Task 6 - Safety Review and Licensing
On the Job Training on Stress Analysis

Fracture Mechanics: Linear Elastic Fracture Mechanics 2/2

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BEYOND THE RESEARCH

UNIVERSITÀ DI PISA

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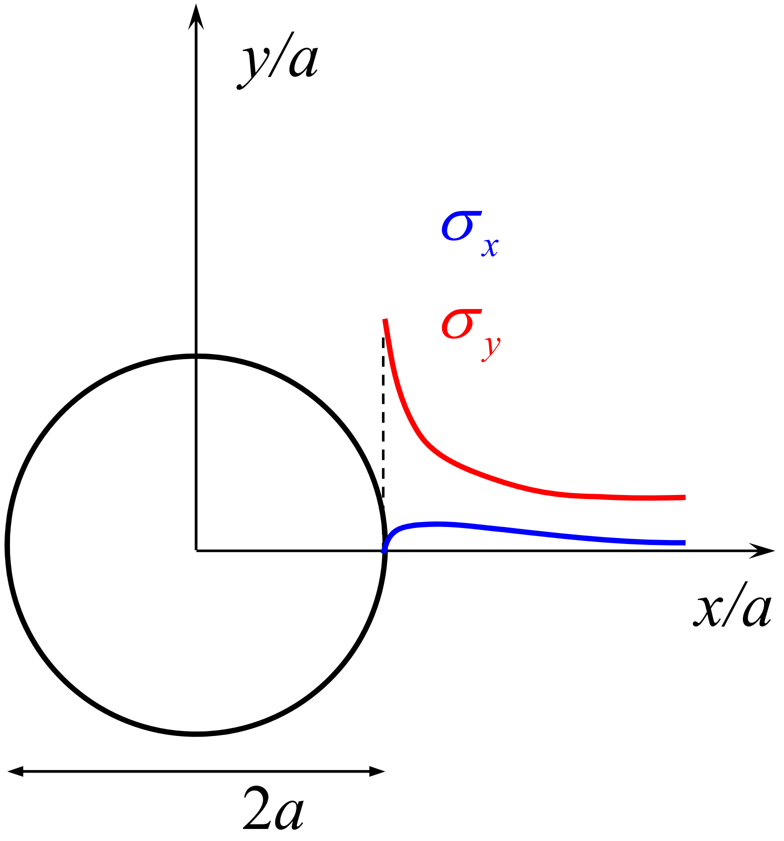
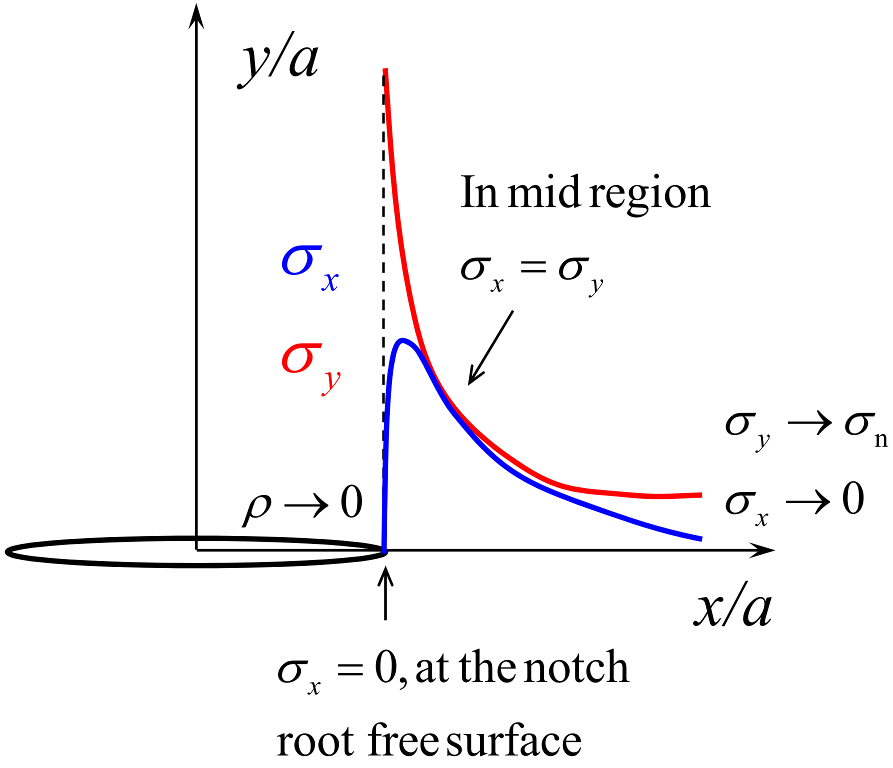
From notch to crack

In-plane stresses



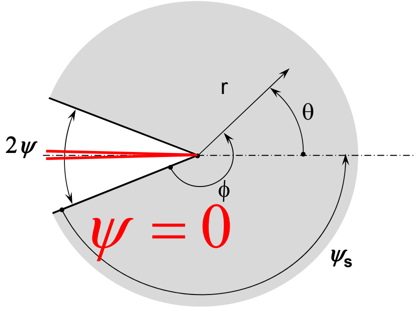


From notch to crack

Westergaard stress function

$$\lambda = 1/2$$

$$\sigma_{ij} \propto r^{\lambda - 1} = r^{-1/2} = \frac{1}{\sqrt{r}}$$



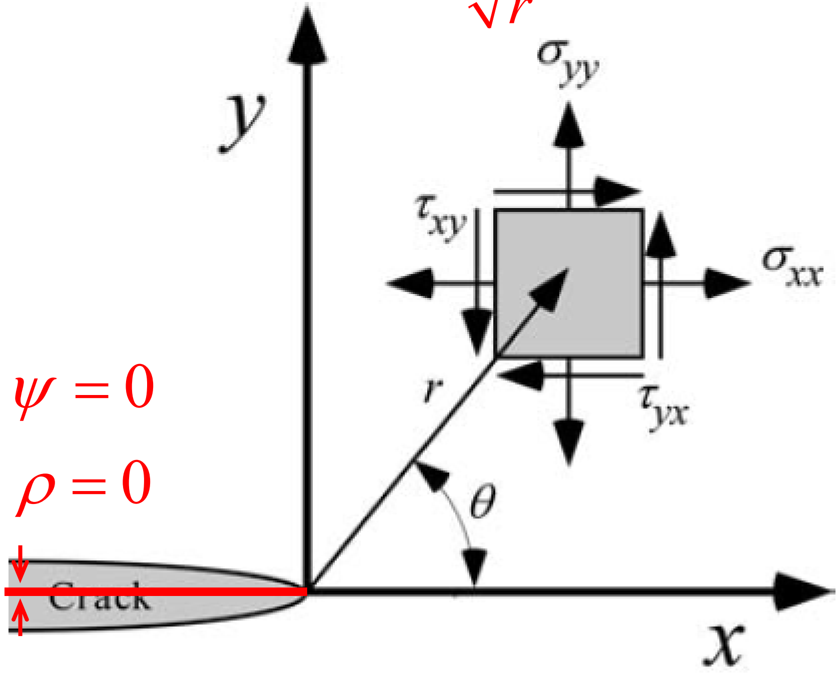
Other nonsingular

terms

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \ldots$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \ldots$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + \ldots$$



$$\psi = 0$$

$$\rho = 0$$

Westergaard/ Irwin stresses

Cartesian/ Cylindrical coordinates

$$\sigma_{xx} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

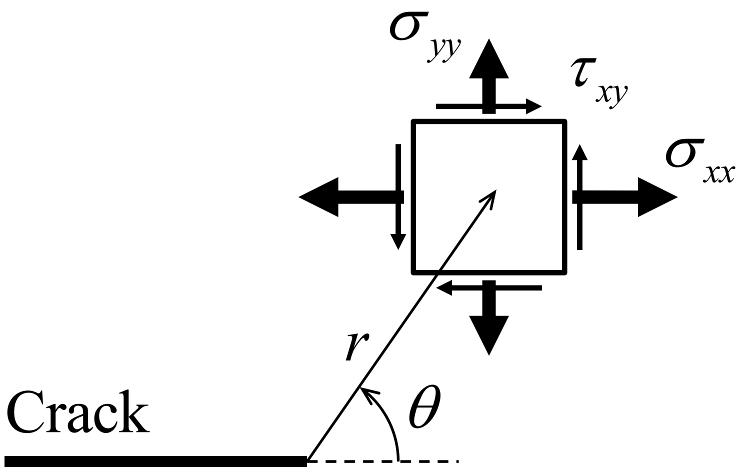
$$\sigma_{yy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

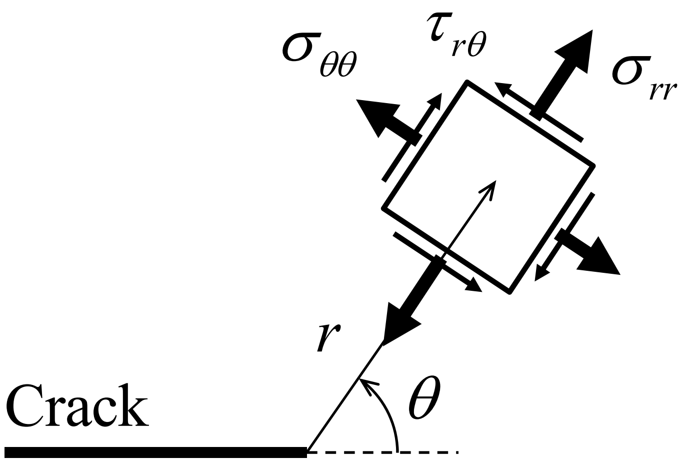
$$\tau_{xy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{5}{4}\cos\left(\frac{\theta}{2}\right)-\frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{3}{4}\cos\left(\frac{\theta}{2}\right)+\frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{1}{4}\sin\left(\frac{\theta}{2}\right)+\frac{1}{4}\sin\left(\frac{3\theta}{2}\right)\right]$$





Stresses ahead of the crack tip

$$\theta = 0:$$

$$\sigma_{xx} = \sigma_{rr} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}$$

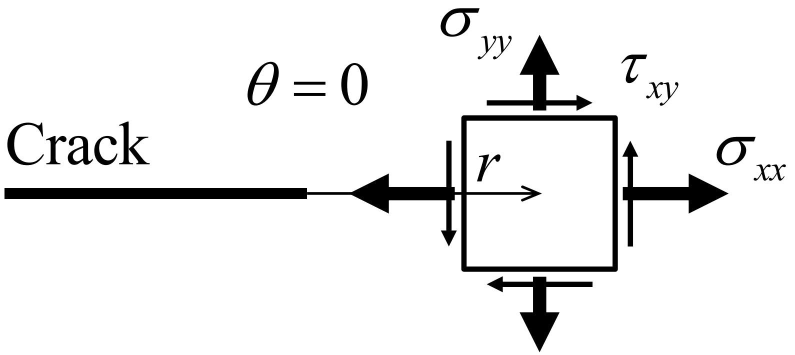
$$\sigma_{yy} = \sigma_{\theta\theta} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}$$

$$\tau_{xy} = \tau_{r\theta} = 0$$

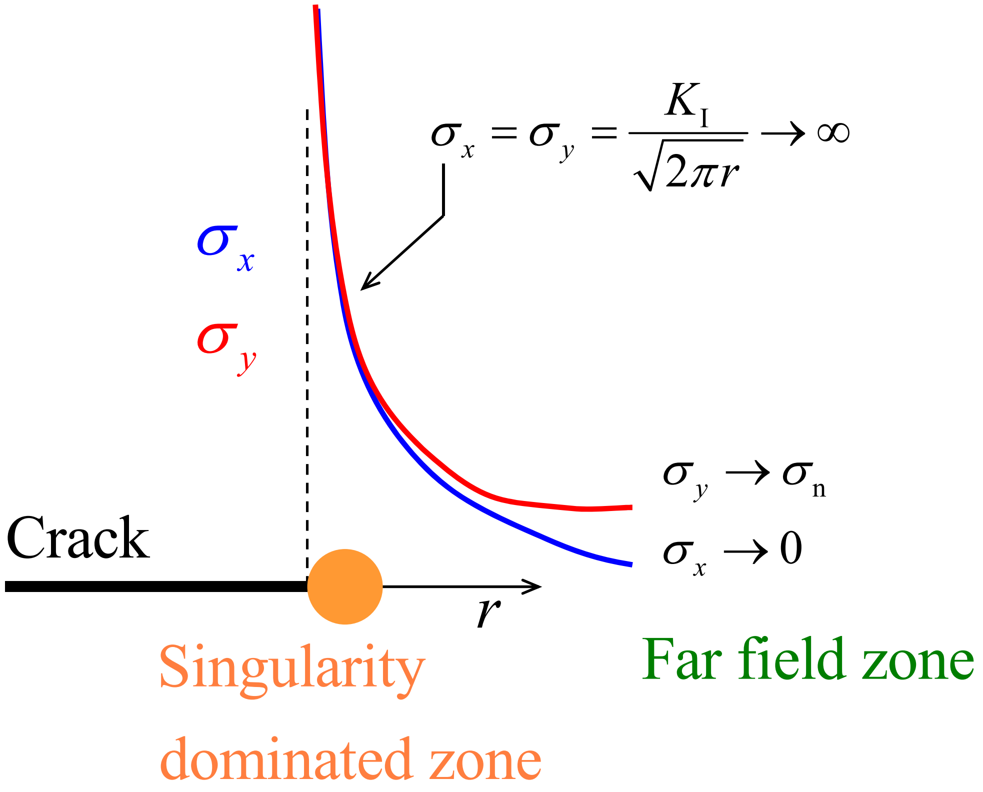
Notes:

$\sigma_{xx} = \sigma_{yy}$ even at the notch root

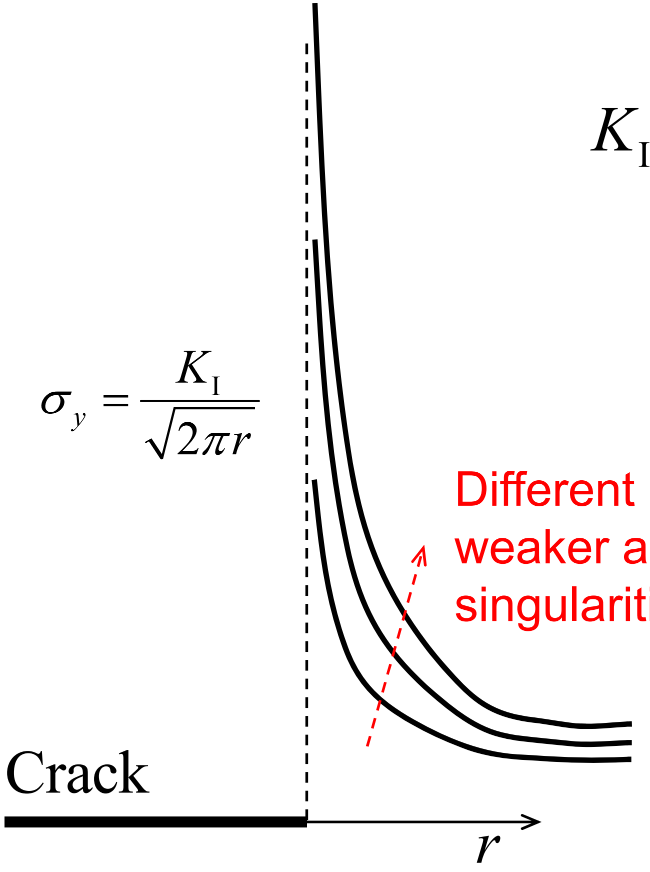
$\tau_{xy} = 0$ symmetry of the problem



Stresses ahead of the crack tip



Stress Intensity Factor (SIF)



K_I is the (first) Stress Intensity Factor

$$\frac{K_\mathrm{I}}{\sqrt{2\pi r}} \leftrightarrow \frac{k_\mathrm{I}}{\sqrt{r}} \text{(same intensification meaning)}$$

What is the reason of 2π ?

Griffith Energy release rate "G"

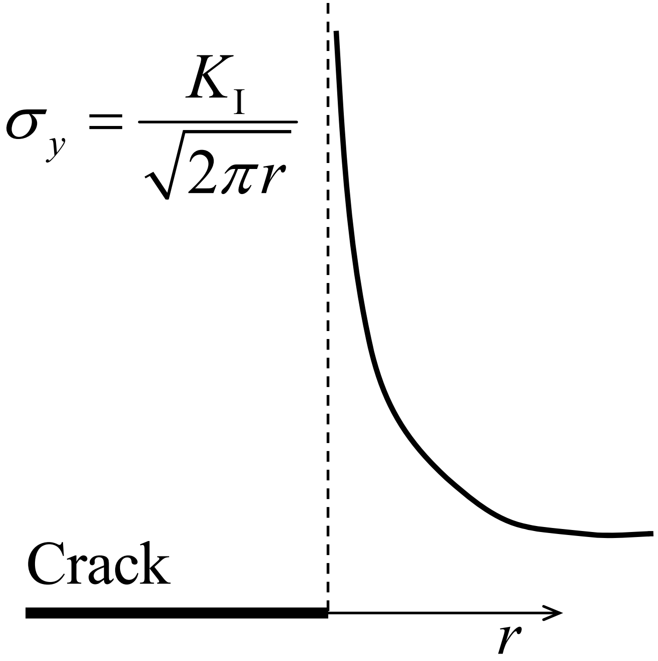
$$\mathcal{G} = \frac{K_I^2}{E}$$

Now this equation holds

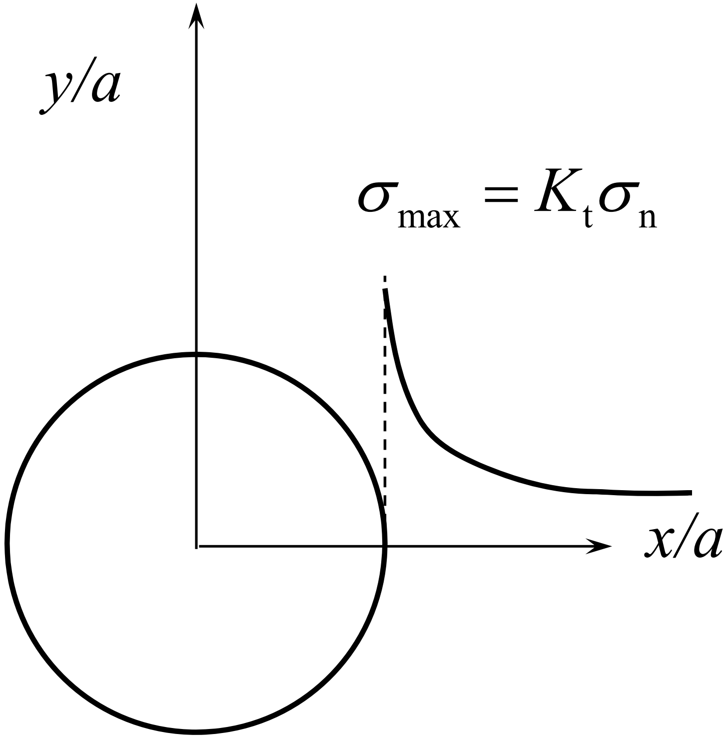
without any 2π term

Stress Intensity Factor

The Stress Intensity Factor is NOT the Stress Concentration Factor



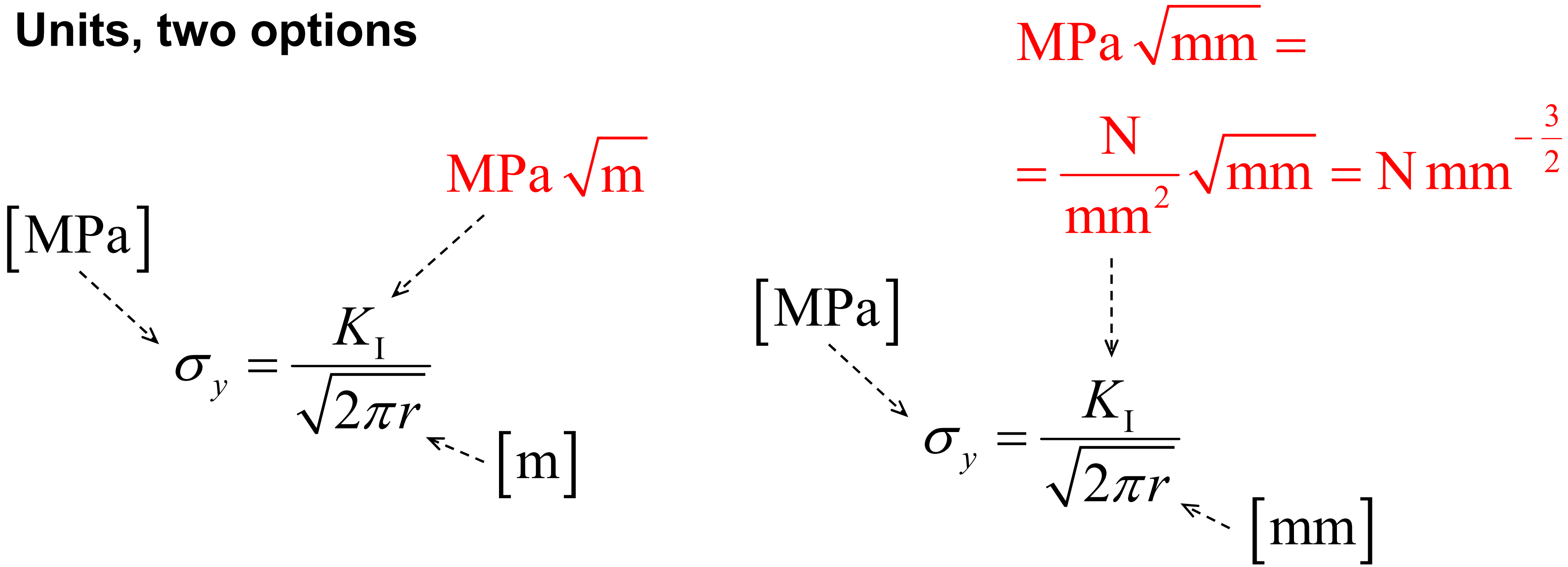
$$\sigma_y = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}$$



$$\sigma_\mathrm{max} = K_\mathrm{t}\sigma_\mathrm{n}$$

Stress Intensity Factor

Units, two options



$$\mathrm{MPa}\sqrt{\mathrm{mm}} =$$

$$= \frac{\mathrm{N}}{\mathrm{mm}^2}\sqrt{\mathrm{mm}} = \mathrm{N\,mm}^{-\frac{3}{2}}$$

$$\mathrm{MPa}\sqrt{\mathrm{m}}$$

$$[\mathrm{MPa}]$$

$$\sigma_y = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}$$

$$[\mathrm{m}]$$

$$[\mathrm{MPa}]$$

$$\sigma_y = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}$$

$$[\mathrm{mm}]$$

Conversion:

$$x\,\mathrm{MPa}\sqrt{\mathrm{m}} = x\,\mathrm{MPa}\sqrt{1000\,\mathrm{mm}} = \sqrt{1000}\,x\,\mathrm{MPa}\sqrt{\mathrm{mm}} \approx 31.6\,x\,\mathrm{MPa}\sqrt{\mathrm{mm}}$$

Example:

$$8.0\,\mathrm{MPa}\sqrt{\mathrm{m}} = 252.8\,\mathrm{MPa}\sqrt{\mathrm{mm}}$$

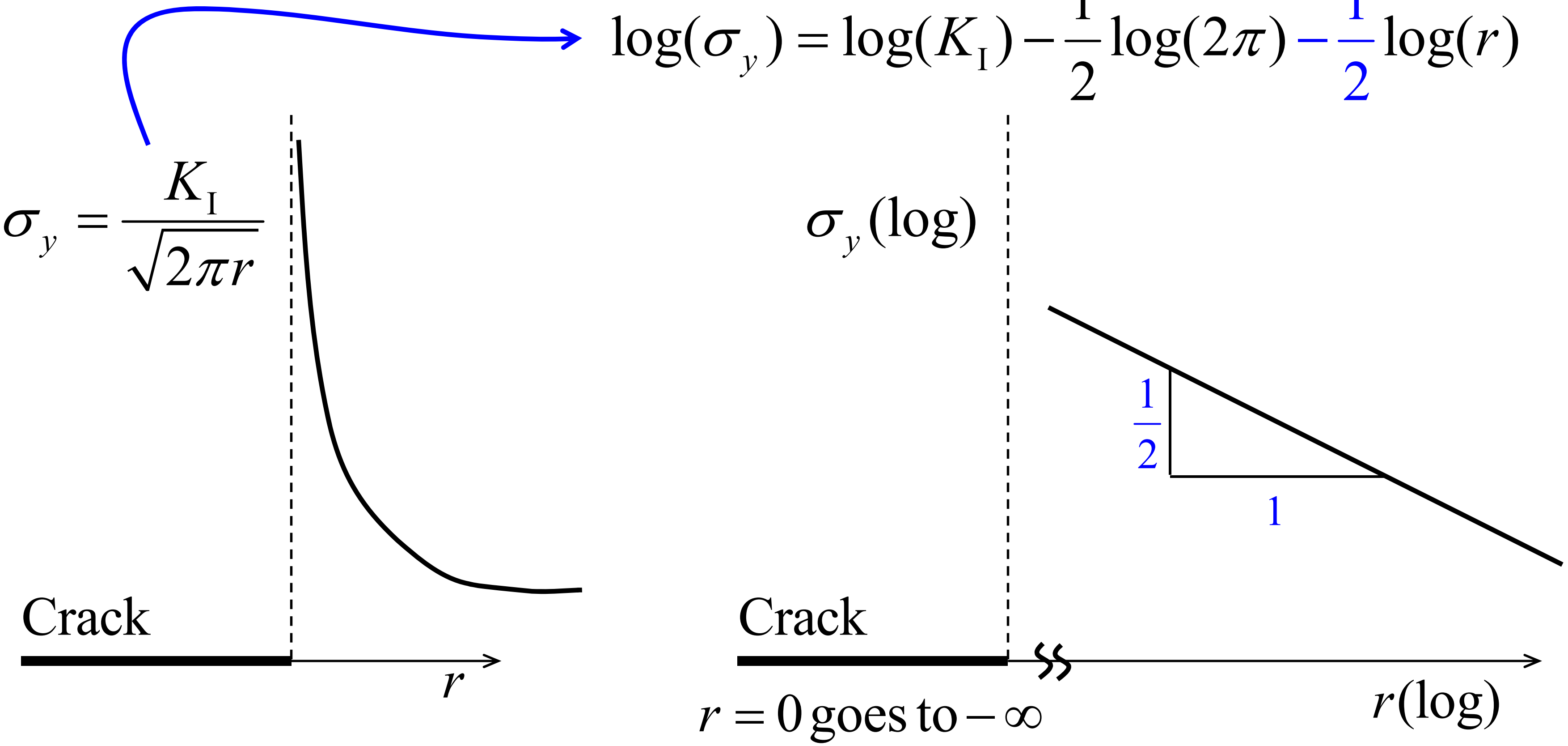
Stress Intensity Factor

Logarithm scale

Let's log both sides...

$$\log(\sigma_y) = \log(K_\mathrm{I}) - \frac{1}{2}\log(2\pi) - \frac{1}{2}\log(r)$$

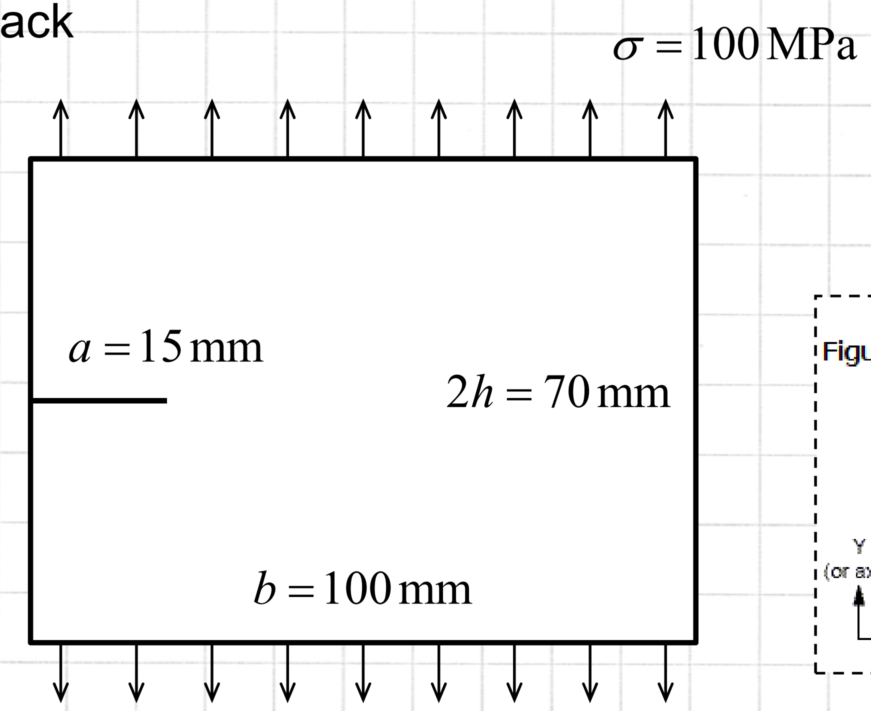
$$\sigma_y = \frac{K_\mathrm{I}}{\sqrt{2\pi r}}$$



Stress Intensity Factor

ANSYS Apdl (classic) – MATLAB:

Verify the -1/2 slope (log-log) and calculate the SIF for a plate with a lateral crack



Element type to be used

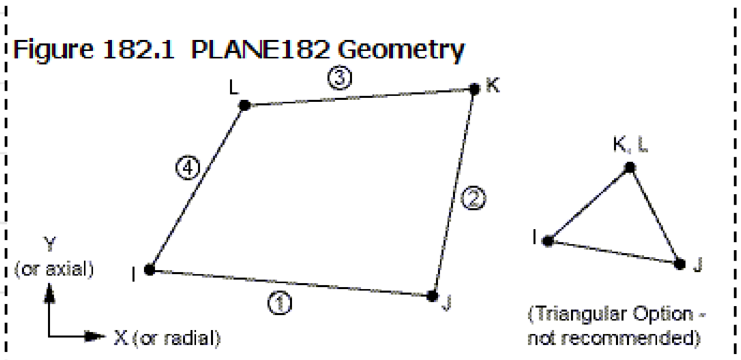
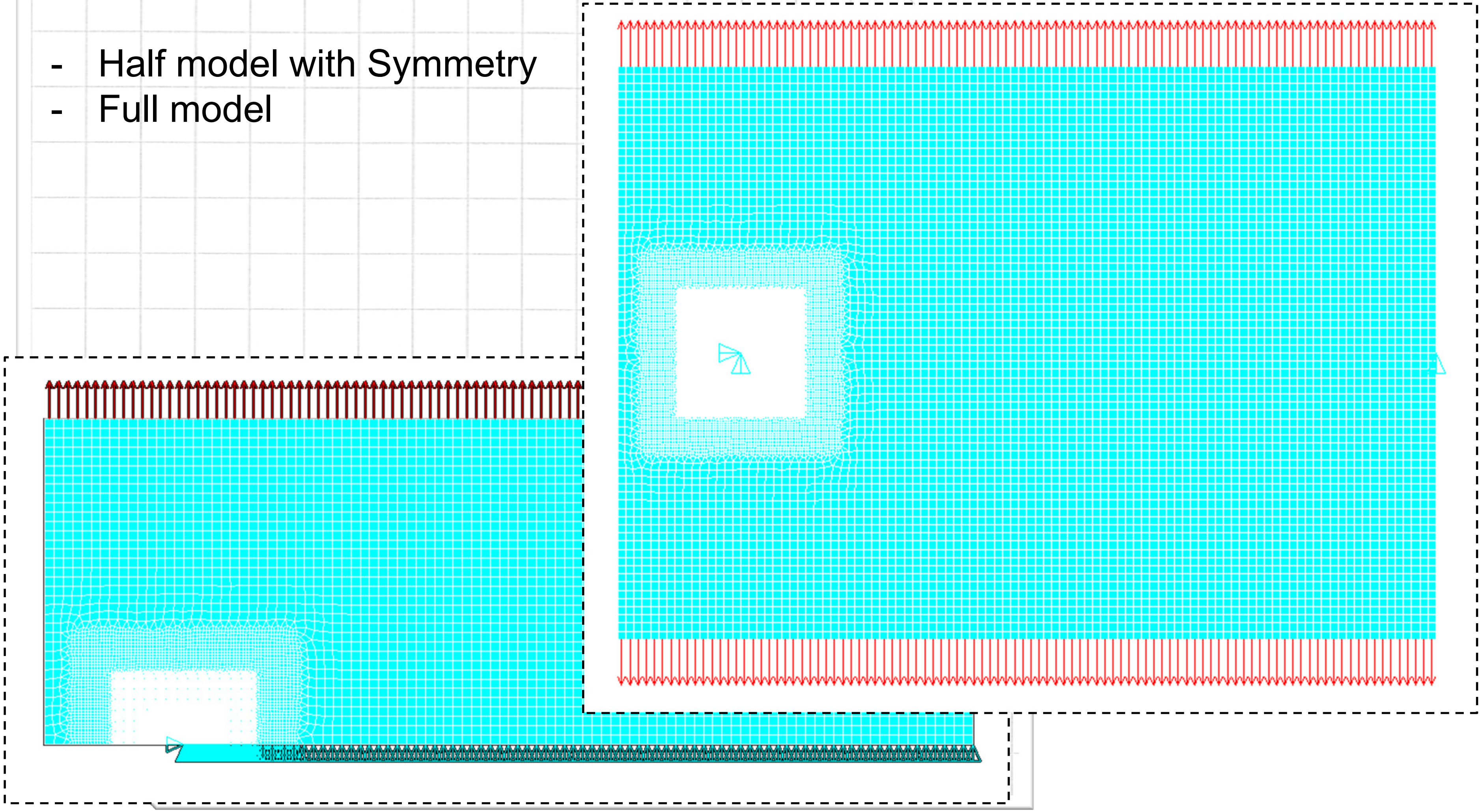


Figure 182.1 PLANE182 Geometry

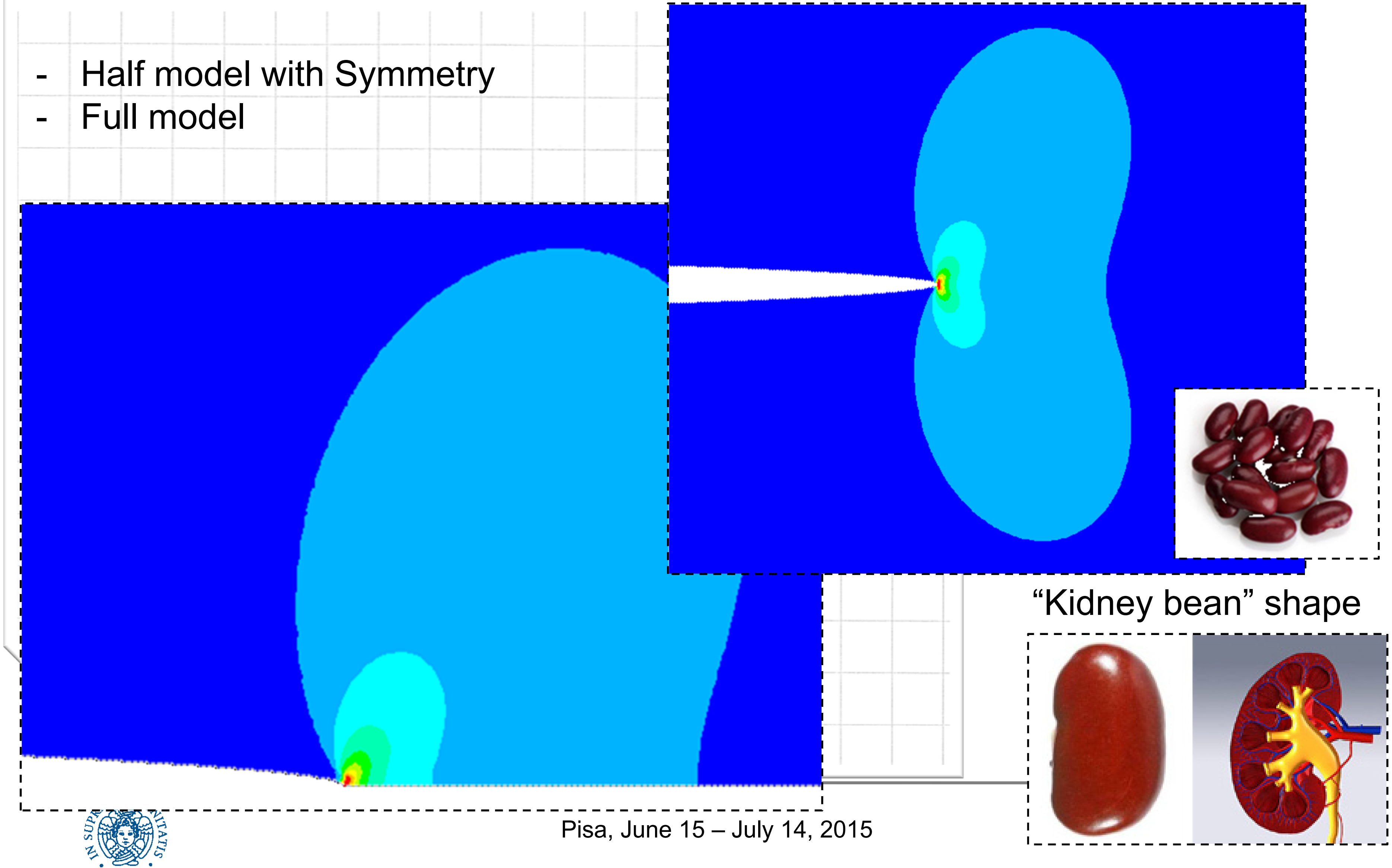
Stress Intensity Factor

- Half model with Symmetry
- Full model



Stress Intensity Factor

- Half model with Symmetry
- Full model

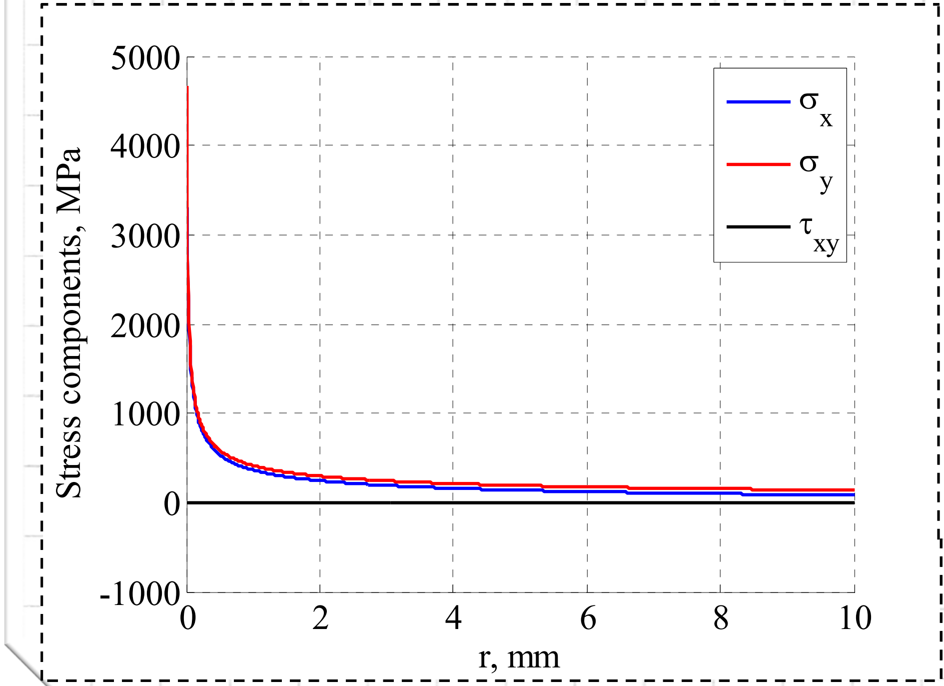


“Kidney bean” shape

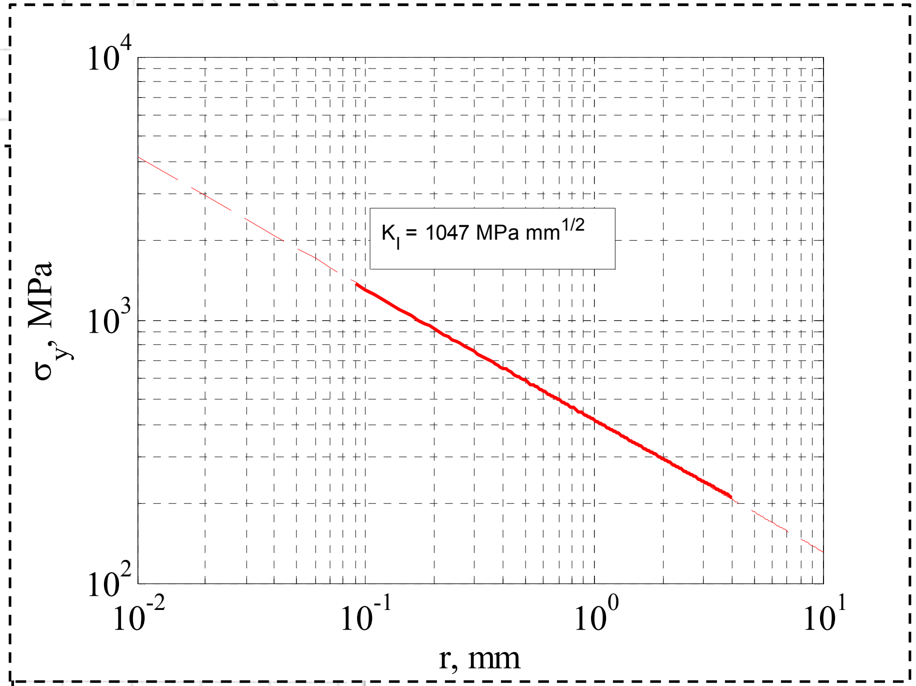
Stress Intensity Factor

MATLAB elaboration

Linear scales



log-log scales



MATLAB "polyfit": $P_1,\ P_2$

$P_1 = -0.5$ (good approx.)

$$K_I = 10^{P_2 + \frac{1}{2}\log_{10}(2\pi)}$$

Stress Intensity Factor

Homework:

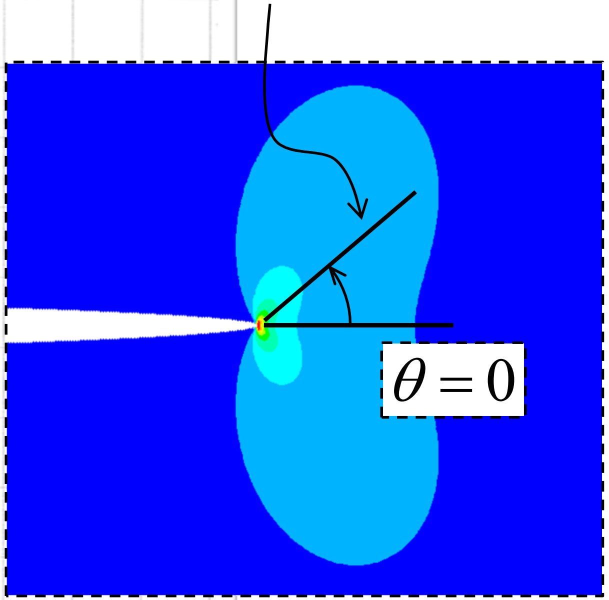
Verify the angle dependency of the stress distribution at the crack tip

$$\sigma_{xx} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

Path at any different angle



$\theta = 0$

In-plane Stress Intensity Factors

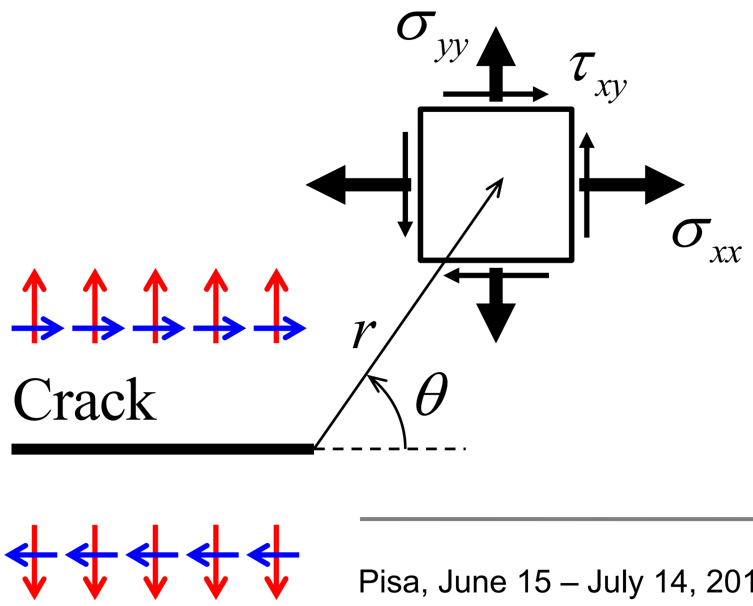
There are two distinct ways to apply <u>in-plane</u> loading

$$\sigma_{xx} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] - \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right)\left[2 + \cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

Cartesian coordinates



Mode I and Mode II
Stress Intensity Factors

Symmetrical and
Nonsymmetrical stress
components

In-plane Stress Intensity Factors

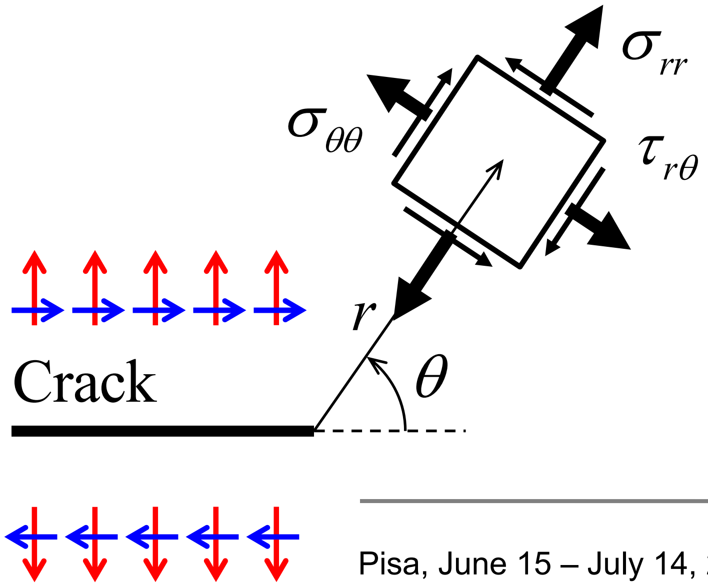
There are two distinct ways to apply in-plane loading

$$\sigma_{rr} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\left[\frac{5}{4}\cos\left(\frac{\theta}{2}\right) - \frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right] + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\left[-\frac{5}{4}\sin\left(\frac{\theta}{2}\right) + \frac{3}{4}\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{\theta\theta} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\left[\frac{3}{4}\cos\left(\frac{\theta}{2}\right) + \frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right] + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\left[-\frac{3}{4}\sin\left(\frac{\theta}{2}\right) - \frac{3}{4}\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{r\theta} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\left[\frac{1}{4}\sin\left(\frac{\theta}{2}\right) + \frac{1}{4}\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\left[\frac{1}{4}\cos\left(\frac{\theta}{2}\right) + \frac{3}{4}\cos\left(\frac{3\theta}{2}\right)\right]$$

Cylindrical coordinates



Mode I and Mode II
Stress Intensity Factors

Symmetrical and
Nonsymmetrical stress
components

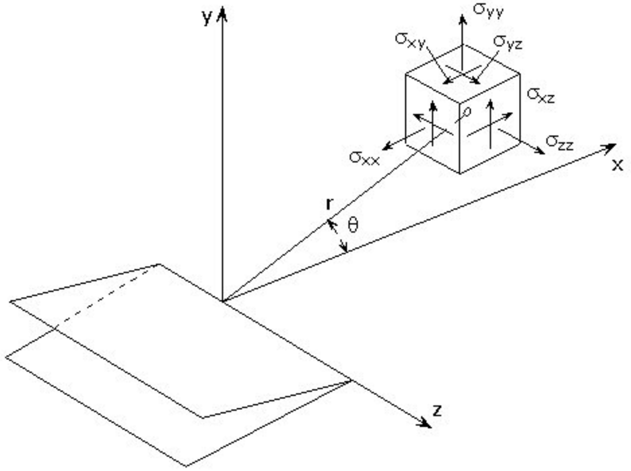
Plane Stress/ Strain

Same solution In-plane

$$\sigma_{xx}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]-\frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\left[2+\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]+\frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

$$\tau_{xy}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)+\frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$



Similarly to the notch,
any point even very close
to the singularity:

$$\sigma_{zz}=\begin{cases}0 & \text{plane stress}\\ \nu\left(\sigma_{xx}+\sigma_{yy}\right) & \text{plane strain}\end{cases}$$

Plane Stress/ Strain

Plane stress, transversal stress/ strain

$$\sigma_{zz} = 0$$

$$\varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) = -\frac{\nu}{E}\left(\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right) - 2\frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\right)$$

Plane strain, transversal stress/ strain

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) = \nu\left(\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right) - 2\frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\right)$$

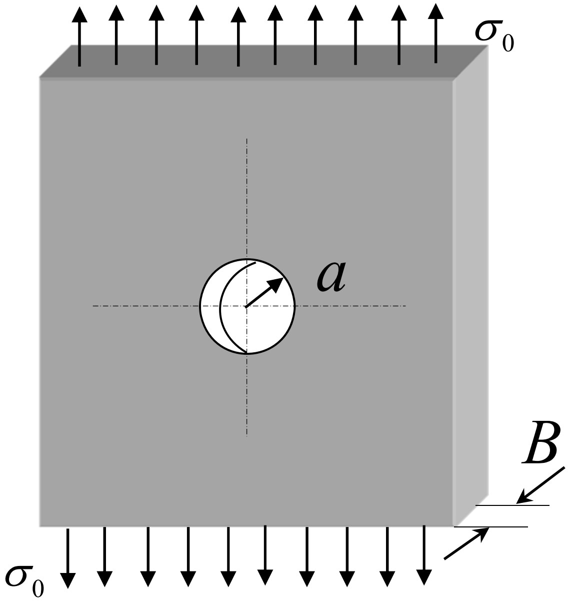
$$\varepsilon_{zz} = 0$$

Plane Stress/ Strain

How plane stress can be possible at the crack tip?

Plane stress solution if *a*>>*B*

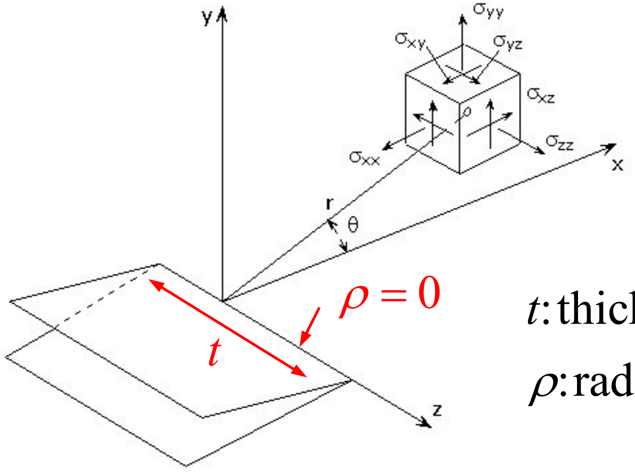
Plane strain if *a*<<*B*



Any thickness, the local radius is “infinitely” smaller being just zero.

Why talking about plane stress for a crack?

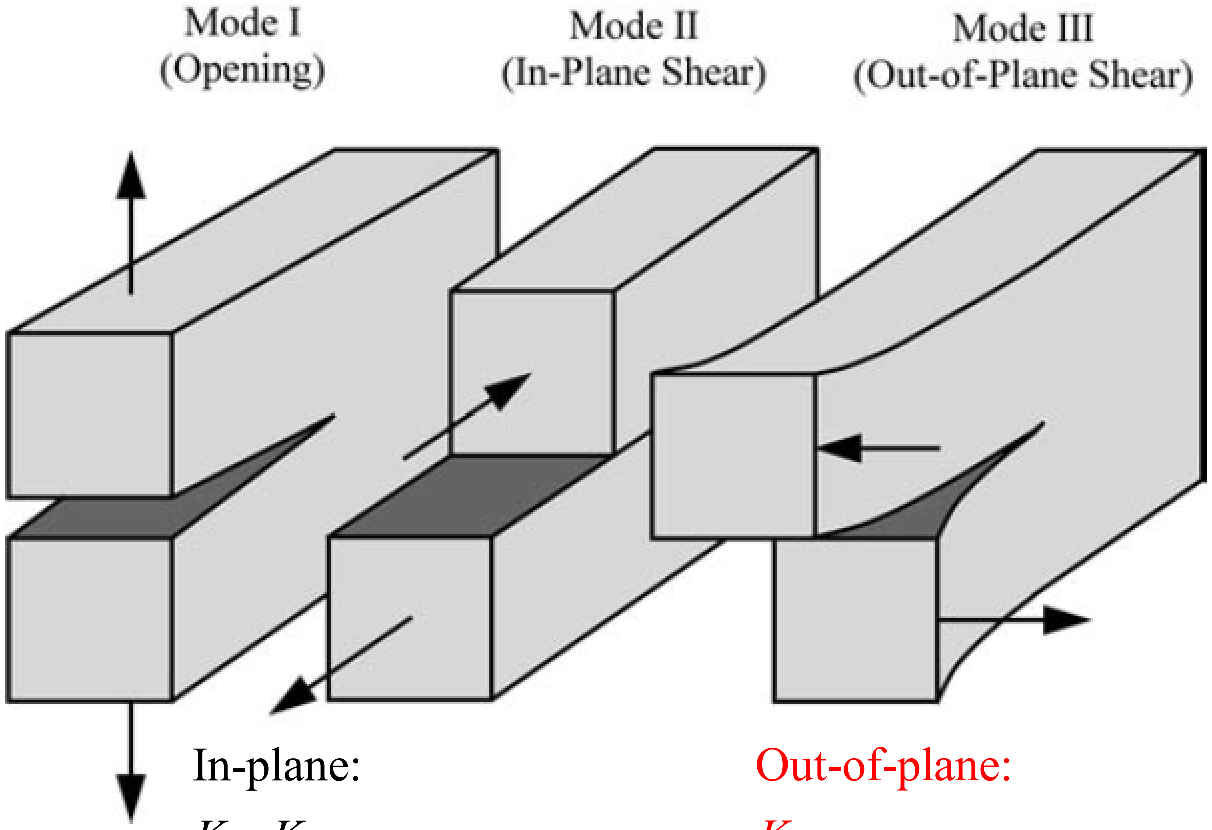
Plastic zone need to introduced.



t: thickness

ρ: radius

The three crack loading modes and related Stress Intensity Factors



In-plane:

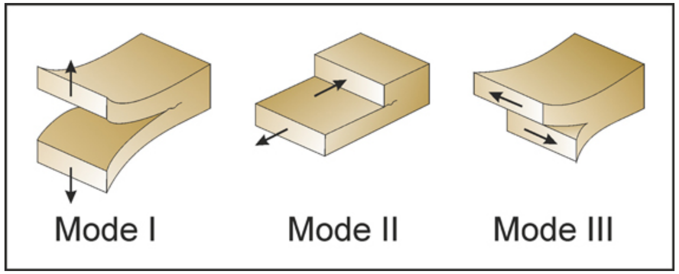
K_{I}, K_{II}

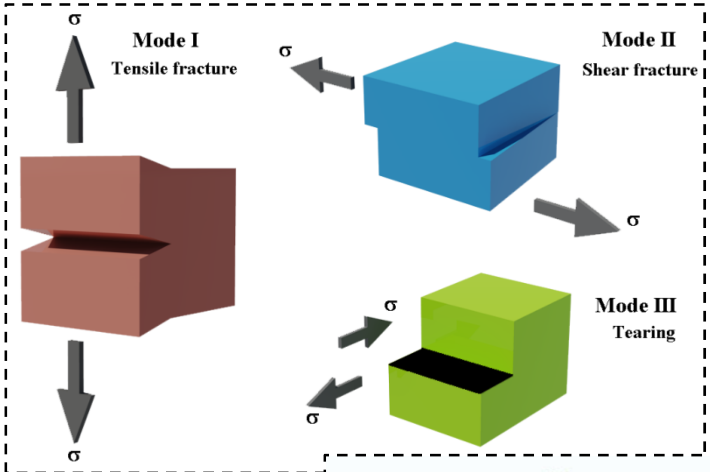
Out-of-plane:

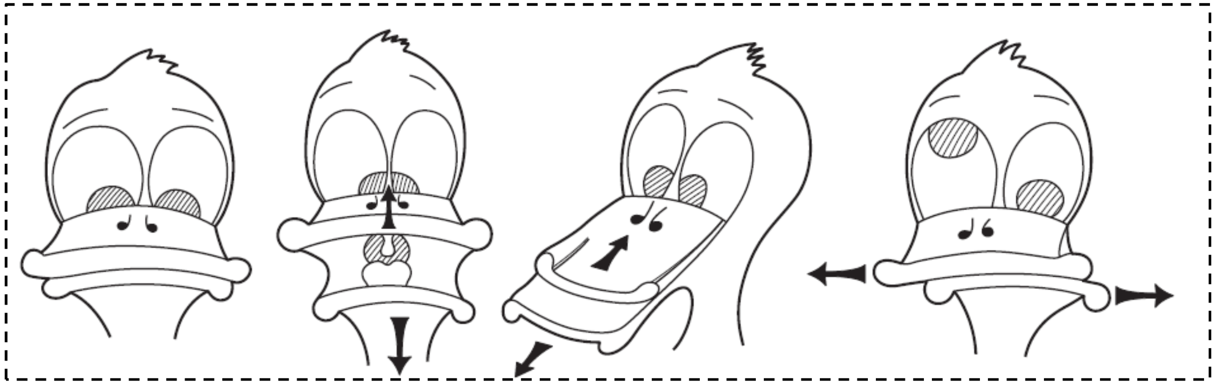
K_{III}

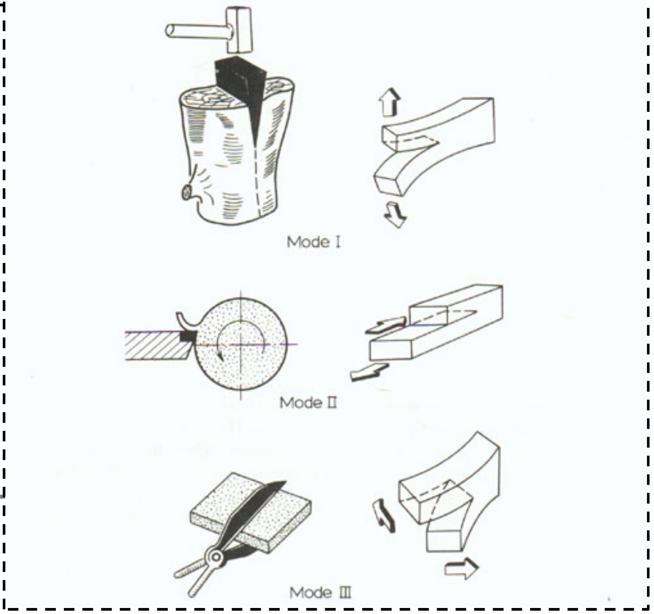
Fully three-dimensional problem

Many figures available to show the three crack modes









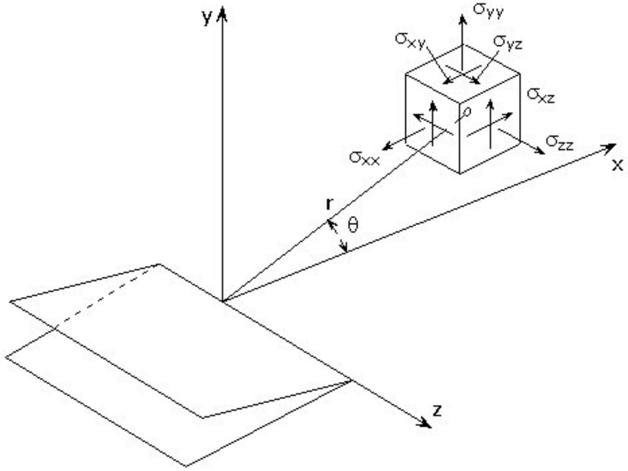
Mode III stress distribution

Out of plane shear stresses

τ_{xz}, τ_{yz} or alternatively σ_{xz}, σ_{yz}

were zero for Mode I and Mode II

either plane Stress or plain Strain



$$\tau_{xz} = -\frac{K_{\mathrm{III}}}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right)$$

$$\tau_{yz} = \frac{K_{\mathrm{III}}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)$$

other stress component are zero

How to calculate SIFs for structures

Different approaches

Dimensional approach, with graph and tabular data

Weight function

Finite elements with different techniques, previously an example with the stress asymptotic approach has been shown

How to calculate SIFs for structures

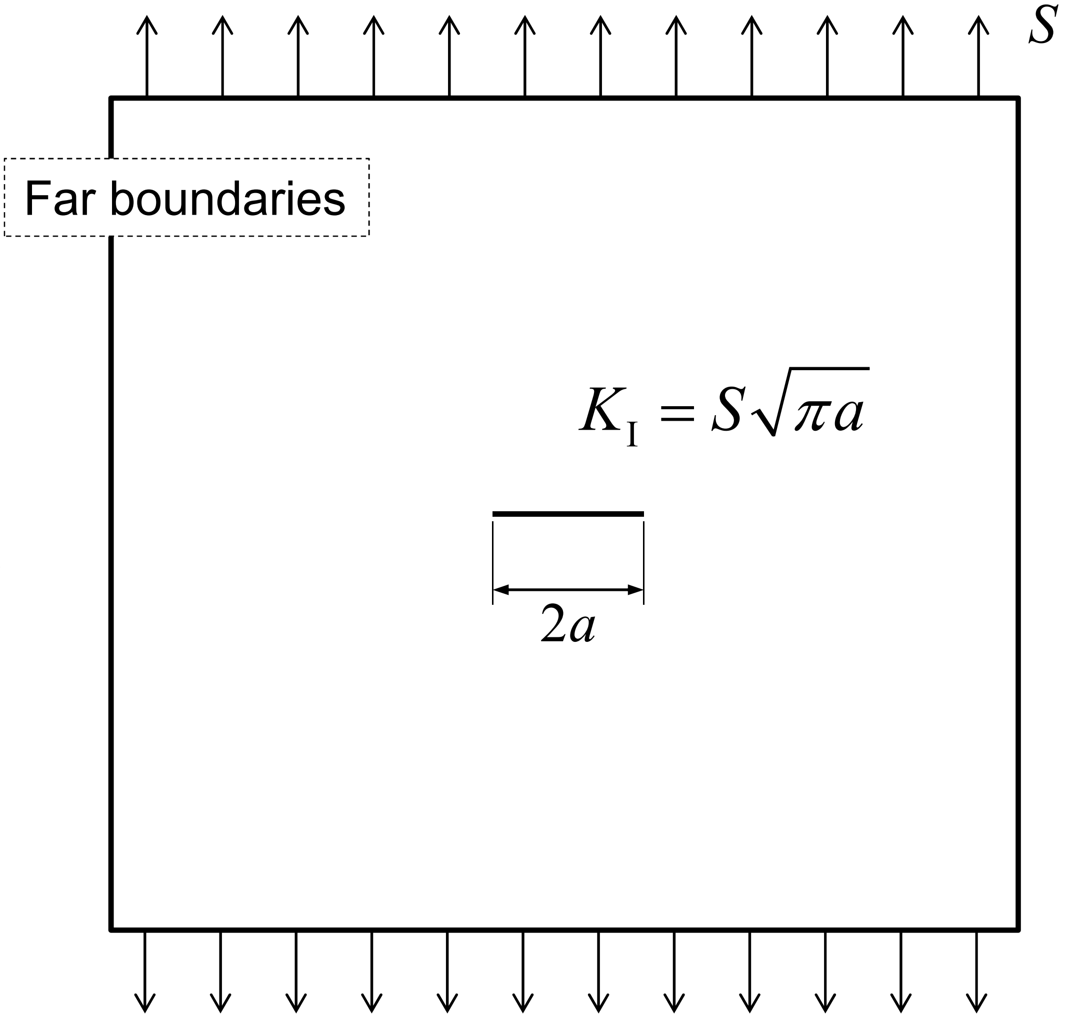
Dimensional approach

$$K_{\mathrm{I}} = FS\sqrt{\pi a}$$

a and S are required just for the dimensional analysis

F is the "shape function" it is dimensionless and depends on the loading configuration and relative dimensions (similarly to K_{t})

For this specific problem:
$F = 1.0$ (theoretical result)



How to calculate SIFs for structures

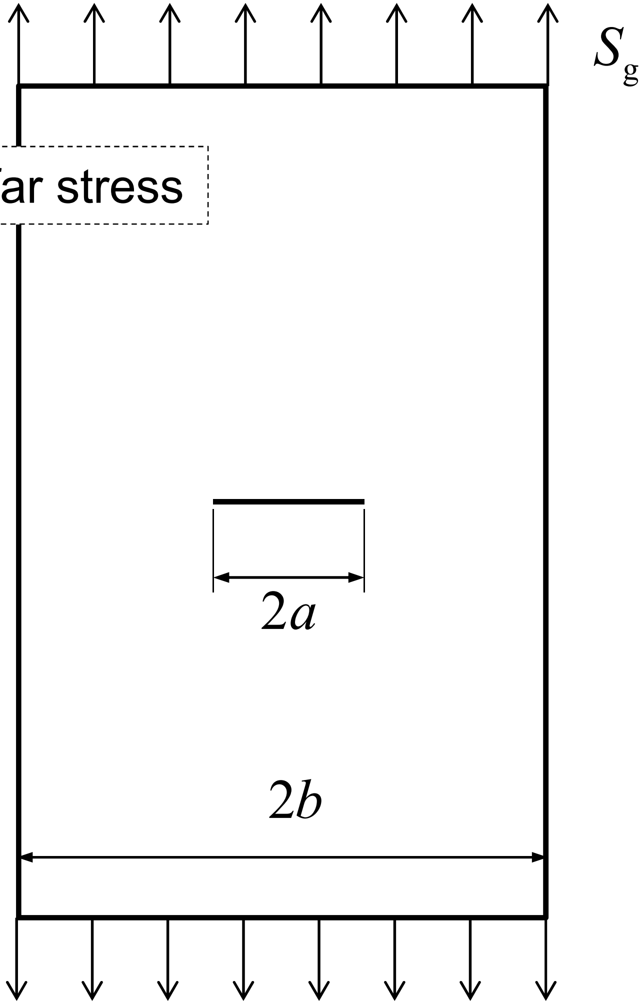
Dimensional approach

$K_{\mathrm{I}} = F S_{\mathrm{g}} \sqrt{\pi a}$

Usually S_{g} gross stress is used
in the formula rather than
S_{n} which is the net stress

For this specific case:
F depends on a / b ratio

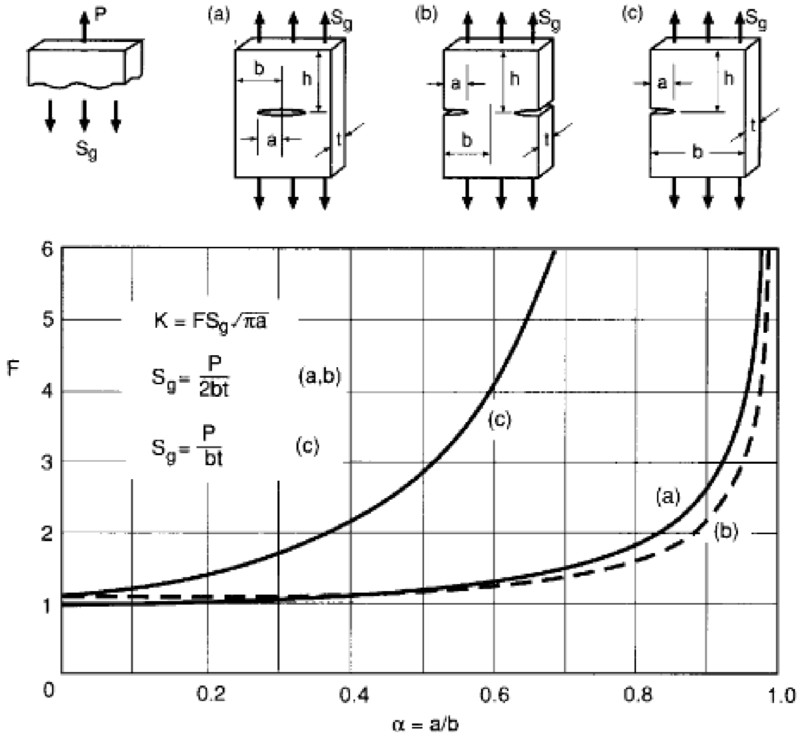
... tabular cases available on textbooks
and atlas to find (approximated)
values for F



Finite with, far stress

How to calculate SIFs for structures

Crack on plate cases



Values for small a/b and limits for 10% accuracy:

(a) $K = S_g\sqrt{\pi a}$ $(a/b \le 0.4)$

(b) $K = 1.12 S_g\sqrt{\pi a}$ $(a/b \le 0.6)$

(c) $K = 1.12 S_g\sqrt{\pi a}$ $(a/b \le 0.13)$

Expressions for any $\alpha = a/b$:

$$\text{(a)}\quad F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} \qquad (h/b \ge 1.5)$$

$$\text{(b)}\quad F = \left(1 + 0.122\cos^4\frac{\pi\alpha}{2}\right)\sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}} \qquad (h/b \ge 2)$$

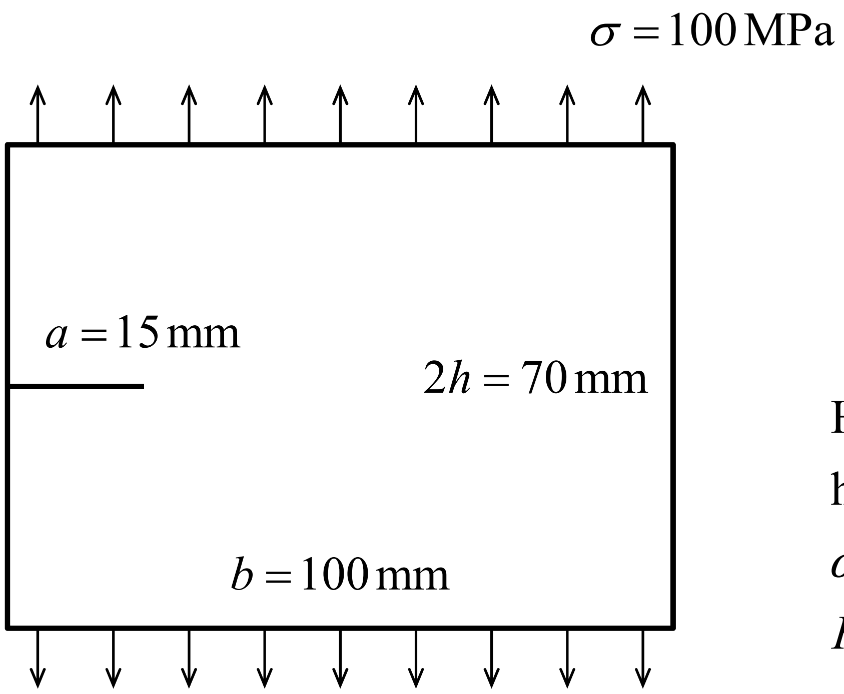
$$\text{(c)}\quad F = 0.265\,(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}} \qquad (h/b \ge 1)$$

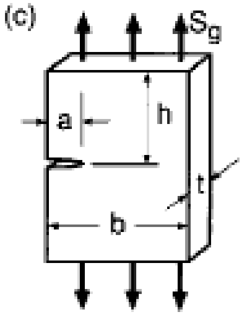
$$F = 1.12$$

for a single-edge-cracked plate with a small crack

How to calculate SIFs for structures

Previous example solved with FE simulation





$$\text{(c)}\quad F = 0.265\,(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}} \qquad (h/b \geq 1)$$

Here $h > b$ is not satisfied,

however, approximately:

$$\alpha = a / b = 0.15$$

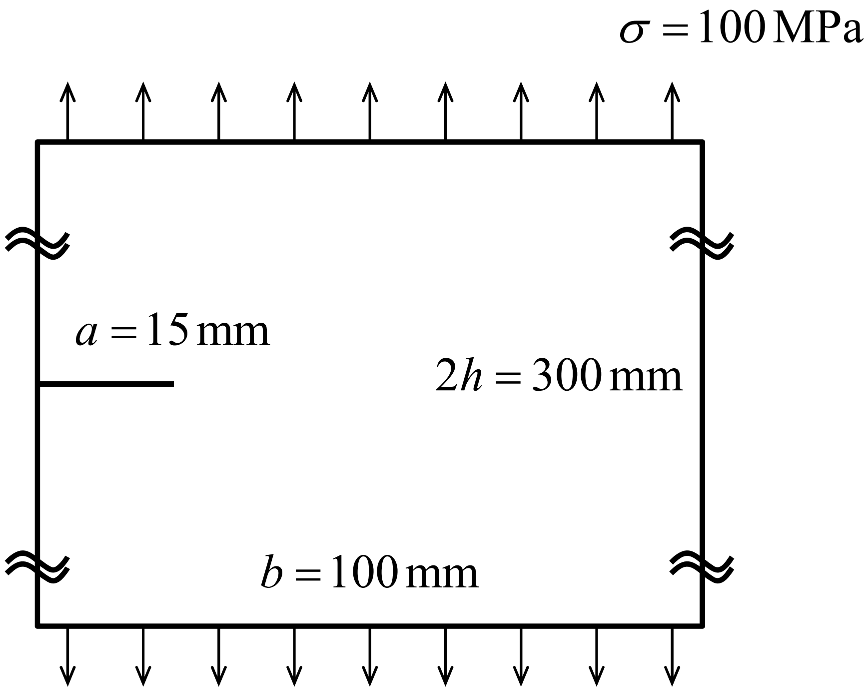
$$F = 1.28\ \text{(not much larger than } 1.12\text{)}$$

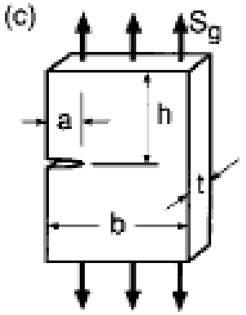
$$K_\text{I} = FS\sqrt{\pi a} = 880\,\text{MPa}\,\text{mm}^{\frac{1}{2}}$$

$$K_\text{I}(\text{ANSYS}) = 1047\,\text{MPa}\,\text{mm}^{\frac{1}{2}}\ (16\%)$$

How to calculate SIFs for structures

Previous example modified, larger height





$$\text{(c)}\ F = 0.265\,(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}} \qquad (h/b \geq 1)$$

Now $h > b$ is satisfied:

$\alpha = a/b = 0.15$

$F = 1.28$ (not much larger than 1.12)

$$K_\mathrm{I} = FS\sqrt{\pi a} = 880\,\text{MPa}\,\text{mm}^{\frac{1}{2}}$$

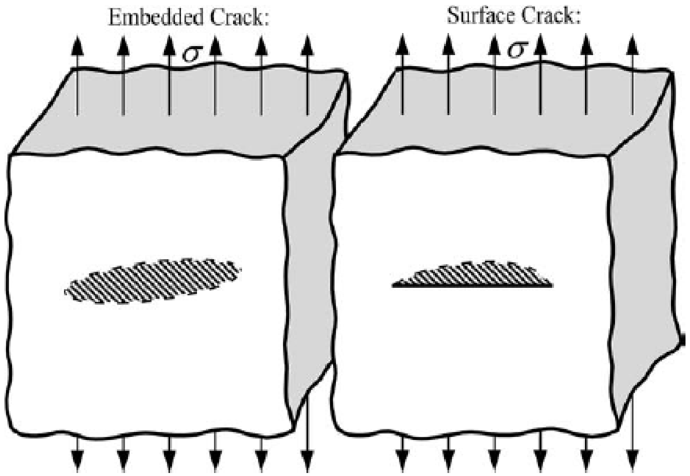
$$K_\mathrm{I}(\text{ANSYS}) = 899\,\text{MPa}\,\text{mm}^{\frac{1}{2}}\ (2\%)$$

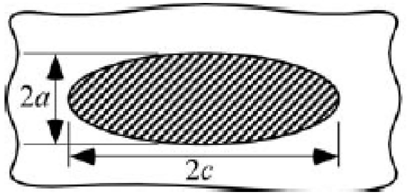
How to calculate SIFs for structures

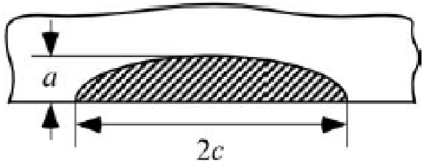
Three-dimensional elliptical or circular (penny-shaped) cracks

Infinite space

Half space







Surf. Ellipt.

Ellipt.

Circ.

$$K_I = \lambda_s \sigma \sqrt{\frac{\pi a}{Q}} f(\phi)$$

$$K_I = \sigma \sqrt{\frac{\pi a}{Q}} f(\phi)$$

$$K_I = \frac{2}{\pi} \sigma \sqrt{\pi a}$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

$$\lambda_s = \left[1.13 - 0.09\left(\frac{a}{c}\right)\right][1 + 0.1(1 - \sin\phi)^2]$$

$$f(\phi) = \left[\sin^2(\phi) + \left(\frac{a}{c}\right)^2 \cos^2(\phi)\right]^{1/4}$$

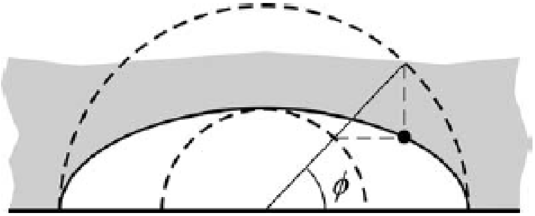


FIGURE 2.19 Mode I stress intensity factors for elliptical and semielliptical cracks. These solutions are valid only as long as the crack is small compared to the plate dimensions and $a \leq c$.

How to calculate SIFs for structures

Many cases available on books and atlas

T.L. Anderson, Fracture Mechanics: Fundamentals and Applications, third edition. CRC Press 2005.

S.A. Laham, R.A. Ainsworth, Stress Intensity Factor and Limit Load Handbook. EPD/GEN/REP/0316/98, ISSUE 2, 1998.

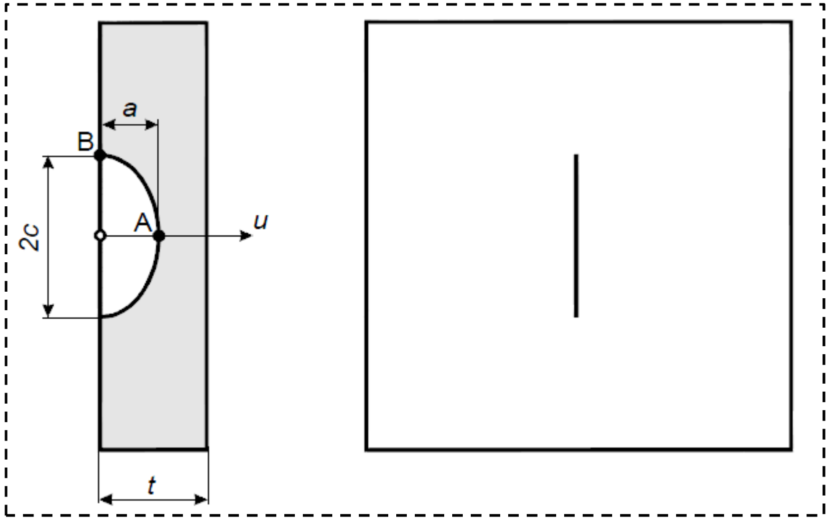
Y. Murakami, Stress Intensity Factors Handbook. Pergamon, 1986.

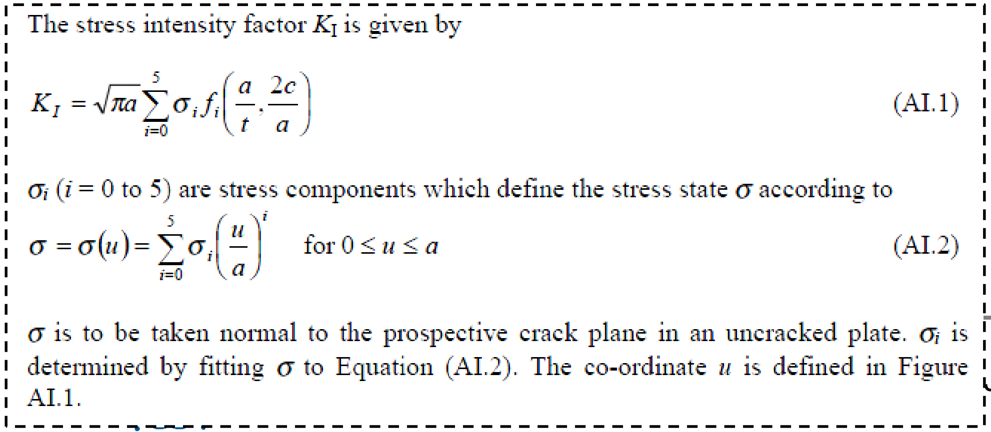
ASTM standards (to be shown next)

… and others

How to calculate SIFs for structures

S.A. Laham, R.A. Ainsworth, Stress Intensity Factor and Limit Load Handbook. EPD/GEN/REP/0316/98, ISSUE 2, 1998.





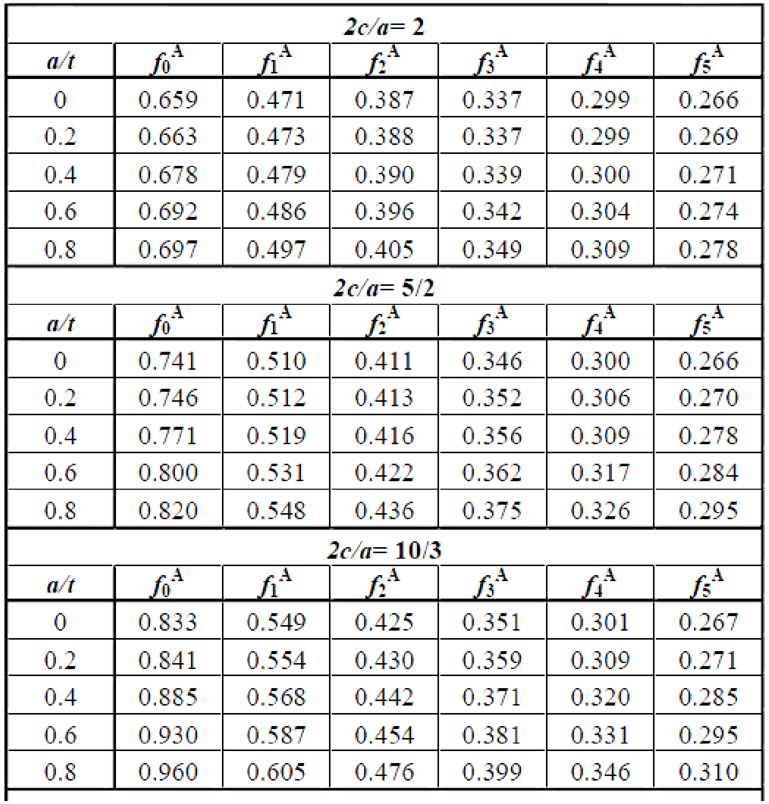
The stress intensity factor K_I is given by

$$K_I = \sqrt{\pi a}\sum_{i=0}^{5}\sigma_i f_i\left(\frac{a}{t},\frac{2c}{a}\right) \qquad \text{(AI.1)}$$

σ_i ($i = 0$ to 5) are stress components which define the stress state σ according to

$$\sigma = \sigma(u) = \sum_{i=0}^{5}\sigma_i\left(\frac{u}{a}\right)^i \quad \text{for } 0 \le u \le a \qquad \text{(AI.2)}$$

σ is to be taken normal to the prospective crack plane in an uncracked plate. σ_i is determined by fitting σ to Equation (AI.2). The co-ordinate u is defined in Figure AI.1.



2c/a= 2						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.659	0.471	0.387	0.337	0.299	0.266
0.2	0.663	0.473	0.388	0.337	0.299	0.269
0.4	0.678	0.479	0.390	0.339	0.300	0.271
0.6	0.692	0.486	0.396	0.342	0.304	0.274
0.8	0.697	0.497	0.405	0.349	0.309	0.278
2c/a= 5/2						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.741	0.510	0.411	0.346	0.300	0.266
0.2	0.746	0.512	0.413	0.352	0.306	0.270
0.4	0.771	0.519	0.416	0.356	0.309	0.278
0.6	0.800	0.531	0.422	0.362	0.317	0.284
0.8	0.820	0.548	0.436	0.375	0.326	0.295
2c/a= 10/3						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.833	0.549	0.425	0.351	0.301	0.267
0.2	0.841	0.554	0.430	0.359	0.309	0.271
0.4	0.885	0.568	0.442	0.371	0.320	0.285
0.6	0.930	0.587	0.454	0.381	0.331	0.295
0.8	0.960	0.605	0.476	0.399	0.346	0.310

How to calculate SIFs for structures

Homework:

Calculate the SIF for a surface crack in a half-space at A and B points:

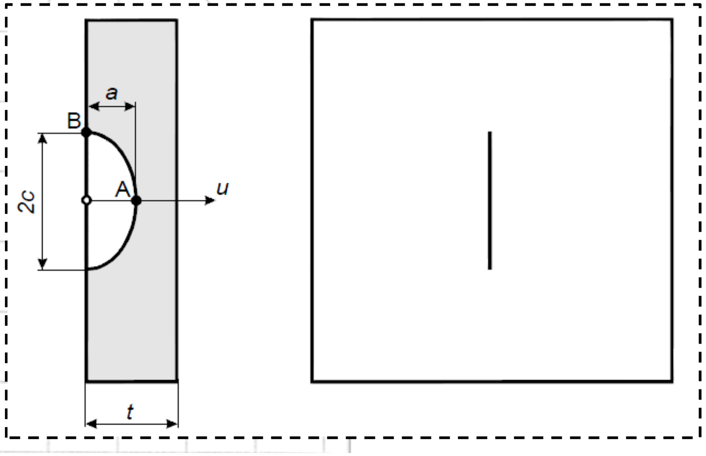
$a << t$

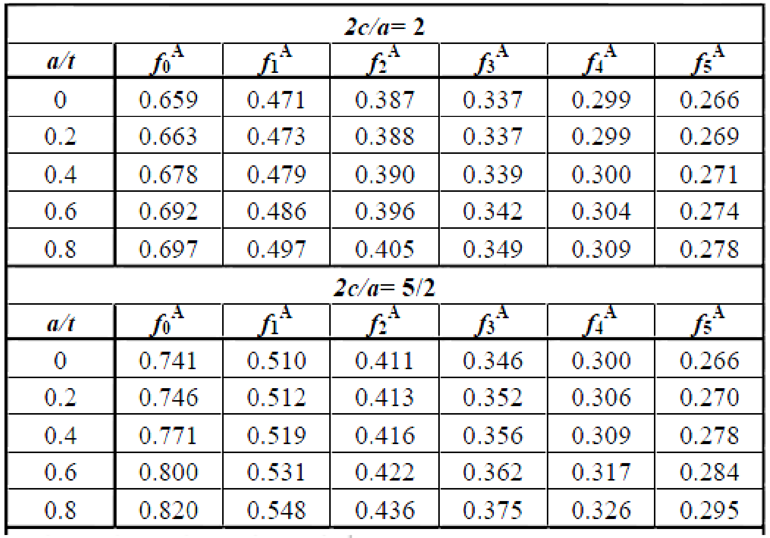
$a = 2\,\text{mm}, c = 2.5\,\text{mm}$

$\sigma = 100\,\text{MPa (uniform)}$

Compare the results

Andreson vs. Laham





2c/a= 2						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.659	0.471	0.387	0.337	0.299	0.266
0.2	0.663	0.473	0.388	0.337	0.299	0.269
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2c/a= 5/2						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
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0.2	0.746	0.512	0.413	0.352	0.306	0.270
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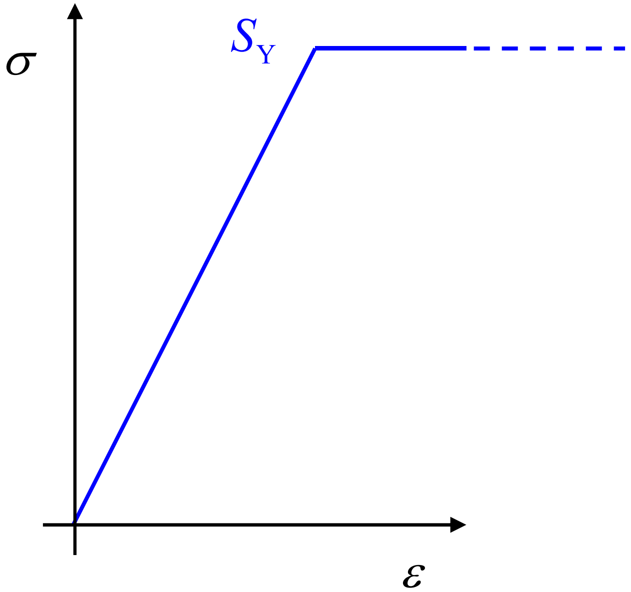
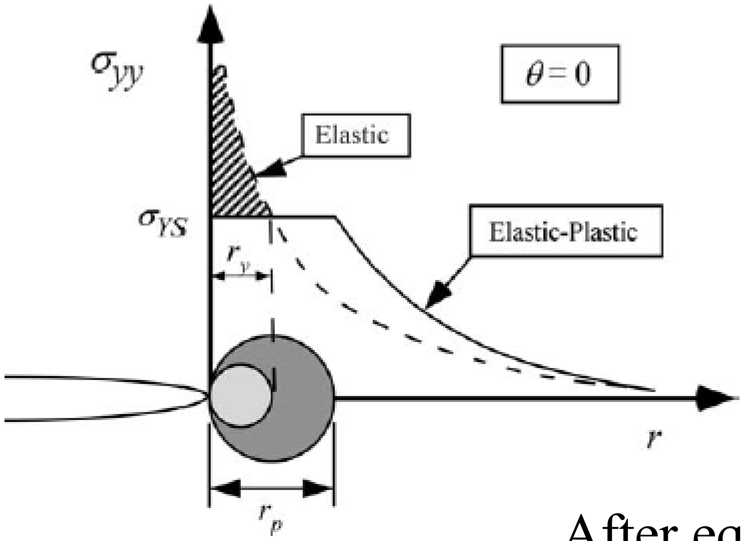
0.6	0.800	0.531	0.422	0.362	0.317	0.284
0.8	0.820	0.548	0.436	0.375	0.326	0.295

Crack tip plastic zone

Material model:

elastic perfectly plastically





Plane stress:

$$\sigma_y = S_Y$$

$$\sigma_y = 0 \rightarrow$$

$$r_{Y\sigma} = \frac{1}{2\pi}\left(\frac{K_I}{S_Y}\right)^2$$

After equlibrium correction:

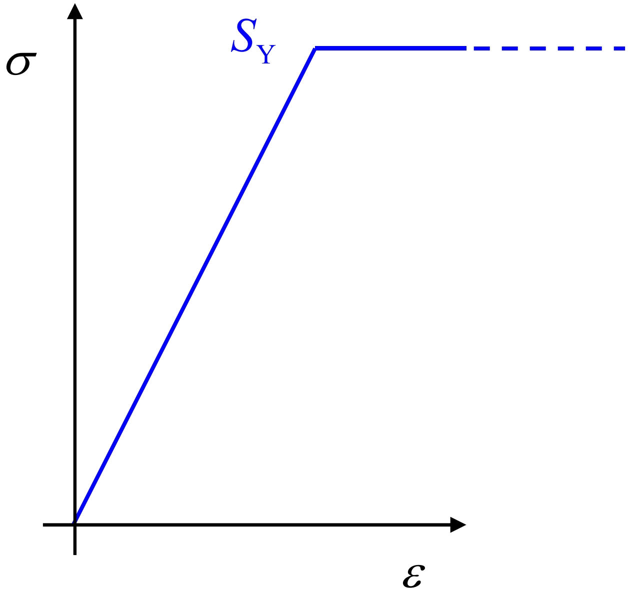
$$r_{p\sigma} = \frac{1}{\pi}\left(\frac{K_I}{S_Y}\right)^2$$

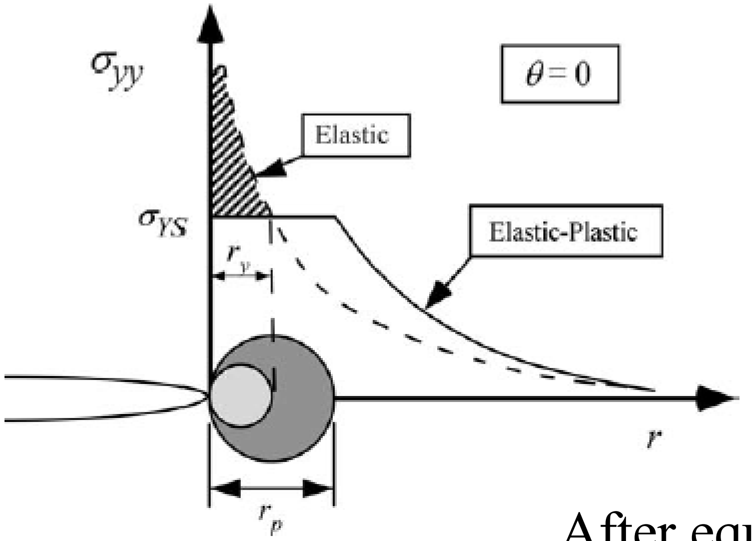
$$(r_{p\sigma} = 2r_{Y\sigma})$$

Crack tip plastic zone

Material model:

elastic perfectly plastically





Plane strain:

$$\sigma_y = S_\mathrm{Y}$$

$$\sigma_y > 0 \rightarrow$$

$$r_{\mathrm{Y}\varepsilon} = \frac{1}{6\pi}\left(\frac{K_\mathrm{I}}{S_\mathrm{Y}}\right)^2$$

After equlibrium correction:

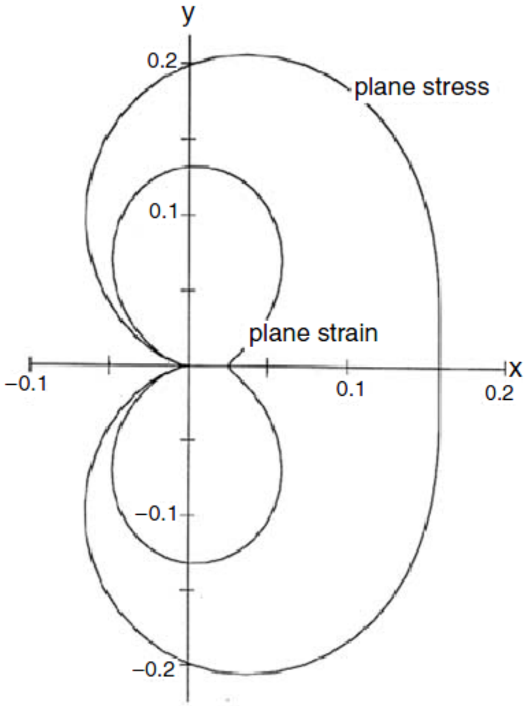
$$r_{\mathrm{p}\varepsilon} = \frac{1}{3\pi}\left(\frac{K_\mathrm{I}}{S_\mathrm{Y}}\right)^2$$

$$(r_{\mathrm{p}\varepsilon} = 2r_{\mathrm{Y}\varepsilon})$$

Crack tip plastic zone

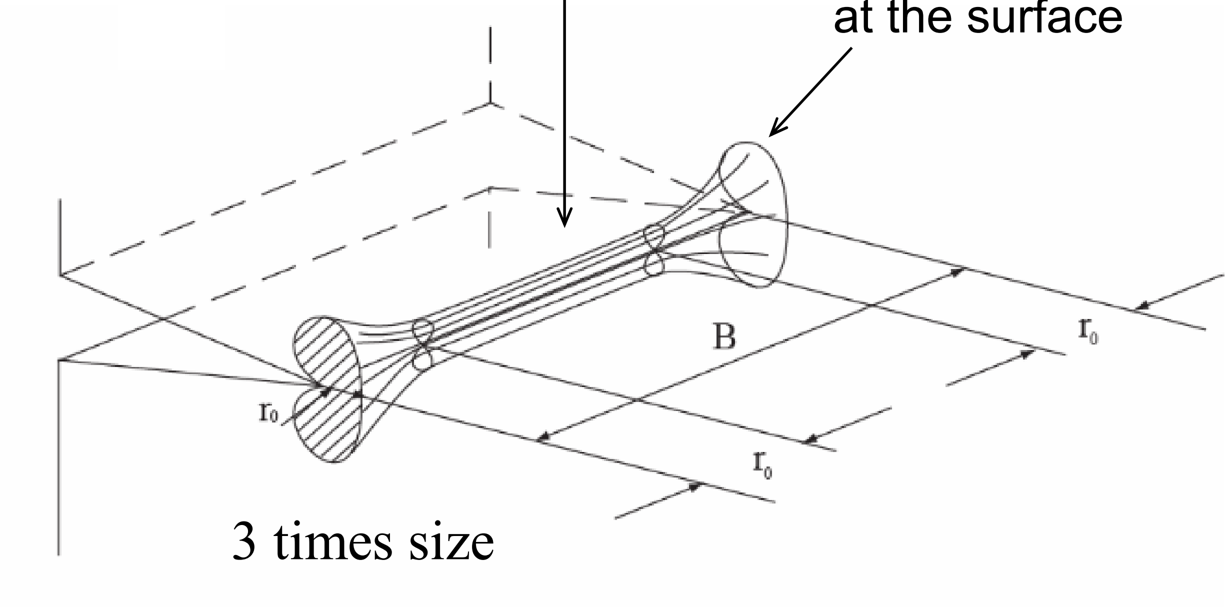
Plane Stress/ Plain Strain

“Kidney bean” shape



Pl. Strain at intermediate thickness, only for a thick plate

Always Pl. Stress at the surface

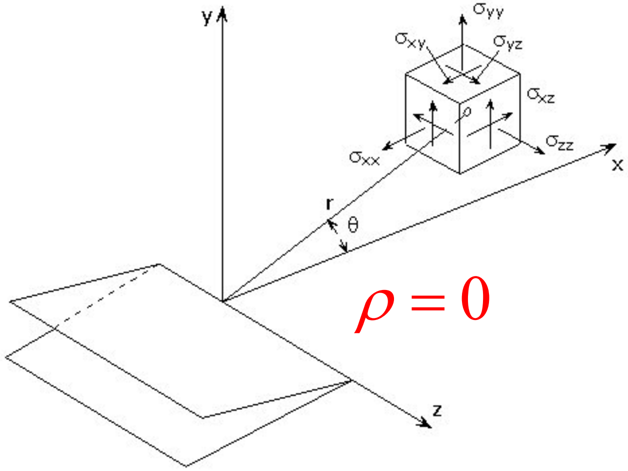


Crack tip plastic zone

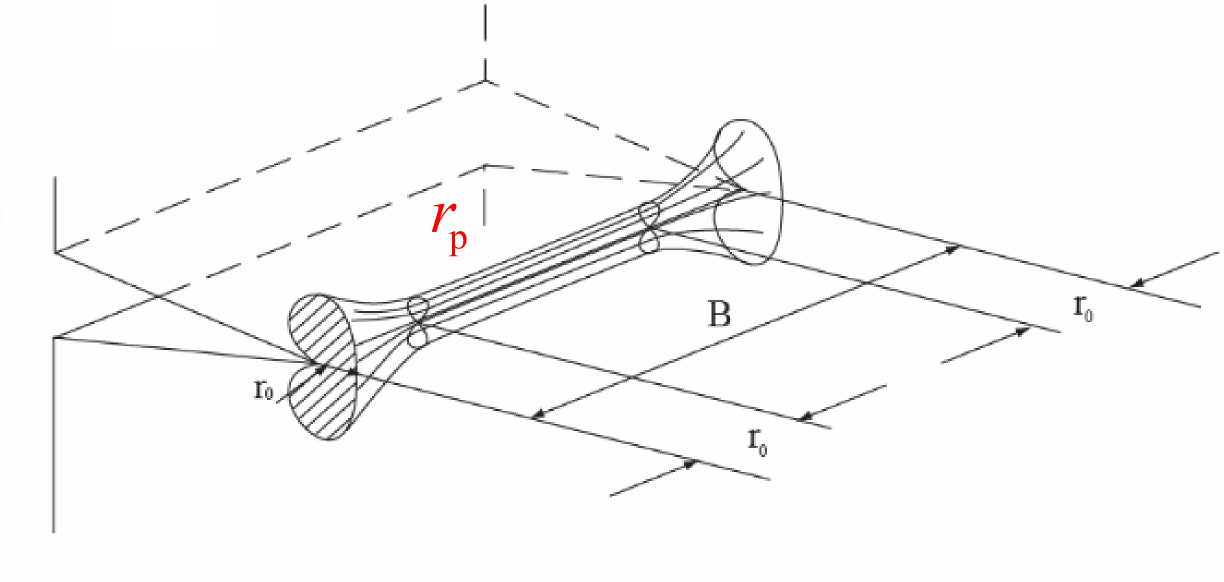
Plane Stress/ Plain Strain

Within the Linear Elastic assumption, always plain strain at the singularity, local radius = 0

After considering the plastic zone, the size to be compared to the thickness is r_p
To distinguish between plane Stress/ Strain



$\rho = 0$

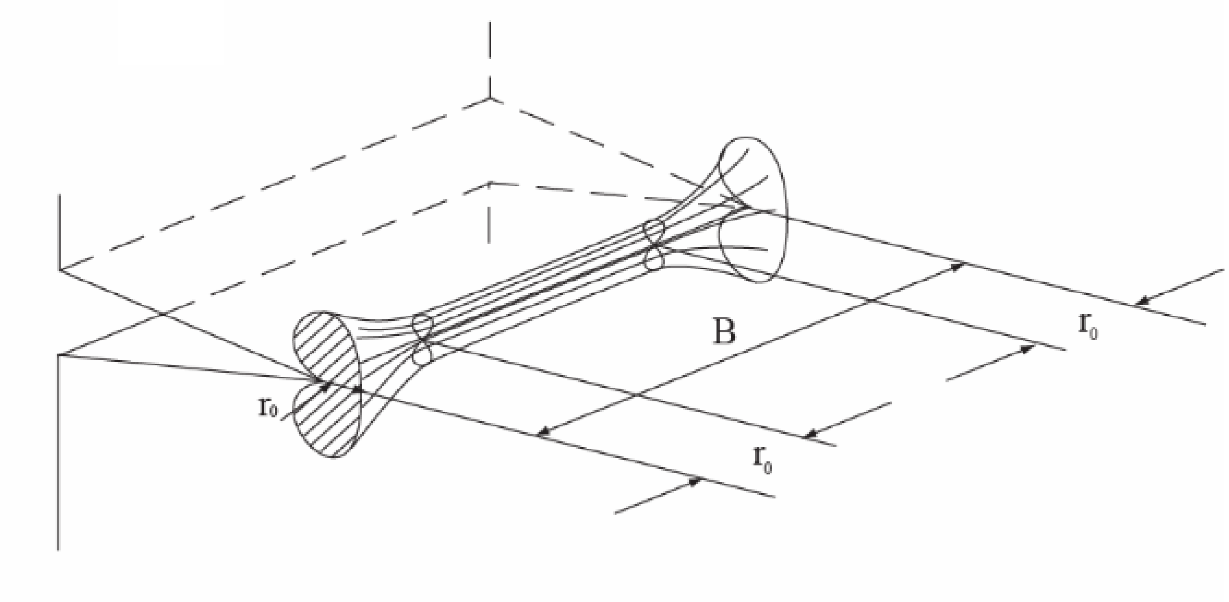


Crack tip plastic zone

Plane Stress/ Plain Strain

To have fully developed plane strain:

$$B > 2.5\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2$$



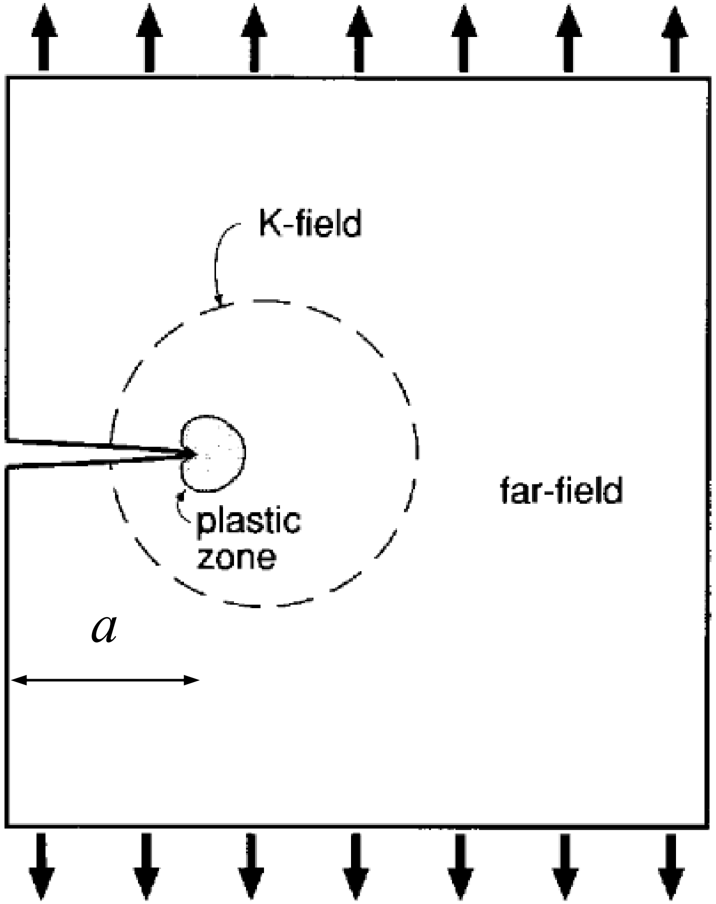
$$2.5\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2 =$$

$$= (2.5\times 6\pi)\frac{1}{6\pi}\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2 =$$

$$= (2.5\times 6\pi)\, r_{\mathrm{Y}\varepsilon} \approx 50\, r_{\mathrm{Y}\varepsilon}$$

Crack tip plastic zone

Small Scale Yielding for LEFM validity



To have LEFM validity:

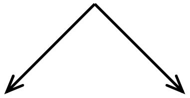
$a >> r_{\mathrm{p}}$

this way the plastic zone is dominated by the K-field.

Usually it is stated that:

$a > 8r_{Y\sigma}$ (for pl.stress)

$a > 8r_{Y\varepsilon}$ (for pl.stress)



for Pl. Stress: for Pl. Strain:

$$a > \frac{4}{\pi}\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2 \qquad a > \frac{4}{3\pi}\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2$$

Crack tip plastic zone

Small Scale Yielding for LEFM validity

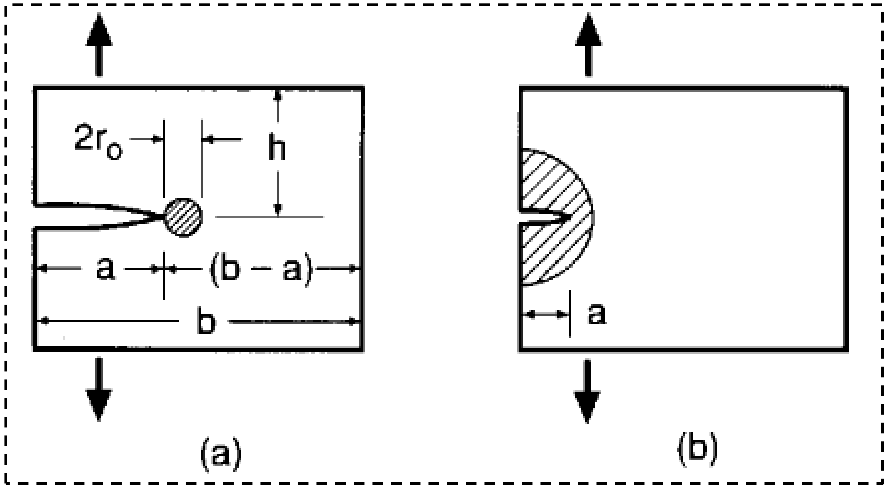
$r_{\mathrm{Y}}\,(r_{\mathrm{p}})$ has to be limited also with respect to the geometry boundaries

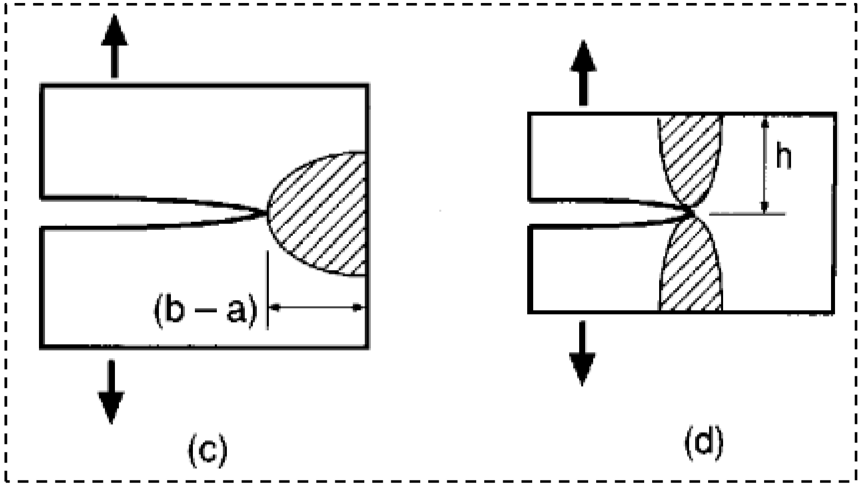
for Pl. Stress:

$$a,(b-a),h > \frac{4}{\pi}\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2$$

for Pl. Strain:

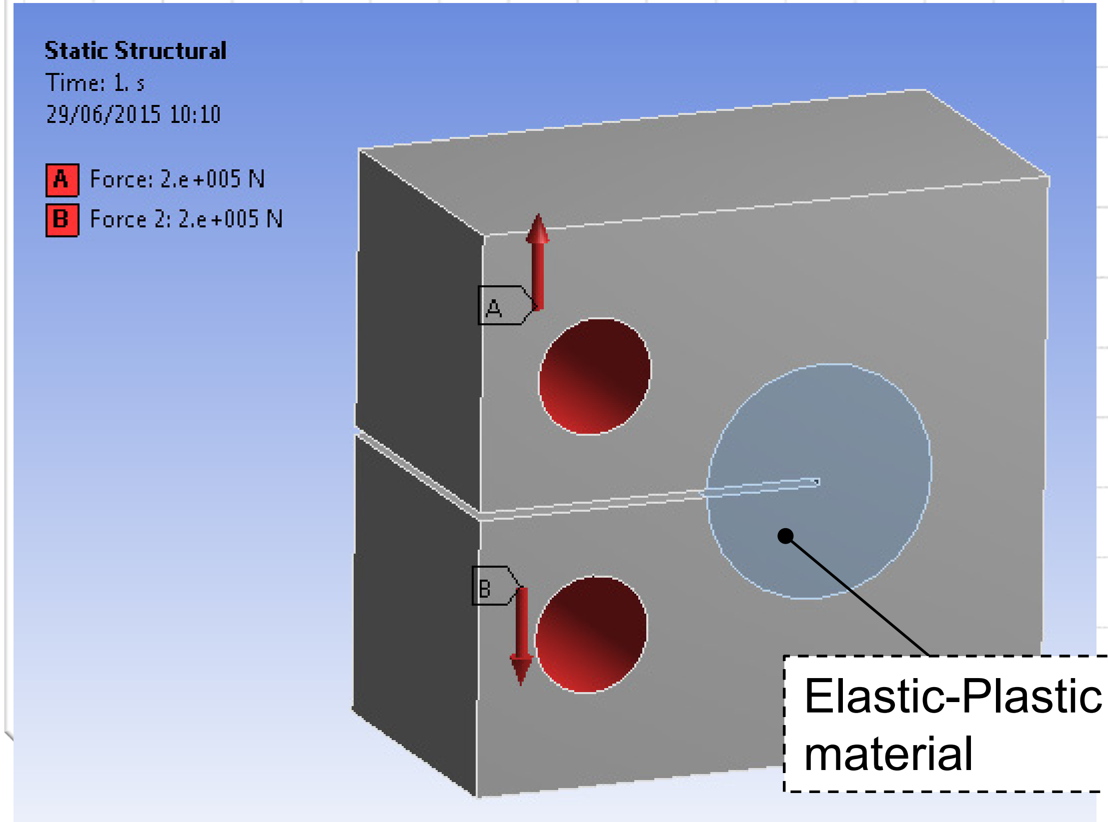
$$a,(b-a),h > \frac{4}{3\pi}\left(\frac{K_{\mathrm{I}}}{S_{\mathrm{Y}}}\right)^2$$





Crack tip plastic zone

ANSYS Wb – ASTM CT specimen:

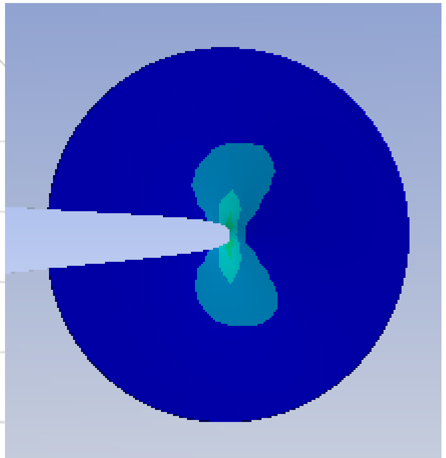


Static Structural
Time: 1. s
29/06/2015 10:10

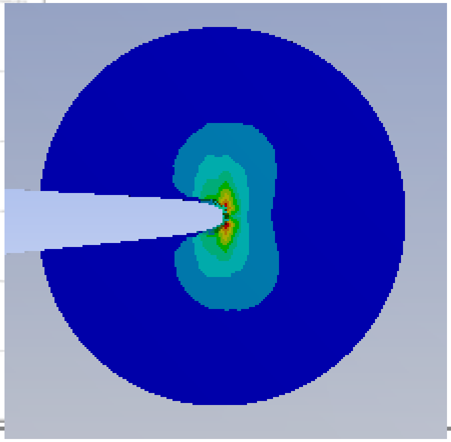
A Force: 2.e+005 N
B Force 2: 2.e+005 N

Elastic-Plastic material

At surface (Pl. Stress)

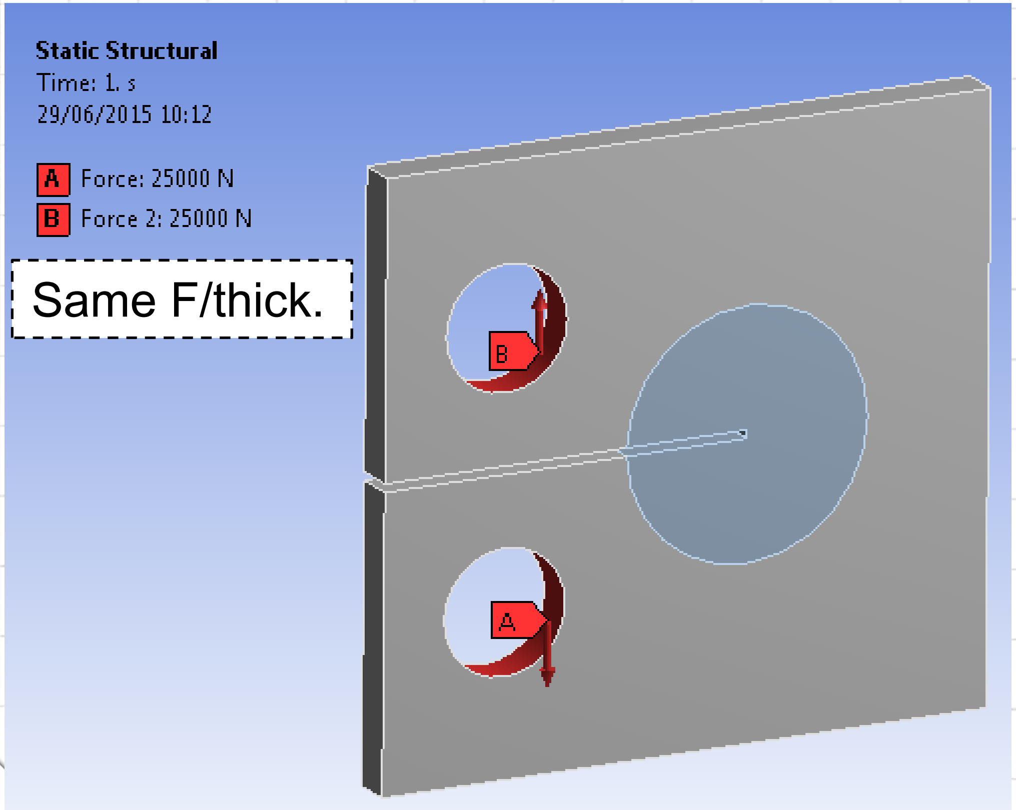


Interior section (Pl. Strain)



Crack tip plastic zone

ANSYS Wb – ASTM CT specimen:

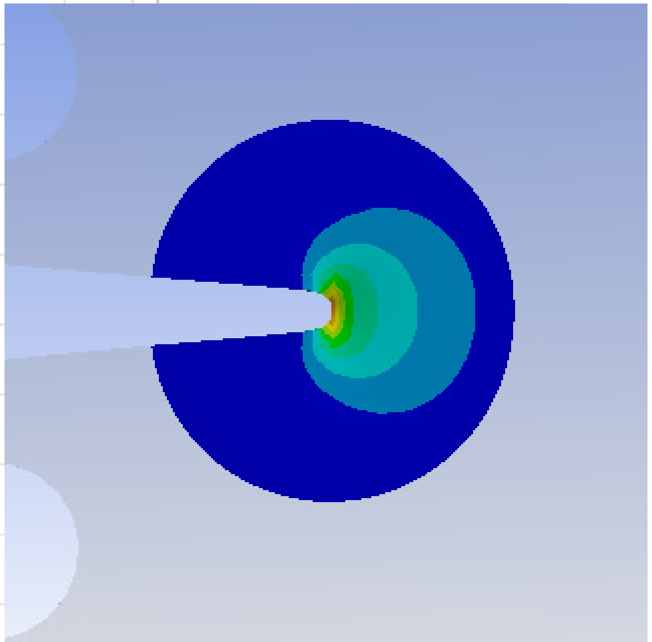


Static Structural
Time: 1. s
29/06/2015 10:12

A Force: 25000 N
B Force 2: 25000 N

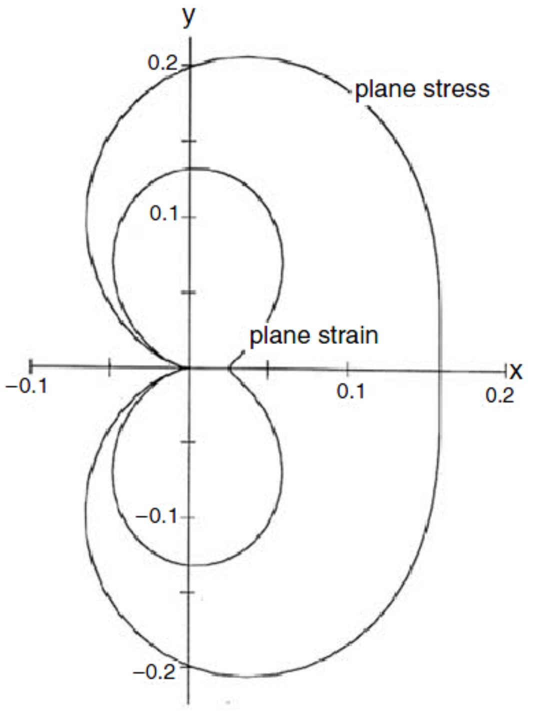
Same F/thick.

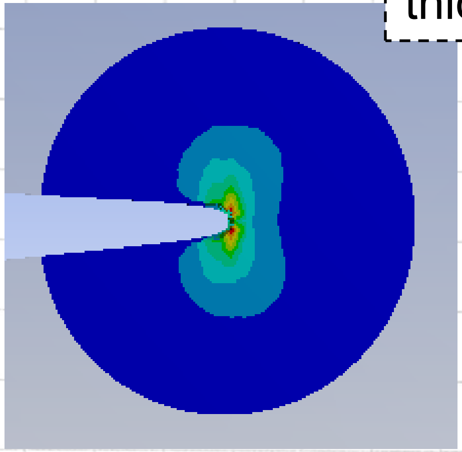
Plane Stress (small thickness with respect to the plastic zone)



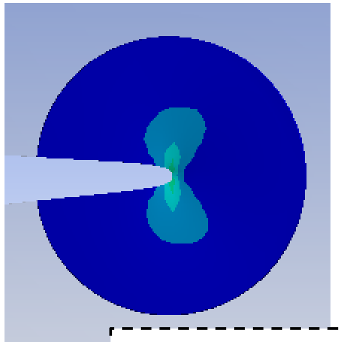
Crack tip plastic zone

ANSYS Wb – ASTM CT specimen:

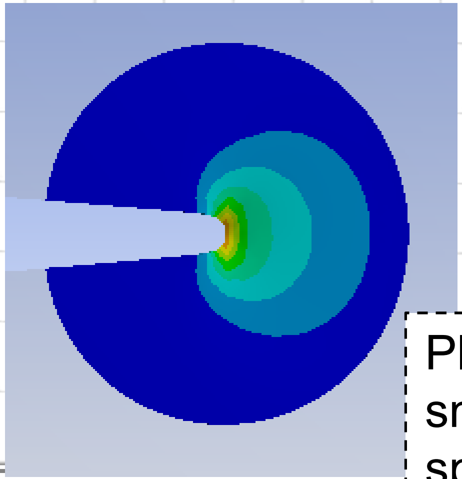




Plane Strain, interior thick specimen



Plane Stress, at thick specimen surface



Plane Stress, small thickness specimen