## Task 6 - Safety Review and Licensing
## On the Job Training on Stress Analysis

# Static strength and High and Low-Cycle Fatigue at room temperature 1/2

Davide Mazzini – **Ciro Santus**

BEYOND THE RESEARCH

IN SUPREMÆ DIGNITATIS · 1343 ·

UNIVERSITÀ DI PISA

**Pisa (Italy)**
**June 15 – July 14, 2015**

## Teaching

Fundamental of Machine Design (Bachelor, Mechanical Engineering)

Computer-Aided Engineering, FE (Master, Mechanical Engineering)

## Research

Fatigue of Materials and Structures

Contact Mechanics

Dynamics

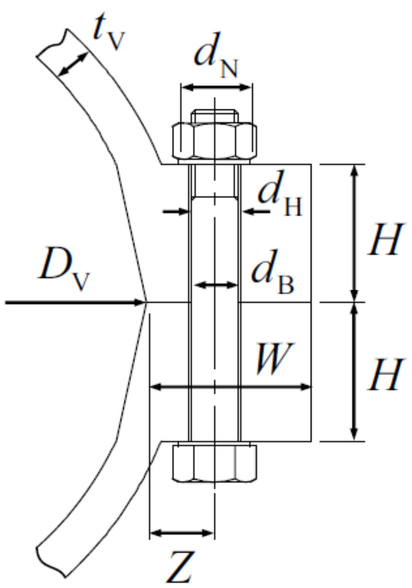
…

# *My latest paper – Eng. Fr. Mechanics, Elsevier*

## Partially open crack model for leakage pressure analysis of bolted metal-to-metal flange

M. Beghini[a], L. Bertini[a], C. Santus[a,*], A. Guglielmo[b], G. Mariotti[b]

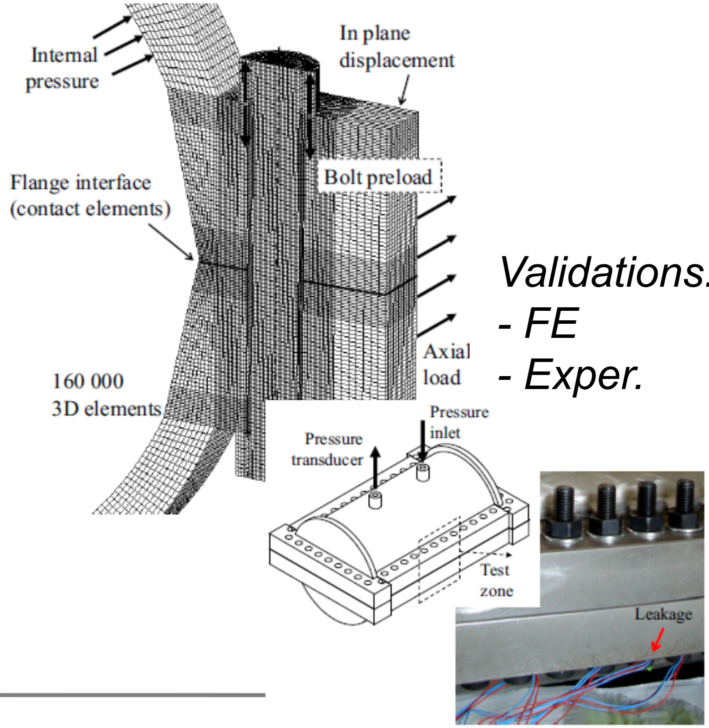
[a] *University of Pisa. DICI - Department of Civil and Industrial Engineering. Largo L. Lazzarino 2, 56122 Pisa, Italy.*
[b] *General Electric, Oil & Gas. Nuovo Pignone, Via F. Matteucci 2, 50127 Florence, Italy.*




$$K(L) = \int_0^L \sigma_n(x)\ h(x,L)\mathrm{d}x = 0$$

$$p_L = \sigma_B \frac{|\sigma_{n,B1}(0)| + C_\beta\ |\sigma_{n,B1}(L)|}{\sigma_{n,p1}(0) + C_\beta\ \sigma_{n,p1}(L) + C_\beta + 1}$$

Flange leakage pressure deduced from a Weight Function application




*Validations:*
*- FE*
*- Exper.*

# *Other paper – Eng. Fr. Mechanics, Elsevier*

Engineering Fracture Mechanics 98 (2013) 153–168

Contents lists available at SciVerse ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

An application of the weight function technique to inclined surface cracks under rolling contact fatigue, assessment and parametric analysis

M. Beghini, C. Santus *

University of Pisa, Department of Civil and Industrial Engineering, Largo L. Lazzarino 2, 56126 Pisa, Italy

Analytical/ Numerical procedure to calculate the Stress Intensity Factors for Rolling Contact Fatigue

*FE validation*

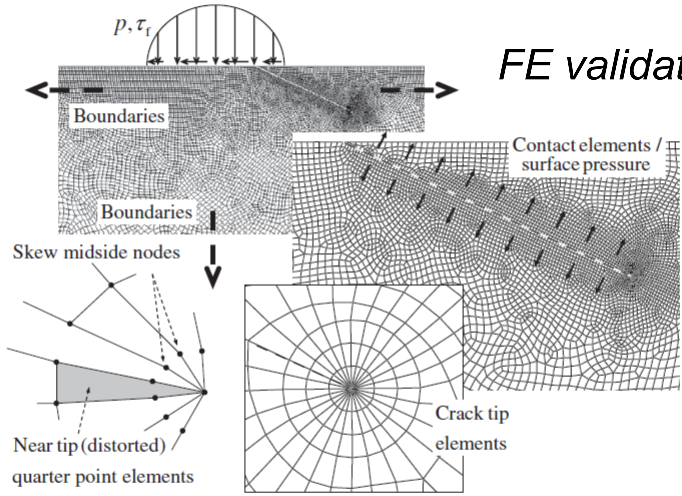
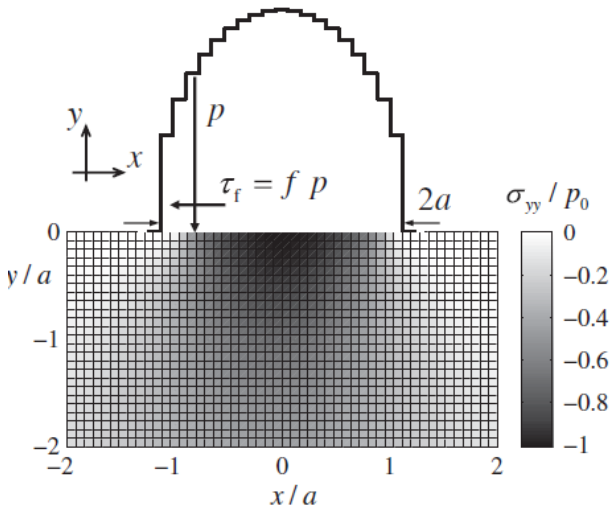



Fig. 7. FE model to calculate the SIFs with the quarter point technique.




$$\begin{bmatrix} K_{\mathrm{I}} \\ K_{\mathrm{II}} \end{bmatrix} = \int_0^c \begin{bmatrix} h_{11}(x', c, \theta) & h_{12}(x', c, \theta) \\ h_{21}(x', c, \theta) & h_{22}(x', c, \theta) \end{bmatrix} \begin{bmatrix} \sigma_{y'y'}(x') + p' \\ \sigma_{x'y'}(x') \end{bmatrix} \mathrm{d}x'$$

## Content

- Static strength of metals, Ductile/ Brittle
  - Tensile test
  - Plastic collapse vs. Brittle fracture notched components
- Fatigue of metals
  - Stress/ Strain approaches
  - Low/ High Cycle Fatigue
  - Fatigue notch sensitivity

## Books on Material mechanical properties

W. D. Callister, D. G. Rethwisch. Fundamentals of Materials Science and Engineering An Integrated Approach. Wiley 2007.

N. E. Dowling. Mechanical Behavior of Materials. Prentice Hall 1999.

## Books specifically on Fatigue

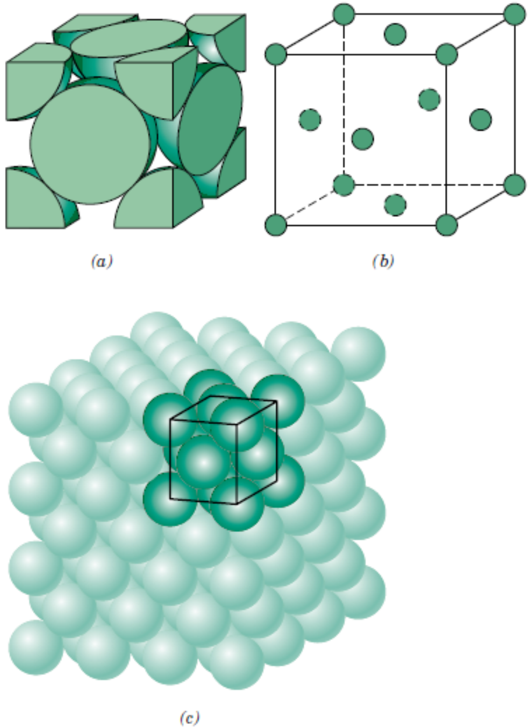
S. Suresh. Fatigue of Materials. Cambridge University Press 1998.

H. E. Boyer. Atlas of Fatigue Curves. ASM International 2003.

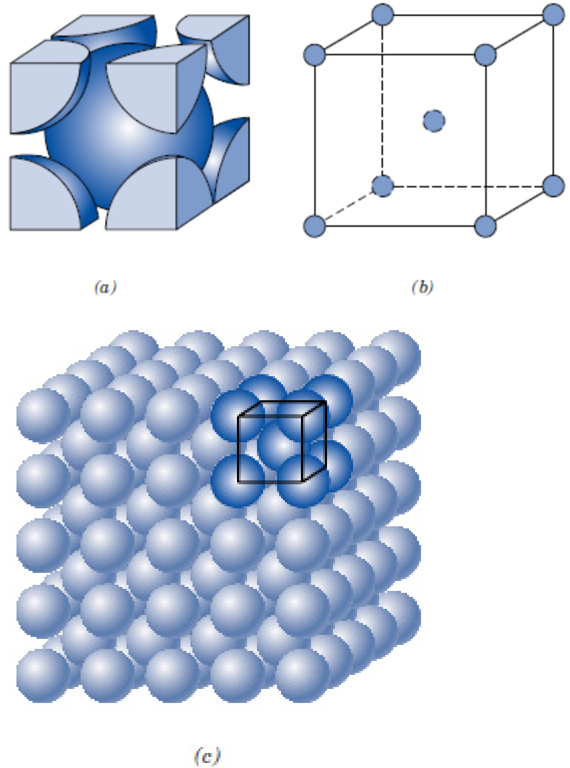
*… and many many others*

## Most usual metal crystal structures

FCC – Face Centered Cubic

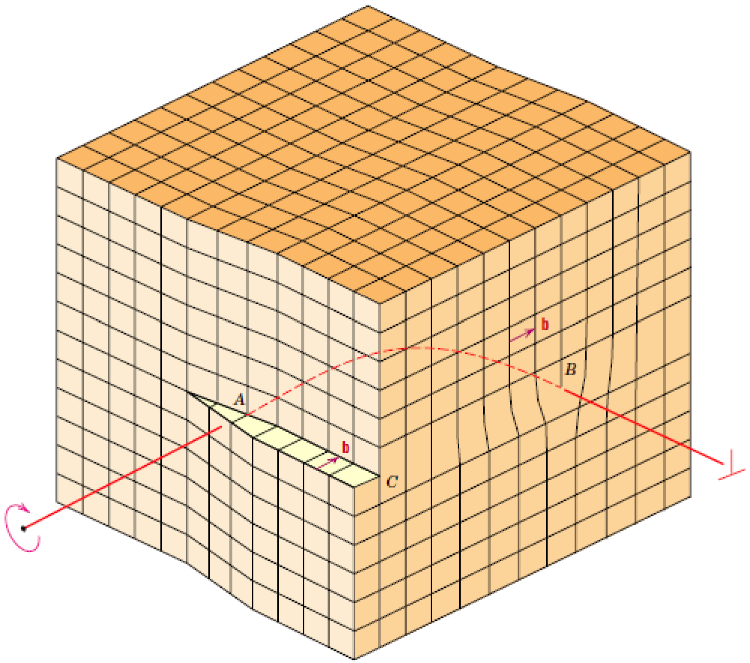


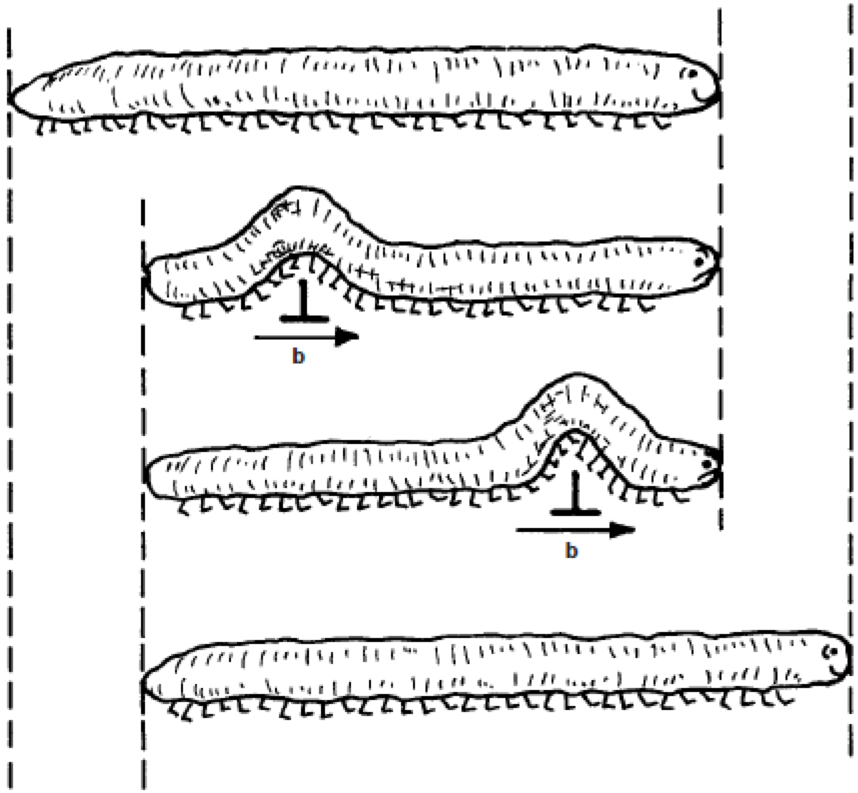

BCC – Body Centered Cubic



## Dislocation mechanics

The dislocation mobility is the basic for Metals ductility



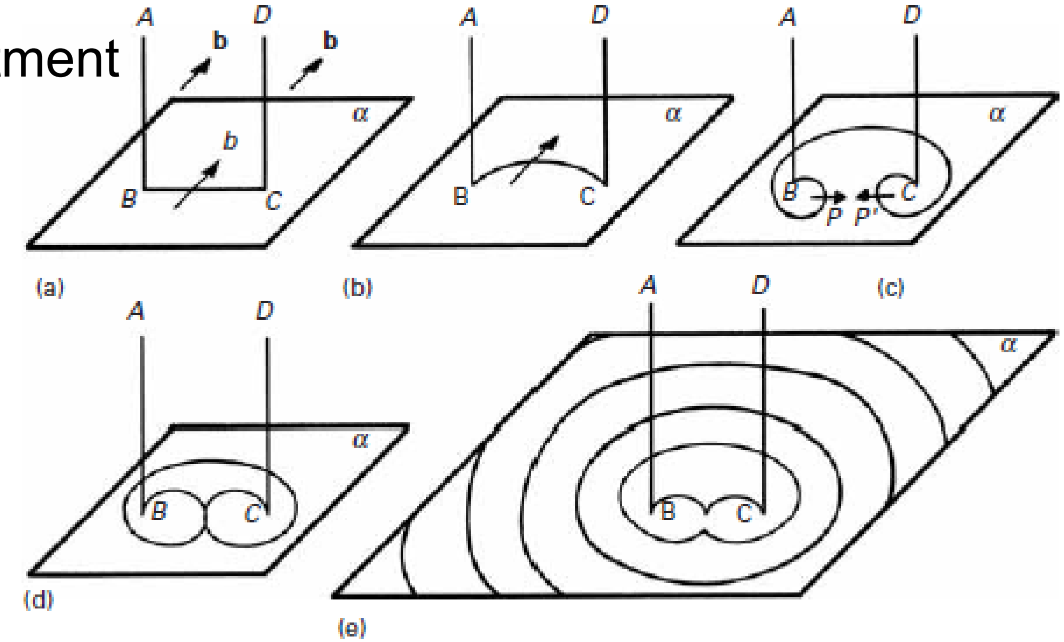



**Dislocation interactions**

Other dislocation previousy accumulated → work hardening

Other defect → alloy composition

Grain boundaries → heat treatment



**Static, quasi-static, or monotonic tests**

Tensile tests

Hardness tests

Fracture Toughness tests

Charpy tests

*… and others*

## Specifications

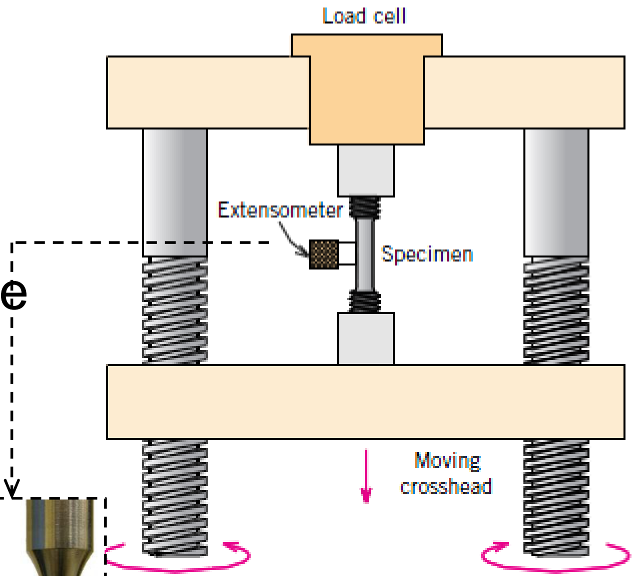
Uniform section of the specimen

Imposed constant (low) Strain rate up to fracture

Measurements:

Load Cell and

Extensometer Displacement






## Material properties tested

Bulk strength “without any gradient” (unnotched specimen)

Ductility up to fracture

## ASTM Standard E8/E8M – 11

Definition of the test, specimen sizes, recommendations, etc.

INTERNATIONAL

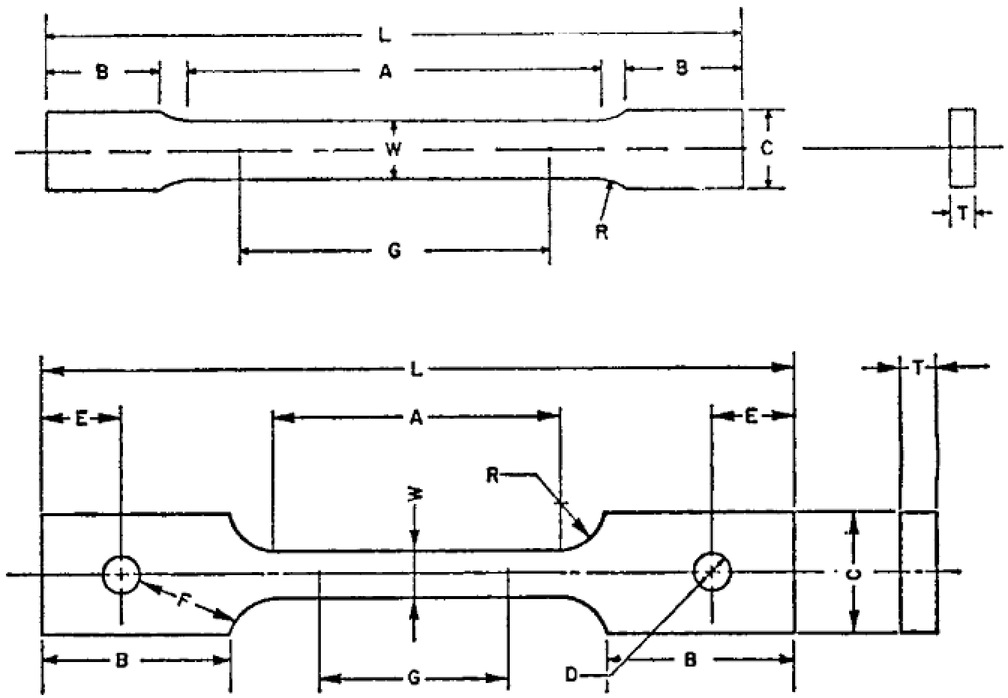
**Designation: E8/E8M – 11**

### Standard Test Methods for
### Tension Testing of Metallic Materials[1]

## ASTM Standard E8/E8M – 11

Specimen:

- Flat specimen
- Round specimen



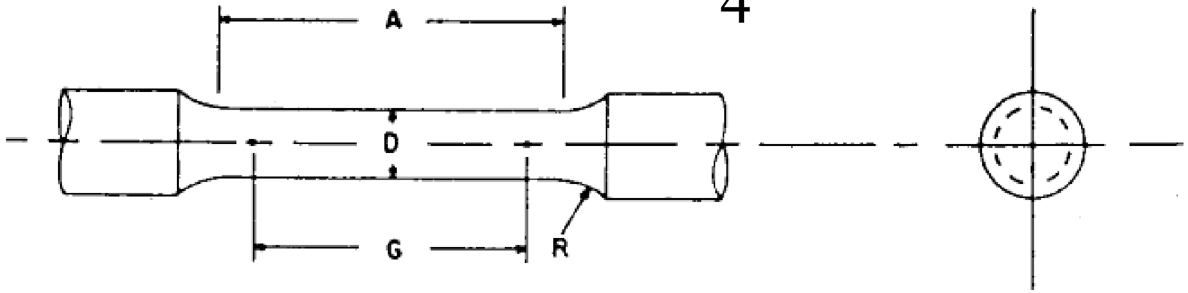
## ASTM Standard E8/E8M – 11

Initial section and lenght:

$$A_0 = \frac{\pi}{4} D^2, \ L_0 = G$$

Specimen:

- Flat specimen
- Round specimen




Dimensions, mm [in.]
**For Test Specimens with Gage Length Four times the Diameter [E8]**

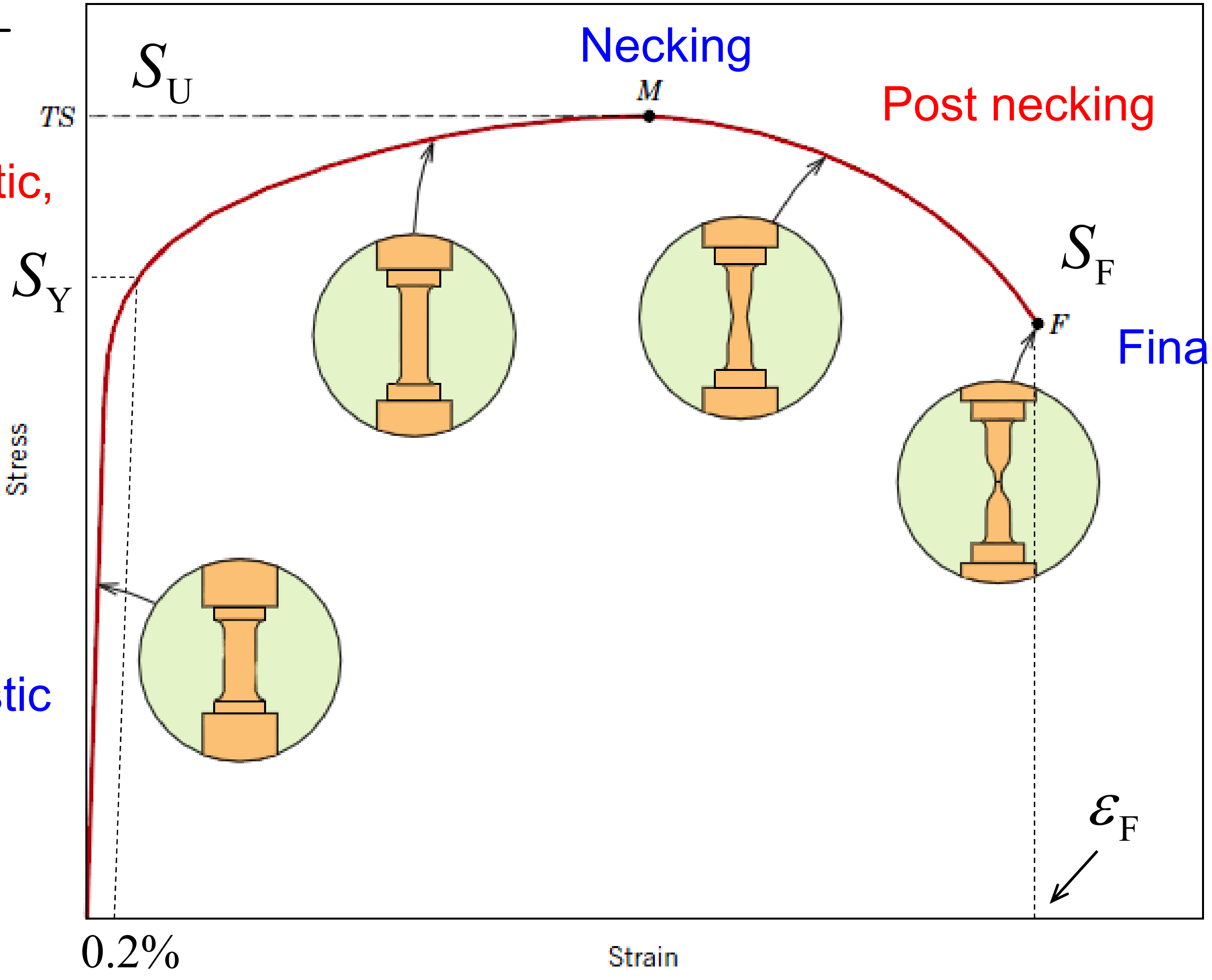
| | Standard Specimen | Small-Size Specimens Proportional to Standard | | | |
|---|---|---|---|---|---|
| | Specimen 1 | Specimen 2 | Specimen 3 | Specimen 4 | Specimen 5 |
| *G*—Gage length | 50.0 ± 0.1 [2.000 ± 0.005] | 36.0 ± 0.1 [1.400 ± 0.005] | 24.0 ± 0.1 [1.000 ± 0.005] | 16.0 ± 0.1 [0.640 ± 0.005] | 10.0 ±0.1 [0.450 ± 0.005] |
| *D*—Diameter (Note 1) | 12.5 ± 0.2 [0.500 ± 0.010] | 9.0 ±0.1 [0.350 ± 0.007] | 6.0 ± 0.1 [0.250 ± 0.005] | 4.0 ± 0.1 [0.160 ± 0.003] | 2.5 ± 0.1 [0.113 ± 0.002] |
| *R*—Radius of fillet, min | 10 [0.375] | 8 [0.25] | 6 [0.188] | 4 [0.156] | 2 [0.094] |
| *A*—Length of reduced section, min (Note 2) | 56 [2.25] | 45 [1.75] | 30 [1.25] | 20 [0.75] | 16 [0.625] |

*Most used*

$$\sigma = \frac{F}{A_0}$$

Elastic-plastic, post yield

Linear elastic behavior




Final Fracture

$$\varepsilon = \frac{\Delta L}{L_0}$$

$F$ Load as measured by the load cell

$\Delta L$ Elongation as measured by the extensometer

$$\sigma = \frac{F}{A_0} \text{ Engineering stress}$$

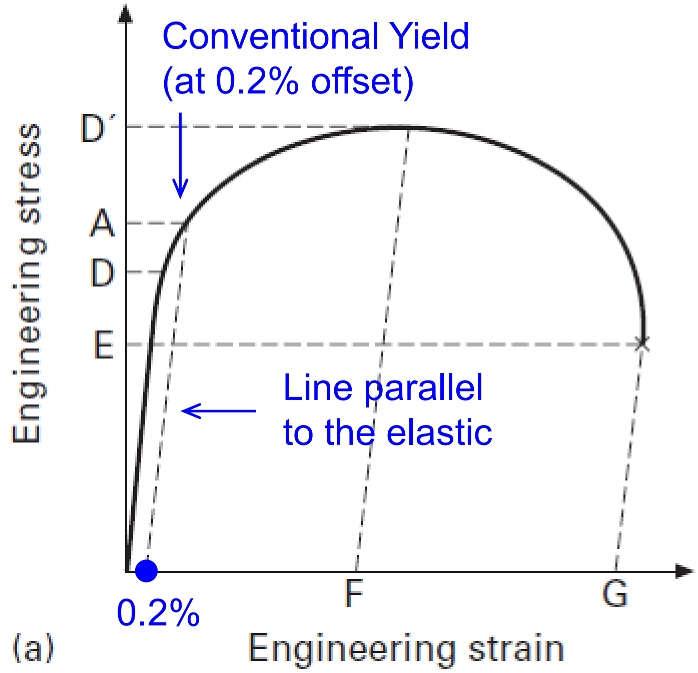
$$\varepsilon = \frac{\Delta L}{L_0} \text{ Engineering strain}$$

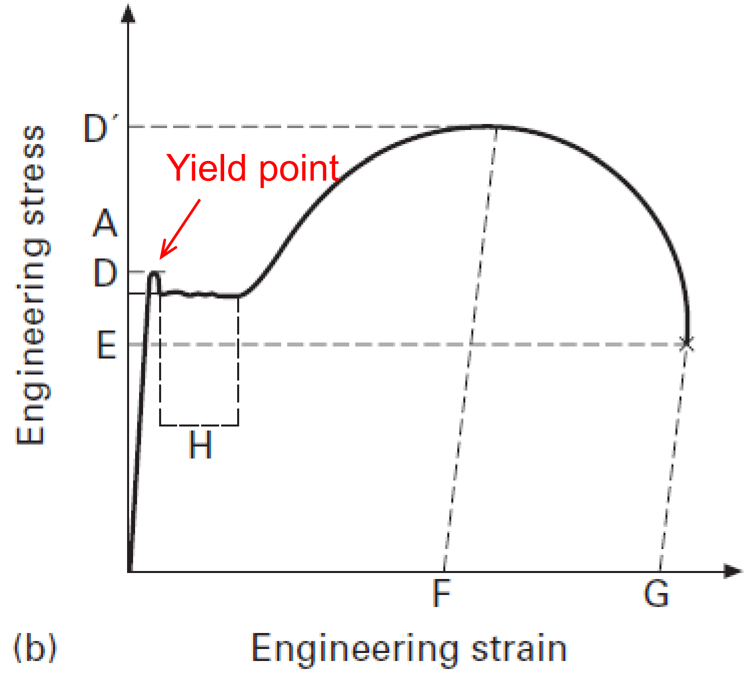
$$E = \frac{\Delta \sigma}{\Delta \varepsilon} \text{(before yield) Young's modulus}$$

$S_{\mathrm{Y}}, S_{\mathrm{U}}, S_{\mathrm{F}}$ Yield, Ultimate, Fracture strength values

$\varepsilon_{\mathrm{F}}$ Elongation at Fracture (usually in %)

## Yield point




Mild/ high-carbon steel, C≥0.2%
And all the other metals

Low-carbon steel, C 0.05-0.15%

**Engineering/ True curve**

$\sigma, \varepsilon$ Engineering

$\tilde{\sigma}, \tilde{\varepsilon}$ True

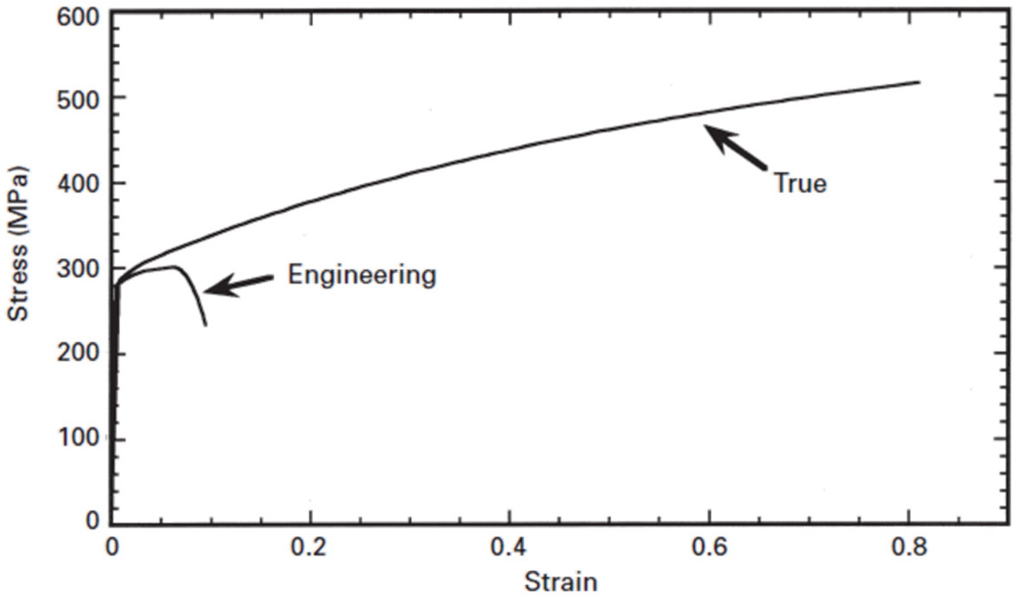
$$\tilde{\sigma} = \frac{F}{A} \quad \text{A is the current area}$$

$$\tilde{\varepsilon} = \frac{\Delta L'}{L_0} + \frac{\Delta L''}{L_0 + \Delta L'} + \ldots = \int \frac{\mathrm{d}L}{L}$$

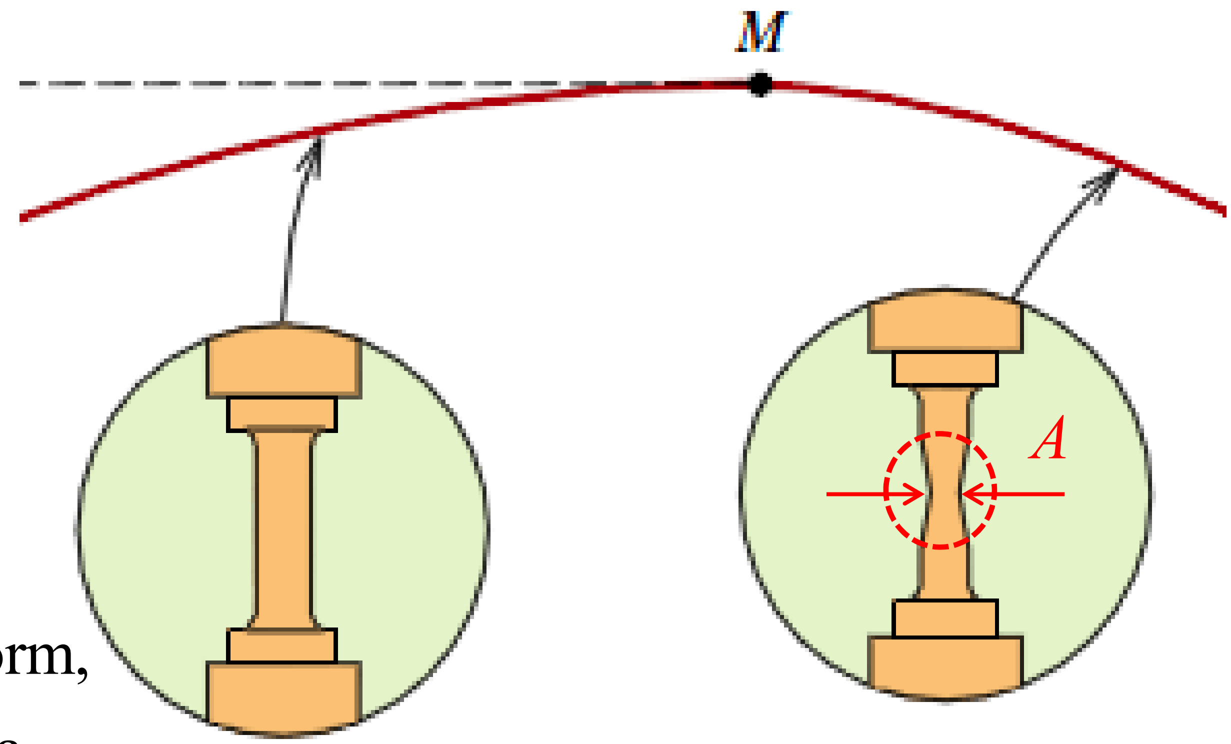
Before necking:

$$\tilde{\sigma} = \sigma(1+\varepsilon)$$

$$\tilde{\varepsilon} = \varepsilon \ln(1+\varepsilon)$$



**After necking**




$A$ is no more uniform, the test reduces to a portion of the specimen

**True curve Stress/ Strain at final fracture**

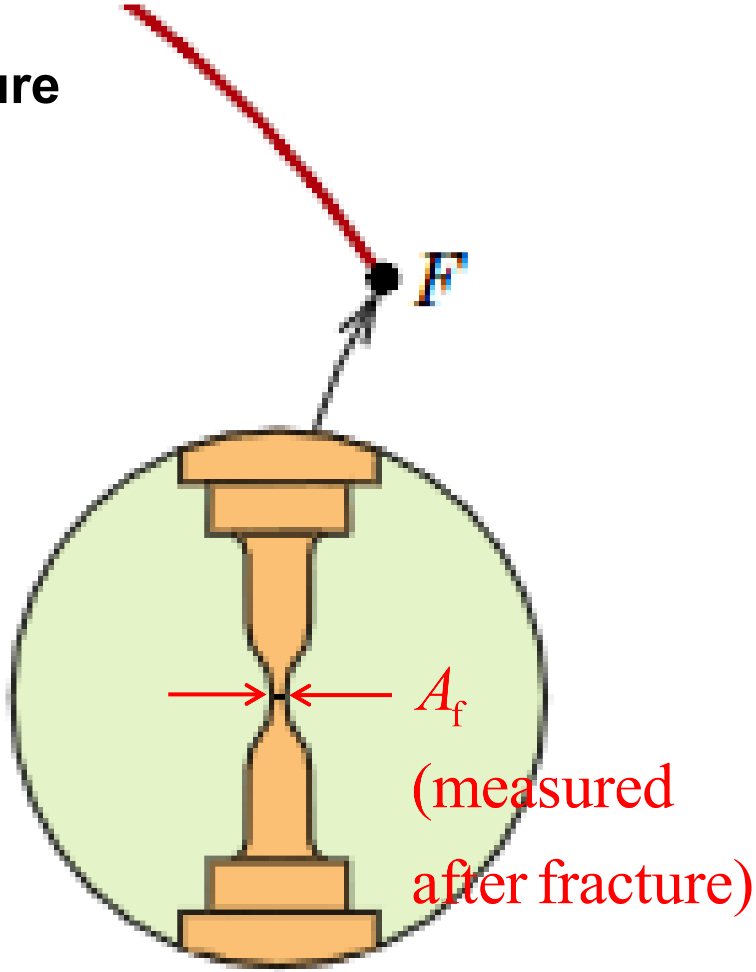
At least at fracture $A_f$ is known:

$$\tilde{\sigma}_F = S_F \frac{A_0}{A_F}$$

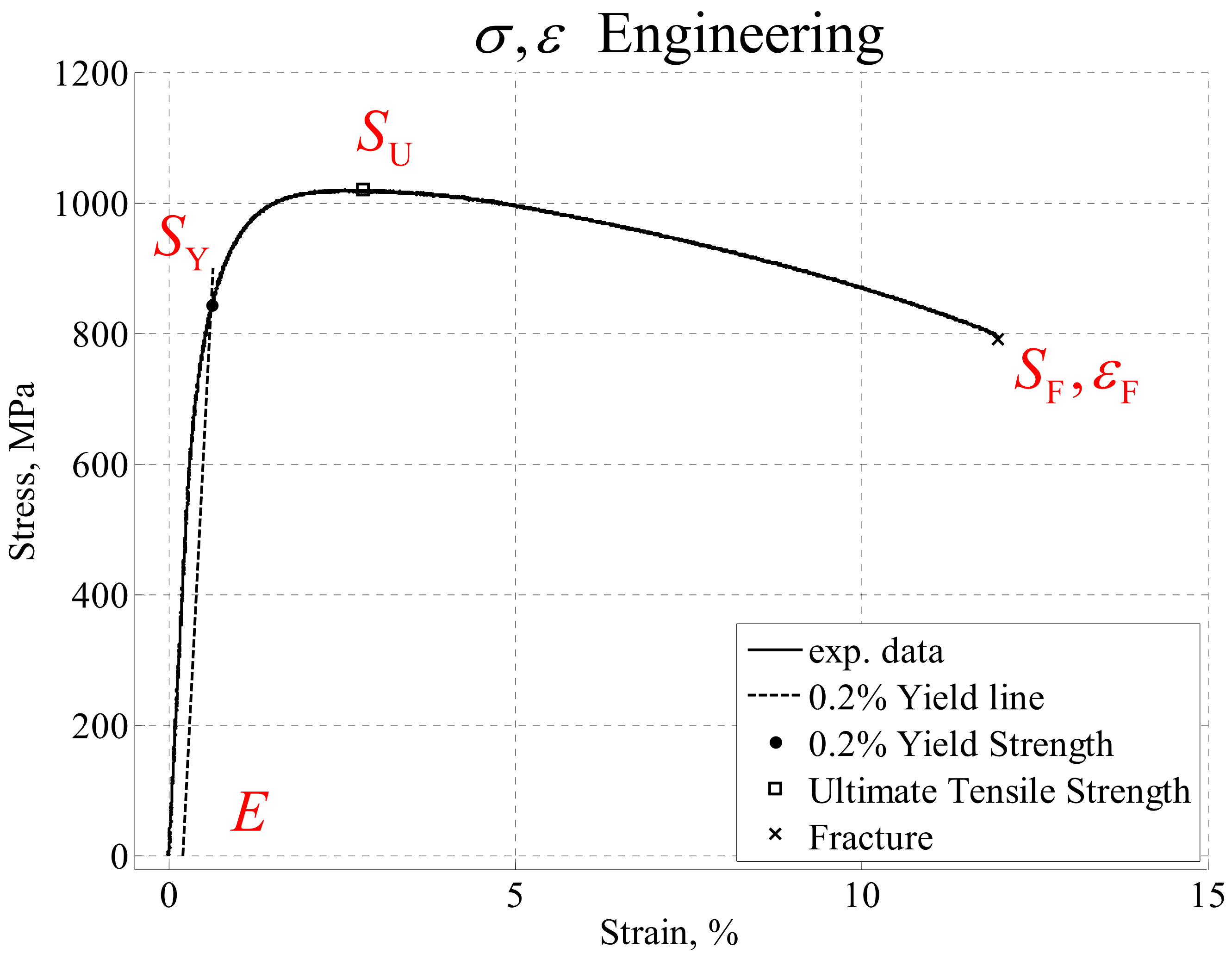
$$\tilde{\varepsilon}_F = \ln\left(\frac{A_0}{A_F}\right)$$

Instead of $\tilde{\varepsilon}_F$, Reduction of Area

$$\%RA = 100\frac{A_0 - A_F}{A_0}$$

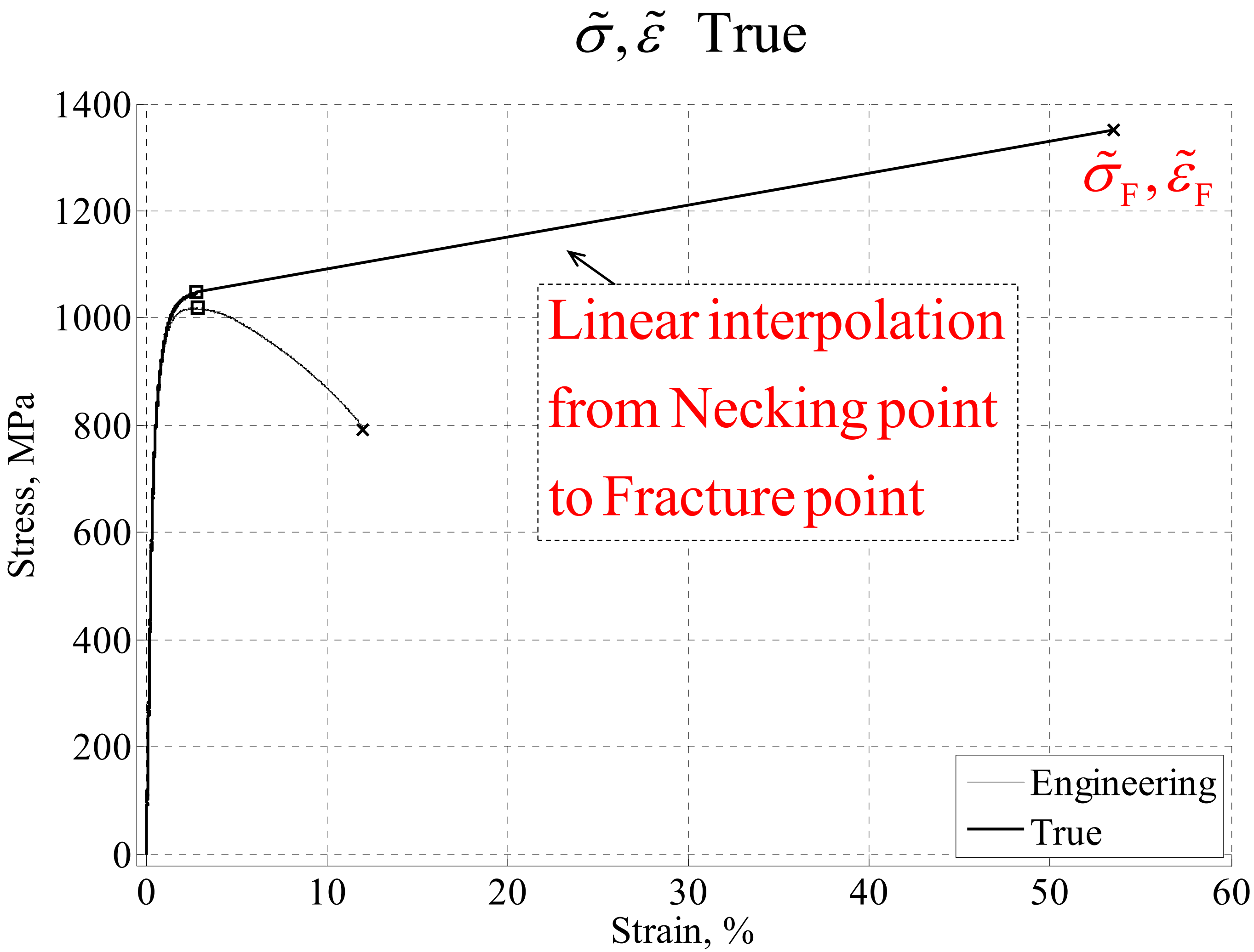


**AISI 4340**



σ, ε Engineering
1200
1000
800
600
400
200
0
Stress, MPa
0
5
10
15
Strain, %
S_U
S_Y
S_F, ε_F
E
exp. data
0.2% Yield line
0.2% Yield Strength
Ultimate Tensile Strength
Fracture

**AISI 4340**



$\tilde{\sigma}, \tilde{\varepsilon}$ True
1400
1200
1000
800
600
400
200
0
Stress, MPa
0
10
20
30
40
50
60
Strain, %
$\tilde{\sigma}_F, \tilde{\varepsilon}_F$
Linear interpolation
from Necking point
to Fracture point
Engineering
True

**Write a MATLAB script to find both Engineering and True curve and find the Stress and elongation parameters**

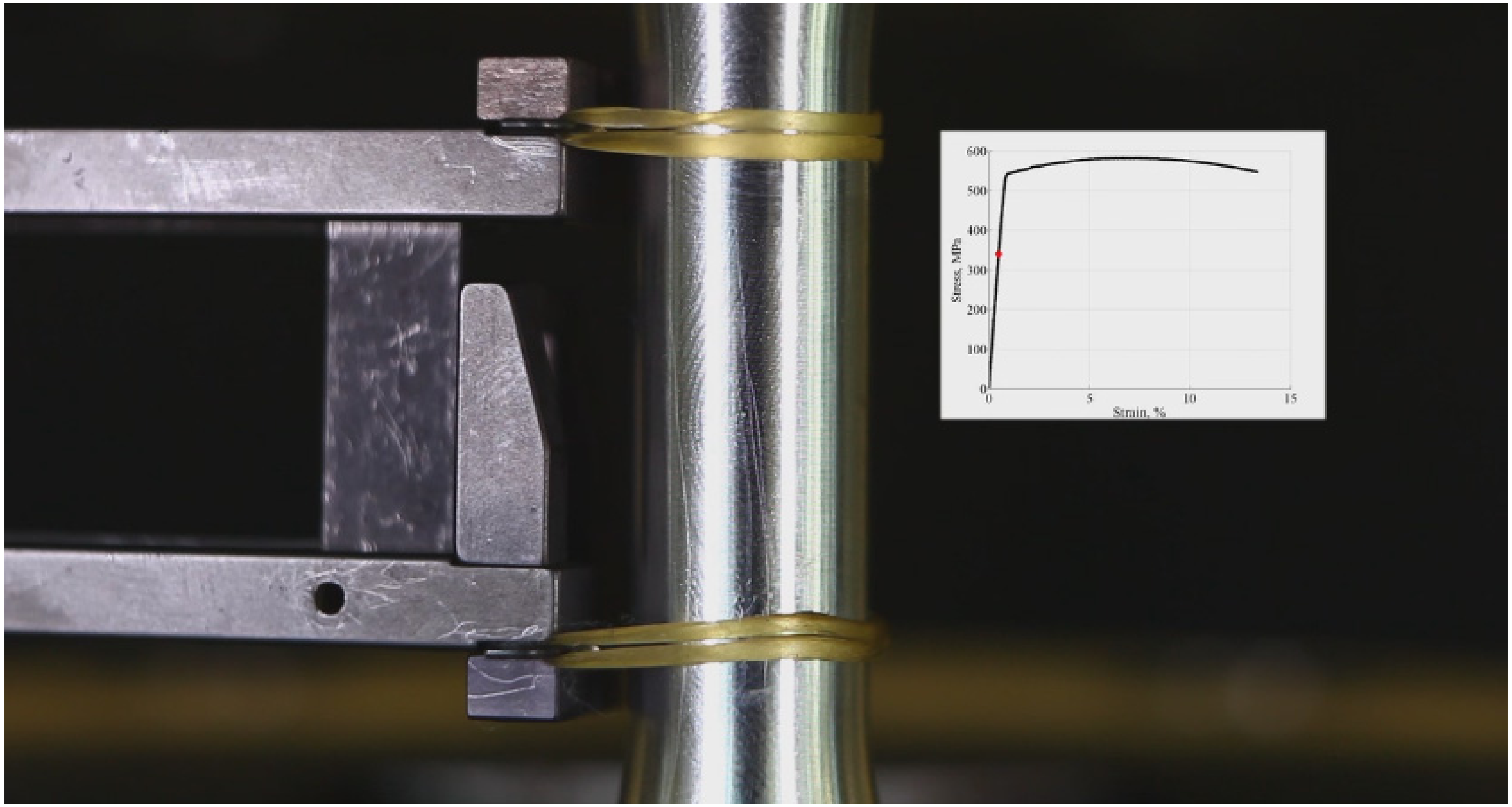
```
clc
close all
clear all
set(0,'DefaultAxesFontName','Times')
set(0,'DefaultAxesFontSize',20)


d = 5.75;  % mm
d_f = 4.4;  % mm
A_0 = pi/4*d^2;
A_f = pi/4*d_f^2;
L_0 = 25.0;  % mm

test = load('Test1.dat');
eps_ing = test(:,2)/L_0;
sig_ing = test(:,4)*1e3/A_0;
```

Test1.dat

```
0.00000 0.0 0.0 0.0
0.02000 0.00095 0.01500 0.00425
0.04000 0.00224 0.01200 0.00224
0.06000 0.00268 0.00600 0.02197
0.08000 0.00056 0.01500 0.03199
0.10000 0.00012 0.02100 0.02798
0.12000 0.00139 0.01200 0.03199
0.14000 0.00011 0.01200 0.03596
0.16000 -0.00287    0.00900 0.05187
0.18000 0.00097 0.01800 0.05793
0.20000 -0.00116    0.01500 0.07600
0.22000 0.00140 0.01800 0.05999
```

# YouTube video

https://www.youtube.com/watch?v=NrIErdXvjRQ



**$S_U$ is not a strength parameter, Necking is a point of instability onset**

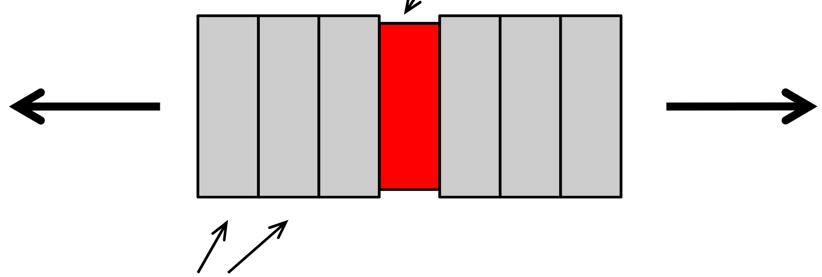
$$F = \tilde{\sigma} A$$

Positive  Negative

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}t} A + \tilde{\sigma} \frac{\mathrm{d}A}{\mathrm{d}t}$$

Chain model of the tensile specimen

Weakest link
It goes into Necking



At necking $\mathrm{d}F / \mathrm{d}t = 0$:

$$\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}t} A = -\tilde{\sigma} \frac{\mathrm{d}A}{\mathrm{d}t}$$

After necking $\mathrm{d}A / \mathrm{d}t$ is

predominant until fracture

thus $\mathrm{d}F / \mathrm{d}t < 0$

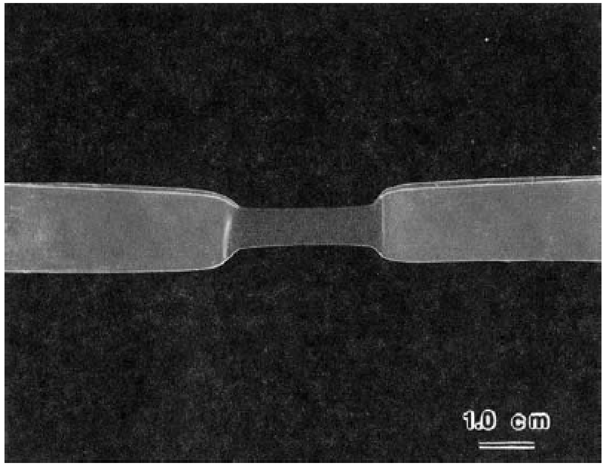
The other links experience unloading before reaching their necking condition, so necking does not extend to the stronger links

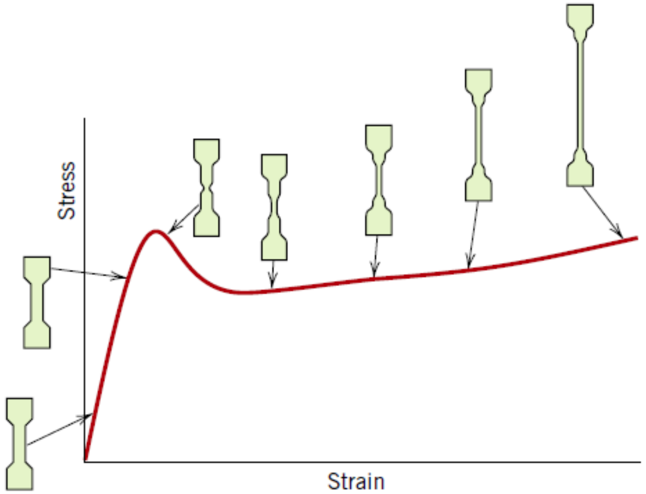
**Other materials (not metal) may have necking distributed on the entire specimen**

At necking $\mathrm{d}F / \mathrm{d}t = 0$ :

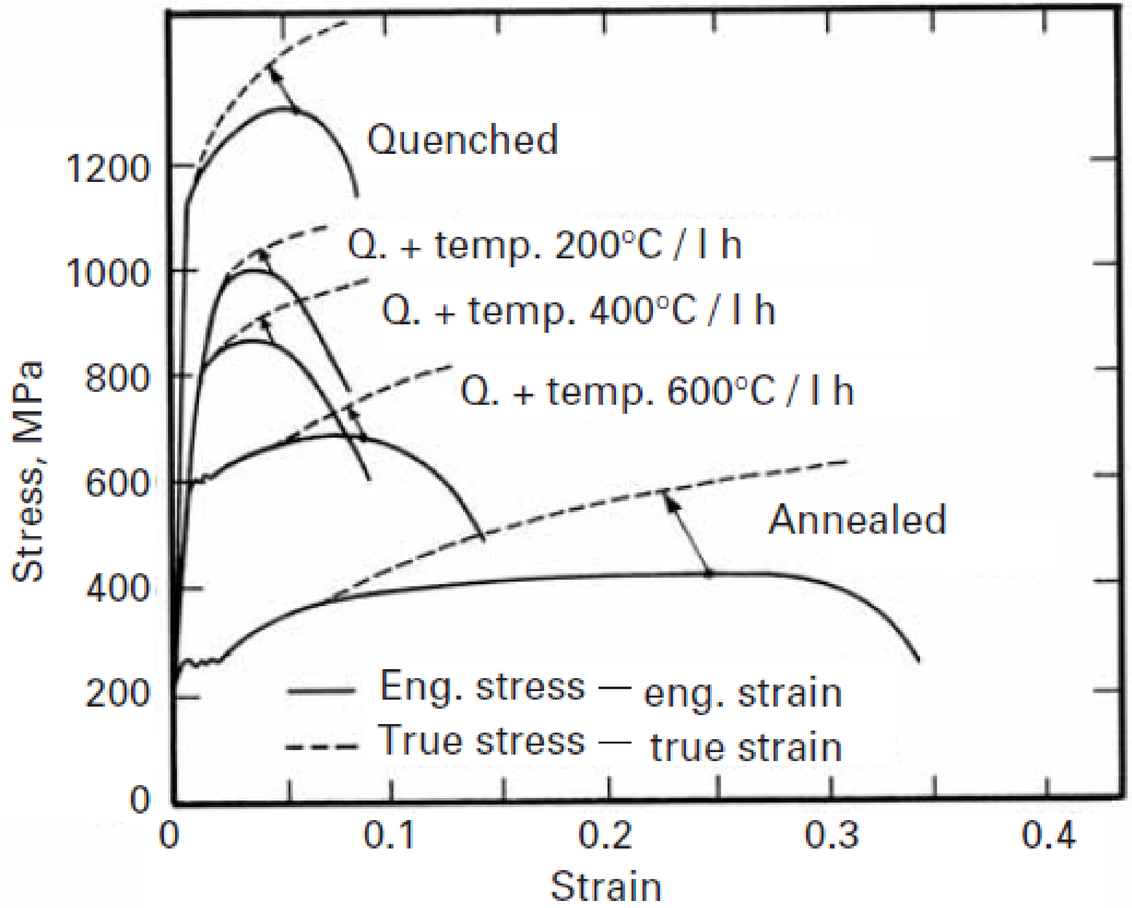
$$\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}t} A = -\tilde{\sigma} \frac{\mathrm{d}A}{\mathrm{d}t}$$

After necking $\mathrm{d}A / \mathrm{d}t$ is predominant thus the load drops, but before fracture, $\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}t}$ becomes predominant again so necking extends to the entire specimen



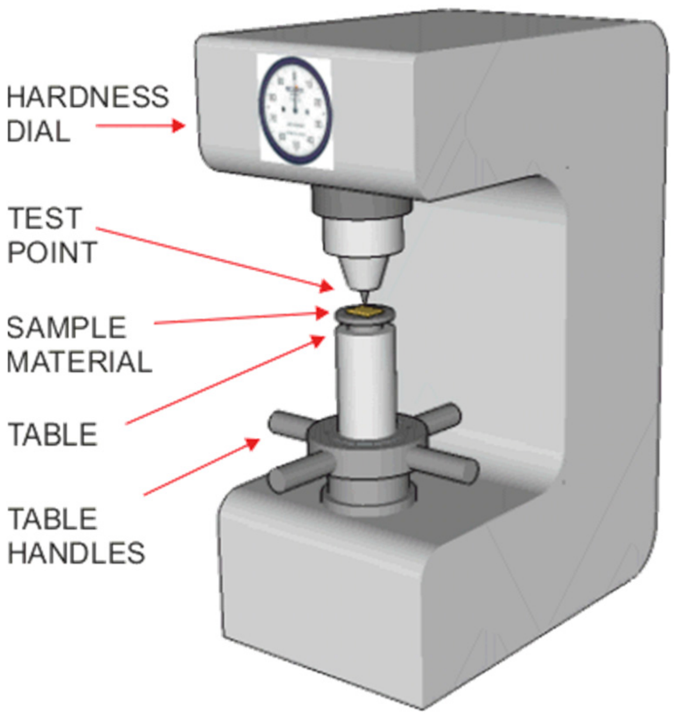



**Tempering after quenching at different temperatures (Es. AISI 4340)**



## Hardness tests

Resistance to the penetration / scratch



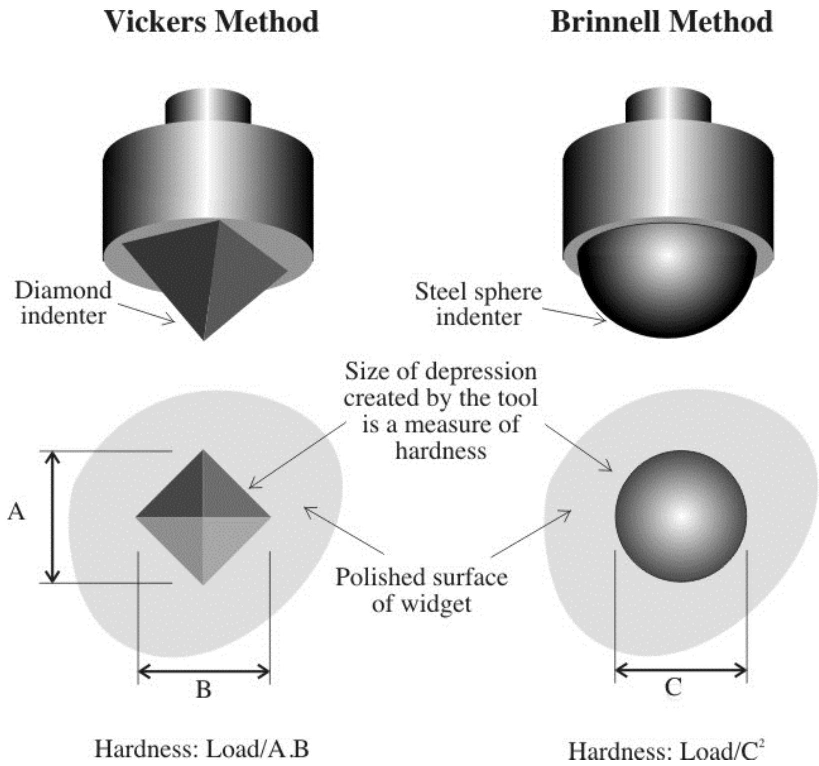
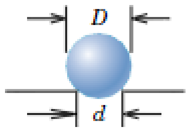
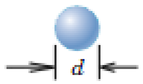
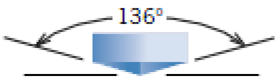
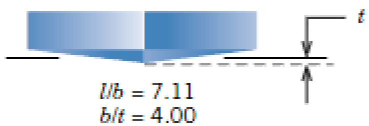
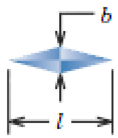
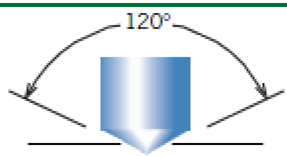
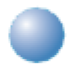
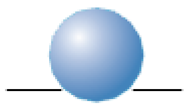
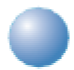



**Table 7.5 Hardness-Testing Techniques**

| Test | Indenter | Shape of Indentation: Side View | Shape of Indentation: Top View | Load | Formula for Hardness Number[a] |
|---|---|---|---|---|---|
| Brinell | 10-mm sphere of steel or tungsten carbide |  |  | $P$ | $HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$ |
| Vickers microindentation | Diamond pyramid |  |  | $P$ | $HV = 1.854P/d_1^2$ |
| Knoop microindentation | Diamond pyramid |  |  | $P$ | $HK = 14.2\,P/l^2$ |
| Rockwell and Superficial Rockwell | Diamond cone; $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres |  |  | 60 kg, 100 kg, 150 kg: Rockwell | |
| | |  |  | 15 kg, 30 kg, 45 kg: Superficial Rockwell | |

**Hardness tests**

Differences with respect to the Tensile test:

- Compressive rather than Tensile
- Plastic deformation and No fracture
- Multiaxial (stress) instead of Uniaxial
- Small surface portion of material instead of bulk material
- Result dependent on the Standard definition of load and indenter size

Nevertheless a linear relationship is remarkably accurate (only for steels):

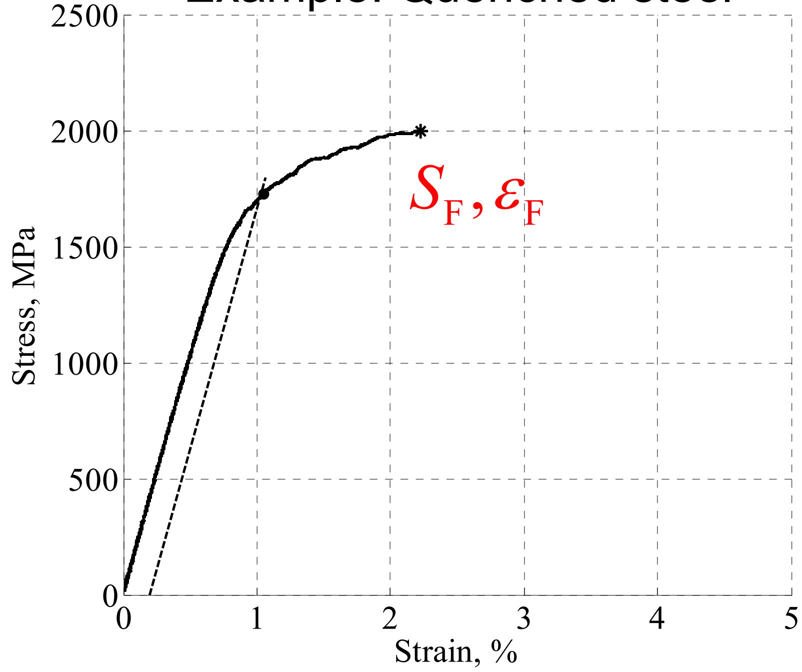
$$S_U = 3.45\,\mathrm{HB}$$

**Metals can be (broadly) distinguished into:**

- Ductile, elongation at fracture > 5%
- Brittle, elongation at fracture < 5%

*Usually brittle metals do not reach the Necking*

Example: Quenched steel



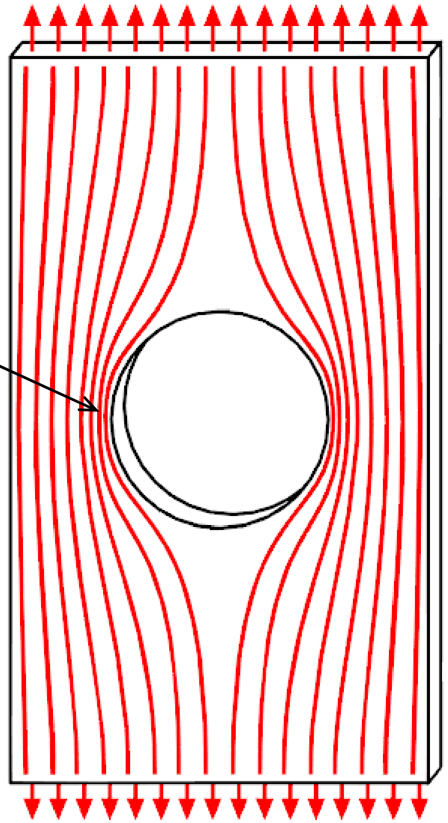
## Different criteria for Ductile/ Brittle metals

- Ductile:
  - Plastic collapse
  - Ductility exhaustion
- Brittle:
  - Fracture

## Stress Concentration – Force flux

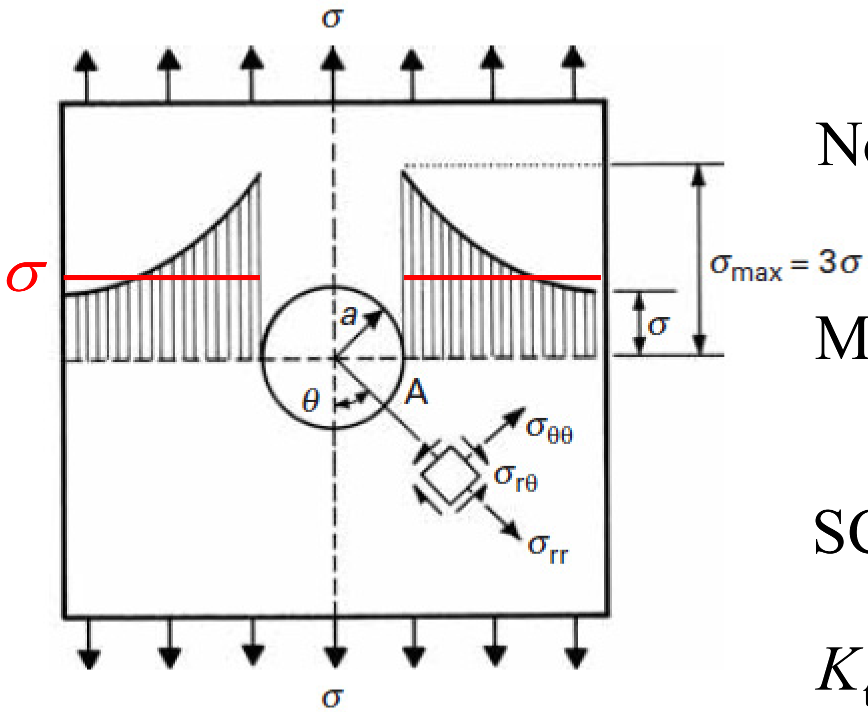
Central hole in a plate

Stress concentrates at the notch apex
either a circle or any other concave shape



## Stress Concentration Factor

Central hole in a plate




Nominal stress: $\sigma, \sigma_n, \sigma_0$ (force/area)

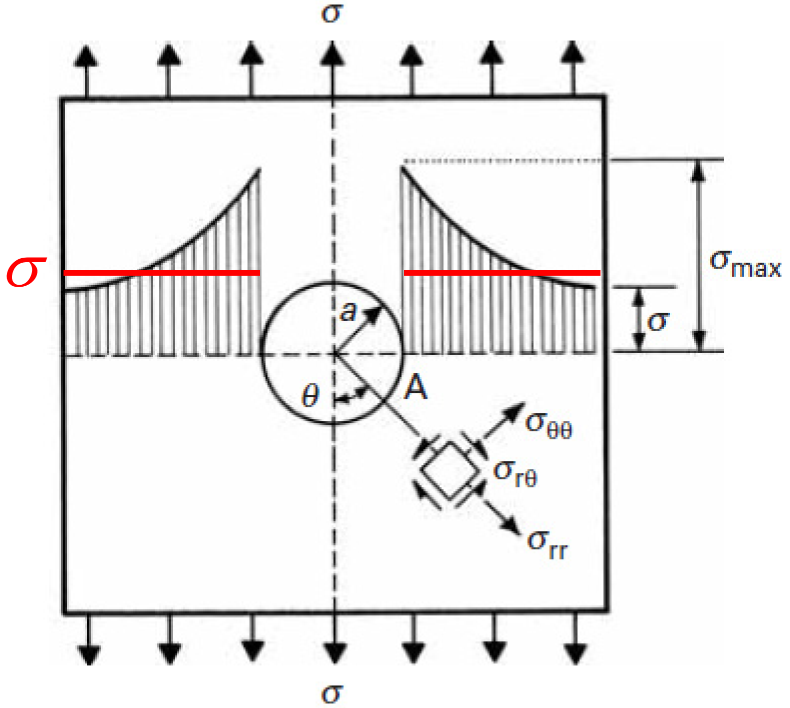
Maximum stress: $\sigma_{max}$ (peak value)

SCF:

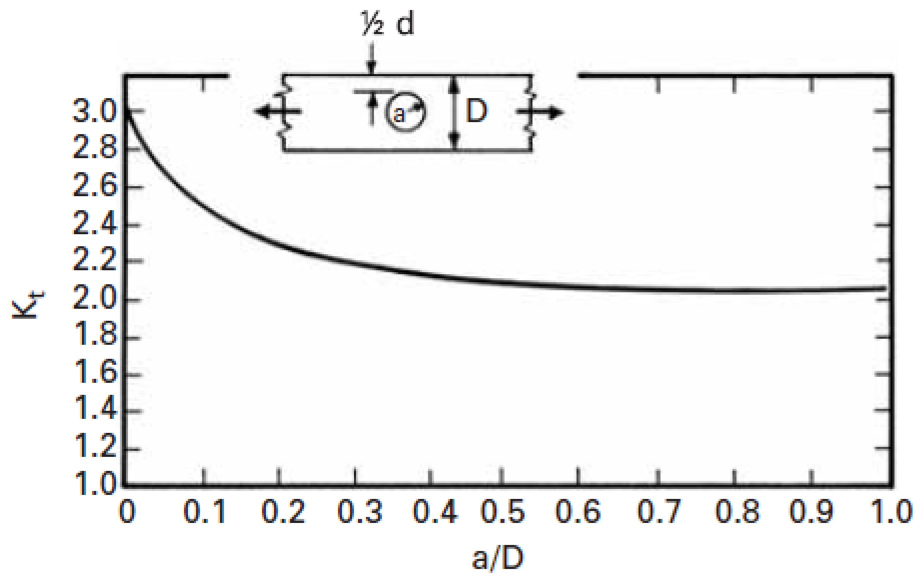
$$K_t = \frac{\sigma_{max}}{\sigma}$$

## Stress Concentration Factor

Central hole in a plate

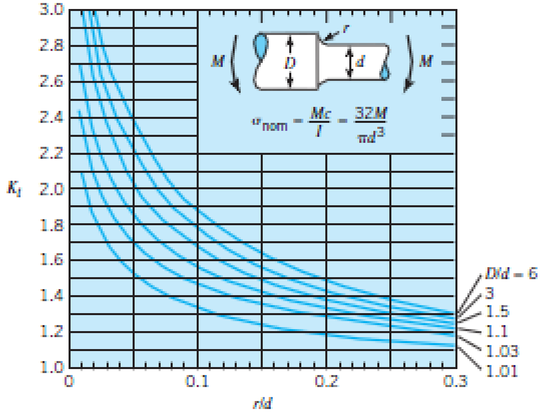


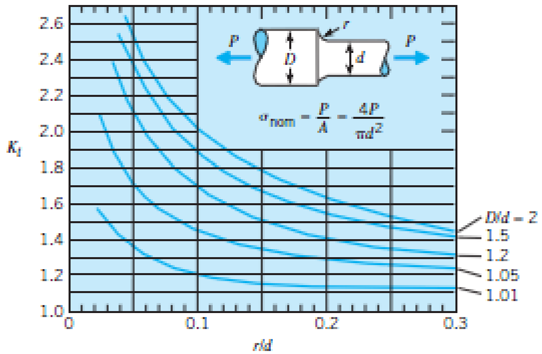

$$K_t = \frac{\sigma_{max}}{\sigma}$$

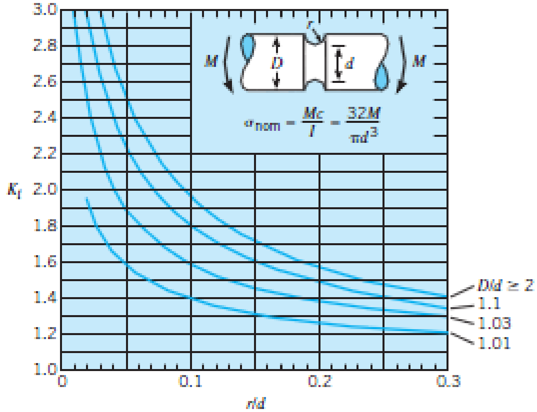


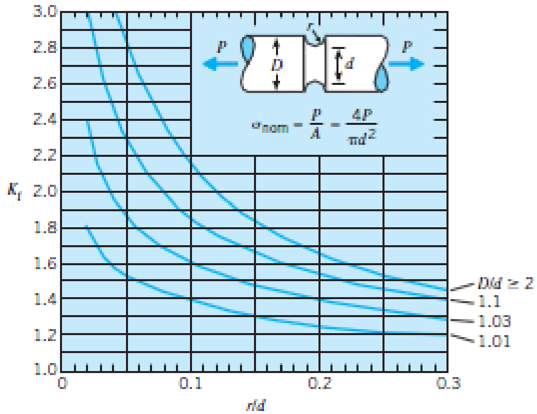
## Stress Concentration Factor

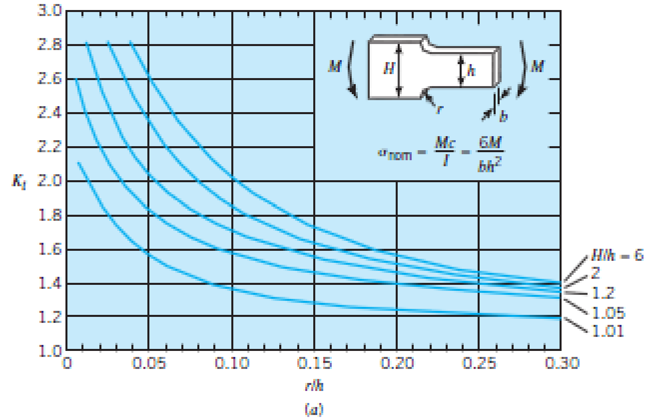
Many tables and graph for several cases

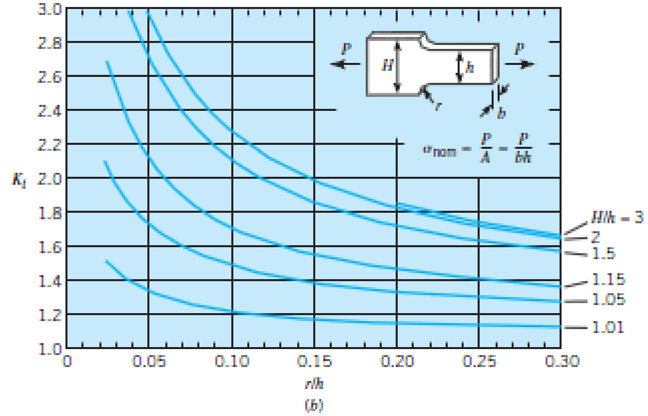


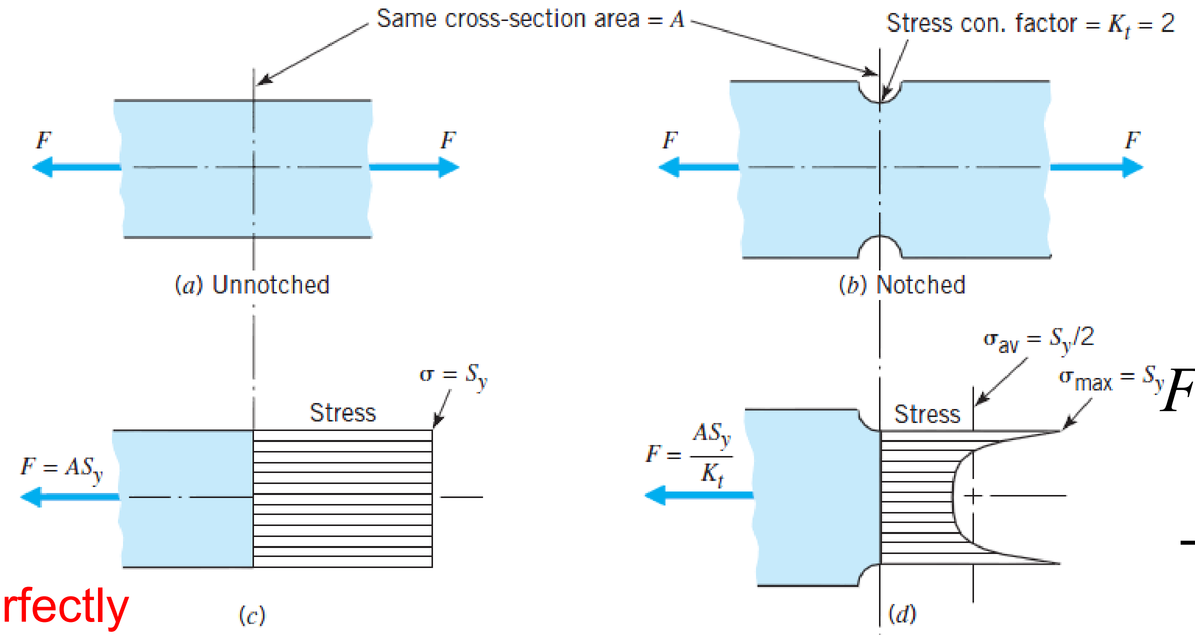



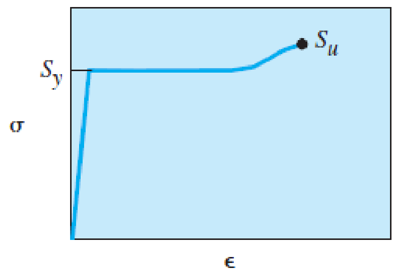



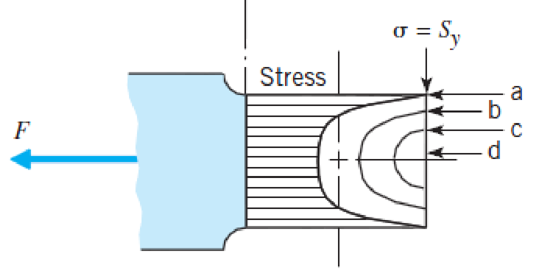



## Plastic collapse

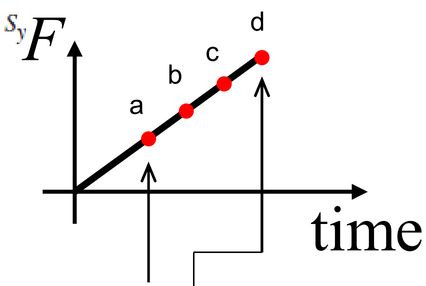



Elastic perfectly plastically Model



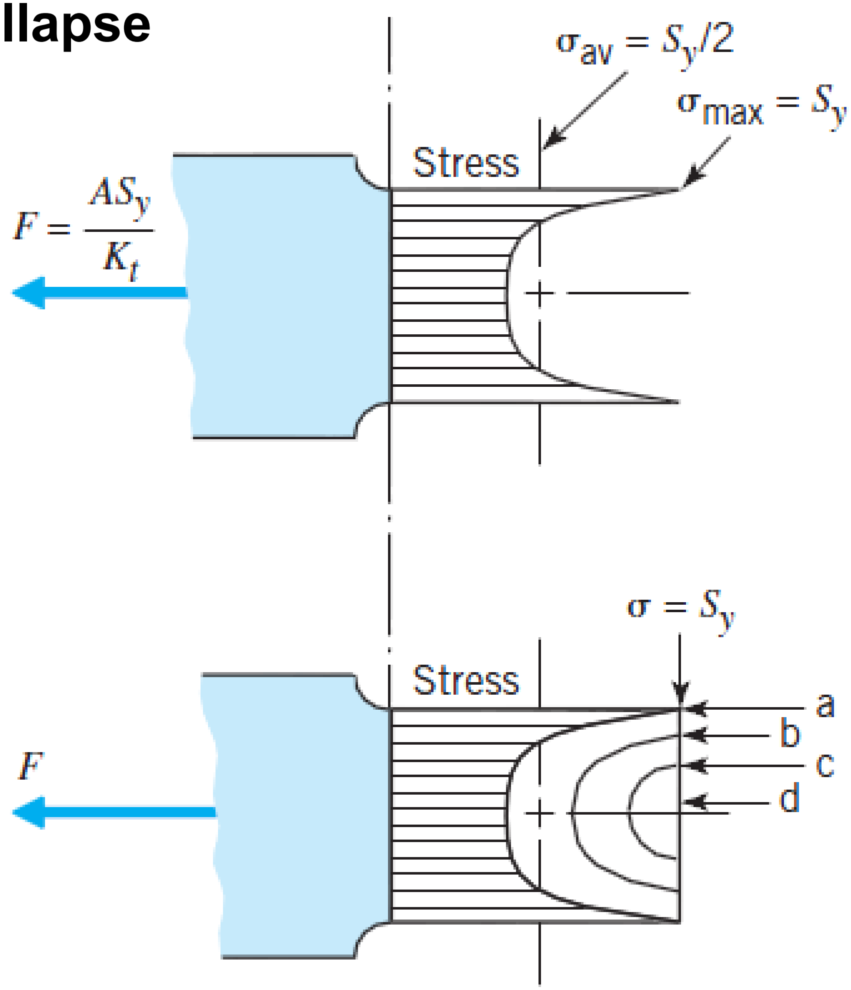

Different stages of the load




Plasticity onset

Plastic collapse

## Plastic collapse

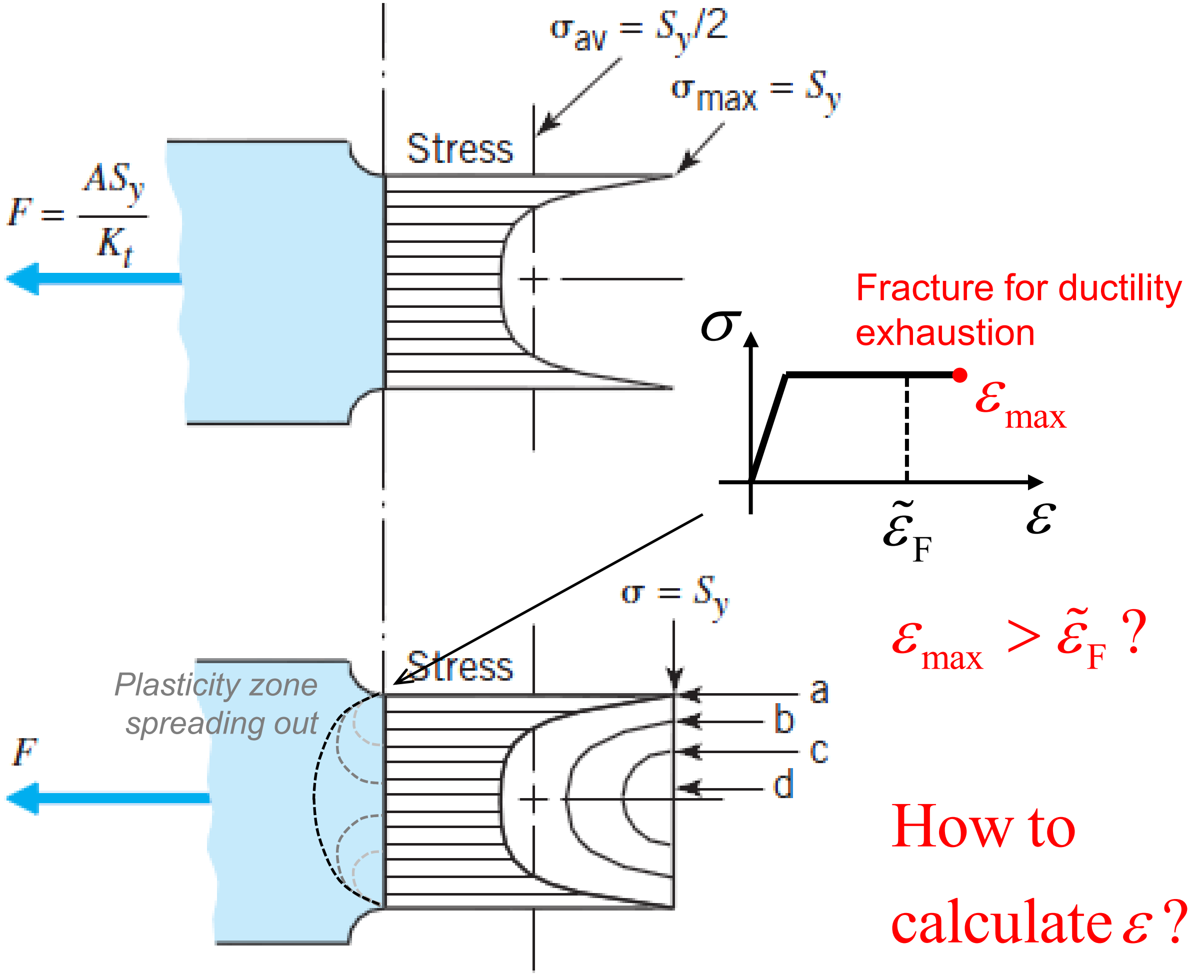



$$F = A\,S_Y$$

At plastic collapse the ultimate force does not depend on the Stress Concentration Factor

## Ductility exhaustion

The fracture could happen before the Plastic Collapse, if the strain reaches the (true) elongation at fracture

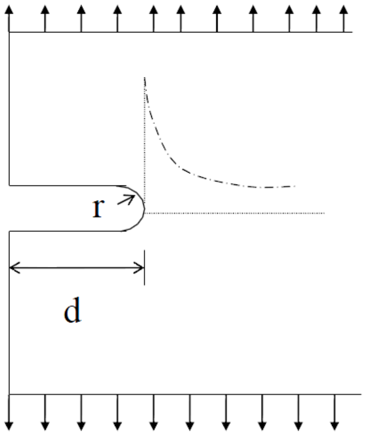


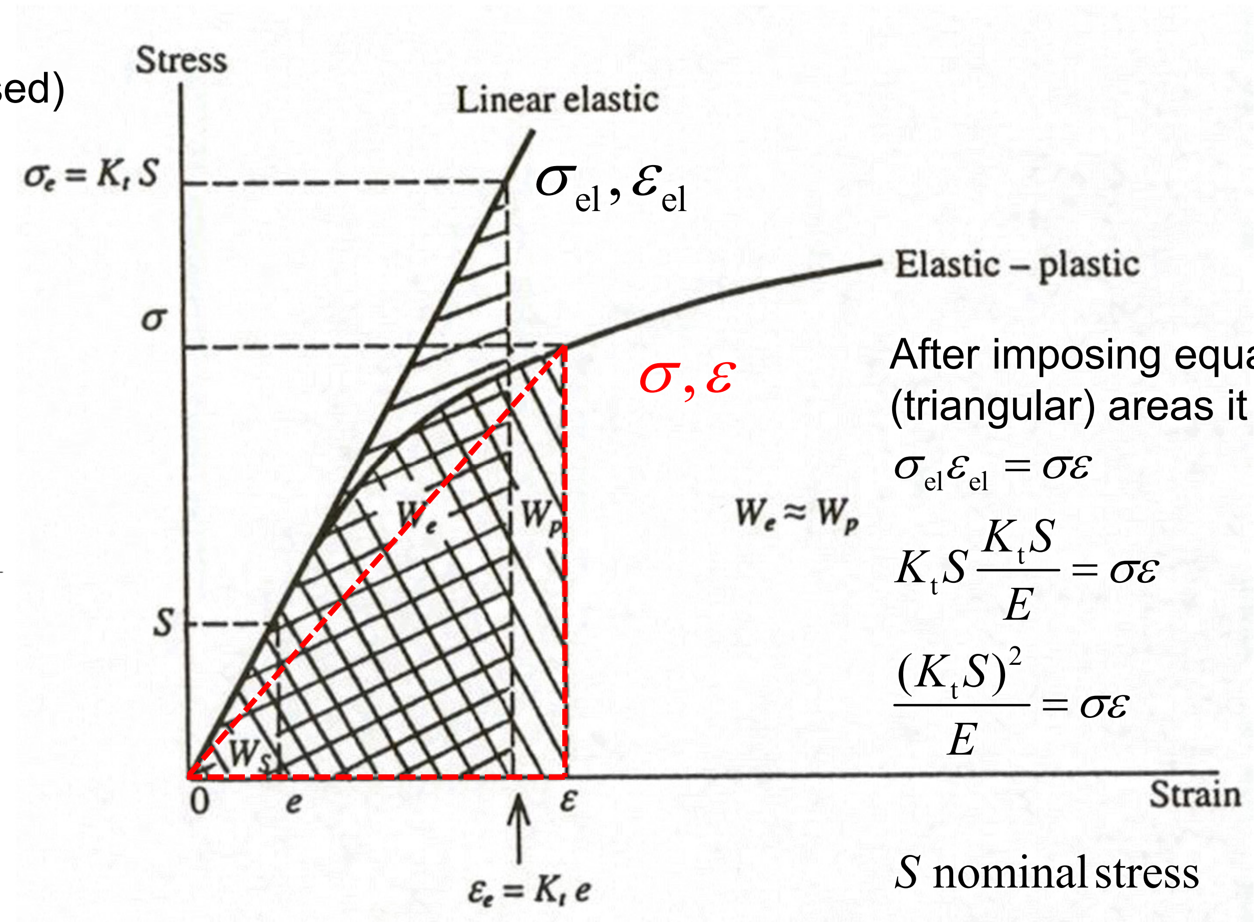

$\varepsilon_{max} > \tilde{\varepsilon}_F$ ?

How to calculate $\varepsilon$ ?

## The Neuber's rule (1946)

Any kind of (radiused) notch




After imposing equal the (triangular) areas it follows:

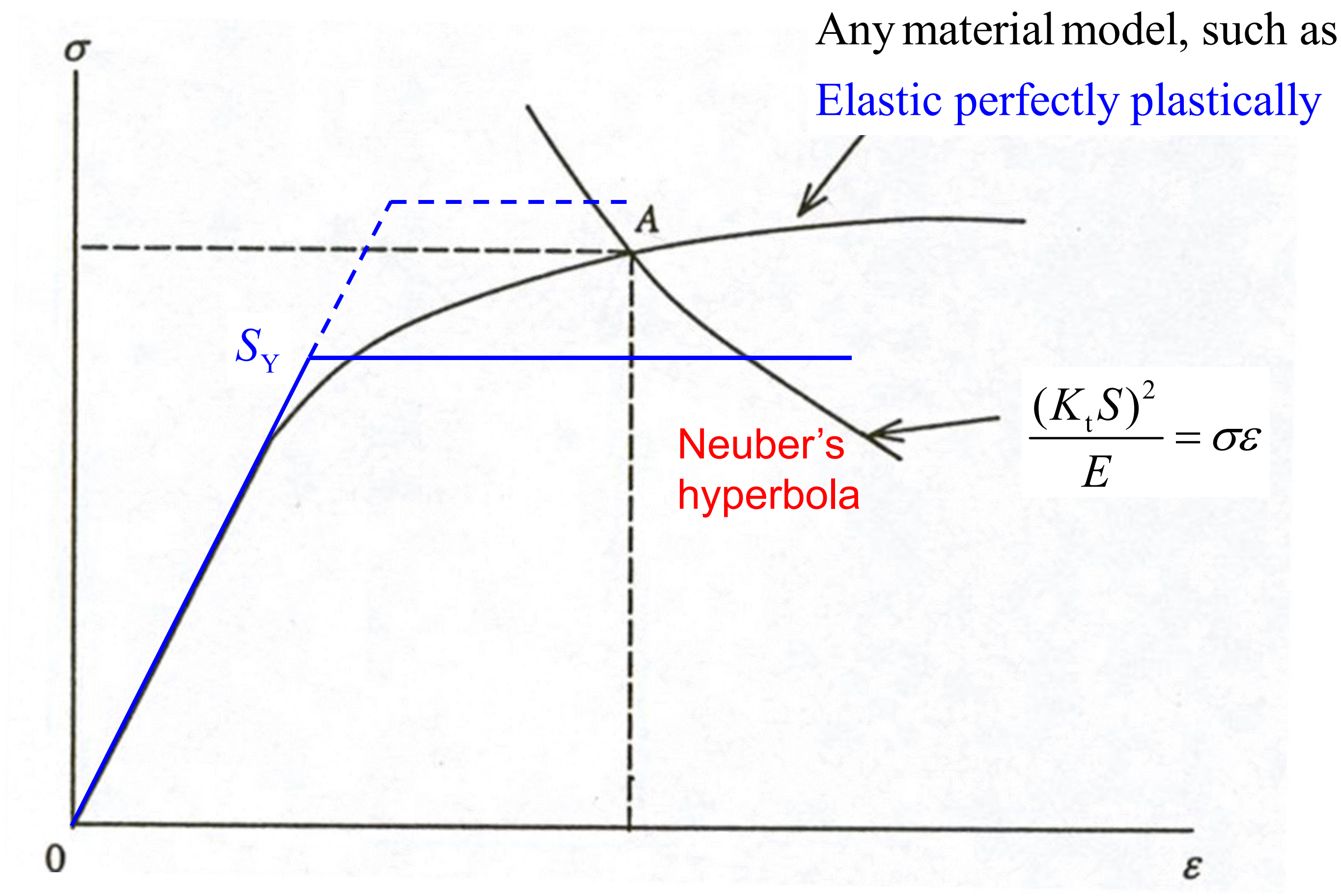
$$\sigma_{\mathrm{el}}\varepsilon_{\mathrm{el}} = \sigma\varepsilon$$

$$K_t S \frac{K_t S}{E} = \sigma\varepsilon$$

$$\frac{(K_t S)^2}{E} = \sigma\varepsilon$$

$S$ nominal stress

## The Neuber's rule (1946)



## Plastic collapse/ Neuber's rule example

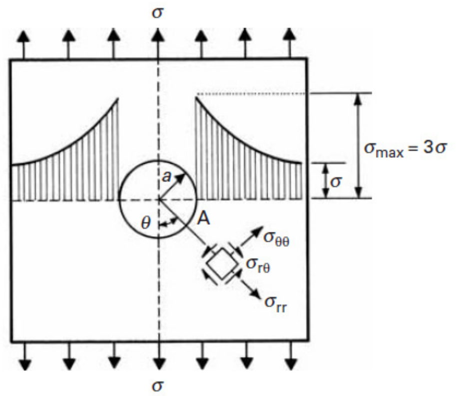
Steel Fe360-S235

$S_Y = 235\,\text{MPa}$

$E = 205\,\text{GPa}$

$RA\% = 50\%$

$K_t = 5.0\,(\text{any shape})$




By increasing the load, what happens first:

Plastic collapse or

Ductility exhaustion?

## Plastic collapse/ Neuber's rule example

Steel Fe360-S235

$S_Y = 235\,\text{MPa}$

$E = 205\,\text{GPa}$

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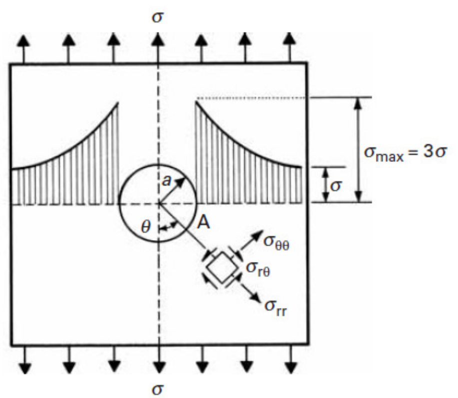
$K_t = 5.0\,(\text{any shape})$




$$\tilde{\varepsilon}_F = \ln\left(\frac{1}{1 - RA\%/100}\right) = 0.69$$

Assuming to have plastixc collapse first:

$$\sigma = S_Y$$

Neuber:

$$\sigma_{max}\varepsilon_{max} = \frac{(K_t\sigma)^2}{E}$$

**Plastic collapse/ Neuber's rule example**

Steel Fe360-S235

$S_Y = 235\,\text{MPa}$

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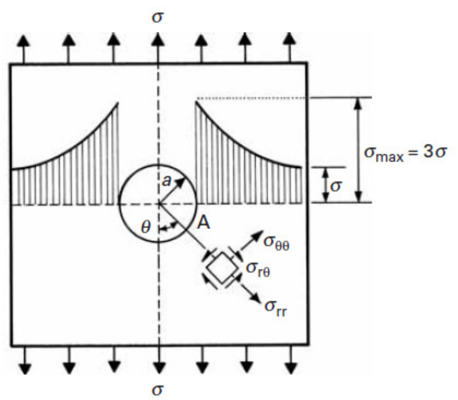


then, assuming elastic prefectly plastic material model:

$\sigma_{max} = S_Y$

finally, $\varepsilon_{max}$ can be solved:

$$\varepsilon_{max} = \frac{K_t^2 S_Y}{E} = 0.029$$

being $\varepsilon_{max} < \tilde{\varepsilon}_F$

plastic collapse happens first

## Plastic collapse/ Neuber's rule example

Steel Fe360-S235

$S_Y = 235\,\text{MPa}$

$E = 205\,\text{GPa}$

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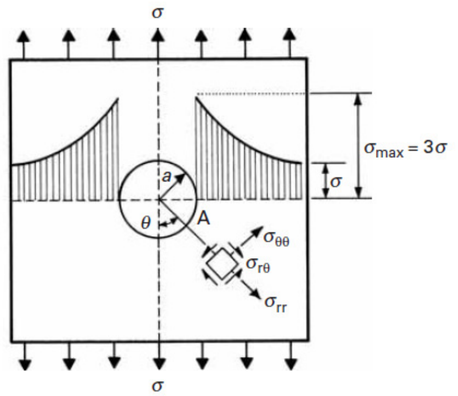



Homework:

1. What if a different steel is considered:

$S_Y = 1700\ \text{MPa and}\ \tilde{\varepsilon}_F = 0.08$

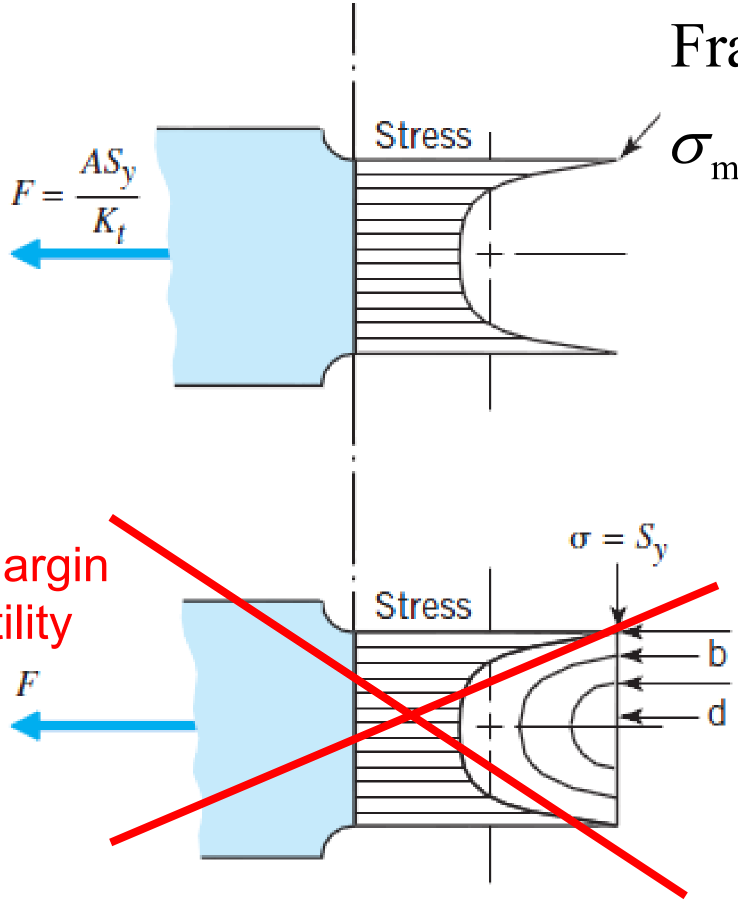
2. Which is the (minimum) $K_t$ to have ductility exhaustion first?

**The maximum stress just induces fracture**

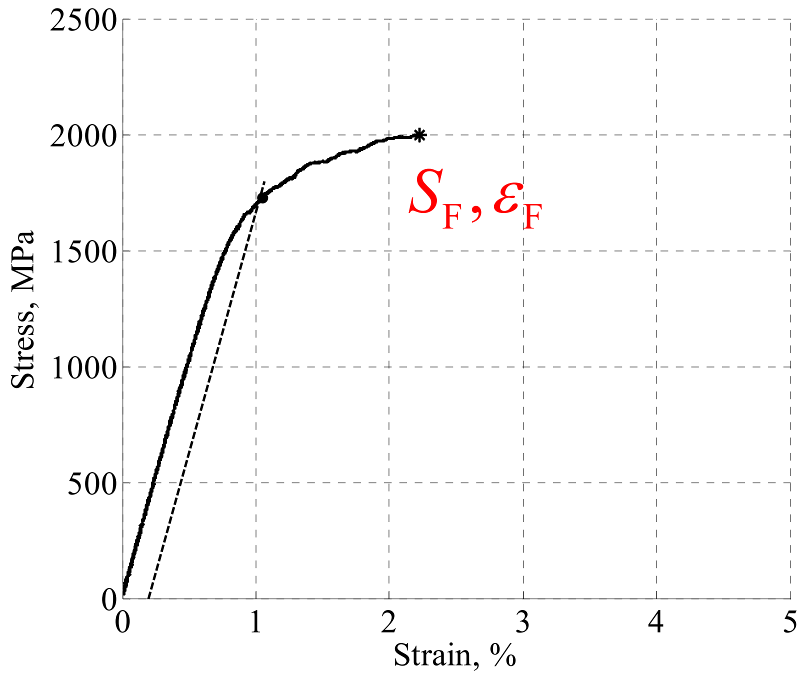
The SCF has a direct effect on fracture. Ductile metals are usually preferred than Brittle

Fracture:

$$\sigma_{max} = K_t \sigma > S_F$$

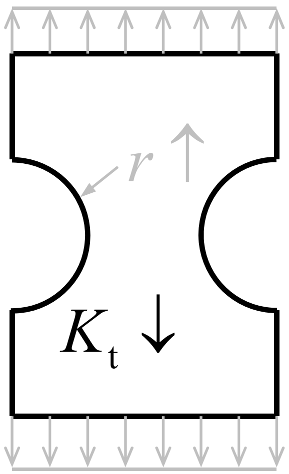



No more margin due to ductility

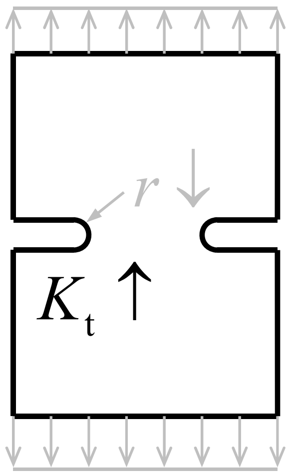


## Different levels of stress concentration severity

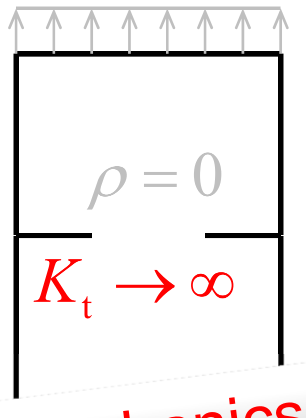
Ductile,
<u>blunt</u> notch
Plastic
collapse




Ductile,
<u>sharp</u> notch
Plastic
Collapse or
Ductility
exhaustion

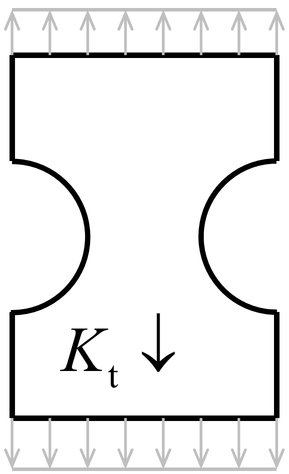



Ductile,
<u>Crack</u> notch
How to
predict the
strength?

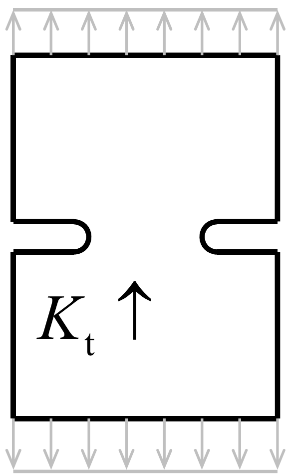



**Fracture Mechanics !**

Brittle,
<u>blunt</u> notch
Fracture




Brittle,
<u>sharp</u> notch
Fracture




Brittle,
<u>Crack</u> notch
How to
predict the
strength?