Efficient procedure for the identification of the Chaboche isotropic-kinematic hardening model parameters

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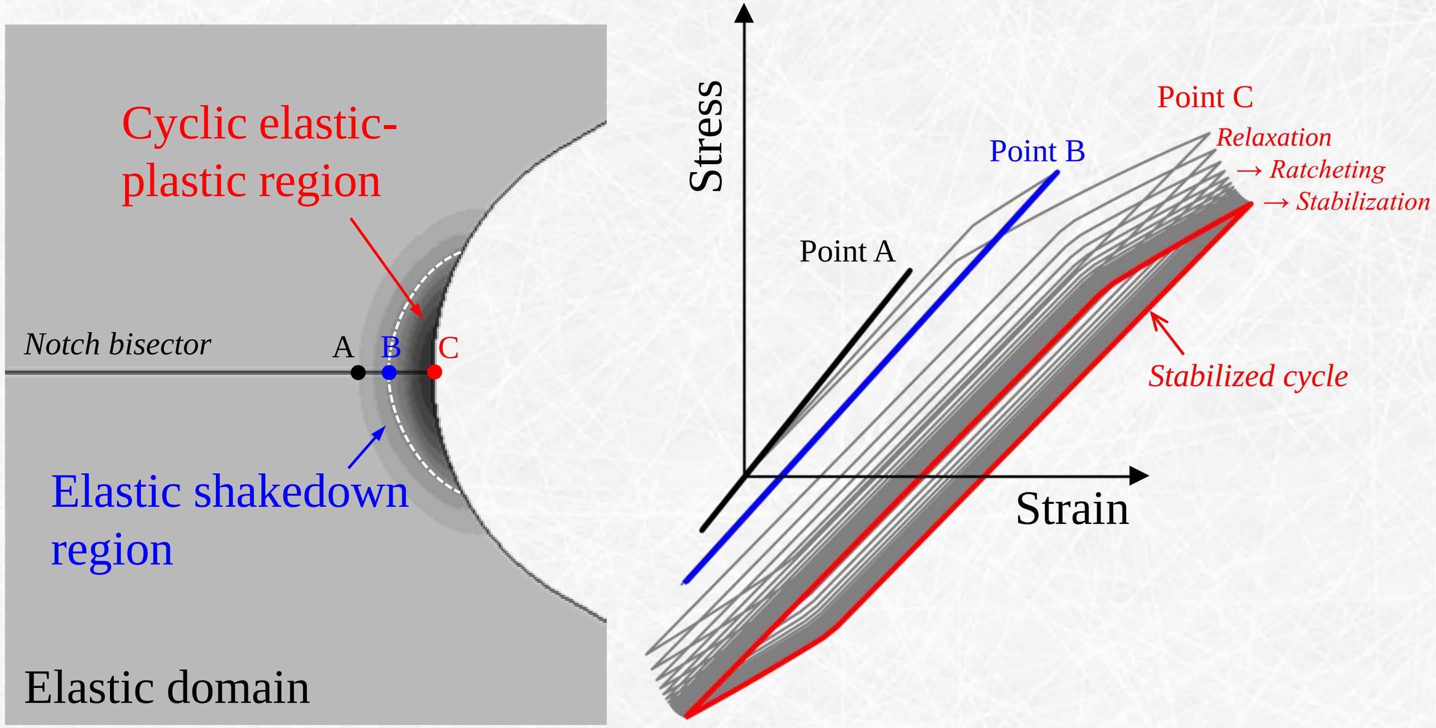
Why?

Cyclic plasticity may occur in the area around the notch, even in HCF



Chaboche backstress model:
(KINEMATIC model)

$$\chi = \sum_{i=1}^{n} \chi_i$$

$$\mathrm{d}\chi_i = C_i \mathrm{d}\varepsilon_\mathrm{p} - \gamma_i \chi_i \left|\mathrm{d}\varepsilon_\mathrm{p}\right|$$

Voce hardening law:
(ISOTROPIC model)

$$p = \int \left|\mathrm{d}\varepsilon_\mathrm{p}\right|$$

$$\sigma_\mathrm{Y} = \sigma_0 + Q\left(1 - e^{-bp}\right)$$

$$\sigma_\mathrm{Y}(p \to \infty) = \sigma_\mathrm{L}$$

A brief review of the Literature…

Examples of complex algorithms

"Sensitivity and optimisation of the Chaboche plasticitymodel parameters in strain-life fatigue predictions", Kourousis et. al., Materials and Design 118 (2017) 107–121

"Parameter determination of Chaboche kinematic hardening model using a multi objective Genetic Algorithm", Mahmoudi et. al. Computational Materials Science 50 (2011) 1114–1122

Genetic Algorithms

Objective function: **pointwise** optimizazion between the modeled curve and the experimental

$$Rss1 = Min \frac{1}{K} \sum_{i=1}^{K} \left[\frac{\left(\sigma_i^{exp} - \sigma_i^{model}\right)}{\sigma_i^{exp}} \right]^2$$

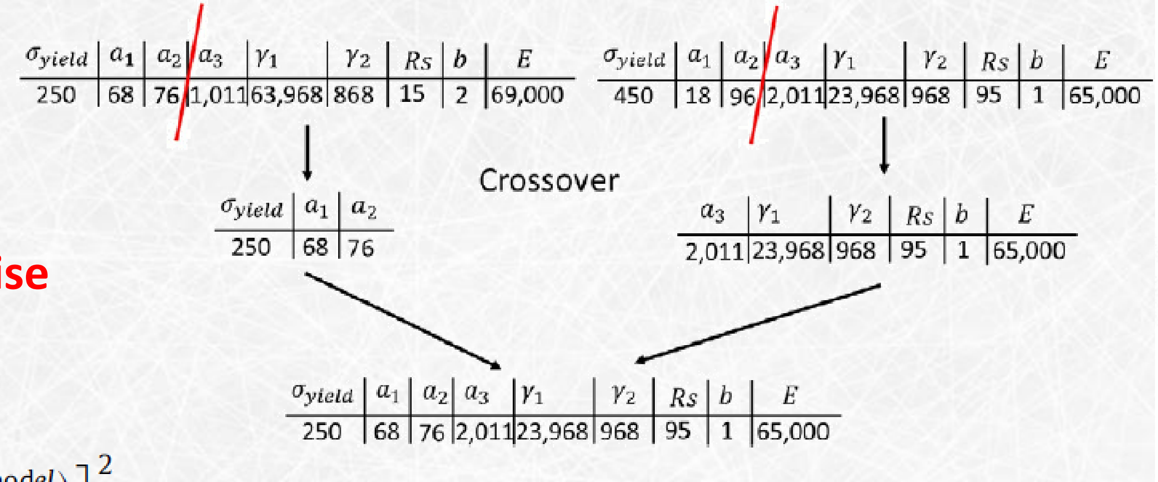
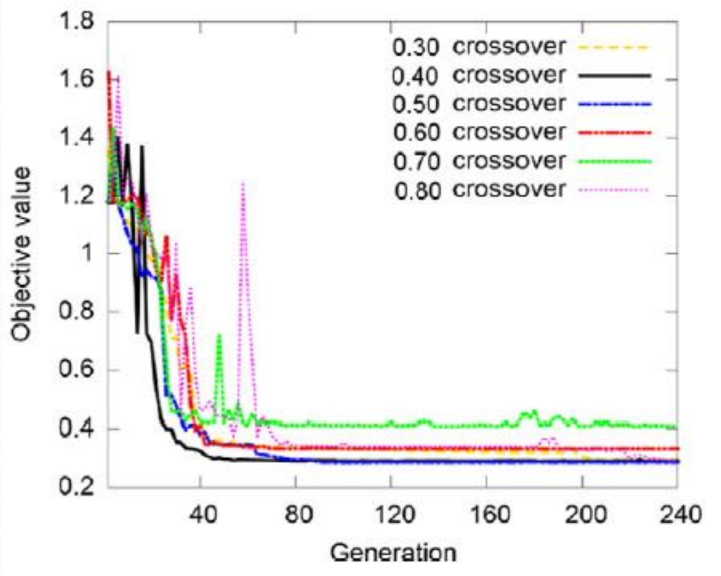


Fig. 10. Crossover of genes in two parameter sets.



Why consider a different approach?

Advantages of the proposed procedure:

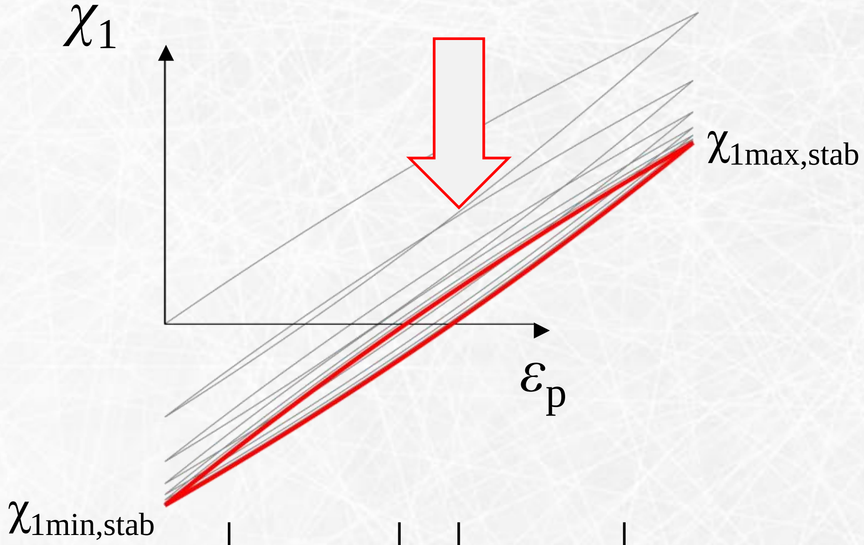
- No complex optimizazions
- Focused on stabilized cycles (more interesting for fatigue analysis)
- It aims to reproduce the global properties of the stabilized cycle (peak to peak stress, average stress, slope at the inversion points, hysteresis area) and ratcheting-rate too

Experimental data needed:

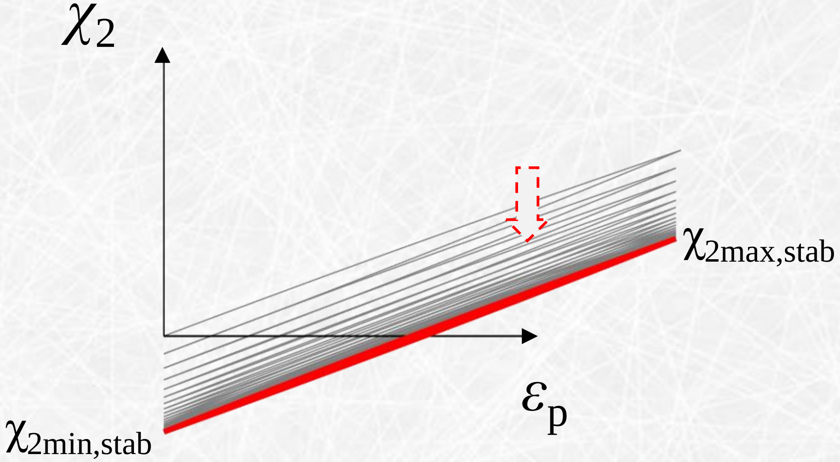
- At least two stabilized cycles obtained by strain-controlled tests with different strain amplitude and average strain (relaxation tests)
- One stress-controlled test (ratcheting test)

Chaboche Kinematic Hardening model with 3 Backstresses

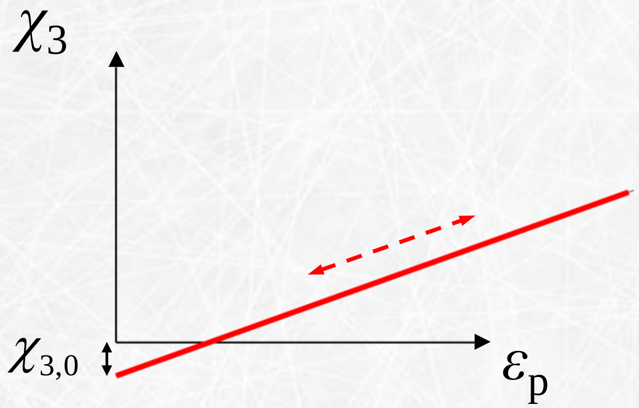
Example of a strain-controlled test with $R_{\varepsilon p}=0$



$$\left|\chi_{1max,stab}\right| = \left|\chi_{1min,stab}\right|$$



$$\left|\chi_{2max,stab}\right| = \left|\chi_{2min,stab}\right|$$



Backstress with rapid dynamics

γ_1

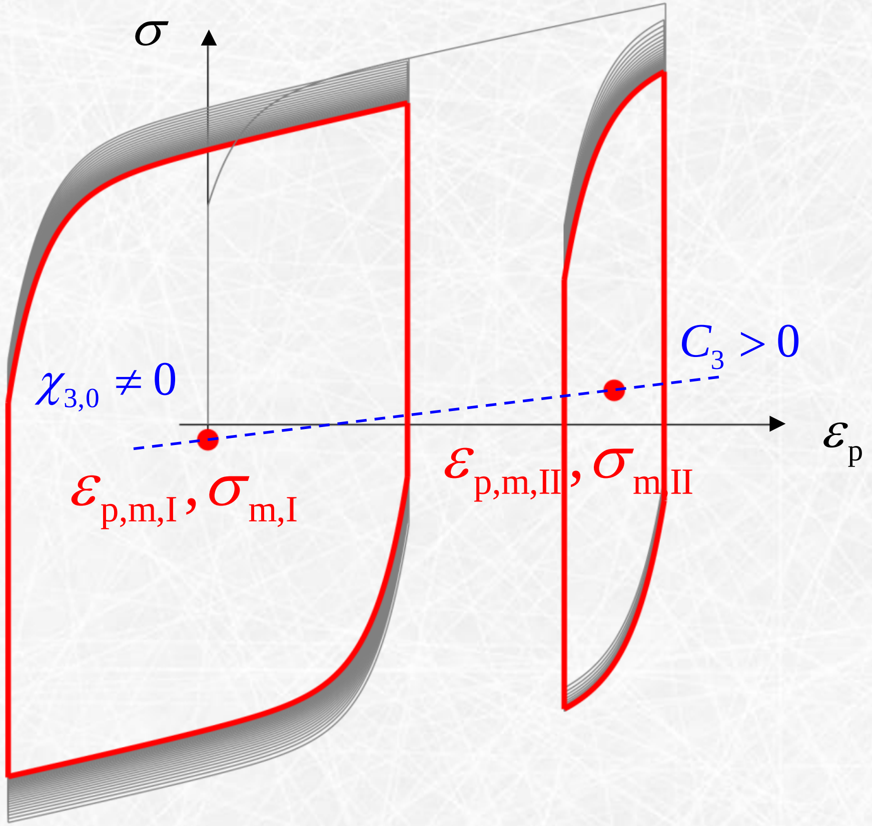
Backstress with slow dynamics

$\gamma_2 << \gamma_1$

Linear Backstress

$\gamma_3 = 0$

How to calculate C_3 and $\chi_{3,0}$



Experimental stabilized cycles in red

$$\sigma_{max,stab} = \sigma_L + \chi_{1max,stab} + \chi_{2max,stab} + \chi_{3max,stab}$$

$$\sigma_{min,stab} = -\sigma_L + \chi_{1min,stab} + \chi_{2min,stab} + \chi_{3min,stab}$$

⇓

$$\sigma_m = \frac{\chi_{3max,stab} + \chi_{3min,stab}}{2}$$

⇓

$$\chi_{3,0} + C_3\, \varepsilon_{p,m,I} = \sigma_{m,I}$$

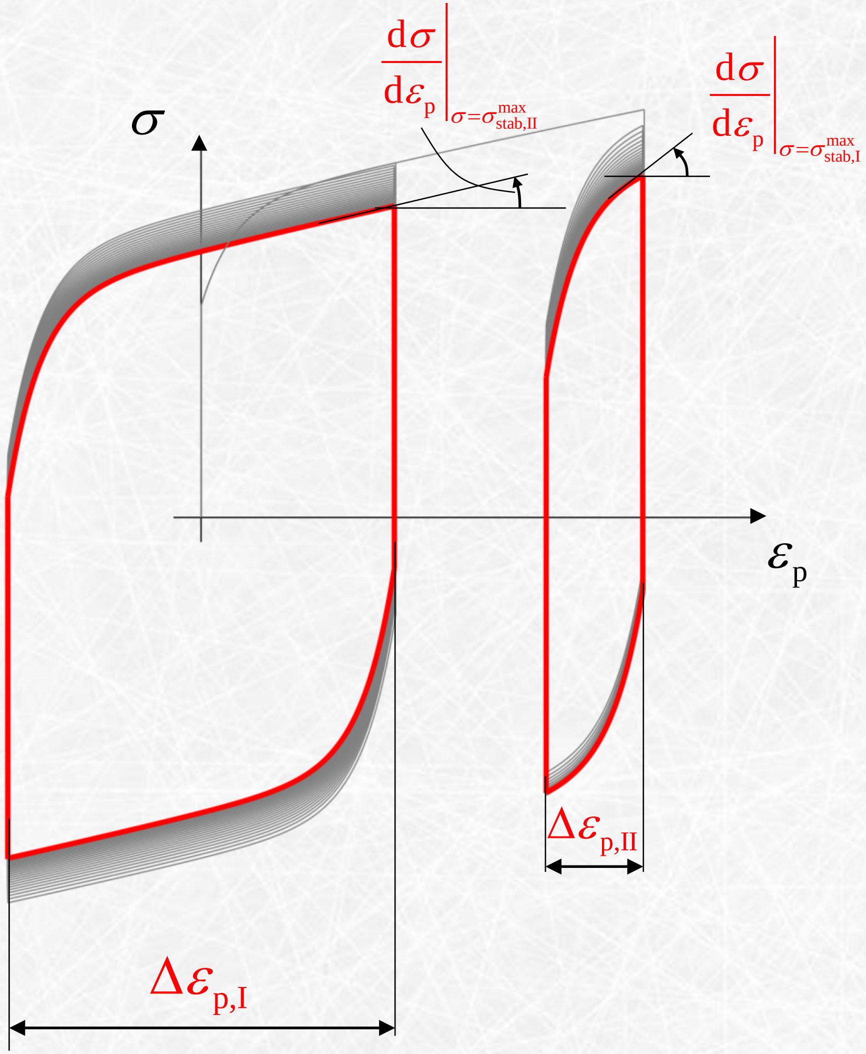
$$\chi_{3,0} + C_3\, \varepsilon_{p,m,II} = \sigma_{m,II}$$

⇓

$$C_3, \chi_{3,0}$$

The C_3 value is generally *much lower* than C_1 and C_2

How to calculate C_1 and C_2



Assuming $\gamma_2 \Delta\varepsilon_p << 1$

$$C_1\left(1-\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{p,I}}{2}\right)\right)+C_2 = -C_3 + \frac{d\sigma}{d\varepsilon_p}\Big|_{\sigma=\sigma_{stab,I}^{max}}$$

$$C_1\left(1-\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{p,II}}{2}\right)\right)+C_2 = -C_3 + \frac{d\sigma}{d\varepsilon_p}\Big|_{\sigma=\sigma_{stab,II}^{max}}$$

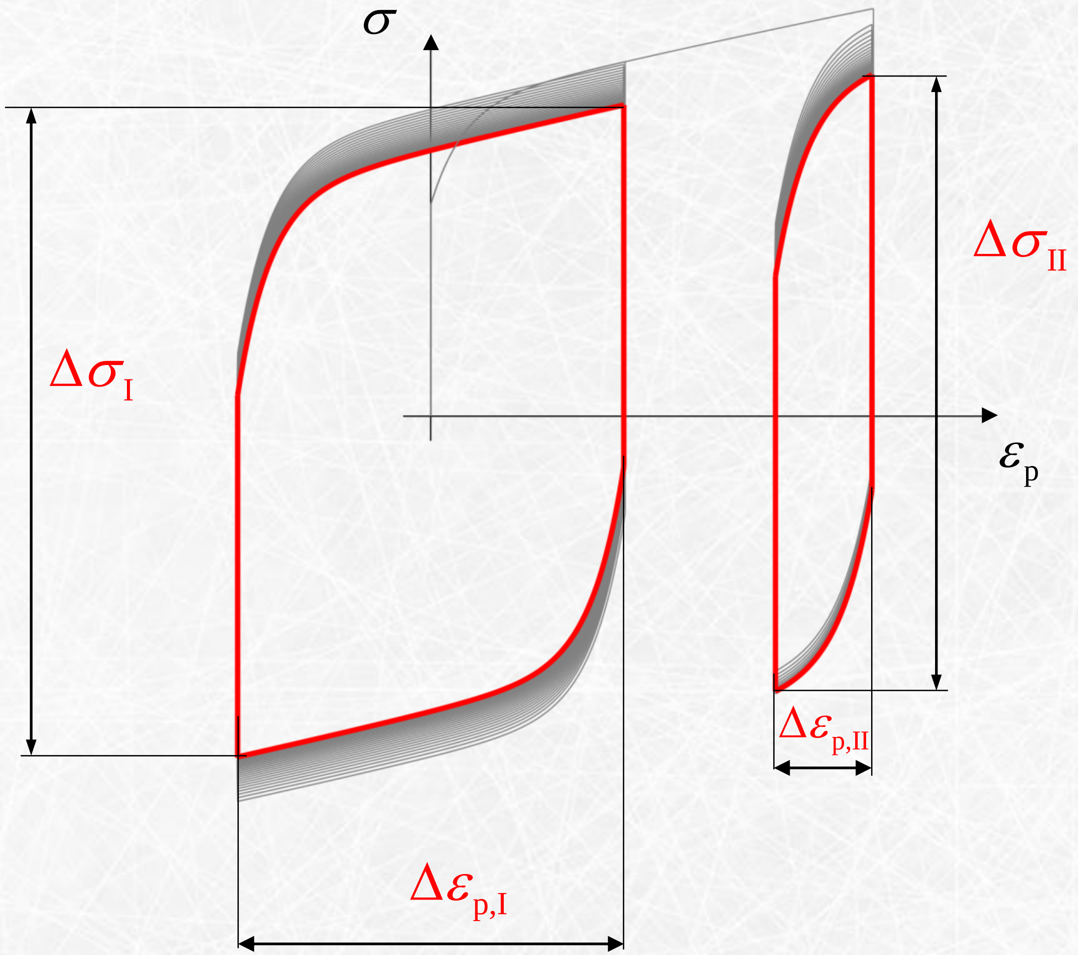
⇩

2×2 system depending on γ_1:

$$C_1 = f(\gamma_1)$$
$$C_2 = g(\gamma_1)$$

γ_2 not involved

How to calculate σ_L



$$\Delta\sigma = \sum_{i=1}^{2} 2\frac{C_i}{\gamma_i}\tanh\left(\frac{\gamma_i\,\Delta\varepsilon_\mathrm{p}}{2}\right) + C_3\,\Delta\varepsilon_\mathrm{p} + 2\sigma_\mathrm{L}$$

Assuming $\gamma_2\,\Delta\varepsilon_\mathrm{p} \ll 1$

⇓

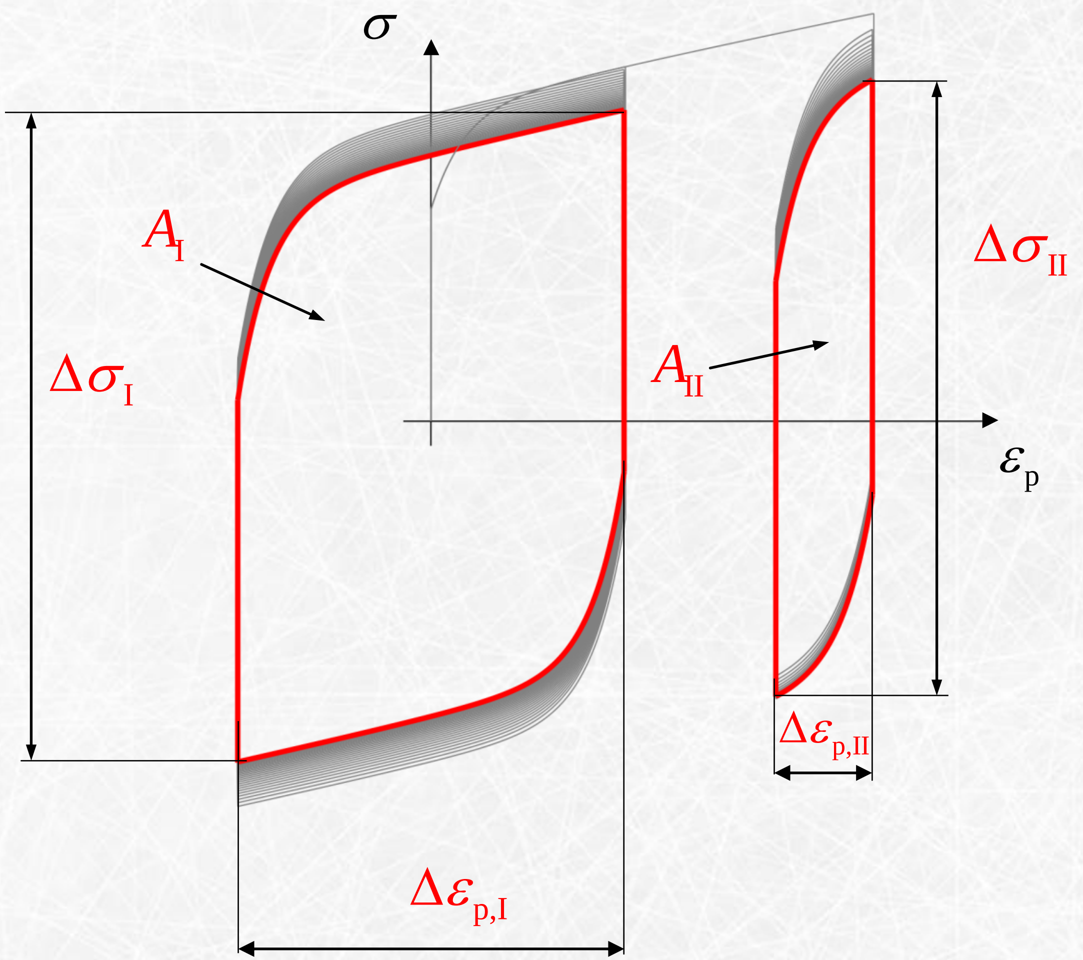
$$\sigma_\mathrm{L,I} = \frac{\Delta\sigma_\mathrm{I}}{2} - \frac{C_1}{\gamma_1}\tanh\left(\frac{\gamma_1\,\Delta\varepsilon_\mathrm{p,I}}{2}\right) - \frac{C_2 + C_3}{2}\Delta\varepsilon_\mathrm{p,I}$$

$$\sigma_\mathrm{L,II} = \frac{\Delta\sigma_\mathrm{II}}{2} - \frac{C_1}{\gamma_1}\tanh\left(\frac{\gamma_1\,\Delta\varepsilon_\mathrm{p,II}}{2}\right) - \frac{C_2 + C_3}{2}\Delta\varepsilon_\mathrm{p,II}$$

⇓

$$\sigma_\mathrm{L} = \frac{\sigma_\mathrm{L,I} + \sigma_\mathrm{L,II}}{2} = h(\gamma_1)$$

How to calculate the hysteresis area of the stabilized cycle



$$A = \oint \sigma \, \mathrm{d}\varepsilon_p = 2\sigma_{\mathrm{L}}\Delta\varepsilon_{\mathrm{p}} + 2\sum_{i=1}^{2}\left(\frac{C_i}{\gamma_i}\Delta\varepsilon_{\mathrm{p}} - 2\frac{C_i}{\gamma_i^2}\tanh\left(\frac{\gamma_i\Delta\varepsilon_{\mathrm{p}}}{2}\right)\right)$$

Assuming $\gamma_2 \, \Delta\varepsilon_{\mathrm{p}} << 1$

⇩

$$A_{\mathrm{I}}^{\mathrm{mod}} = 2\sigma_{\mathrm{L}}\Delta\varepsilon_{\mathrm{p,I}} + 2\left(\frac{C_1}{\gamma_1}\Delta\varepsilon_{\mathrm{p,I}} - 2\frac{C_1}{\gamma_1^2}\tanh\left(\frac{\gamma_1\Delta\varepsilon_{\mathrm{p,I}}}{2}\right)\right)$$

$$A_{\mathrm{II}}^{\mathrm{mod}} = 2\sigma_{\mathrm{L}}\Delta\varepsilon_{\mathrm{p,II}} + 2\left(\frac{C_1}{\gamma_1}\Delta\varepsilon_{\mathrm{p,II}} - 2\frac{C_1}{\gamma_1^2}\tanh\left(\frac{\gamma_1\Delta\varepsilon_{\mathrm{p,II}}}{2}\right)\right)$$

$A_{\mathrm{I}}^{\mathrm{mod}}$ and $A_{\mathrm{II}}^{\mathrm{mod}}$ depend both on γ_1

How to calculate γ_1 1/2

Three functions are defined:

- An error function that measures the error in predicting the cycle amplitudes
- The relative error on the prediction of the hysteresis area of stabilized cycle I
- The relative error on the prediction of the hysteresis area of stabilized cycle II

All of these depend on the parameter γ_1

$$\Sigma(\gamma_1) = \left|\frac{\sigma_{\mathrm{L,I}} - \sigma_{\mathrm{L,II}}}{\sigma_{\mathrm{L}}}\right| = \frac{\frac{1}{2}(\Delta\sigma_{\mathrm{I}} - \Delta\sigma_{\mathrm{II}}) - \frac{C_1}{\gamma_1}\left(\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{\mathrm{p,I}}}{2}\right) - \tanh\left(\frac{\gamma_1 \Delta\varepsilon_{\mathrm{p,II}}}{2}\right)\right) - \frac{C_2 + C_3}{2}(\Delta\varepsilon_{\mathrm{p,I}} - \Delta\varepsilon_{\mathrm{p,II}})}{\frac{1}{4}(\Delta\sigma_{\mathrm{I}} - \Delta\sigma_{\mathrm{II}}) - \frac{C_1}{2\gamma_1}\left(\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{\mathrm{p,I}}}{2}\right) + \tanh\left(\frac{\gamma_1 \Delta\varepsilon_{\mathrm{p,II}}}{2}\right)\right) - \frac{C_2 + C_3}{4}(\Delta\varepsilon_{\mathrm{p,I}} - \Delta\varepsilon_{\mathrm{p,II}})}$$

$$\Lambda_{\mathrm{I}}(\gamma_1) = \frac{A_{\mathrm{I}}^{\mathrm{mod}} - A_{\mathrm{I}}}{A_{\mathrm{I}}} = \frac{2\sigma_{\mathrm{L}}\Delta\varepsilon_{\mathrm{p,I}} + 2\left(\frac{C_1}{\gamma_1}\Delta\varepsilon_{\mathrm{p,I}} - 2\frac{C_1}{\gamma_1^2}\tanh\left(\frac{\gamma_1\Delta\varepsilon_{\mathrm{p,I}}}{2}\right)\right)}{A_{\mathrm{I}}} - 1$$

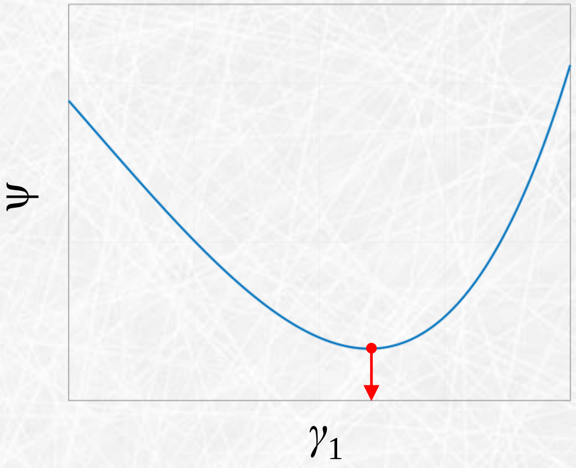
$$\Lambda_{\mathrm{II}}(\gamma_1) = \frac{A_{\mathrm{II}}^{\mathrm{mod}} - A_{\mathrm{II}}}{A_{\mathrm{II}}} = \frac{2\sigma_{\mathrm{L}}\Delta\varepsilon_{\mathrm{p,II}} + 2\left(\frac{C_1}{\gamma_1}\Delta\varepsilon_{\mathrm{p,II}} - 2\frac{C_1}{\gamma_1^2}\tanh\left(\frac{\gamma_1\Delta\varepsilon_{\mathrm{p,II}}}{2}\right)\right)}{A_{\mathrm{II}}} - 1$$

How to calculate γ_1 2/2

A global error function is defined by combining them:

$$\Psi(\gamma_1) = (1-\alpha)\Sigma^2 + \alpha(\Lambda_{\mathrm{I}}^2 + \Lambda_{\mathrm{II}}^2)$$

$$0 \le \alpha \le 1$$



By finding γ_1 that minimizes ψ (γ1), σ_L, C_1, C_2 can be updated

$$C_1\left(1-\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{p,\mathrm{I}}}{2}\right)\right) + C_2 = -C_3 + \left.\frac{d\sigma}{d\varepsilon_p}\right|_{\sigma=\sigma_{\mathrm{stab,I}}^{\max}}$$

$$C_1\left(1-\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{p,\mathrm{II}}}{2}\right)\right) + C_2 = -C_3 + \left.\frac{d\sigma}{d\varepsilon_p}\right|_{\sigma=\sigma_{\mathrm{stab,II}}^{\max}}$$

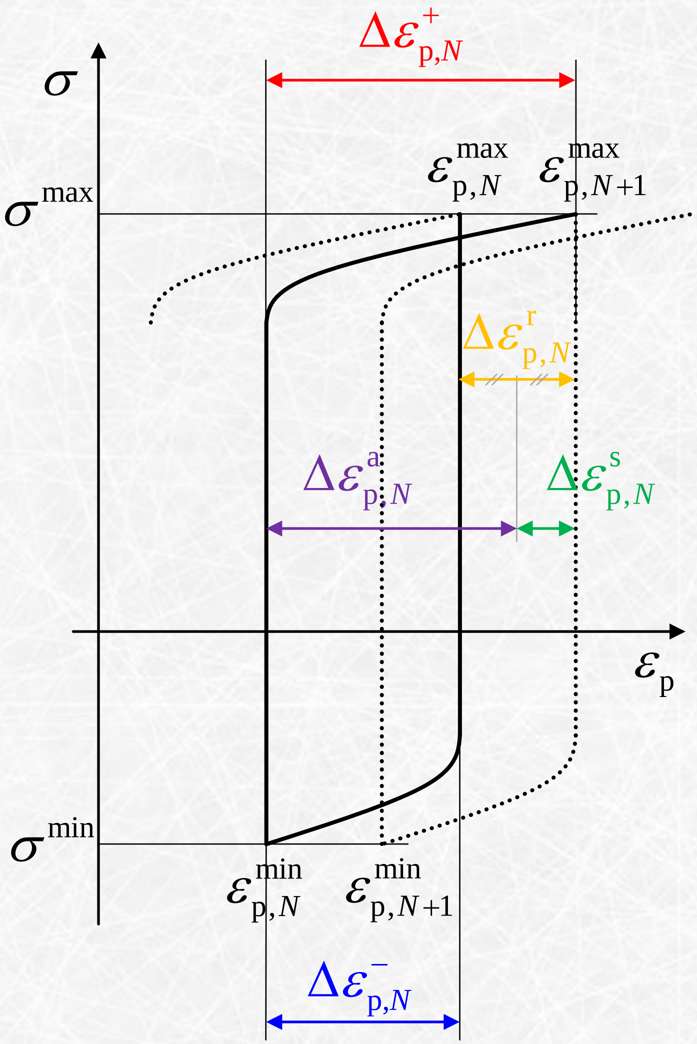
2×2 system

$$\sigma_{L,\mathrm{I}} = \frac{\Delta\sigma_{\mathrm{I}}}{2} - \frac{C_1}{\gamma_1}\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{p,\mathrm{I}}}{2}\right) - \frac{C_2 + C_3}{2}\Delta\varepsilon_{p,\mathrm{I}}$$

$$\sigma_{L,\mathrm{II}} = \frac{\Delta\sigma_{\mathrm{II}}}{2} - \frac{C_1}{\gamma_1}\tanh\left(\frac{\gamma_1 \Delta\varepsilon_{p,\mathrm{II}}}{2}\right) - \frac{C_2 + C_3}{2}\Delta\varepsilon_{p,\mathrm{II}}$$

$$\rightarrow \sigma_L = \frac{\sigma_{L,\mathrm{I}} + \sigma_{L,\mathrm{II}}}{2}$$

How to calculate γ_2 1/3



$$\begin{cases} \Delta\varepsilon^{a}_{p,N} = \dfrac{\Delta\varepsilon^{+}_{p,N} + \Delta\varepsilon^{-}_{p,N}}{2} \\ \Delta\varepsilon^{s}_{p,N} = \dfrac{\Delta\varepsilon^{+}_{p,N} - \Delta\varepsilon^{-}_{p,N}}{2} \\ \Delta\varepsilon^{r}_{p,N} = 2\Delta\varepsilon^{s}_{p,N} \end{cases}$$

According to the Chaboche model

$\Delta\varepsilon^{a}_{p,N}$ and $\Delta\varepsilon^{s}_{p,N}$ are constant

after a certain number of cycles (Lee et al., 2014 Int. J. Plast. , Paul 2012 Mater. Sci. Eng, Zhang et al. Int. J. Fatigue)

$$\Delta\varepsilon^{a}_{p,N} \rightarrow \Delta\varepsilon^{a}_{p}$$

$$\Delta\varepsilon^{s}_{p,N} \rightarrow \Delta\varepsilon^{s}_{p}$$

$$\Delta\varepsilon^{r}_{p,N} \rightarrow \Delta\varepsilon^{r}_{p}$$

How to calculate γ_2 2/3

With various rearrangements (assuming $C_3 = 0$):

$$\sigma_{\max} = \sum_{i=1}^{2} \chi_{i,\text{stab}}^{\max} + \sigma_{\text{L}} = \sum_{i=1}^{2} \frac{C_i}{\gamma_i}\left(1-\exp\left(-\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{s}}\right)\right)\operatorname{csch}\left(\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}}\right) + \frac{C_i}{\gamma_i}\tanh\left(\frac{\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}}}{2}\right) + \sigma_{\text{L}}$$

$$\sigma_{\min} = \sum_{i=1}^{2} \chi_{i,\text{stab}}^{\min} - \sigma_{\text{L}} = \sum_{i=1}^{2} \frac{C_i}{\gamma_i}\left(\exp\left(\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{s}}\right)-1\right)\operatorname{csch}\left(\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}}\right) - \frac{C_i}{\gamma_i}\tanh\left(\frac{\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}}}{2}\right) - \sigma_{\text{L}}$$

$\Downarrow$

$$\sigma_{\text{m}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \sum_{i=1}^{2} \frac{C_i}{\gamma_i}\frac{\sinh\left(\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{s}}\right)}{\sinh\left(\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}}\right)}$$

Plastic shakedown only if $\sigma_{\text{m}} = 0$

$\Downarrow$

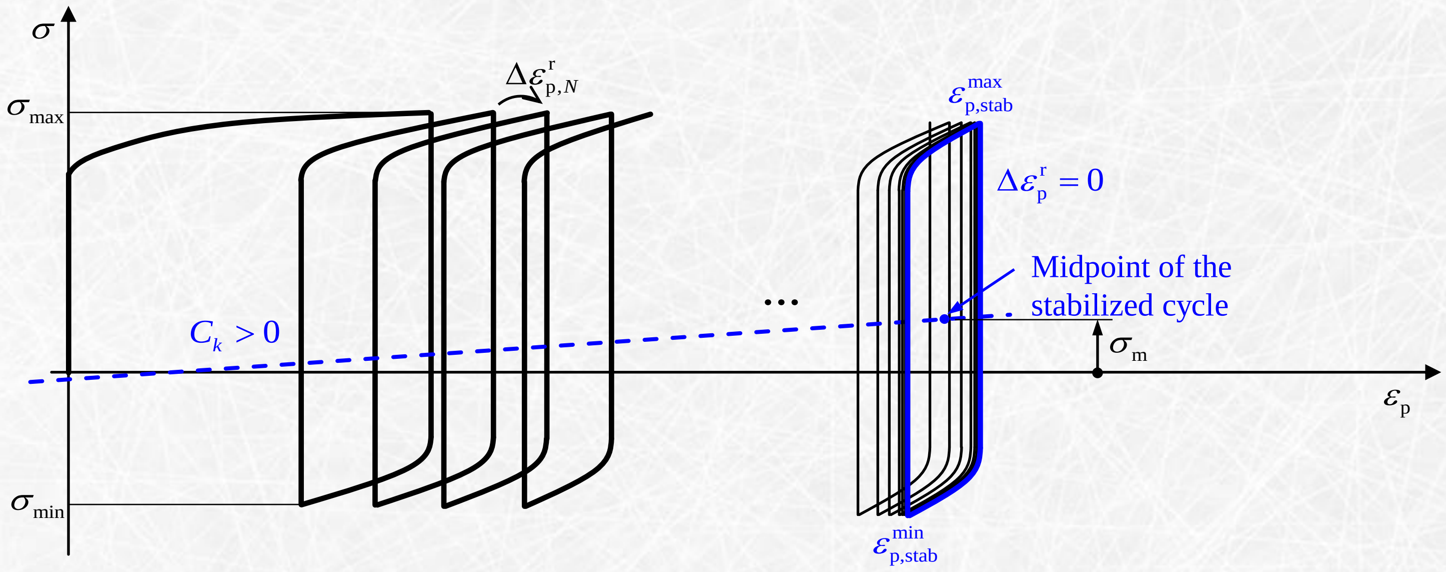
- If $\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{s}} \ll 1$ for each backstress (fairly common case due to small plastic strain increment per cycle) $\sigma_{\text{m}} = \frac{\Delta\varepsilon_{\text{p}}^{\text{r}}}{2}\sum_{i=1}^{2}\frac{C_i}{\sinh\left(\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}}\right)}$
- If $\gamma_i\,\Delta\varepsilon_{\text{p}}^{\text{a}} \ll 1$ for each backstress, σ_{m} can be further approximated with $\sigma_{\text{m}} = \frac{\Delta\varepsilon_{\text{p}}^{\text{r}}}{2\,\Delta\varepsilon_{\text{p}}^{\text{a}}}\sum_{i=1}^{2}\frac{C_i}{\gamma_i}$

Invertible function

- If $\gamma_2 \ll \gamma_1$ and $\frac{C_2}{\gamma_2} \gg \frac{C_1}{\gamma_1}$ $\Rightarrow$ $\sigma_{\text{m}} = \frac{C_2}{\gamma_2}\frac{\Delta\varepsilon_{\text{p}}^{\text{r}}}{2\,\Delta\varepsilon_{\text{p}}^{\text{a}}}$

How to calculate γ_2 3/3

When a linear backstress is considered $\chi_{3,N+1}^{\max} = \chi_{3,N}^{\max} + C_3 \, \Delta\varepsilon_{\mathrm{p}}^{\mathrm{r}}$



The only achievable equilibrium is $\Delta\varepsilon_{\mathrm{p}}^{\mathrm{r}} = 0$

C_3 is generally quite small

⇩

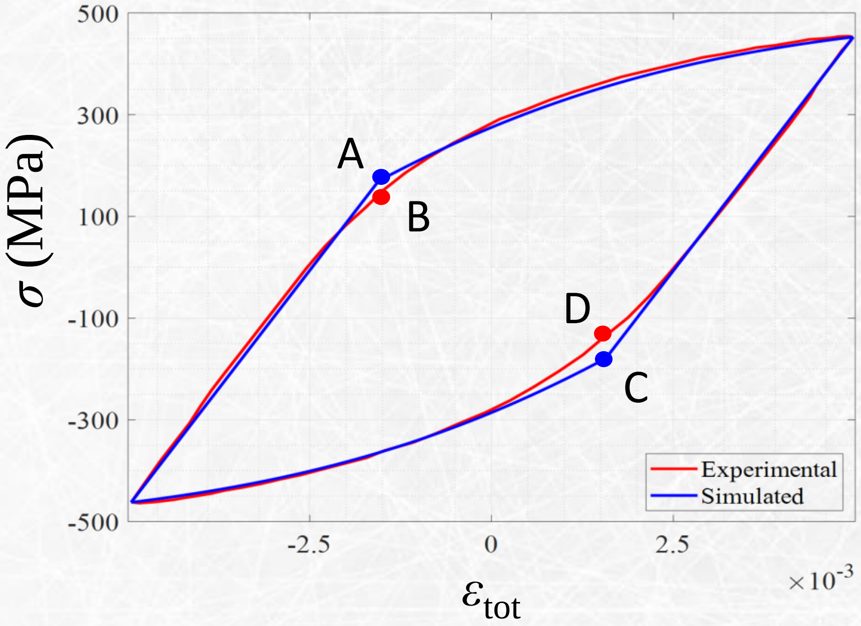
a practically constant ratcheting-rate is often observed in experimental test $\varepsilon_{\mathrm{p,stab}}^{\mathrm{m}} = \dfrac{\sigma_{\mathrm{m}} - \chi_{3,0}}{C_3}$

⇩

a constant ratcheting-rate can be considered for the calibration of γ_2

A fourth backstress can be added...

To improve the prediction near the elastic limit



$\varepsilon_{\text{tot,A}} = \varepsilon_{\text{tot,B}}$ $\varepsilon_{\text{tot,C}} = \varepsilon_{\text{tot,D}}$

$\frac{C_4}{\gamma_4}$ which minimizes $r\left(\frac{C_4}{\gamma_4}\right) = |\sigma_A - \sigma_B|^2 + |\sigma_C - \sigma_D|^2$

Set $\alpha = 0$ in $\Psi(\gamma_1) = (1-\alpha)\Sigma^2 + \alpha(\Lambda_I^2 + \Lambda_{II}^2)$

and obtain $C_1, \gamma_1, C_2, C_3, \chi_{3,0}$

- Generate a matrix of trial values for C_4 and γ_4
- For each matrix entry evaluate $\sigma_L^{'} = \sigma_L - \frac{C_4}{\gamma_4}$
- Minimize $\Lambda_I^2 + \Lambda_{II}^2$ and obtain C_4, γ_4 and the updated value of $\sigma_L^{'}$

Clearly, the initial hysteresis area (with $\alpha = 0$)

must be bigger than the hysteresis area of the experimental cycle

Isotropic Hardening

The elastic limit generally depends on the cumulated plastic strain

⇩

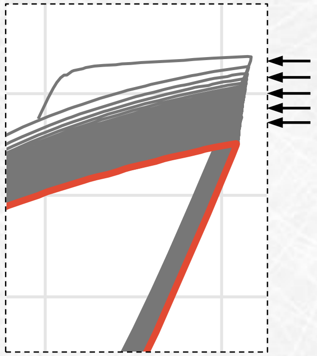
$$p = \int \left| \mathrm{d}\varepsilon_{\mathrm{p}} \right|$$

Thus, an isotropic hardening rule is introducted (Voce) $\sigma_{\mathrm{Y}} = \sigma_0 + Q\left(1 - e^{-bp}\right)$

$$\sigma_{\mathrm{Y}}(p \to \infty) = \sigma_{\mathrm{L}}$$

⇩

the transient during pre-stabilization cycles is used to find *Q* and *b*



⇩

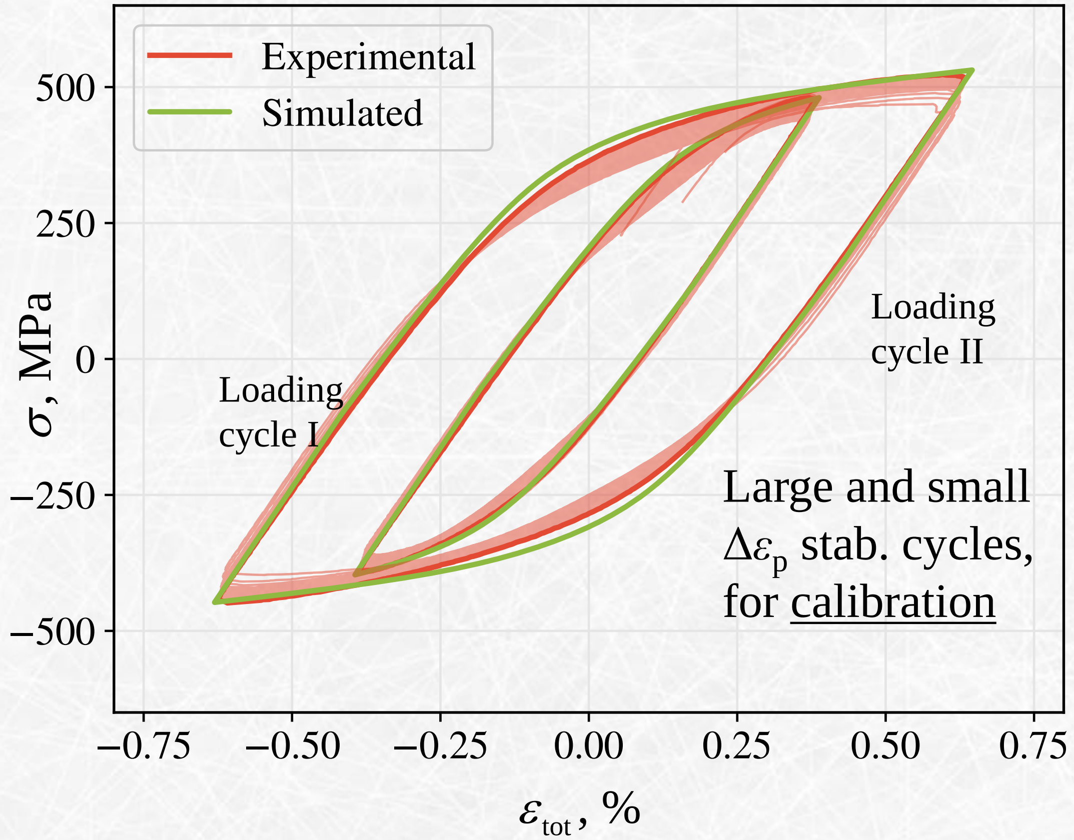
$$\sigma_N^{\max} = \sigma_{\mathrm{Y}} + \sum_{i=1}^{n} \chi_{i,N}^{\max} = \sigma_{\mathrm{L}} - Q\exp(-bp) + \sum_{i=1}^{n} \chi_{i,N}^{\max}$$ so $$\sigma_{\mathrm{Y}} - \sigma_{\mathrm{L}} = -Q\exp(-bp)$$

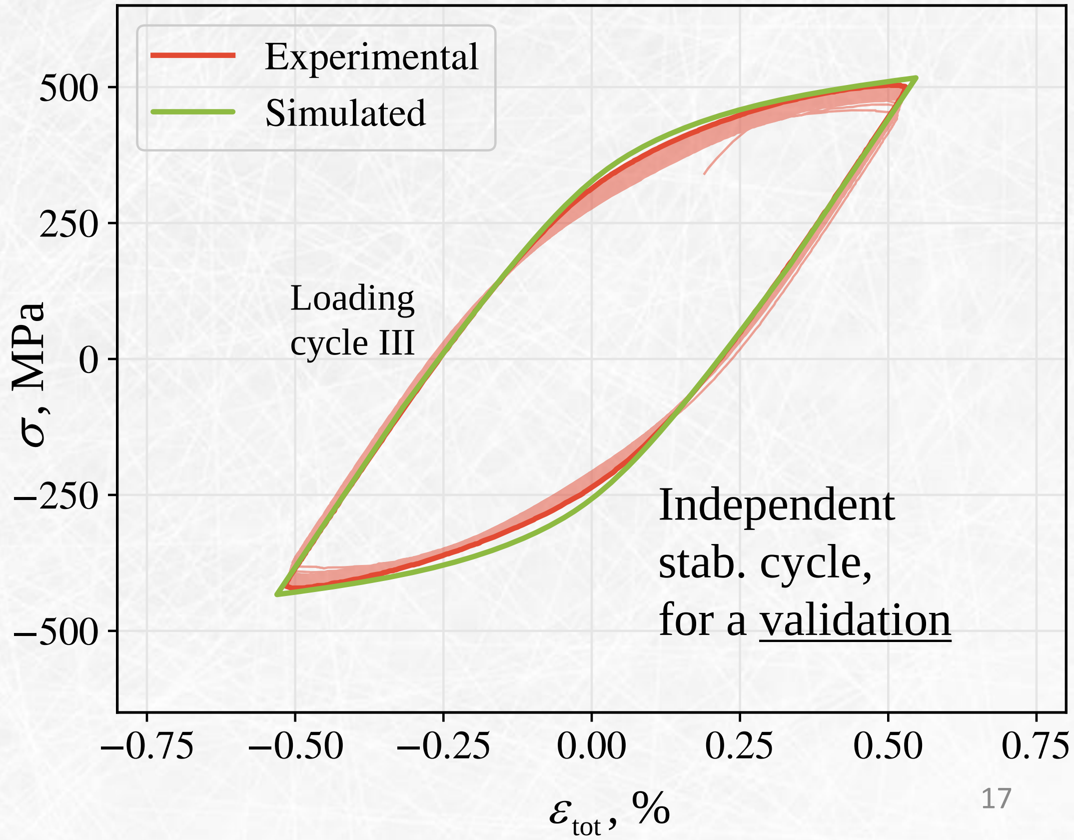
Q and b can be determined with a least-squares fit

Experimental data and validations 1/4

High-silicon ferritic ductile cast iron

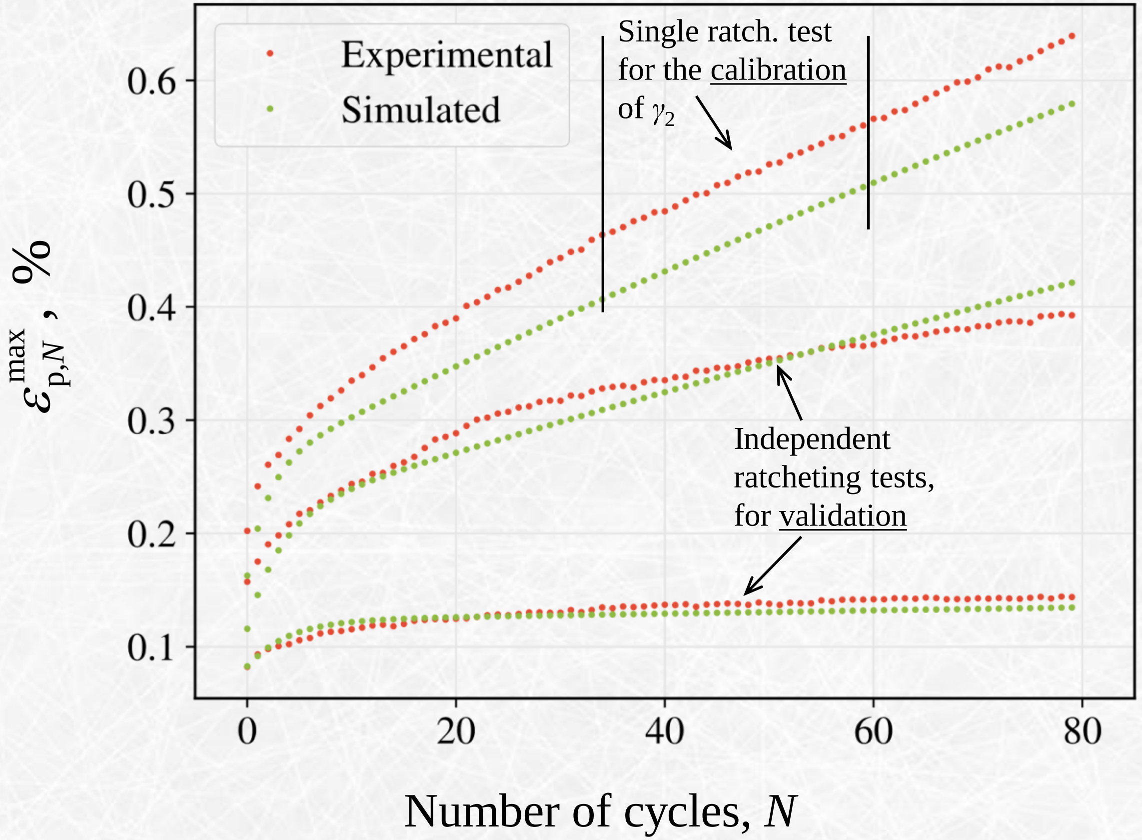
Two strain-controlled tests
with $R_\varepsilon \simeq -1$ used to calculate
the parameters C_1, γ_1, C_2, C_3, $\chi_{3,0}$, C_4, γ_4





Experimental data and validations 2/4

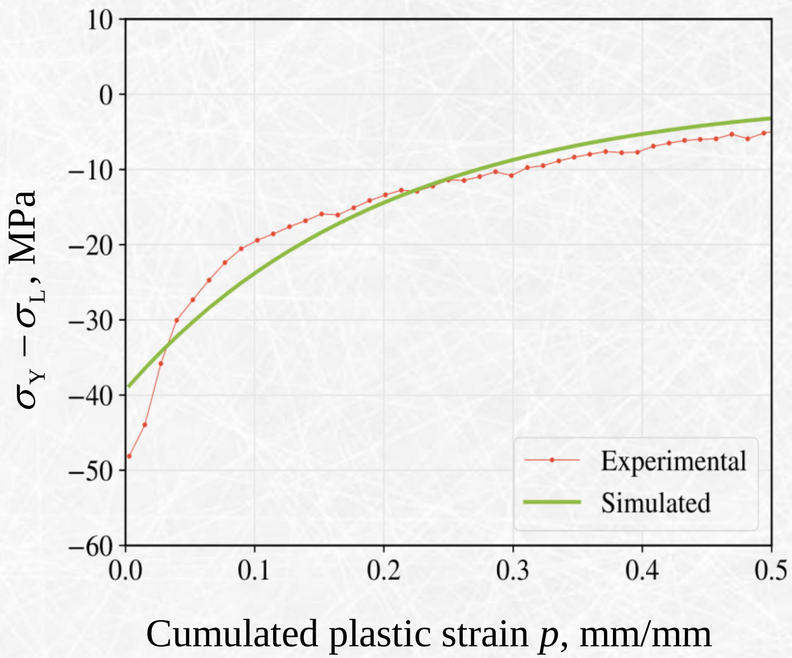
One stress-controlled test used to identify γ_2
Two stress-controlled tests to validate the result



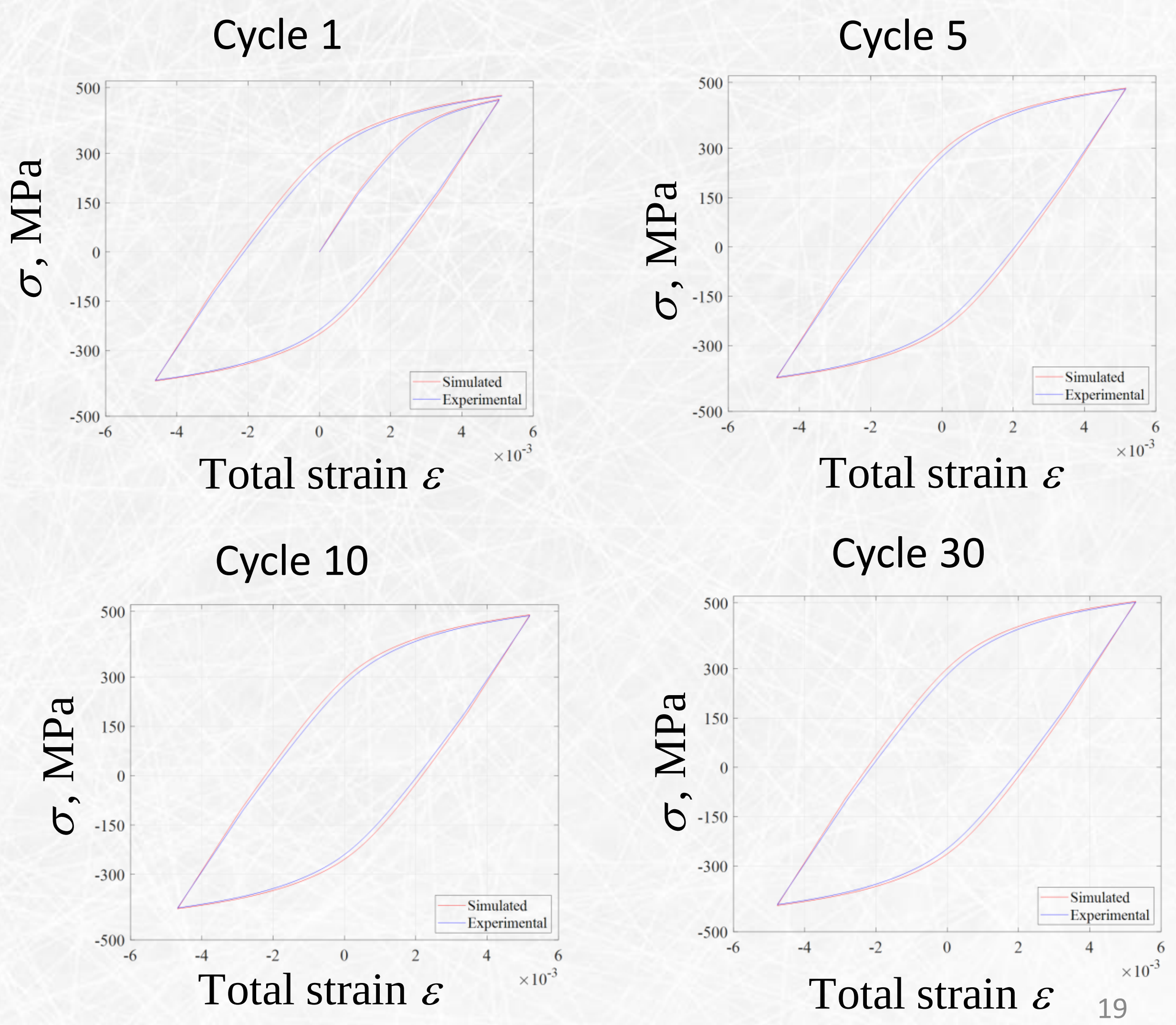
$$\gamma_2 = \frac{\operatorname{asinh}\left(\frac{1}{C_2}\left(\frac{2\sigma_m}{\Delta\varepsilon_p^r} - \frac{C_1}{\sinh\left(\gamma_1 \Delta\varepsilon_p^a\right)}\right)\right)^{-1}}{\Delta\varepsilon_p^a}$$

Experimental data and validations 3/4

Isotropic hardening trend of cycle I, calibration of Voce's parameters

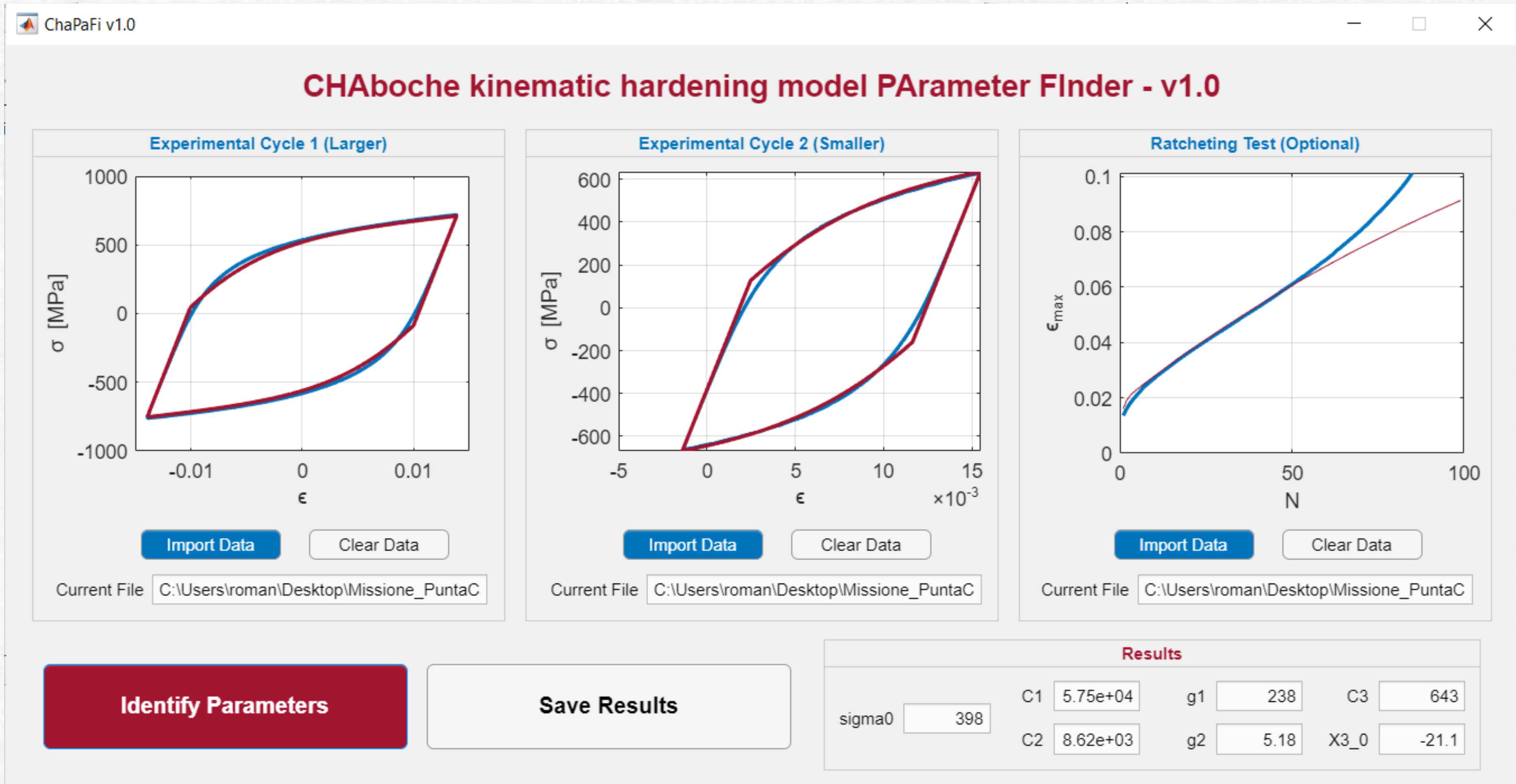


Validation of the overall CIKH model on some of the cycles of cycle loading III (strain-controlled)



Experimental data and validations 4/4

The procedure is resumed in a powerful and simple Matlab Gui to calculate Chaboche Kinematic hardening model parameters



Avaible online with C. Santus, T. Grossi, L. Romanelli et. Al. , Int. J. Plas., December 2022

Conclusion

A procedure to identify Chaboche and Voce parameters

- Two strain-controlled tests and one stress-controlled test to obtain the parameters
- No complex algorithms, analytical closed-form expressions are exploited
- Experimental data used: peak to peak stress, average stress, slope at the inversion points and ratcheting rate
- Potential leverage of a fourth backstress component to improve the prediction accuracy

Results obtained for the high-silicon ferritic ductile cast iron

C_1	γ_1	C_2	γ_2	C_3	$\chi_{3,0}$	C_4	γ_4	σ_L	Q	b
82.1 GPa	841	12.1 GPa	13.5	1124 MPa	42.1 MPa	798 GPa	4538	172 MPa	39.3 MPa	5.02