**).

$$J = -\frac{\mathrm{d}\Pi}{\mathrm{d}A}$$

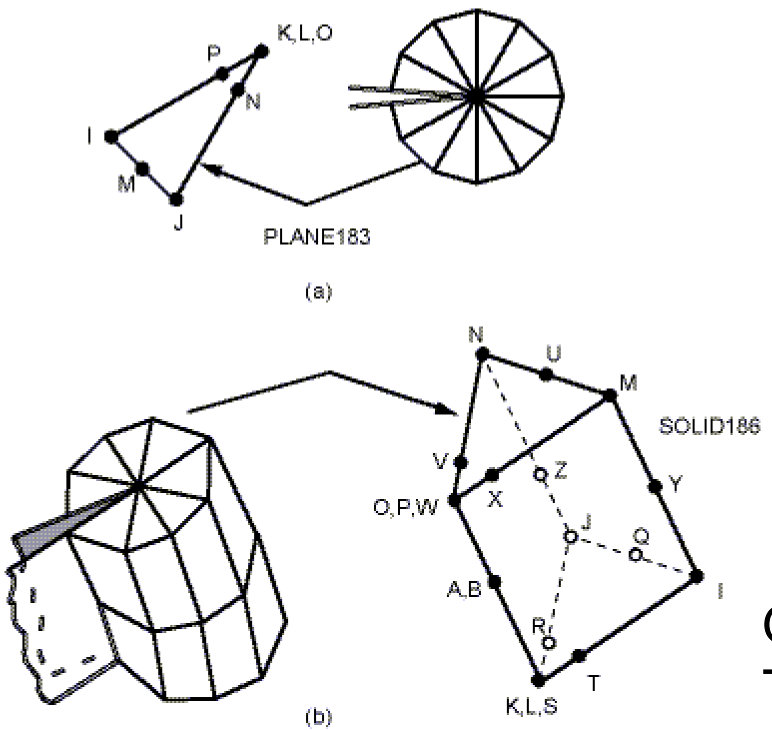
$$K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}$$

$$J \leftarrow K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}$$

$$J \rightarrow K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}$$

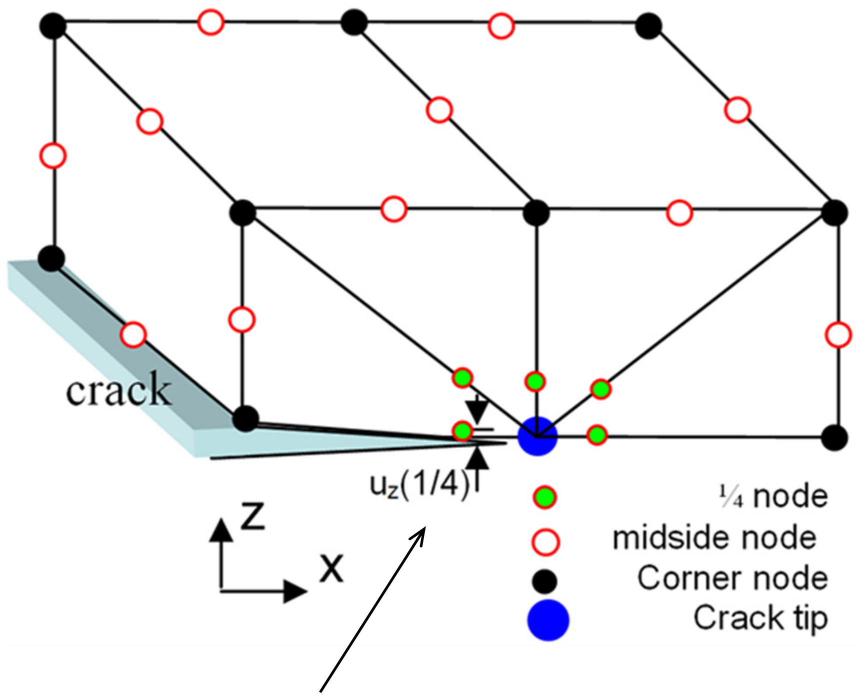
$$\text{(only if 2/3 are zero, or ratios are known)}$$

KCALC command



(a) 2-D models and (b) 3-D models

ANSYS Help



Quarter point technique:
The mid-side node is shifted at ¼ of the element size to reproduce the displacement derivative singularity.
From displacement to (elastic) $K_{I(II,III)}$

KCALC command

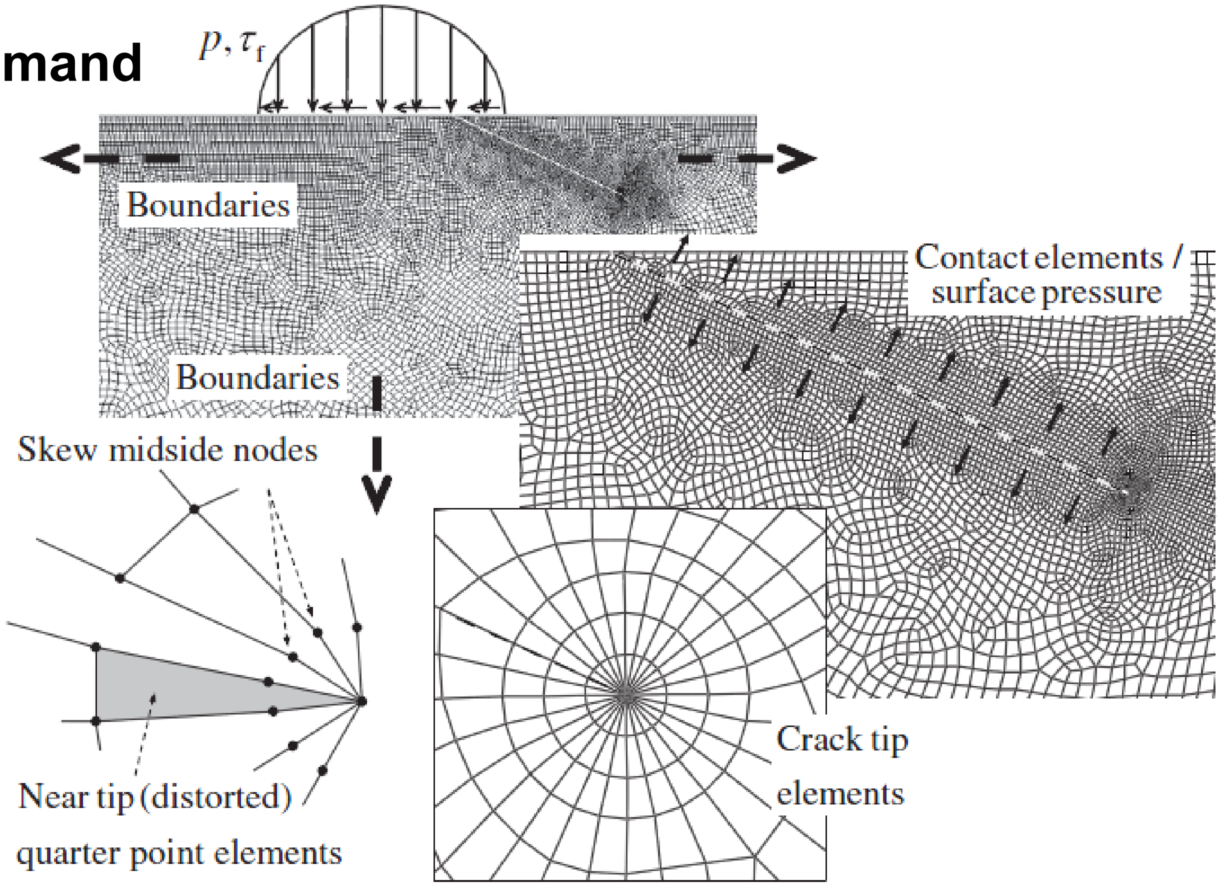


Fig. 7. FE model to calculate the SIFs with the quarter point technique.

M. Beghini, C. Santus. "An application of the weight function technique to inclined surface cracks under rolling contact fatigue, assessment and parametric analysis". Engineering Fracture Mechanics. Vol.98, pp.153–168, 2013. DOI: 10.1016/j.engfracmech.2012.10.024

CINT command options

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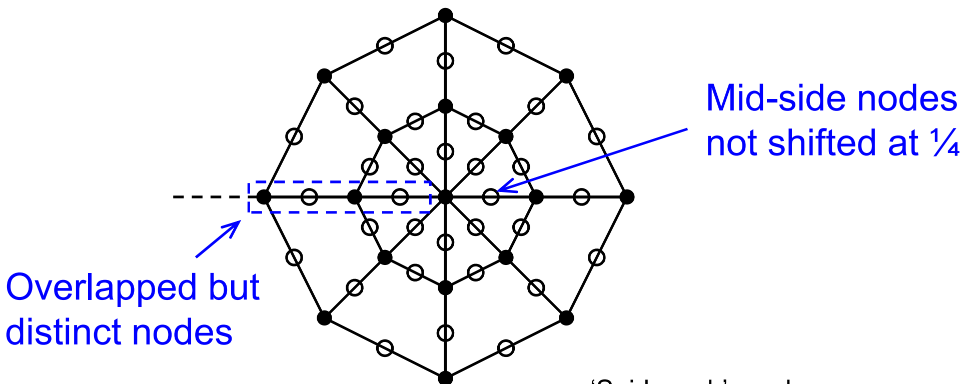
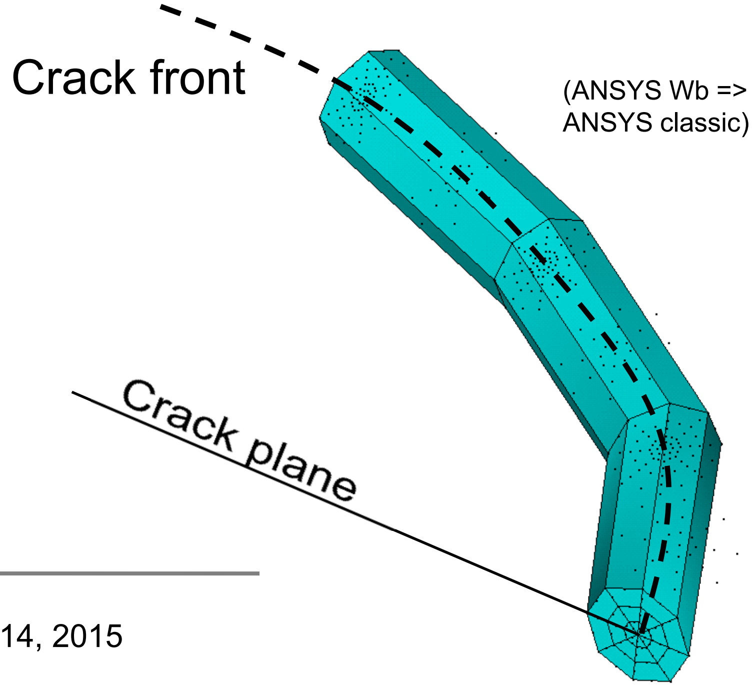


‘Spiderweb’ mesh



CINT command options

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11.3.2.1.3. Element Support, Mesh and Material Behavior

The VCCT method for energy-release rate calculation (accessed via the **CINT** command) supports the following elements:

- PLANE182
- PLANE183
- SOLID185
- SOLID186

Plane elements (not for Workbench)

Workbench default element thus VCCT is not available

In most cases, ANSYS, Inc. recommends using linear elements including PLANE182 and SOLID185.

CINT command options

Several tools are available for evaluating fracture mechanics parameters:

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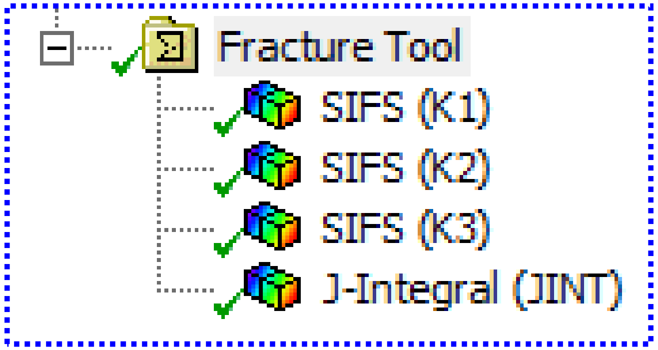
- ✘ Stress-intensity factors calculation with extrapolation during postprocessing (**KCALC**).

Postprocessing menu: only available CINT for J and CINT for K1(2,3)



CINT command options

- Stress-intensity factors calculation with the interaction integral approach during solution (**CINT**)

11.3.3.1.1. Understanding Interaction Integral Formulation

The interaction integral is defined as

$$I = -\int_V q_{i,j}\left(\sigma_{kl}\varepsilon_{kl}^{aux}\delta_{ij} - \sigma_{kj}^{aux}u_{k,i} - \sigma_{kj}u_{k,i}^{aux}\right)dV \Big/ \int_s \delta q_n ds$$

where:

$\sigma_{ij}, \varepsilon_{ij}, u_i$ are the stress, strain and displacement,

$\sigma_{ij}^{aux}, \varepsilon_{ij}^{aux}, u_i^{aux}$ are the stress, strain and displacement of the auxiliary field,

and q_i is the crack-extension vector.

The interaction integral is associated with the stress-intensity factors as

$$I = \frac{2}{E^*}\left(K_1K_1^{aux} + K_2K_2^{aux}\right) + \frac{1}{\mu}K_3K_3^{aux}$$

where:

K_i (i = 1,2,3) = mode I, II, and III stress-intensity factors

K_i^{aux} (i = 1,2,3) = auxiliary mode I, II and III stress-intensity factors

E* = E for plane stress and E* = E / (1 - □v2) for plane strain

E = Young's modulus

v = Poisson's ratio

μ = shear modulus

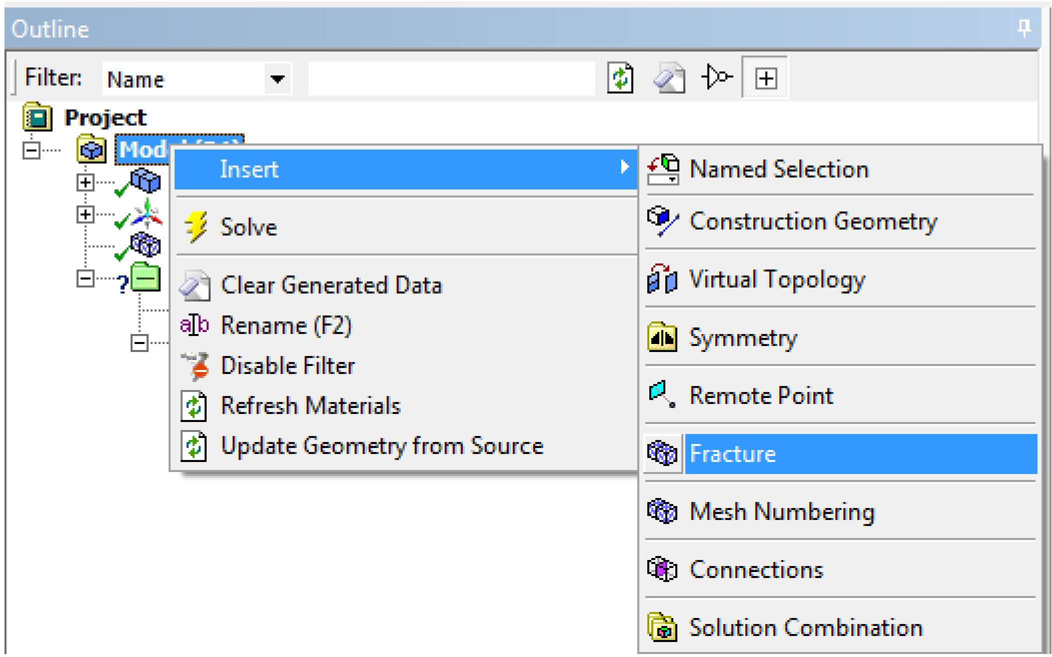
CINT,SIFS

Volume (not path) integration, with the virtual load technique to distinguish K_1, K_2 and K_3

This way the SIFs are obtained as integration, rather than point displacement as KCALC

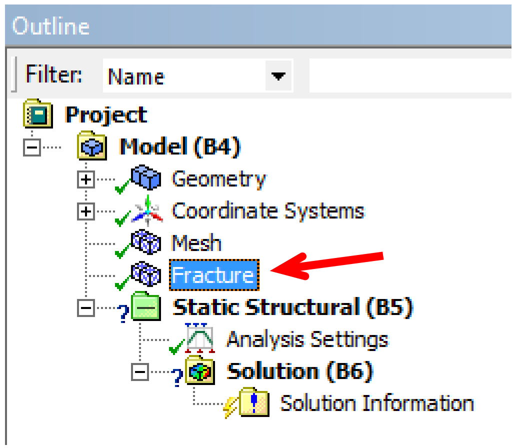
A coarser mesh is enough for an accurate solution

How to introduce a crack

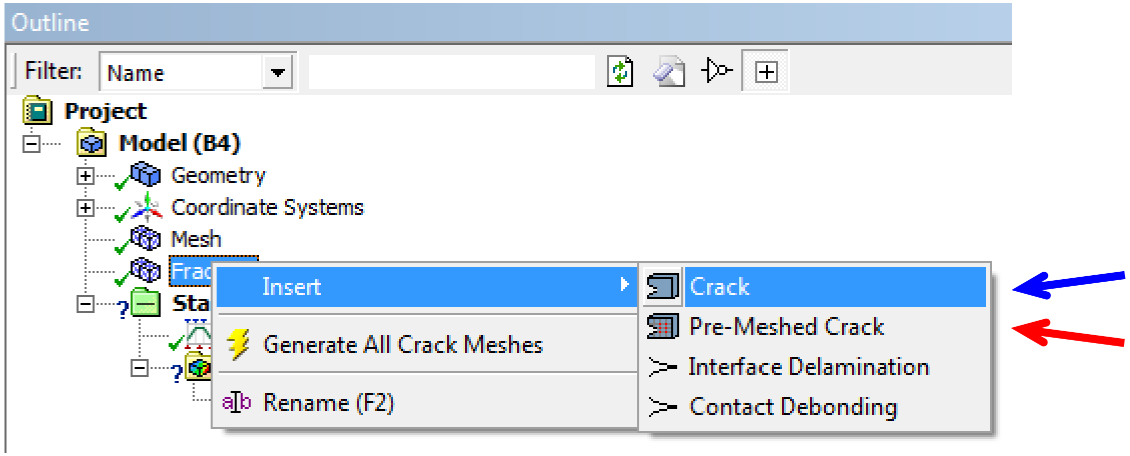


Introduce the 'Fracture' module into the Modeler

(this feature is available only since the 14.5 ANSYS version, the current version is 16.0)



How to introduce a crack



1. It is possible to (easily) introduce a crack starting from a solid uncracked body and automatically generate the spiderweb mesh distribution at the crack front
(Worth noting: this is for a semielliptical crack only)

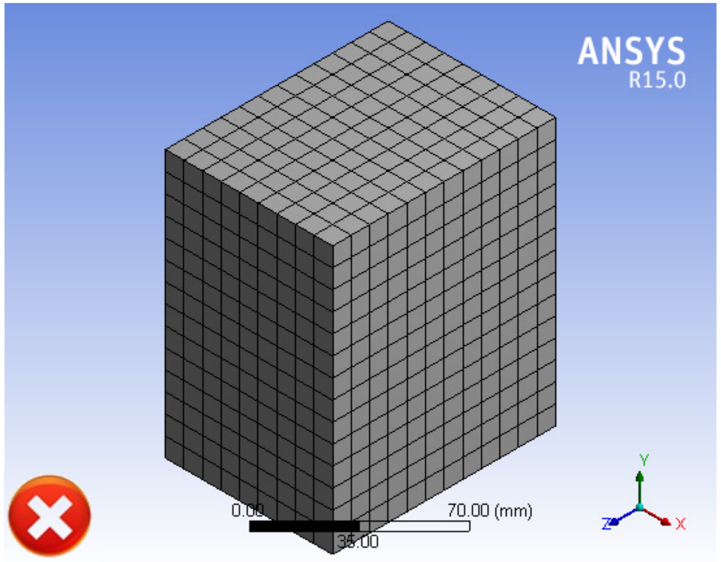
2. Alternatively a (generic) crack front can be prepared in advance and introduced as the crack front

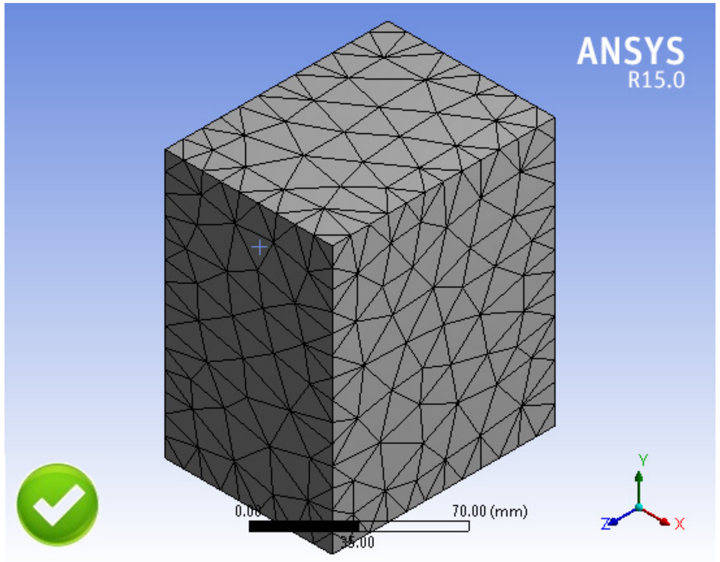
How to introduce a crack

Mesh preparation

Details of "Patch Conforming Method" - Method

Scope	
Scoping Method	Geometry Selection
Geometry	1 Body
Definition	
Suppressed	No
Method	Tetrahedrons
Algorithm	Patch Conforming
Element Midside Nodes	Use Global Setting

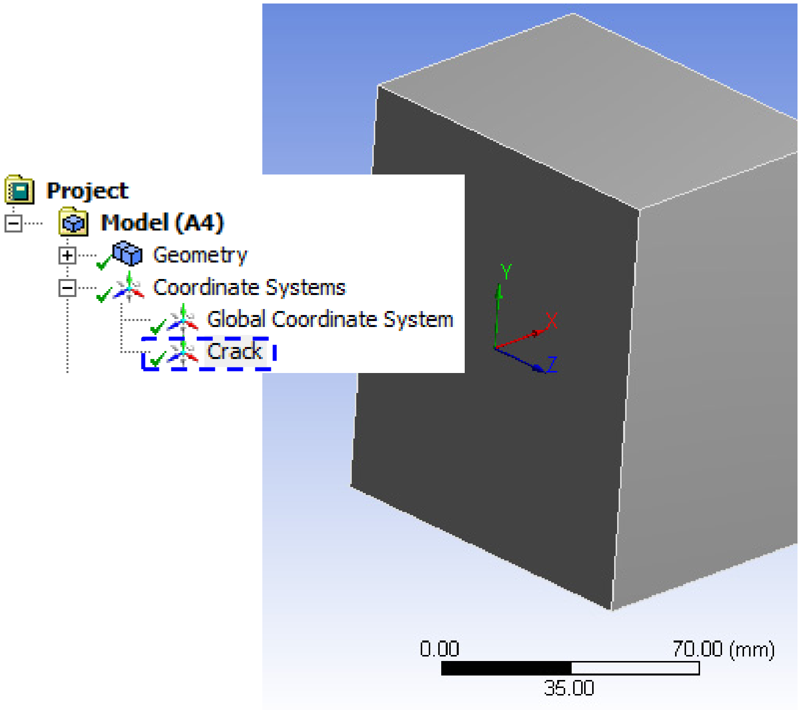




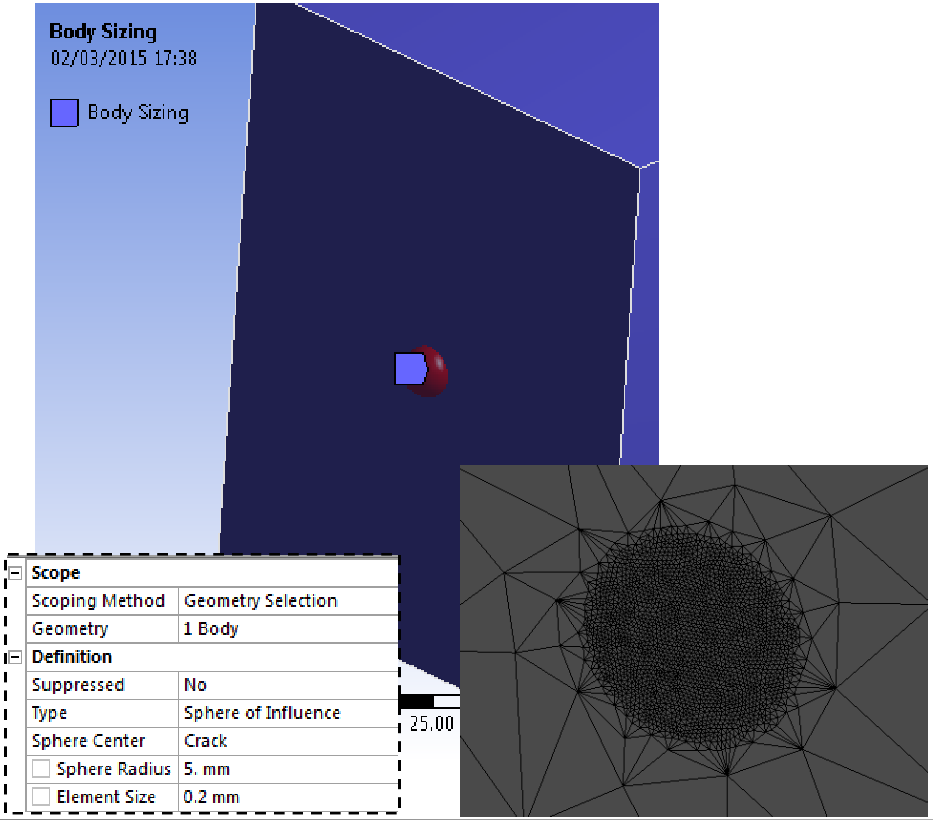
Usually Hexahedrons are to be preferred than Tetrahedrons
Here Tets are necessary for mesh continuity

Mesh preparation

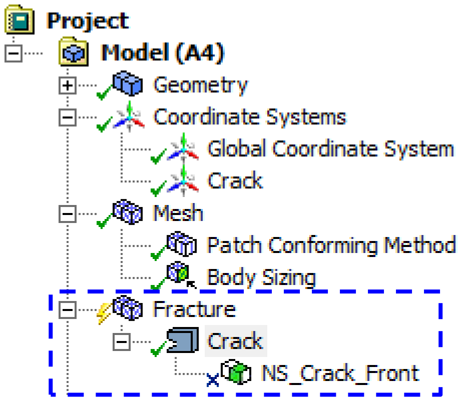
A strong local refinement is required
The Sphere of Influence tool is recommended

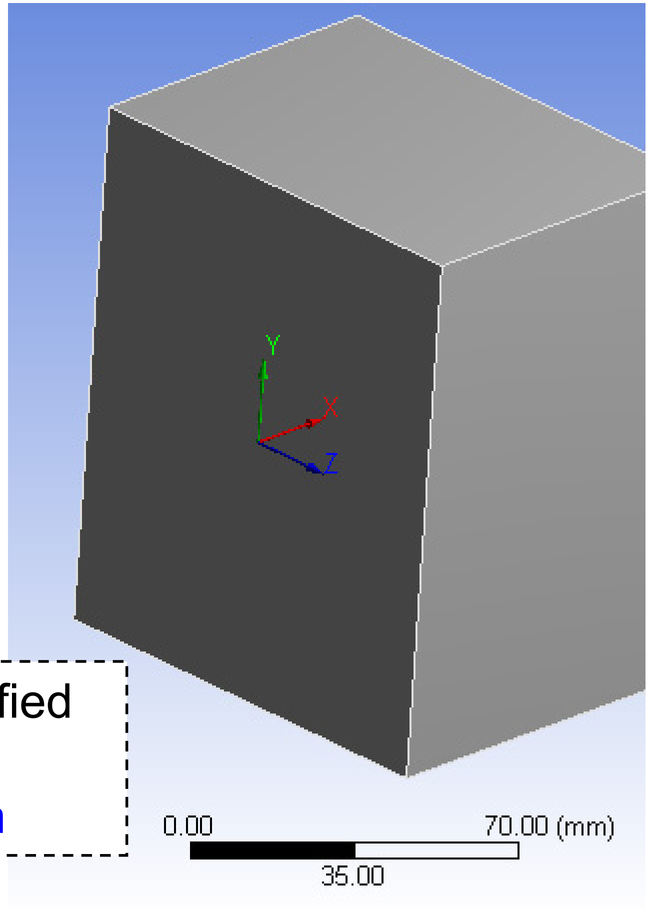


New local Coordinate System, both for mesh and for crack position/ orientation



Crack position and orientation





The Crack is identified by its dedicated Coordinate System

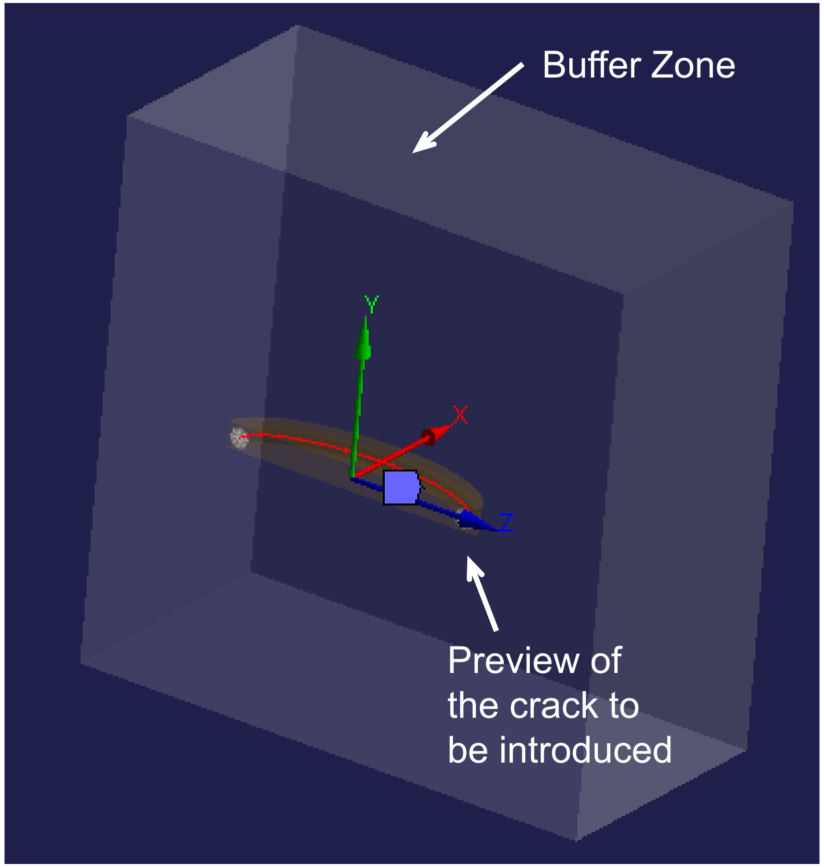
The crack semi-elliptical 'center' is coincident with Coordinate System origin point

X axis has to be inward the component to be cracked and it should be perpendicular

If not the crack plane is going to be perpendicular to the surface anyway

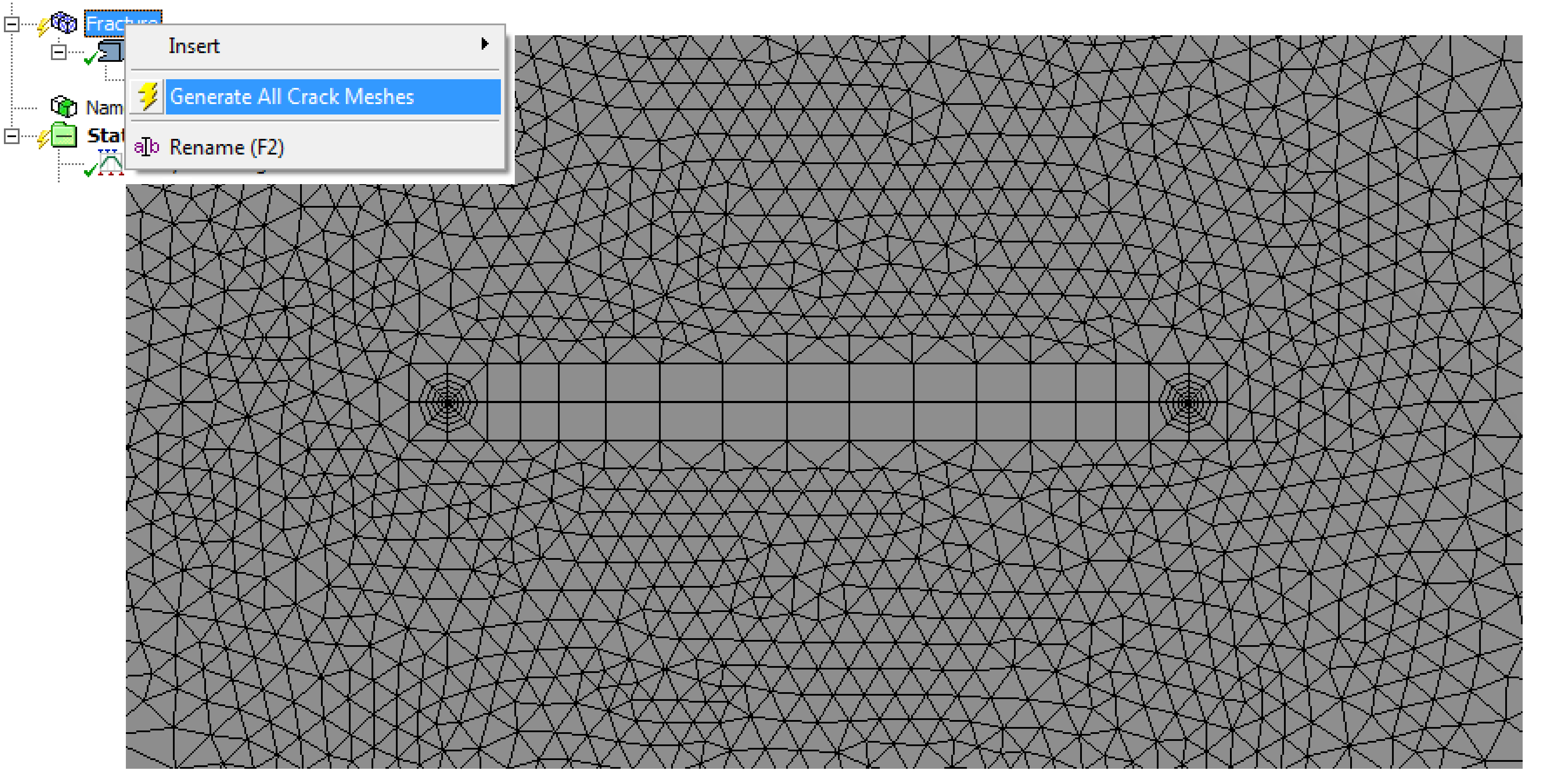
X-Z axes define the crack plane, thus Y is the vertical direction

Mesh and geometry parameters of the crack

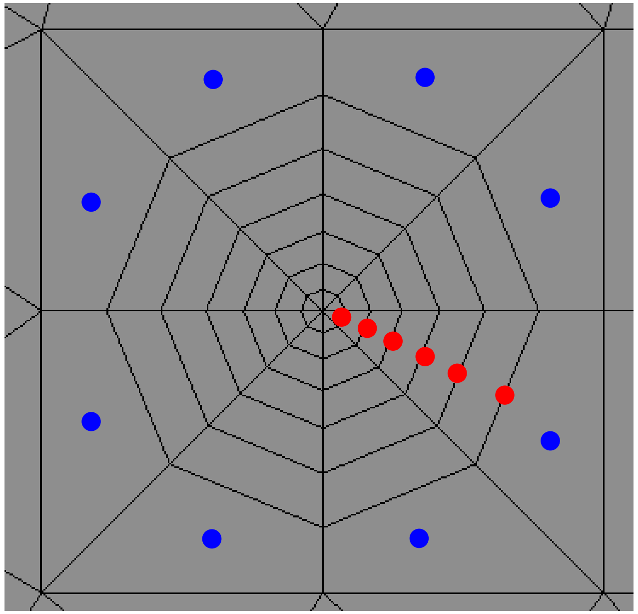


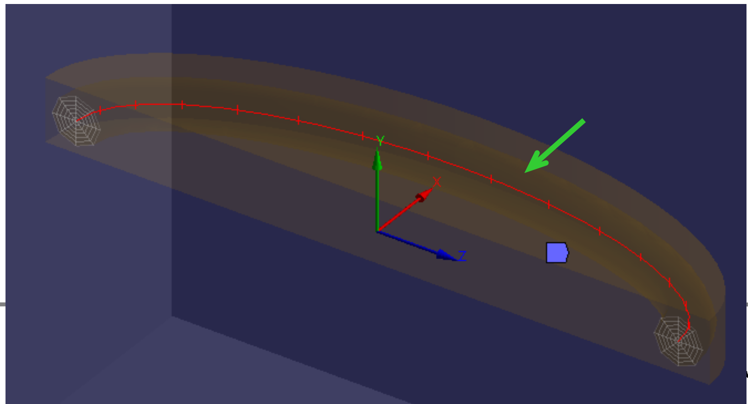
Scope	
Source	Crack
Scoping Method	Geometry Selection
Geometry	1 Body
Definition	
Coordinate System	Crack
Crack Shape	Semi-Elliptical
--Major Radius	2.5 mm ← Dir. Z
--Minor Radius	1. mm ← Dir. X
Fracture Affected Zone	Program Controlled
Fracture Affected Zone Height	0.52028 mm
Largest Contour Radius	0.2 mm
Circumferential Divisions	8
Mesh Contours	6
Crack Front Divisions	15
Solution Contours	Match Mesh Contours
Suppressed	No
Buffer Zone Scale Factors	
X Scale Factor	2.
Y Scale Factor	2.
Z Scale Factor	2.
Named Selections Creation	
Crack Front Nodes	NS_Crack_Front
Crack Faces Nodes	Off
Contact Pairs Nodes	Off

The introduction of the crack is just a matter of mesh



Spiderweb mesh parameters





Scope	
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Scoping Method	Geometry Selection
Geometry	1 Body
Definition	
Coordinate System	Crack
Crack Shape	Semi-Elliptical
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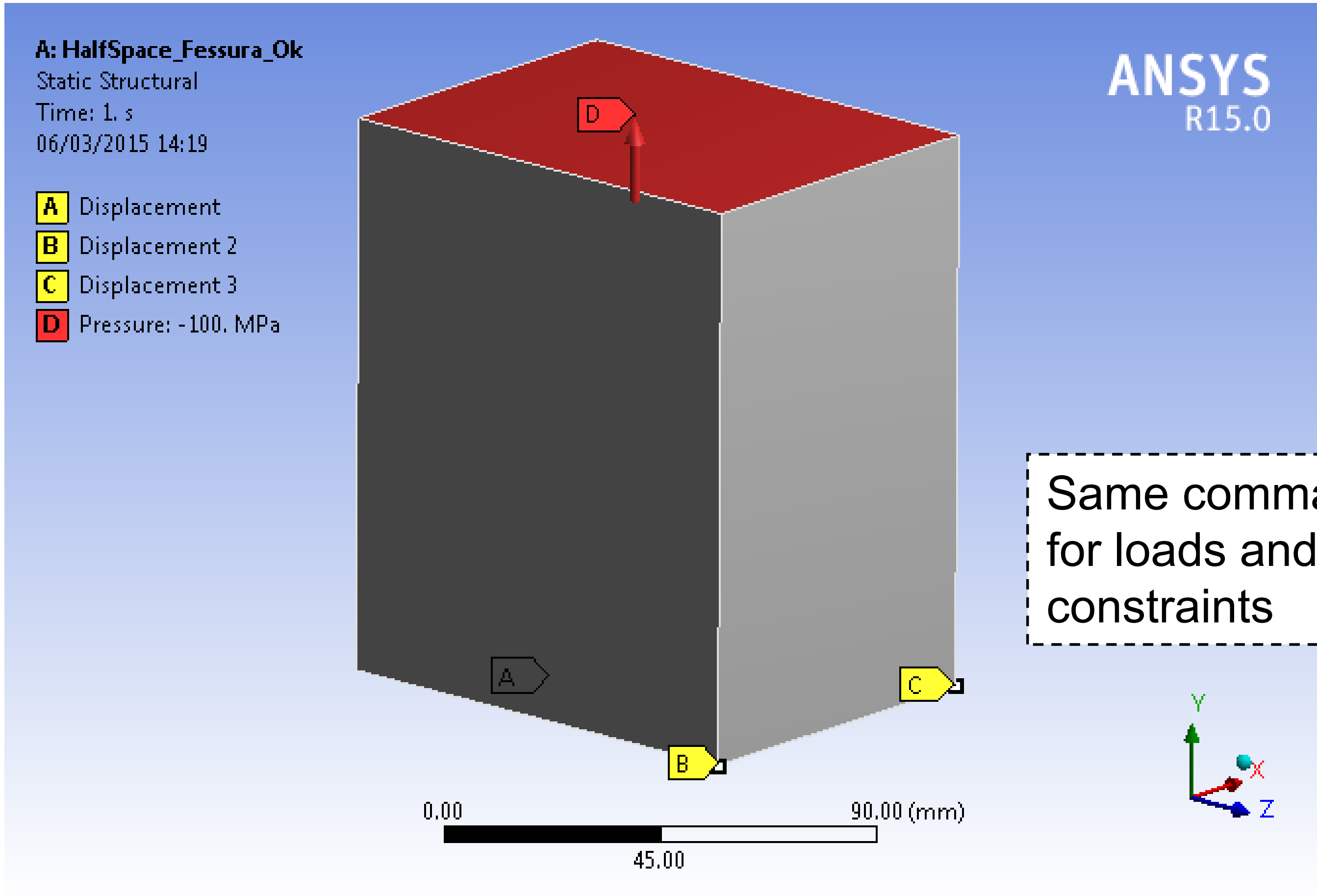
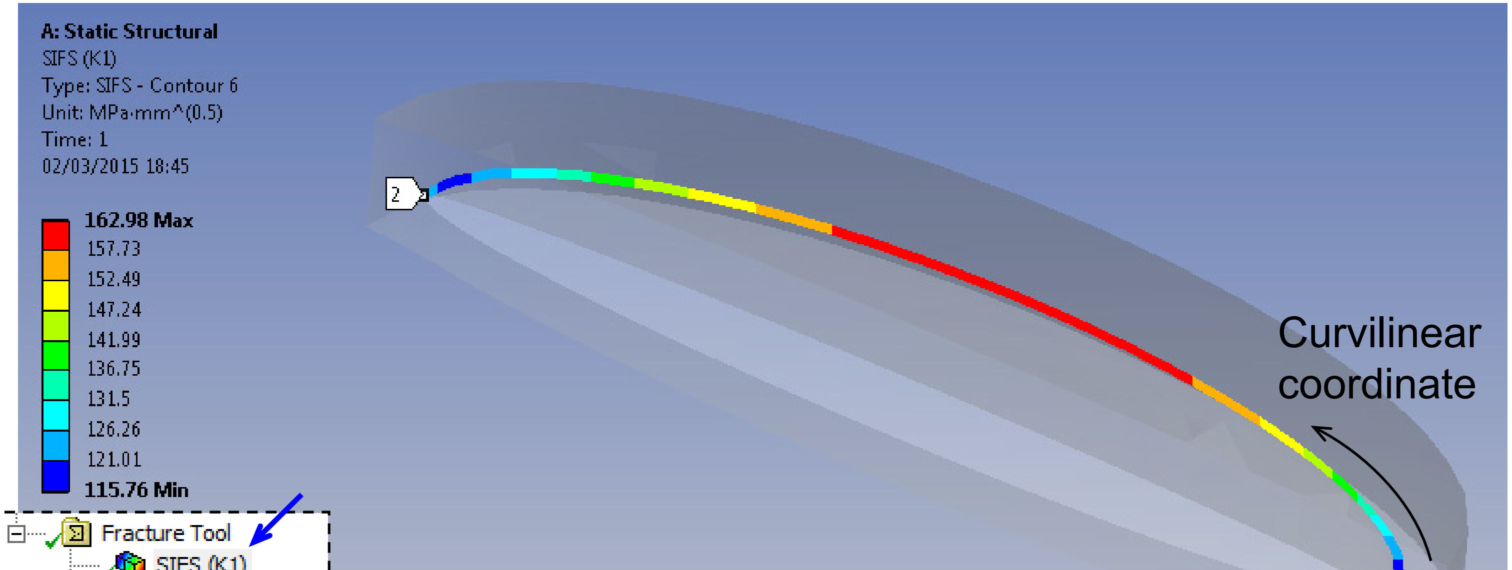
Mesh Contours	6
Crack Front Divisions	15
Solution Contours	Match Mesh Contours
Suppressed	No
Buffer Zone Scale Factors	
X Scale Factor	2.
Y Scale Factor	2.
Z Scale Factor	2.
Named Selections Creation	
Crack Front Nodes	NS_Crack_Front
Crack Faces Nodes	Off
Contact Pairs Nodes	Off

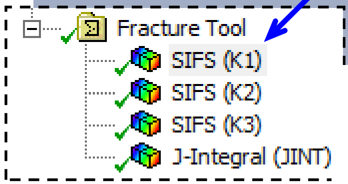
Half-space surface crack basic example

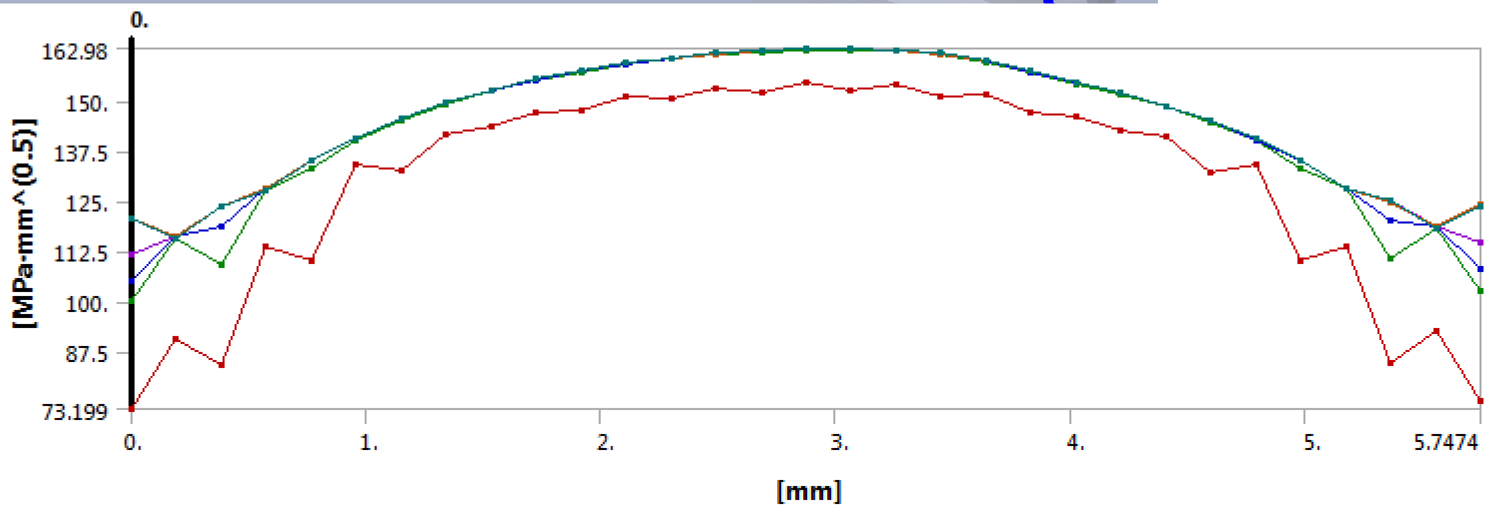


Same commands for loads and constraints

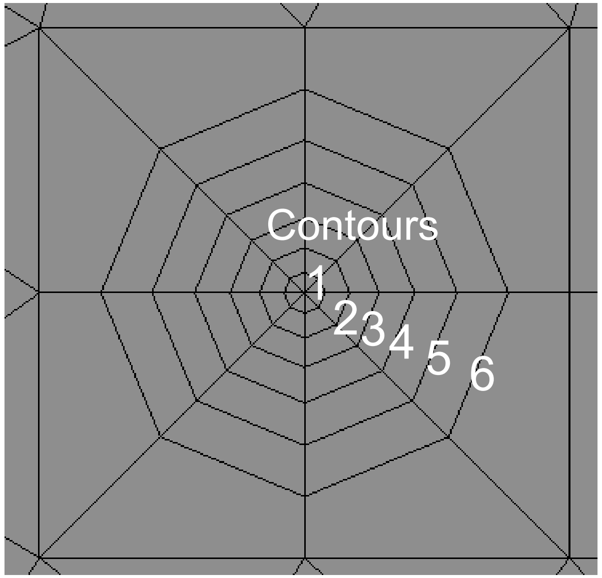
Postprocessing, $K_{1(2,3)}$ and J along the crack front







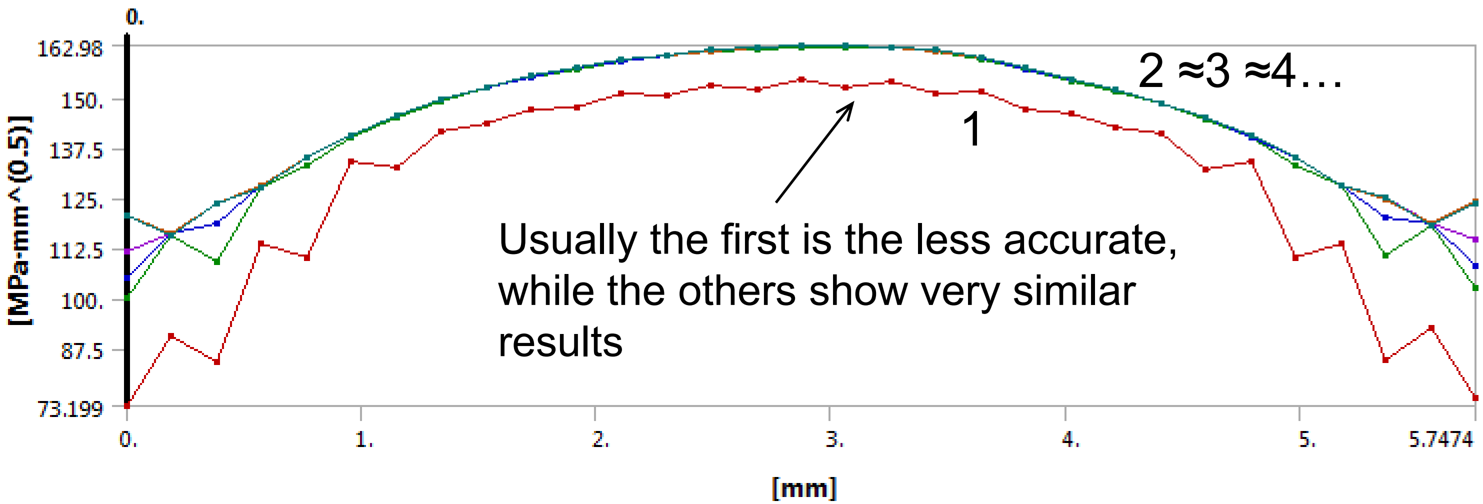
Contour integration results



Tabular values

Tabular Data

	Length [mm]	SIFS (K1) Contour 1 [MPa...	SIFS (K1) Contour
1	0.	73.199	99.981
2	0.19157	90.745	115.71
3	0.38125	83.981	109.26
4	0.57298	113.56	127.68
5	0.7644	110.01	133.25
6	0.95615	134.1	140.17
7	1.1478	132.6	144.91
8	1.3396	141.66	149.12
9	1.5314	143.47	152.33
10	1.7231	147.04	154.97
11	1.9149	147.35	157.21
12	2.1066	150.89	159.04
13	2.2984	150.28	160.34
14	2.4902	153.	161.31
15	2.6819	152.07	162.18

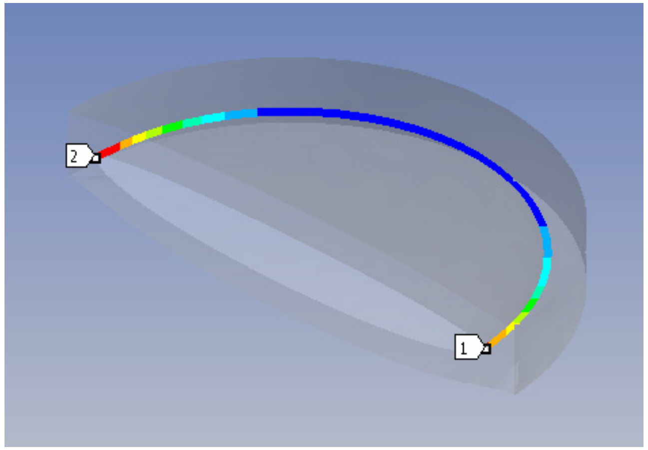


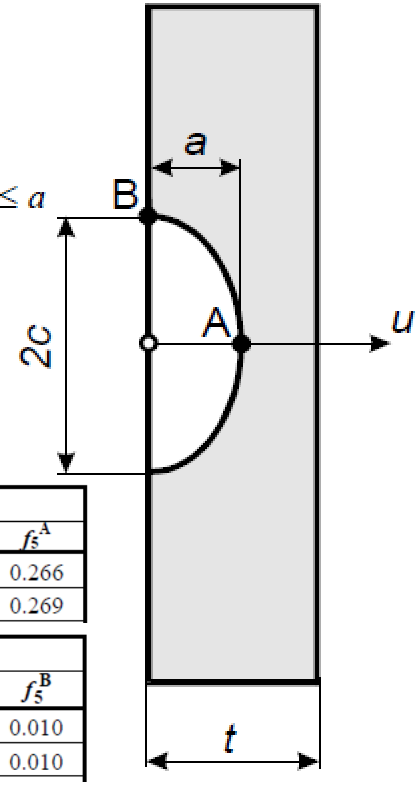
Usually the first is the less accurate, while the others show very similar results

Validation example 1 (half space)

$$K_I = \sqrt{\pi a}\sum_{i=0}^{5}\sigma_i f_i\left(\frac{a}{t},\frac{2c}{a}\right)$$

$$\sigma = \sigma(u) = \sum_{i=0}^{5}\sigma_i\left(\frac{u}{a}\right)^i \quad \text{for } 0 \le u \le a$$





$a = 2\,\text{mm} << t$

$c = 2$ mm

$\sigma_0 = 100$ MPa $(\sigma_i = 0,\ i = 1...5)$

	$2c/a = 2$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.659	0.471	0.387	0.337	0.299	0.266
0.2	0.663	0.473	0.388	0.337	0.299	0.269

	$2c/a = 2$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.716	0.118	0.041	0.022	0.014	0.010
0.2	0.729	0.123	0.045	0.023	0.014	0.010

$a = 2\,\text{mm} << t$

$c = 2$ mm

$\sigma_0 = 100$ MPa

ANSYS Wb:

$K_I(A) = 162\,\text{MPa}\sqrt{\text{mm}} = 5.12\,\text{MPa}\sqrt{\text{m}}$

$K_I(B) = 177\,\text{MPa}\sqrt{\text{mm}} = 5.60\,\text{MPa}\sqrt{\text{m}}$

Laham's SIFs Handbook:

$K_I(A) = 165\,\text{MPa}\sqrt{\text{mm}} = 5.22\,\text{MPa}\sqrt{\text{m}}$

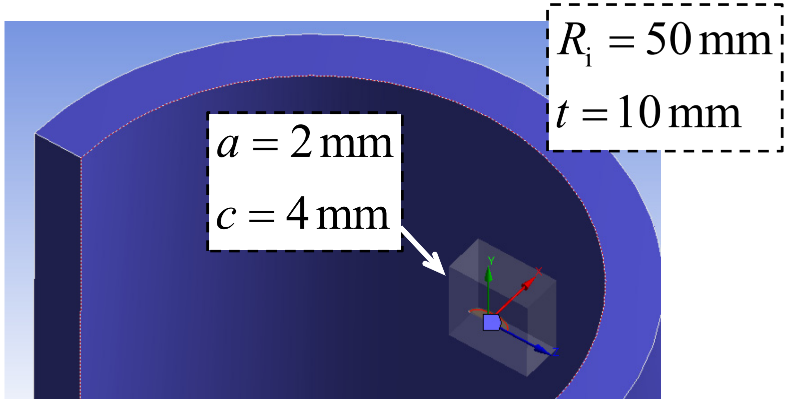
(diff.% = 1.9%)

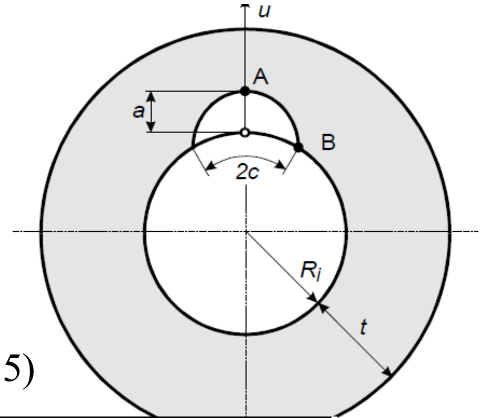
$K_I(B) = 179\,\text{MPa}\sqrt{\text{mm}} = 5.68\,\text{MPa}\sqrt{\text{m}}$

(diff.% = 1.4%)

Validation example 2 (pipe)

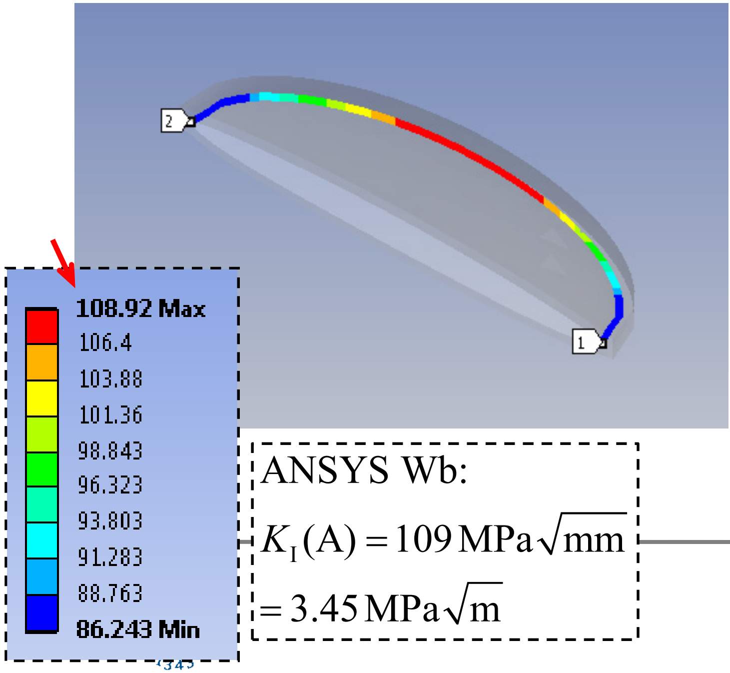
$$K_I = \sqrt{\pi a}\left(\sum_{i=0}^{3}\sigma_i f_i\left(\frac{a}{t},\frac{2c}{a},\frac{R_i}{t}\right)+\sigma_{bg} f_{bg}\left(\frac{a}{t},\frac{2c}{a},\frac{R_i}{t}\right)\right)$$





$\sigma_0 = 50$ MPa $(\sigma_i = \sigma_{bg} = 0,\ i = 1 \ldots 5)$

2c/a= 4, R_i/t = 5					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.886	0.565	0.430	0.352	0.738
0.2	0.890	0.556	0.424	0.347	0.761
0.4	0.934	0.576	0.440	0.362	0.817



ANSYS Wb:

$K_I(A) = 109\,\text{MPa}\sqrt{\text{mm}}$
$= 3.45\,\text{MPa}\sqrt{\text{m}}$

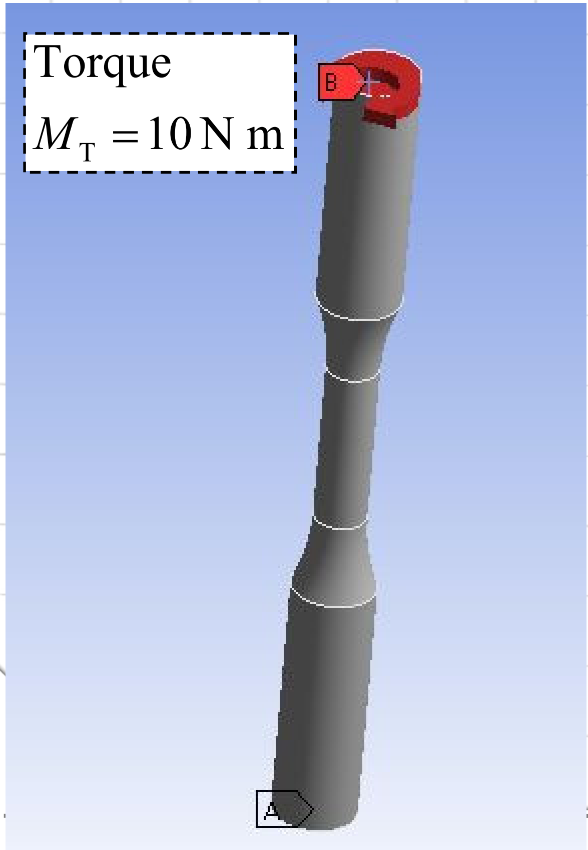
Laham's SIFs Handbook:

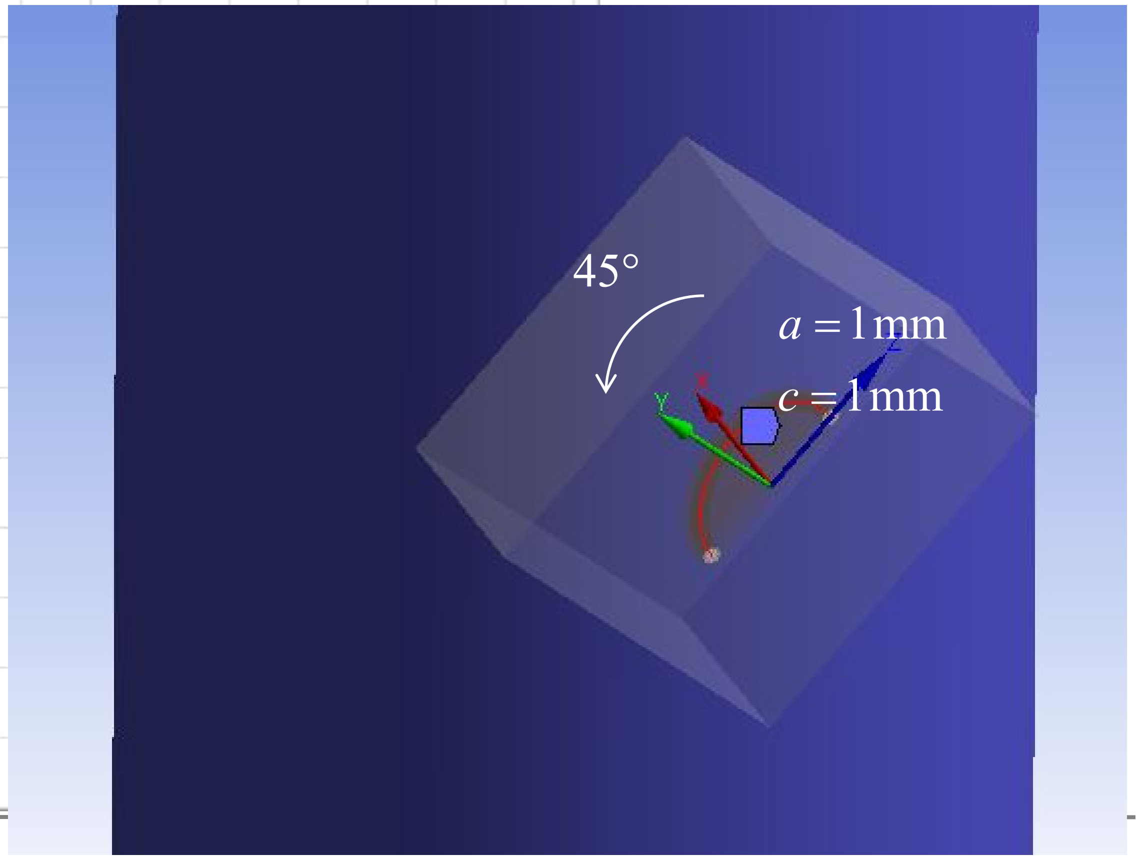
$K_I(A) = 112\text{MPa}\sqrt{\text{mm}} = 3.53\,\text{MPa}\sqrt{\text{m}}$

(diff.% = 2.3%)

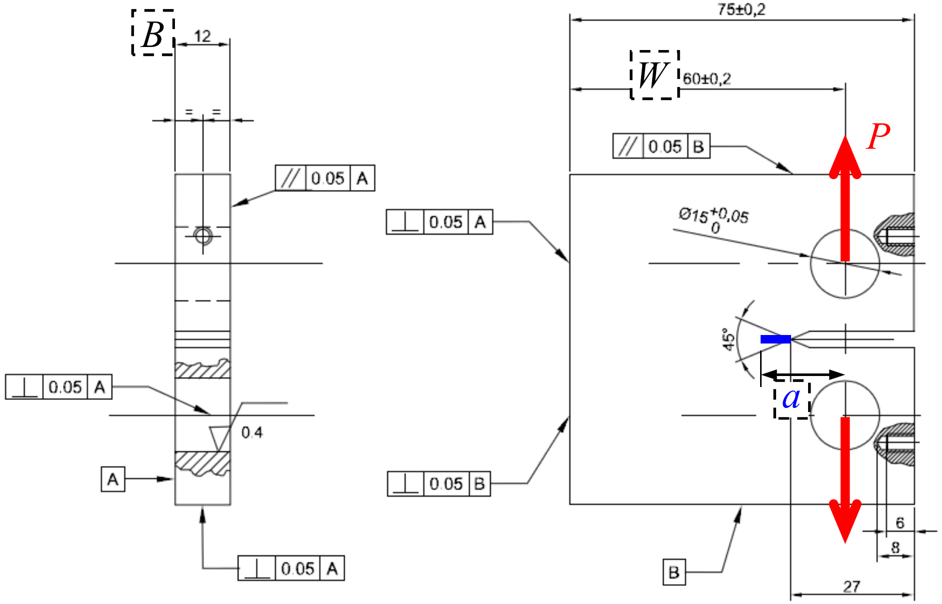
Exercise 1:
Calculate the three SIFs.
Which on is expected to be zero?

Introduce an inclined crack on the specimen surface



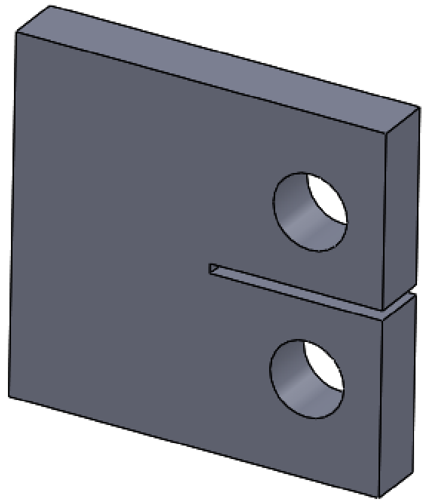


Exercise 2:
Verify the ASTM standard CT specimen formula for K_{I}



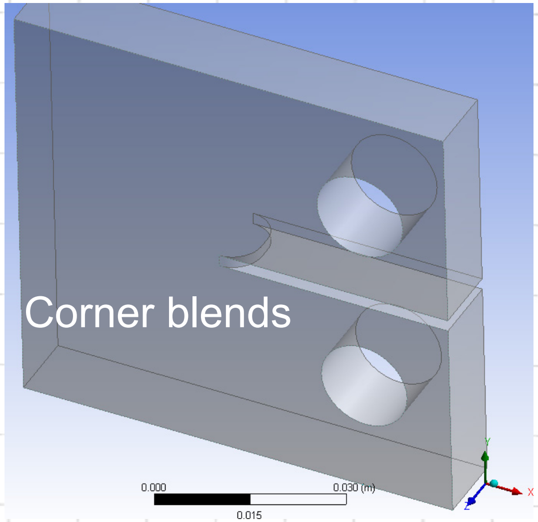
$$\alpha = a / W$$

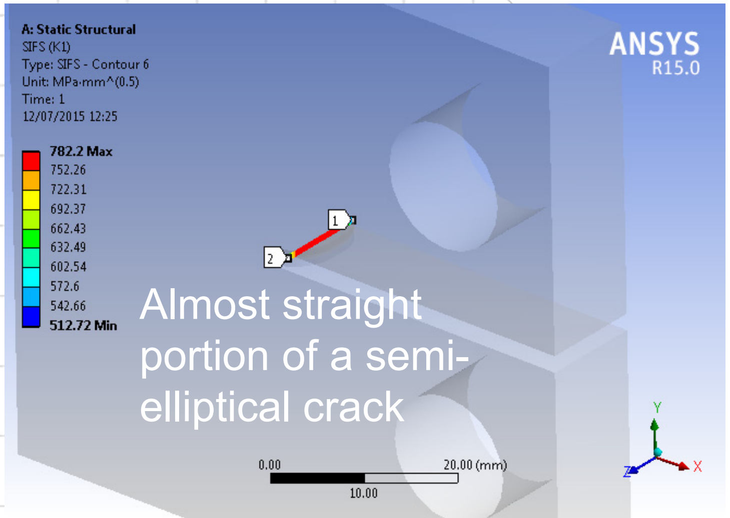
$$K_{\mathrm{I}} = \frac{P}{B\sqrt{W}} \frac{2+\alpha}{(1-\alpha)^{3/2}} (0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.64\alpha^4)$$



How to manage a semi-elliptical crack for this case?

Exercise 2:





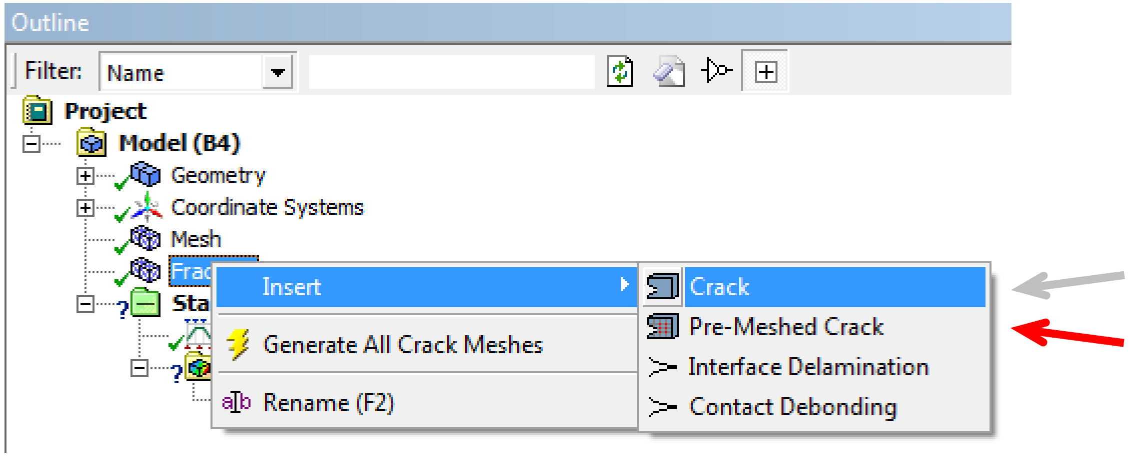
$$P = 10\,\text{kN} \quad a = 22.5\,\text{mm}, \quad W = 60\,\text{mm}, \quad B = 12\,mm$$

$$K_I = \frac{P}{B\sqrt{W}} \frac{2+\alpha}{(1-\alpha)^{3/2}} (0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.64\alpha^4) = 699\,\text{MPa}\sqrt{\text{mm}}$$

$$K_I(\text{ANSYS}) = 782\,\text{MPa}\sqrt{\text{mm}}$$

$$\Delta\% = 12\,\%$$ ← Large percentage difference

How to introduce a crack – Pre-Meshed Crack

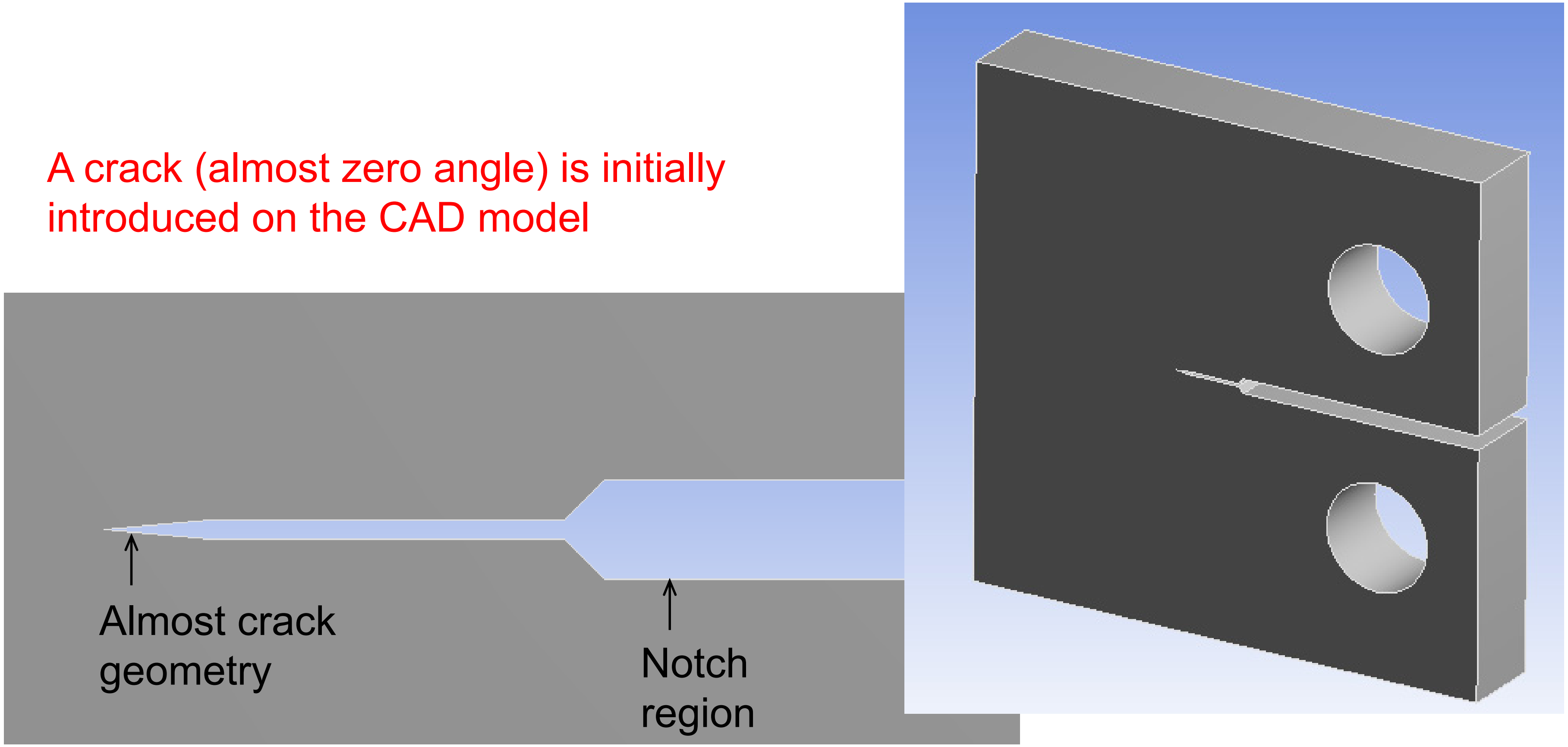


1. It is possible to (easily) introduce a crack starting from a solid uncracked body and automatically generate the spiderweb mesh distribution at the crack front
(Worth noting: this is for a semielliptical crack only)

2. Alternatively a (generic) crack front can be prepared in advance and introduced as the crack front

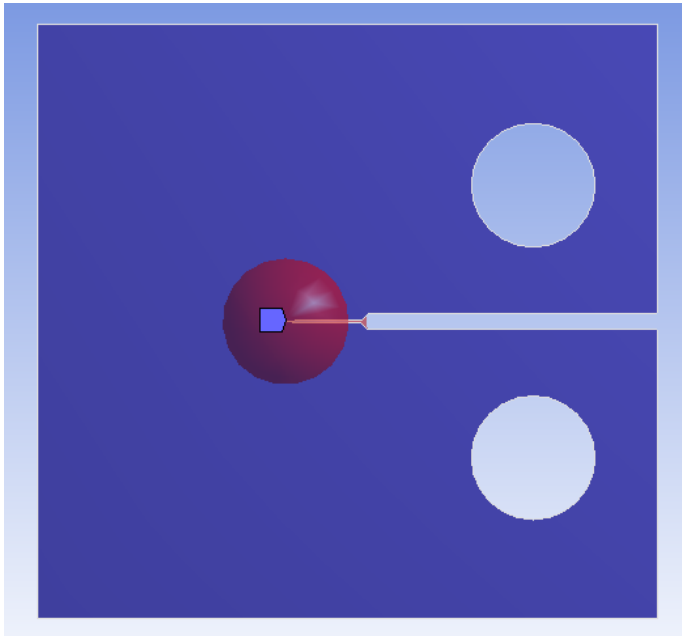
Pre-Meshed Crack, CT specimen example

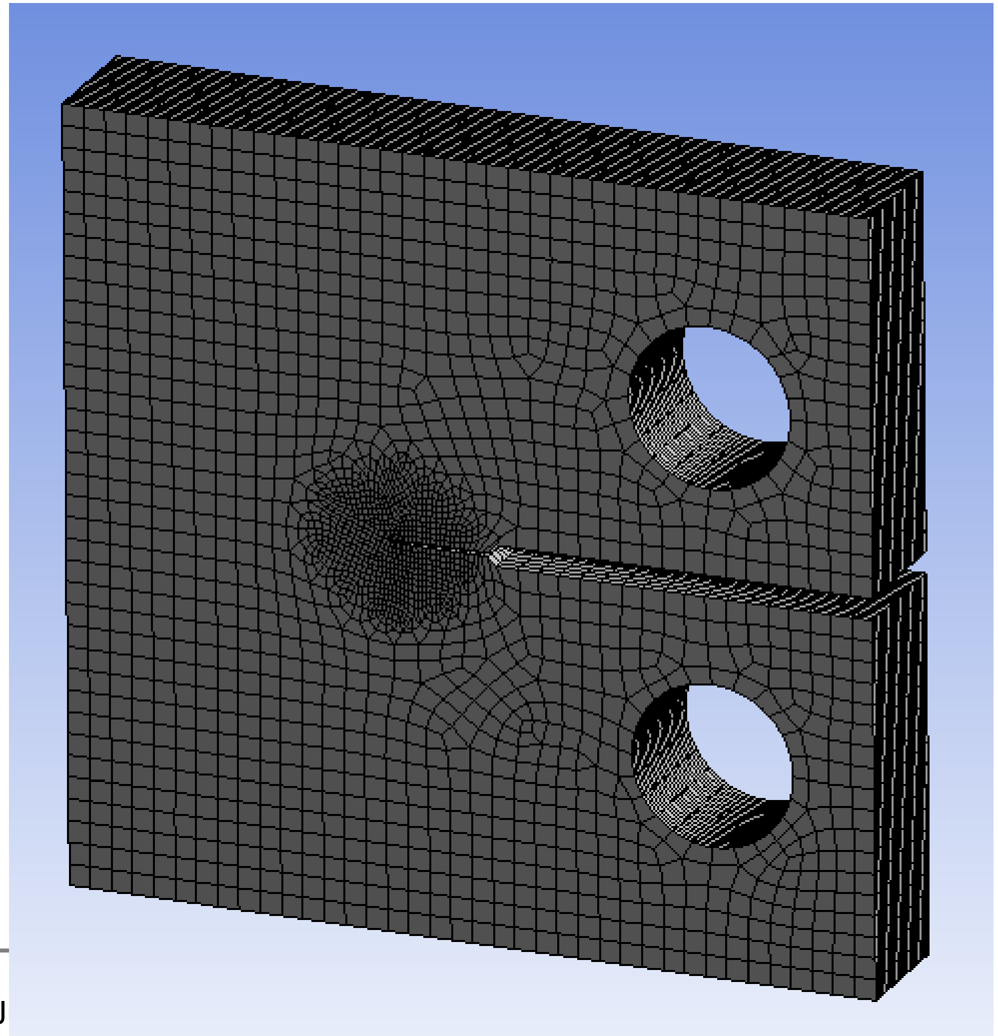
A crack (almost zero angle) is initially introduced on the CAD model



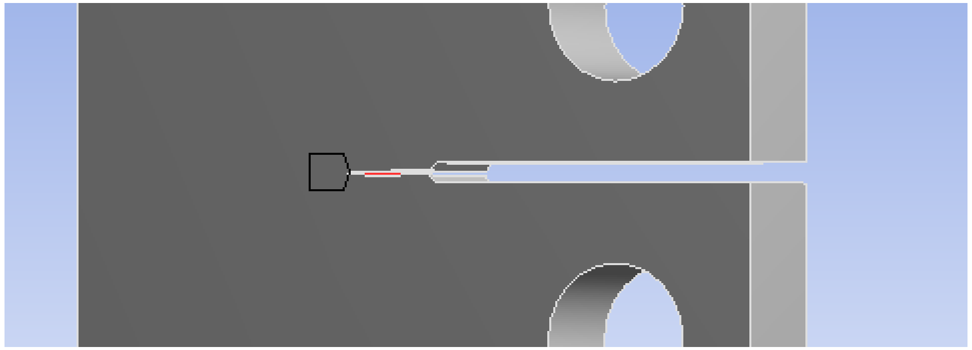
Pre-Meshed Crack, CT specimen example

Hexahedrons mesh is to be preferred in this case





Pre-Meshed Crack, CT specimen example



<u>Named selection:</u>
Crack front

- Named Selections
  - Crack Front after PreCrack
  - Crack Front after PreCrack - nodes

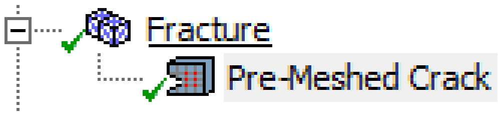
The edge is then converted into the nodes

Crack Front after PreCrack - nodes

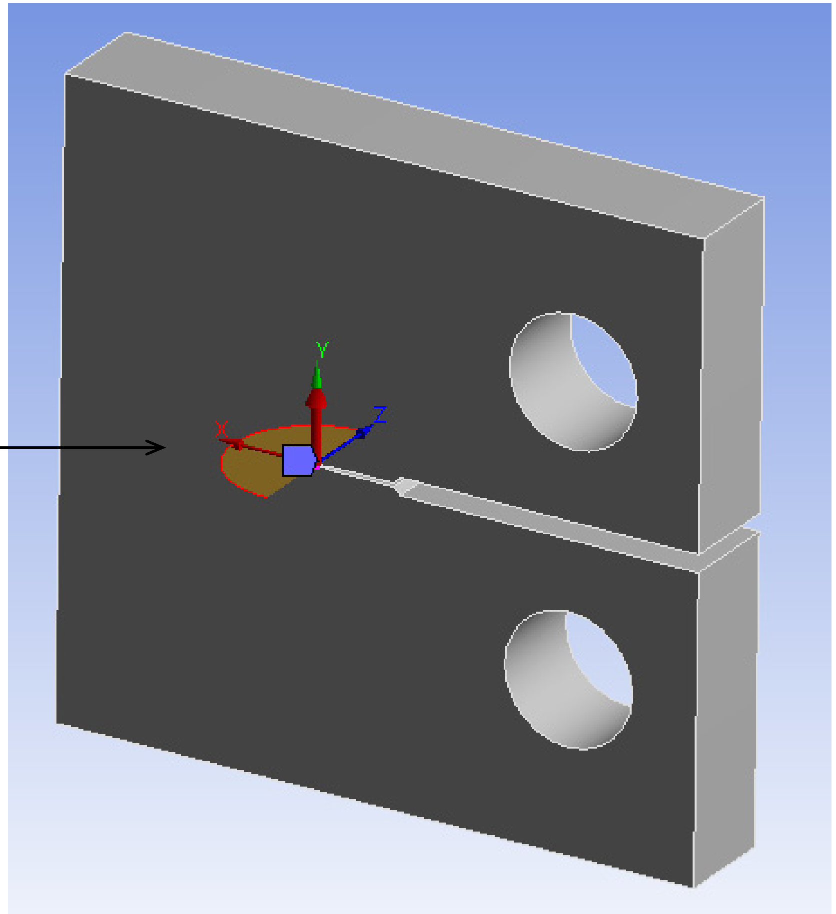
Generate

	Action	Entity Type	Criterion	Operator	Units	Value	Lower Bo...	Upper Bo...	Coordina...
☑	Add	Edge	Named S...	Equal	N/A	Crack Fro...	N/A	N/A	N/A
☑	Convert To	Mesh No...	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Pre-Meshed Crack, CT specimen example



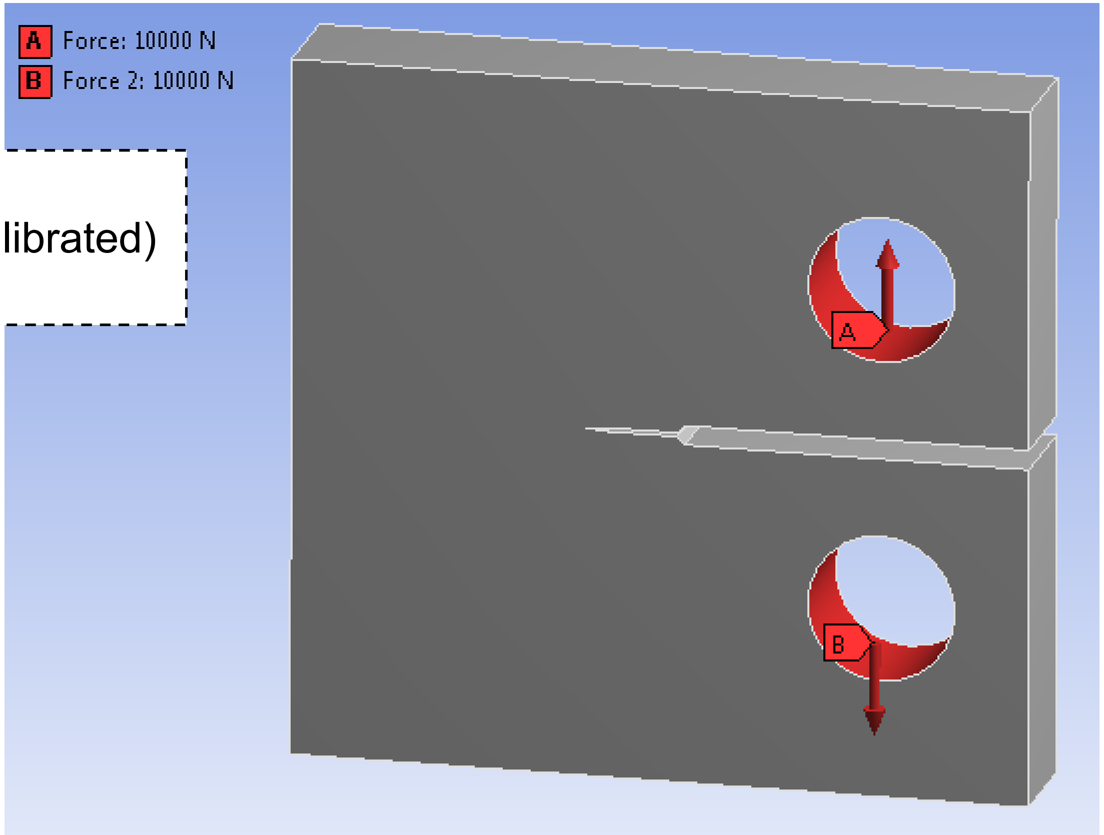
Crack orientation, not crack actual shape and size →



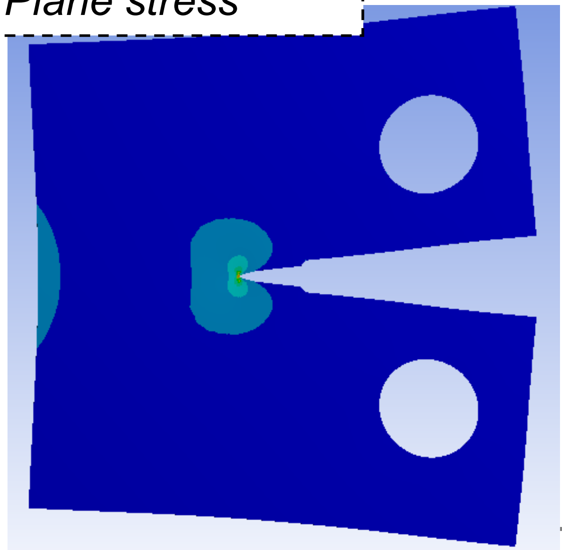
The associated coordinate system is placed at one edge of the crack

Pre-Meshed Crack, CT specimen example

Usual (self-equilibrated) loading



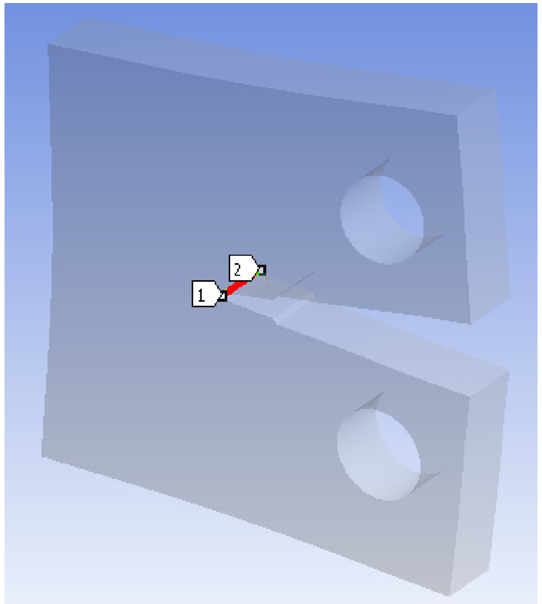
Stress distribution
Plane stress



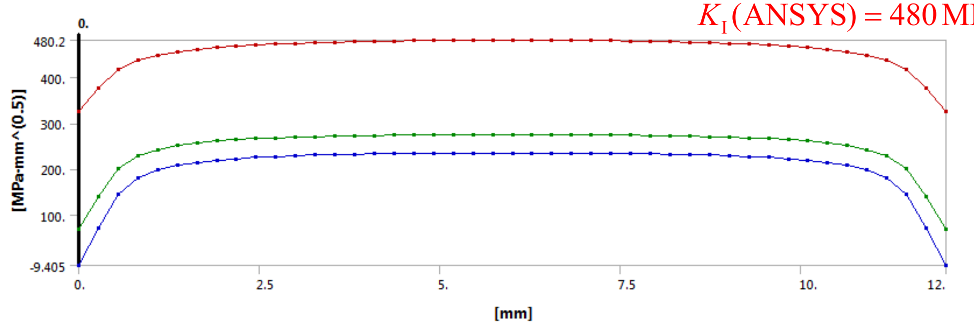
Pre-Meshed Crack, inaccurate results

$$P = 10\,\text{kN} \quad a = 30\,\text{mm},\ W = 60\,\text{mm},\ B = 12\,mm$$

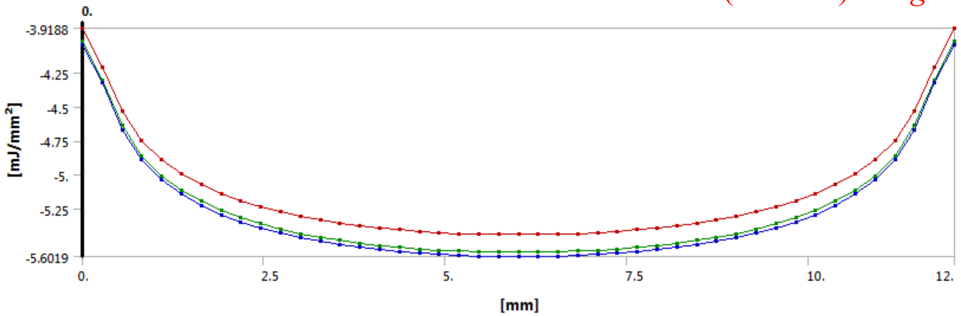
$$K_I = \frac{P}{B\sqrt{W}} \frac{2+\alpha}{(1-\alpha)^{3/2}} (0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.64\alpha^4) = 971\,\text{MPa}\sqrt{\text{mm}}$$



$K_I(\text{ANSYS}) = 480\,\text{MPa}\sqrt{\text{mm}}$



$J(\text{ANSYS}) = negative\,??$



Maybe this technique is intended for future development…