1st Virtual European Conference on Fracture - VECF1
1st Virtual European Conference on Fracture
First day: 29th June 2020
ESIS European Structural Integrity Society

Statistical evaluation of the critical distance in the finite life fatigue regime

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Aims and objectives

- Introduce the optimized V-notched specimen for assessing the (fatigue) critical distance

- Introduce the procedure to find the statistical distribution of the critical distance

- Extend this procedure to the finite life regime

- Assess the fatigue strength of another specimen, with blunter radius, and its statistical distribution

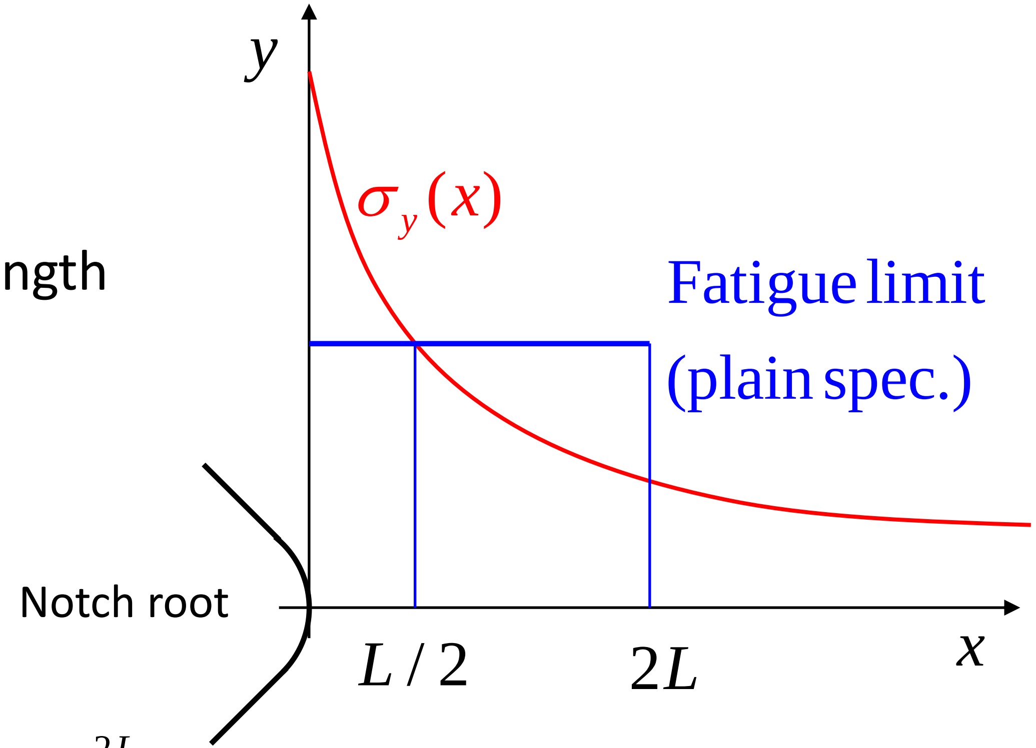
- Application to the Aluminium alloy 7075-T6

The Theory of Critical Distances (TCD), Line Method – Point Method (Fatigue)

- Point Method
- Line Method
- Area Method
- Volume Method
- Other local methods, such as SED

Critical Distance or
Material characteristic length

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\Delta \sigma_{\mathrm{fl}}}\right)^2$$



$$\int_0^{2L} \Delta\sigma_y(x)\,\mathrm{d}x = \Delta\sigma_{\mathrm{fl}} \quad \text{(Line Method, LM)}$$

$$\Delta\sigma_y(L/2) = \Delta\sigma_{\mathrm{fl}} \quad \text{(Point Method, PM)}$$

How to obtain L?
- Literature (similar materials)
- Experiment ($\Delta\sigma_{\mathrm{fl}}$, ΔK_{th})

Critical distance determination with the SIF threshold

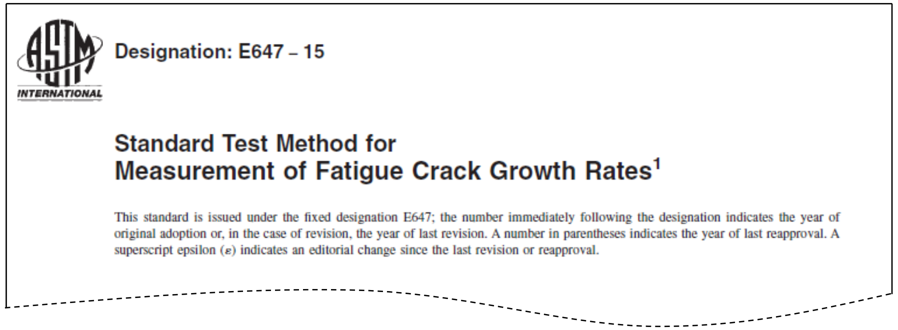
Notch severity

- Plain specimen
- Notched specimen blunt
- Notched specimen sharp
- Notched specimen ultra-sharp
- Crack threshold C(T) / M(T) specimen

$\Delta\sigma_{\mathrm{fl}}$

ΔK_{th}

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\Delta\sigma_{\mathrm{fl}}}\right)^2$$


Designation: E647 – 15

Standard Test Method for
Measurement of Fatigue Crack Growth Rates[1]

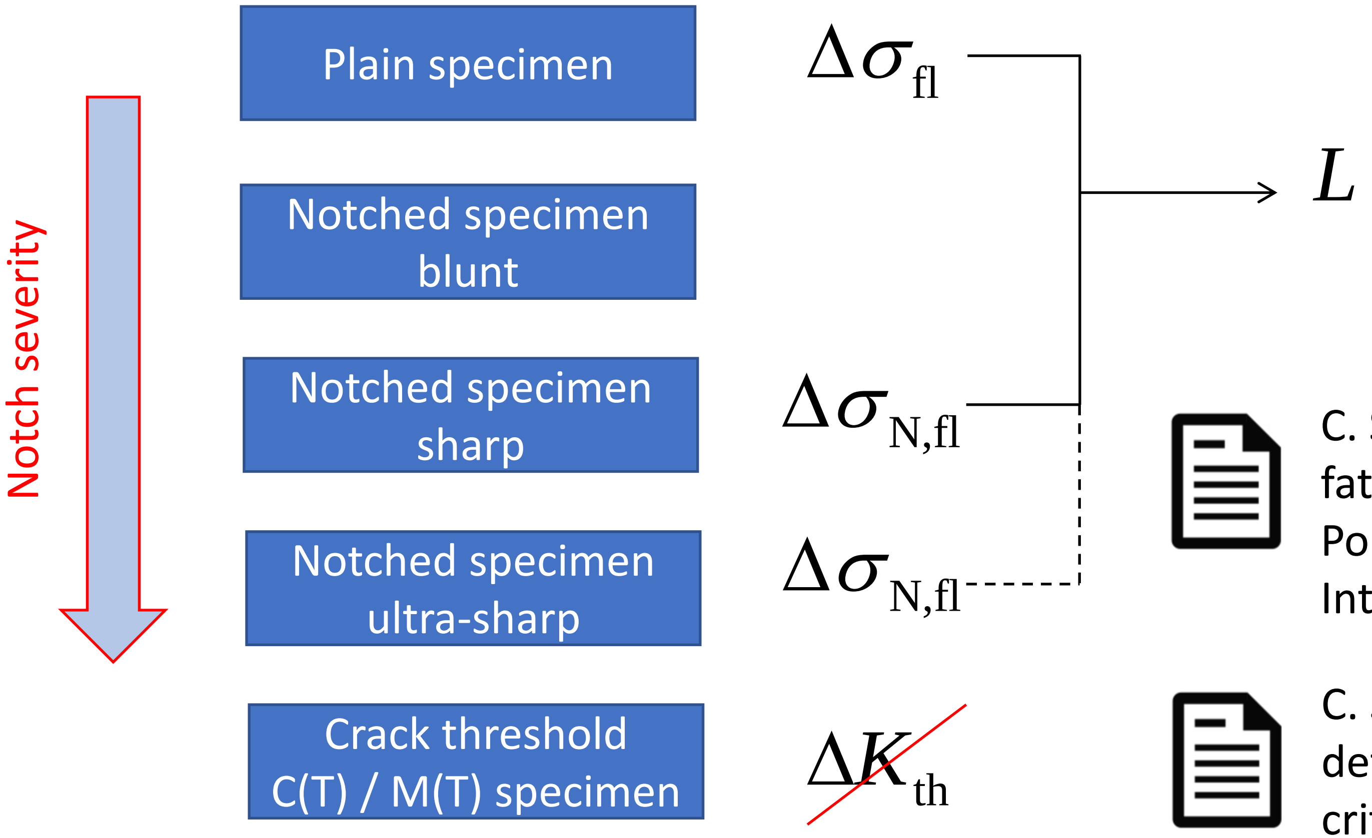
This standard is issued under the fixed designation E647; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

- Challenging though ASTM standard, the crack length measure is required

- Specimen C(T), or M(T), may not fit into the samples, especially for material supply in bars

- Precracking technique dependence

Critical distance determination by combining a notched specimen with the plain specimen

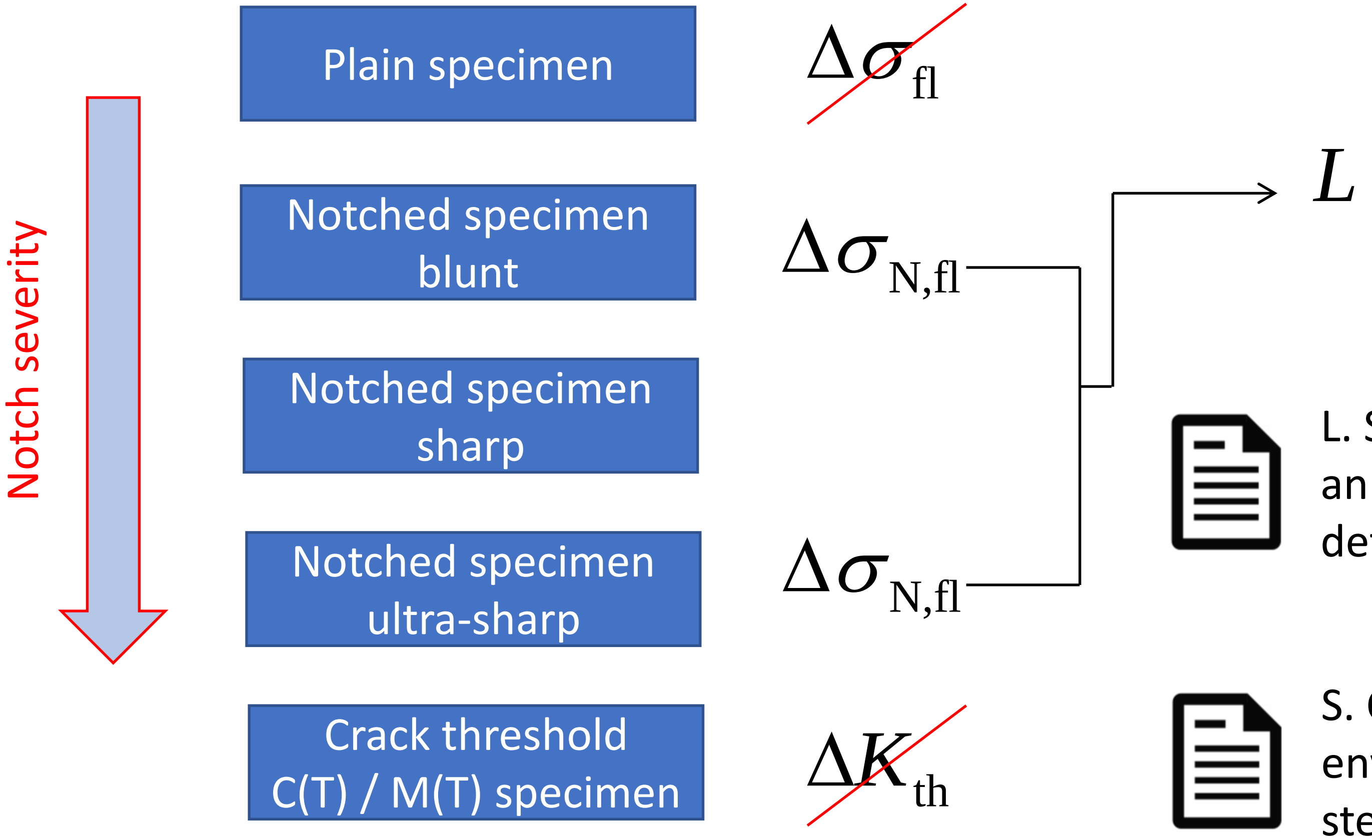


The length which satisfies the TCD criterion, either Line or Point Method

C. Santus, D. Taylor, M. Benedetti. Determination of the fatigue critical distance according to the Line and the Point Methods with rounded V-notched specimen. International Journal of Fatigue, 2018

C. Santus, D. Taylor, M. Benedetti. Experimental determination and sensitivity analysis of the fatigue critical distance obtained with rounded V-notched specimens. International Journal of Fatigue, 2018

Critical distance determination by combining two <u>notched</u> specimens



Two notches with different severity can be used as well, and no referring to the plain specimen.
This is the common approach for the brittle fracture (static) critical distance.

L. Susmel, D. Taylor. The theory of critical distances as an alternative experimental strategy for the determination of KIc and DKth. Eng Fract Mech, 2010

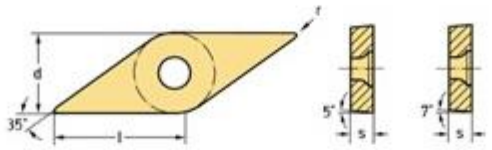
S. Cicero, P. González, B. Arroyo, J.A. Álvarez. Analysis of environmentally assisted cracking processes in notched steels using the point method. Procedia Struct Integrity 2019

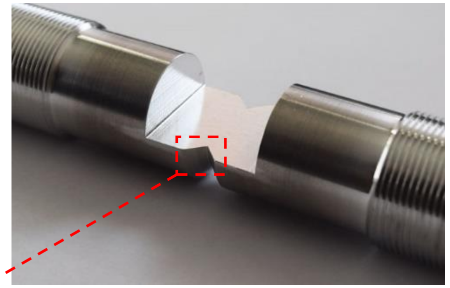
Proposed optimal specimen

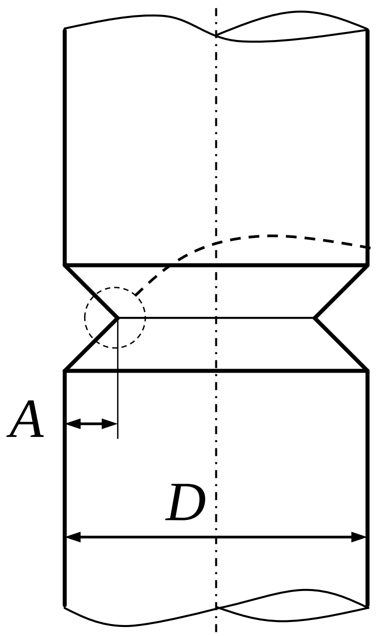
- V-notch axisymmetric specimen:
easy to manufacture, no boundary effects

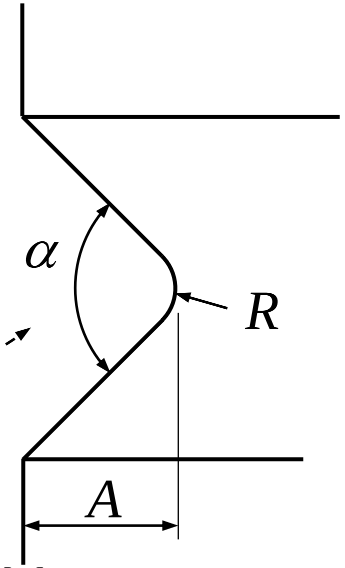
- Relatively open angle: 90°, 60°

- Sharp root radius, at least well controlled: the nose radius of the tool

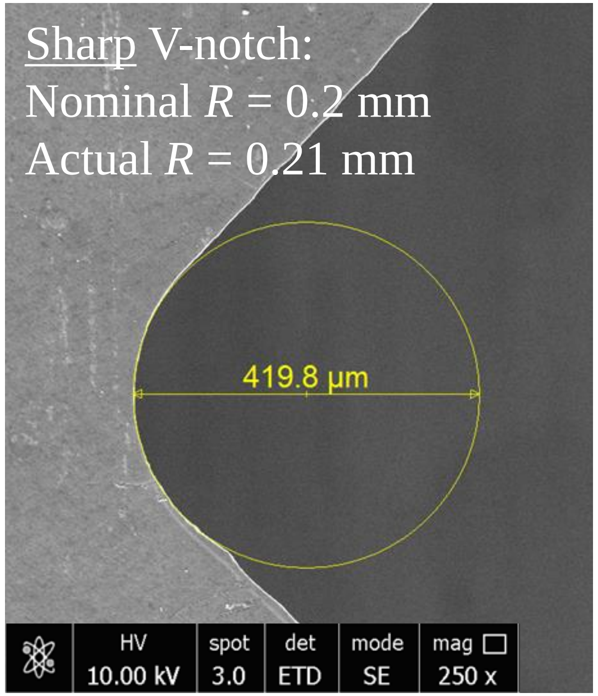




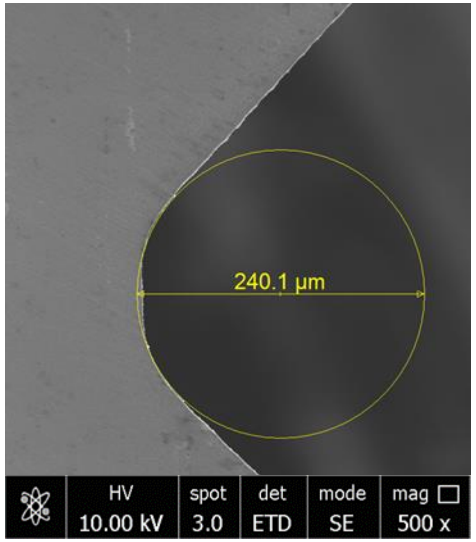




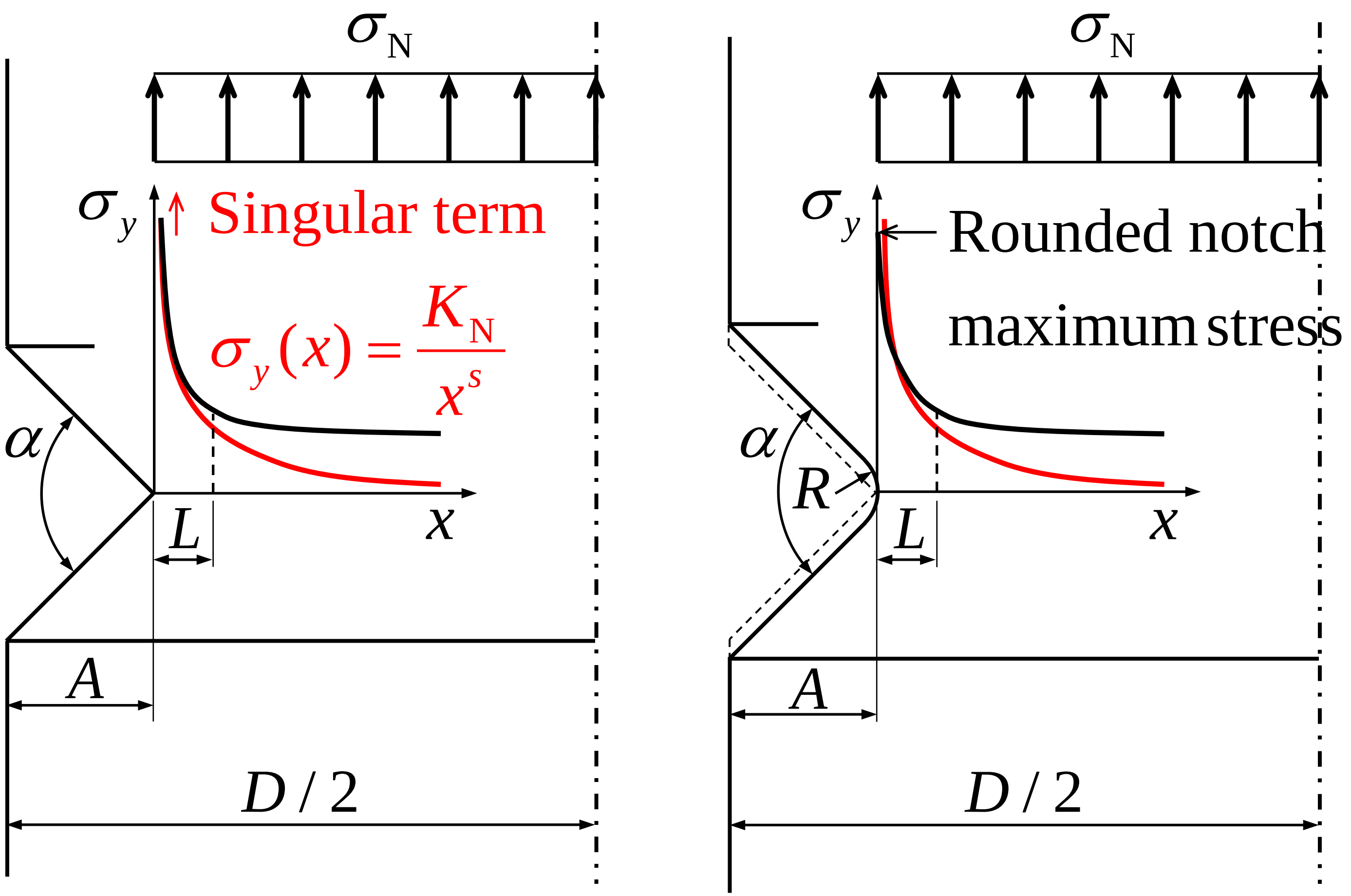
Max stress concentration:
$A = 0.3\ D/2$

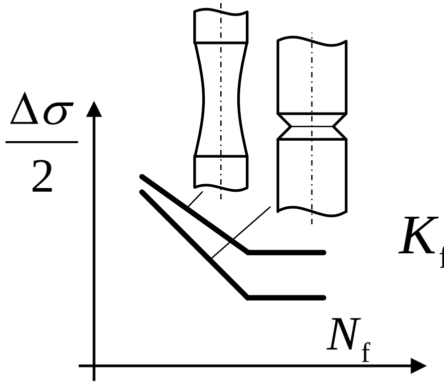


Ultrasharp V-notch
Nominal R = 0.1 mm
Actual R = 0.12 mm



Procedure for the critical distance calculation





$$K_\mathrm{f} = \frac{\Delta\sigma_\mathrm{fl}}{\Delta\sigma_\mathrm{N,fl}}$$

Line Method

$$l_0 = \frac{1}{2}\left(\frac{K_\mathrm{N,UU}}{(1-s)K_\mathrm{f}}\right)^{1/s}$$

$$l = l_\mathrm{min} + \frac{l_0 - \gamma_\mathrm{min}}{\beta}$$

$$L = \frac{D}{2} l$$

Point Method

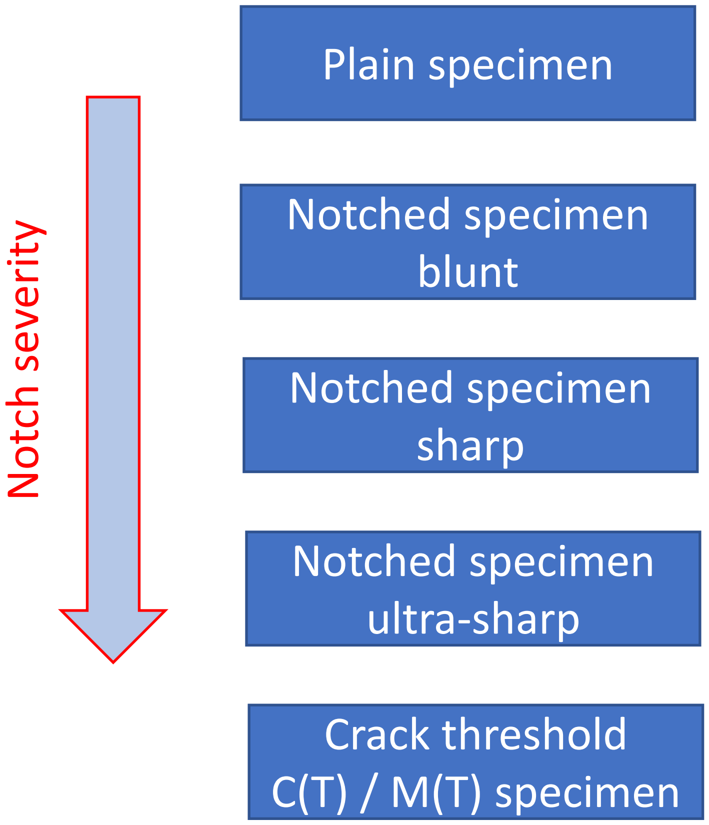
$$l'_0 = 2\left(\frac{K_\mathrm{N,UU}}{K_\mathrm{f}}\right)^{1/s}$$

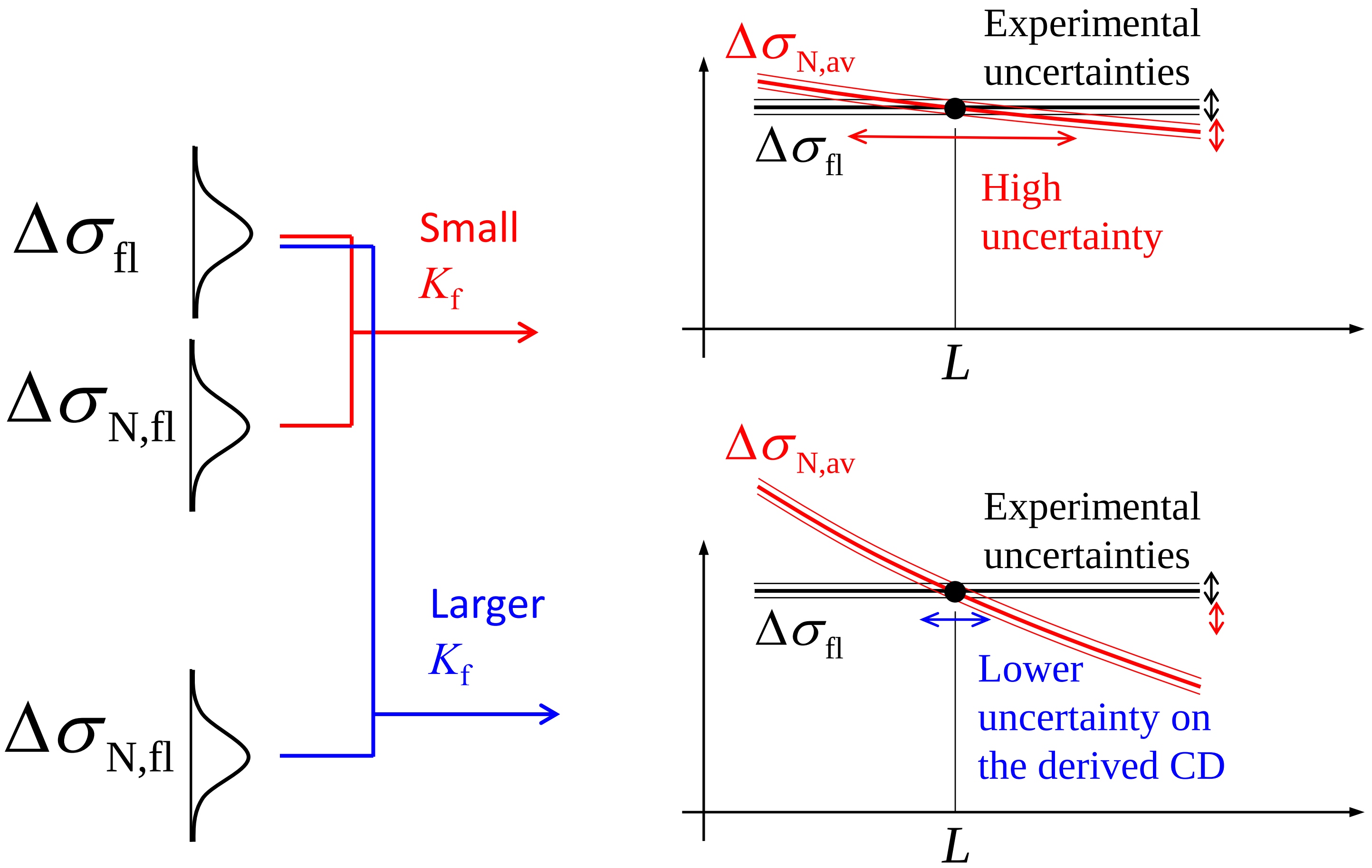
$$l' = \sum_{i=1}^{5} \delta_i \, l_0'^{\,5-i}$$

$$L' = \frac{D}{2} l'$$

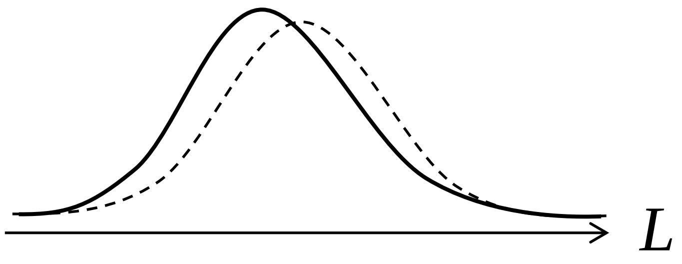
- Determination of the Fatigue stress concentration factor K_f
- Dimensionless and singularity based (Line Method) critical distance l_0
- Conversion from l_0 to the critical distance l with a FE-based analytical procedure (linearity)
- Determination of the actual critical distance L

Critical distance determination, statistical assessment





Monte Carlo simulation: assuming Normal distributions of the specimen strengths, the obtained distribution of the critical distance is Skew-Normal



Proposed procedure for the statistical assessment of the critical distance

Fatigue limits and standard deviations: $\Delta\sigma_{\mathrm{fl}},\ S,\ \Delta\sigma_{\mathrm{N,fl}},\ S_{\mathrm{N}}$

Equivalent coefficient of variation (CV) of the input data:

$$\Sigma = \sqrt{\frac{(S/\Delta\sigma_{\mathrm{fl}})^2 + (S_{\mathrm{N}}/\Delta\sigma_{\mathrm{N,fl}})^2}{2}}$$

Normalized coefficient of variation (NCV) of the dimensionless critical distance:

$$\nu = \frac{1}{\Sigma}\frac{\delta}{\mu}$$

μ, δ: mean and standard deviation of the critical distance

Equations provided for:

$\nu, \mu \rightarrow \delta$

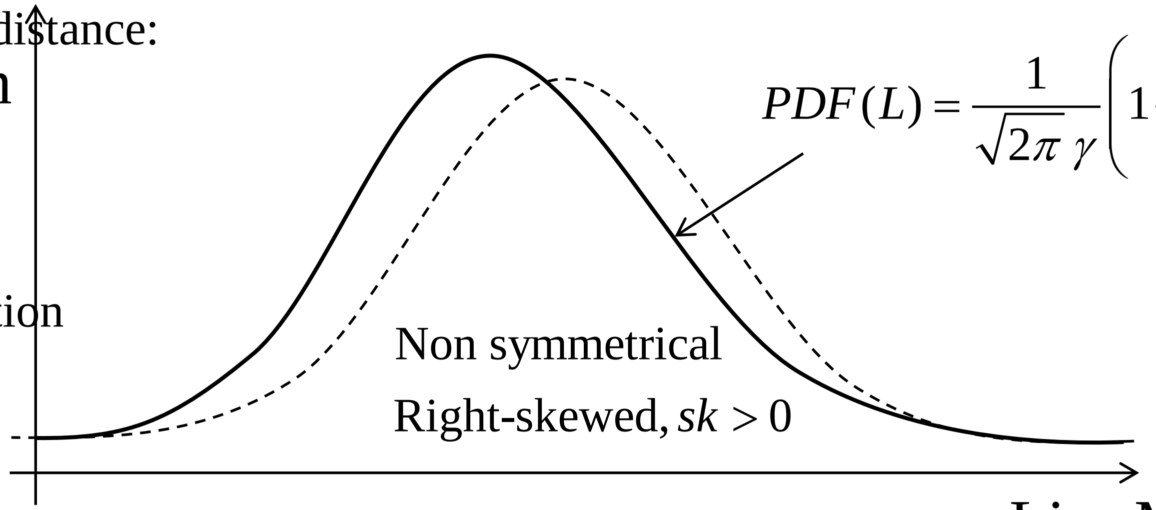
and then for:

sk (skewness)

and finally:

α, β, γ

$$PDF(L) = \frac{1}{\sqrt{2\pi}\,\gamma}\left(1+\mathrm{erf}\left(\frac{\alpha(L-\beta)}{\sqrt{2}\,\gamma}\right)\right)\exp\left(-\frac{(L-\beta)^2}{2\gamma^2}\right)$$

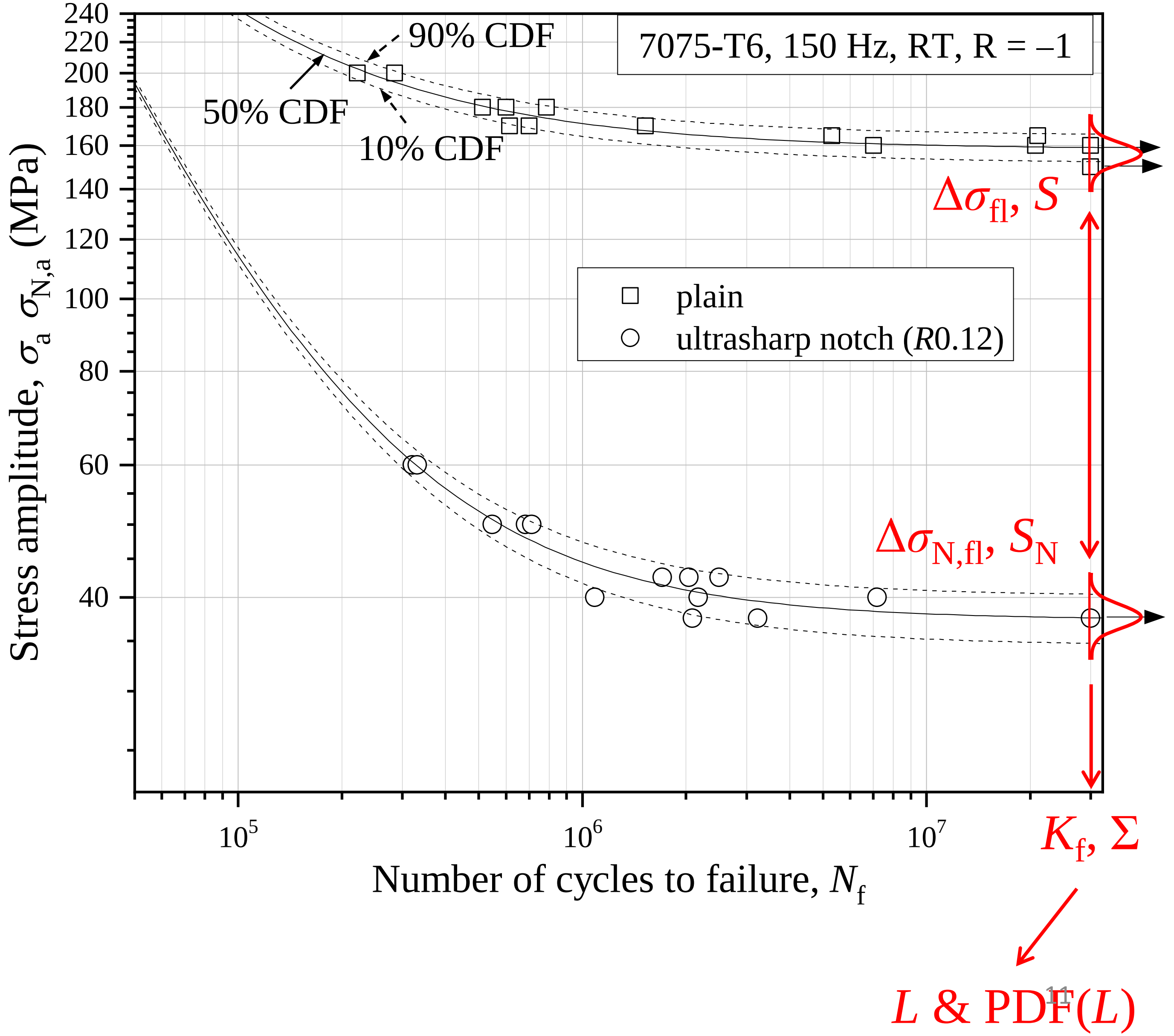


M. Benedetti, C. Santus. Statistical properties of threshold and notch derived estimations of the critical distance according to the line method of the theory of critical distances. International Journal of Fatigue, 2020

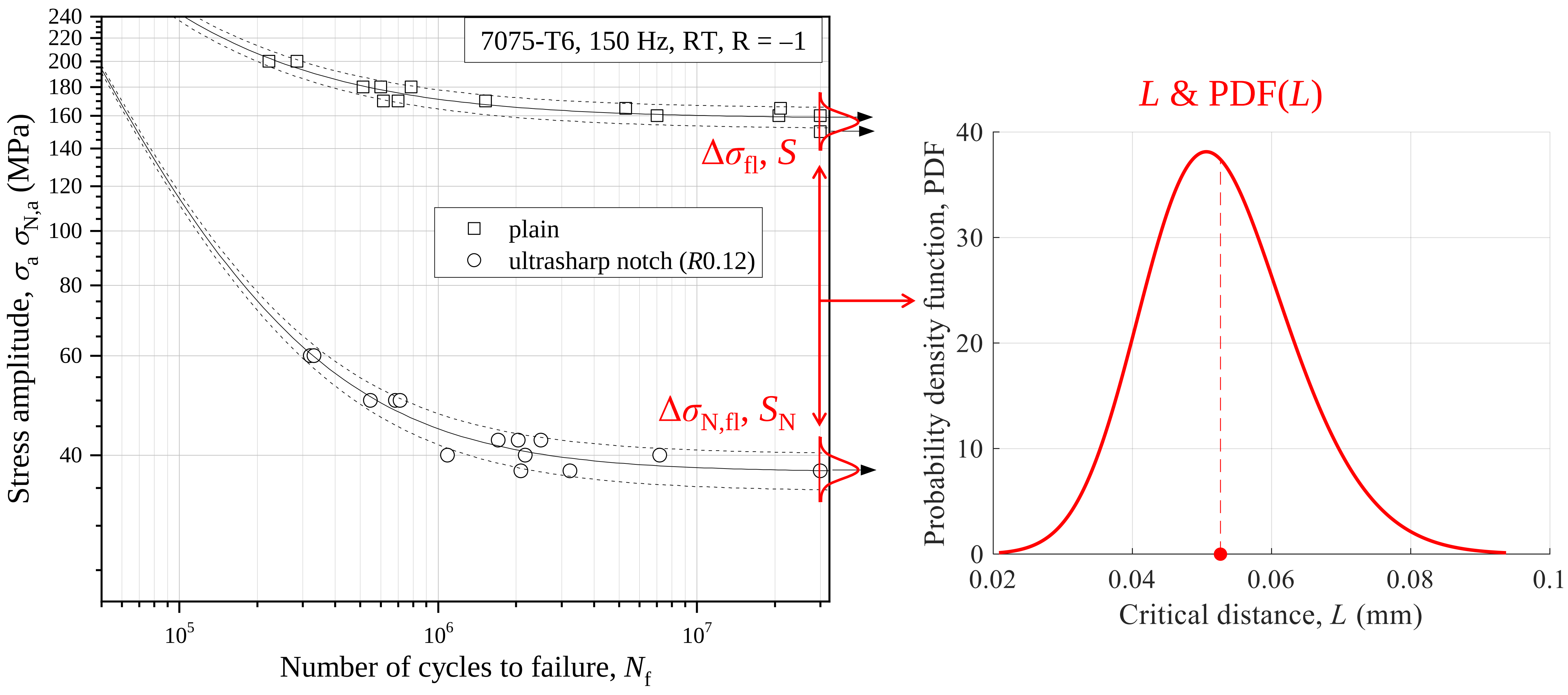
Aluminium alloy 7075-T6 experimental data

Specimens used for L assessment and prob. distr.

- Plain specimen — $R = \infty$
- Notched specimen blunt — $R = 1$ mm
- Notched specimen sharp — $R = 0.21$ mm
- Notched specimen ultra-sharp — $R = 0.12$ mm
- Crack threshold C(T) / M(T) specimen — $R = 0$

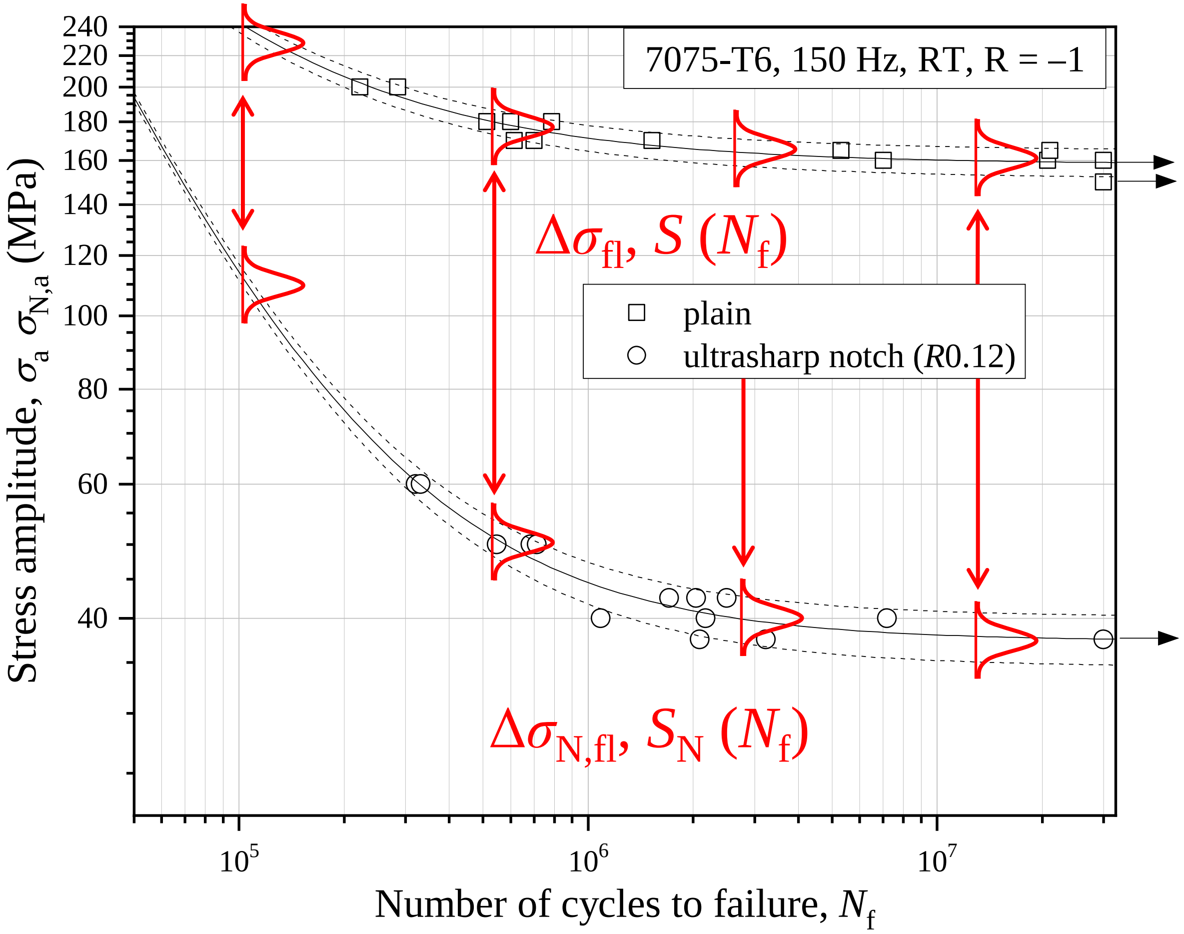


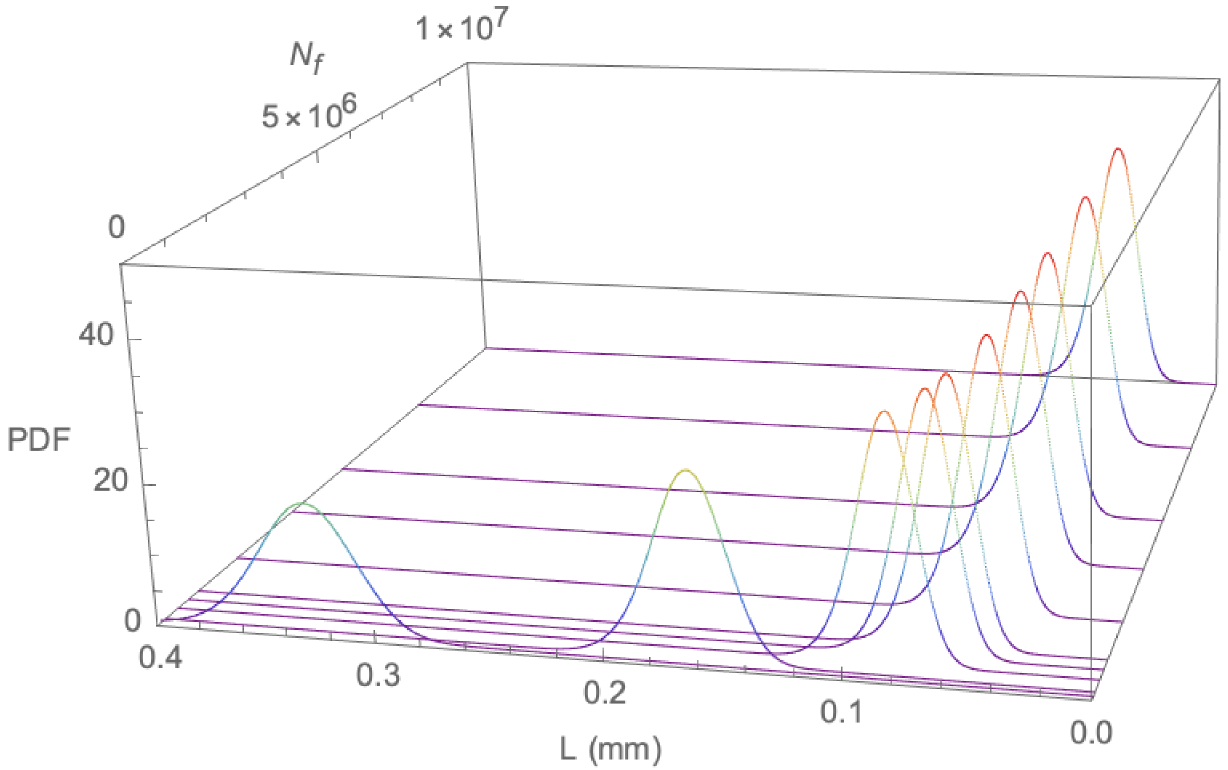
Aluminium alloy 7075-T6 experimental data



Aluminium alloy 7075-T6, critical distance and statistical distribution

Same procedure applied for any value of N_f

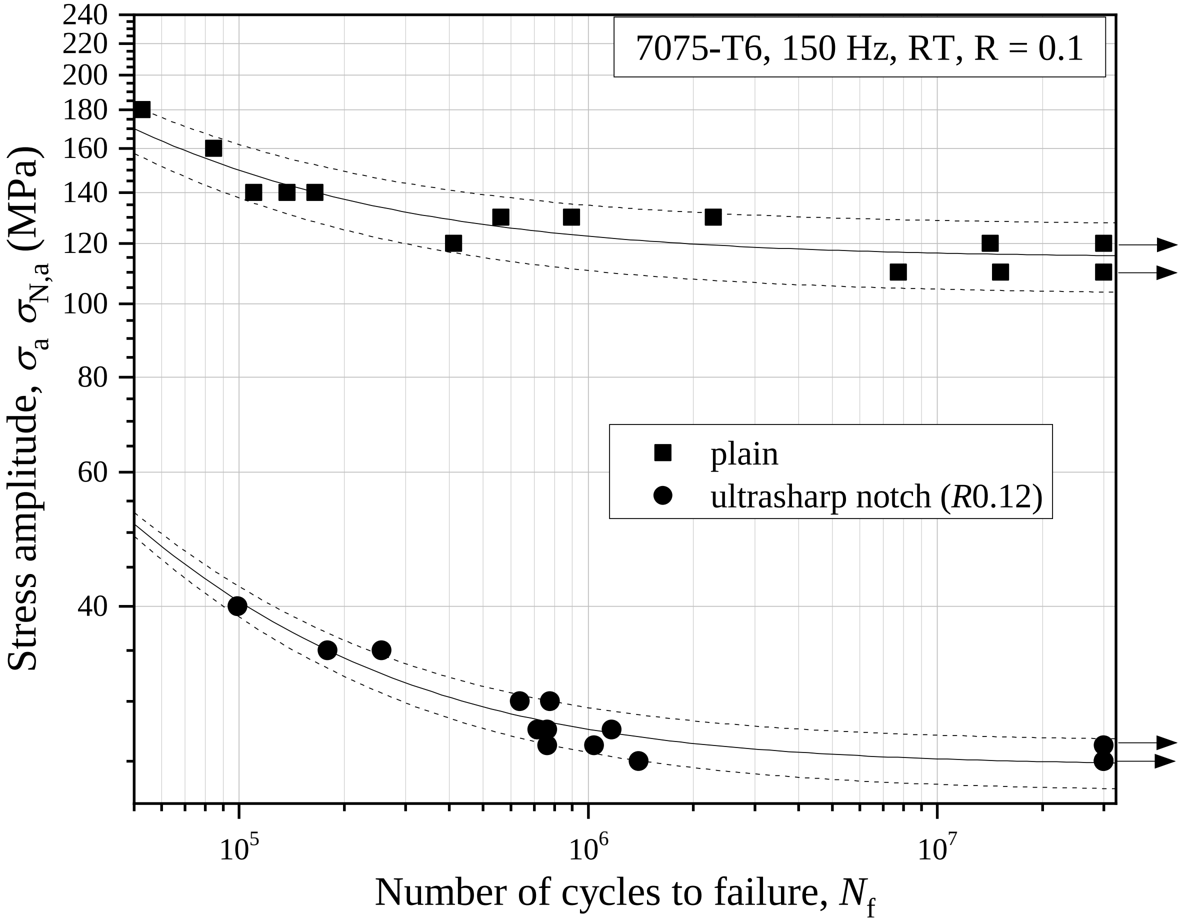


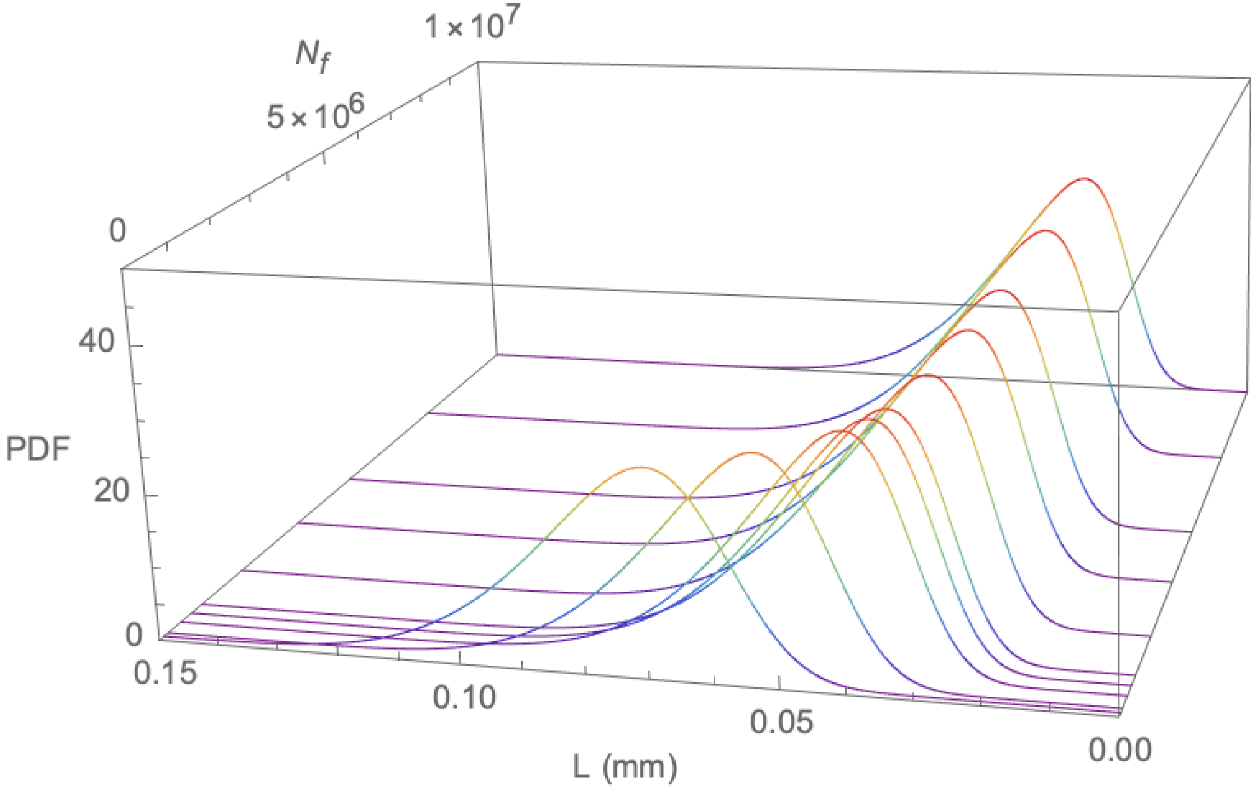


Aluminium alloy 7075-T6, critical distance and statistical distribution

← previously load ratio R = -1

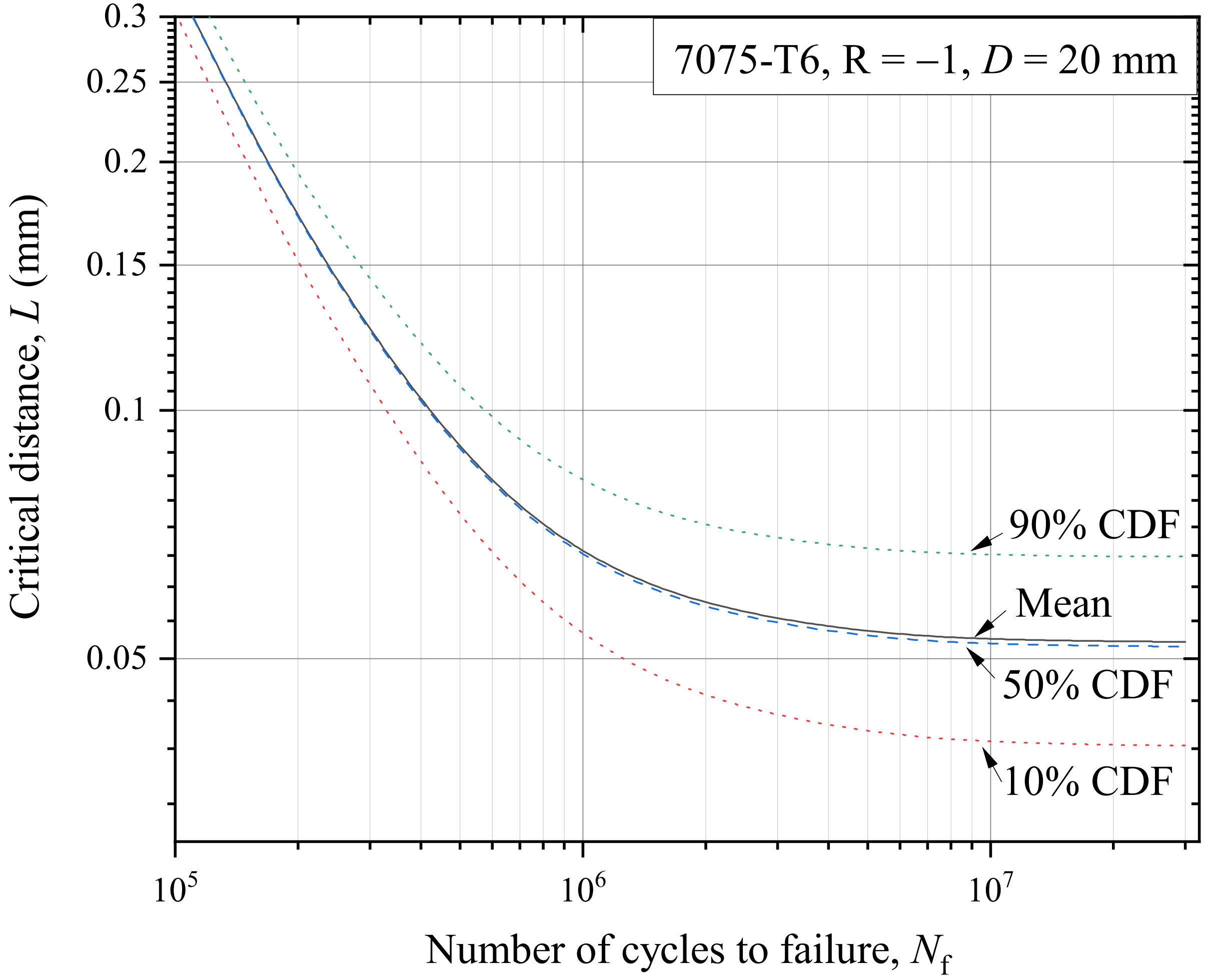
now load ratio R = 0.1





Cumulative density function (CDF) of the critical distance

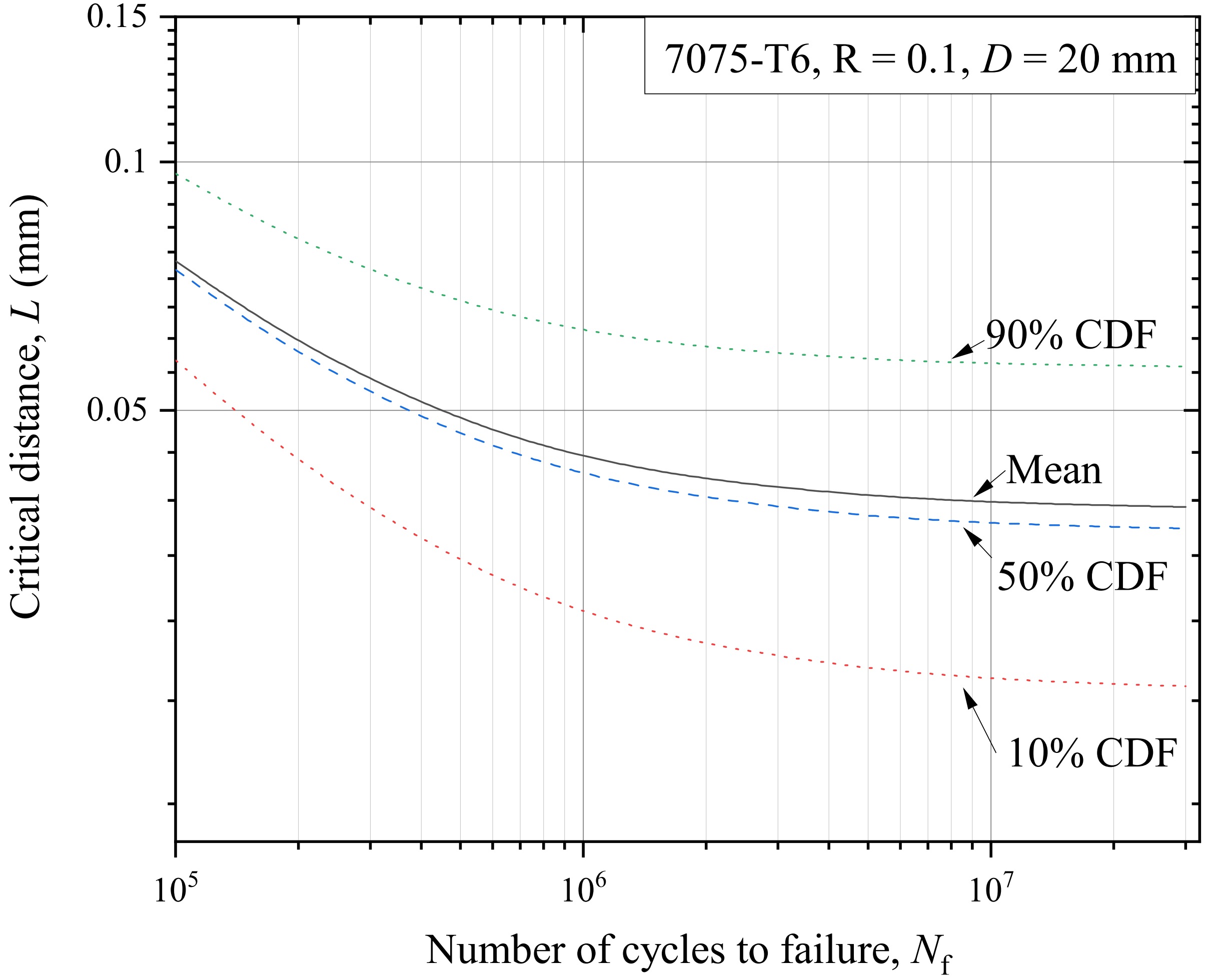
Load ratio R = -1



Low skewness

Higher st. dev. at lower N_f

Load ratio R = 0.1



High skewness

Almost constant st. dev.

Limit notch radius for an accurate assessment of the critical distance

Specimens used for L assessment and prob. distr.

- Plain specimen — $R = \infty$
- Notched specimen blunt — $R = 1$ mm
- Notched specimen sharp — $R = 0.21$ mm
- $R_{\lim}$
- Notched specimen ultra-sharp — $R = 0.12$ mm
- Crack threshold C(T) / M(T) specimen — $R = 0$

Normalized coefficient of variation (NCV):

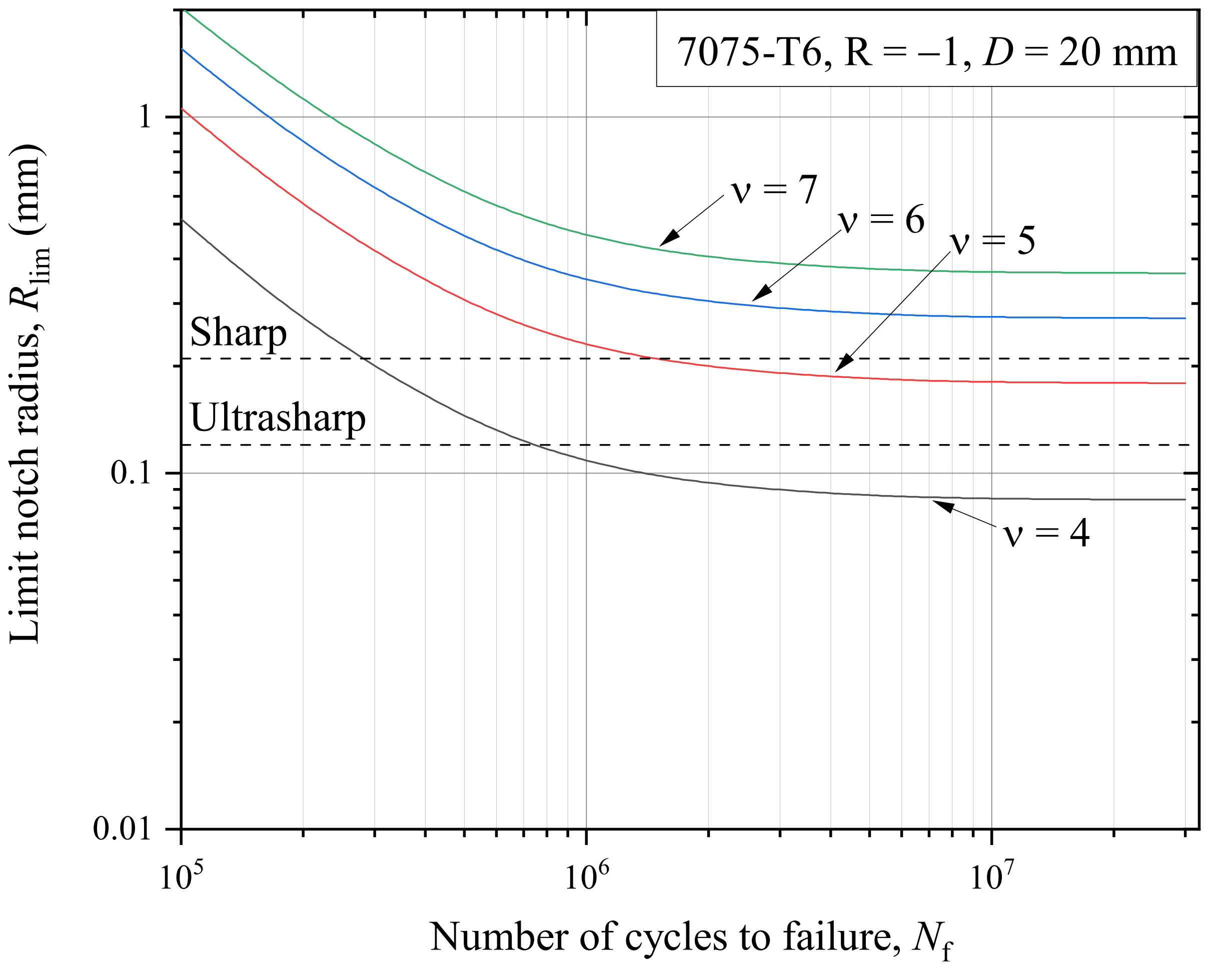
$$\nu = \frac{1}{\Sigma}\frac{\delta}{\mu} = \frac{\text{St.dev. to mean value ratio of the critical distance}}{\text{Equivalent coefficient of variation of the input strenghts}}$$

By imposing a certain level of NCV, the limit radius value can be obtained: $R_{\lim}$

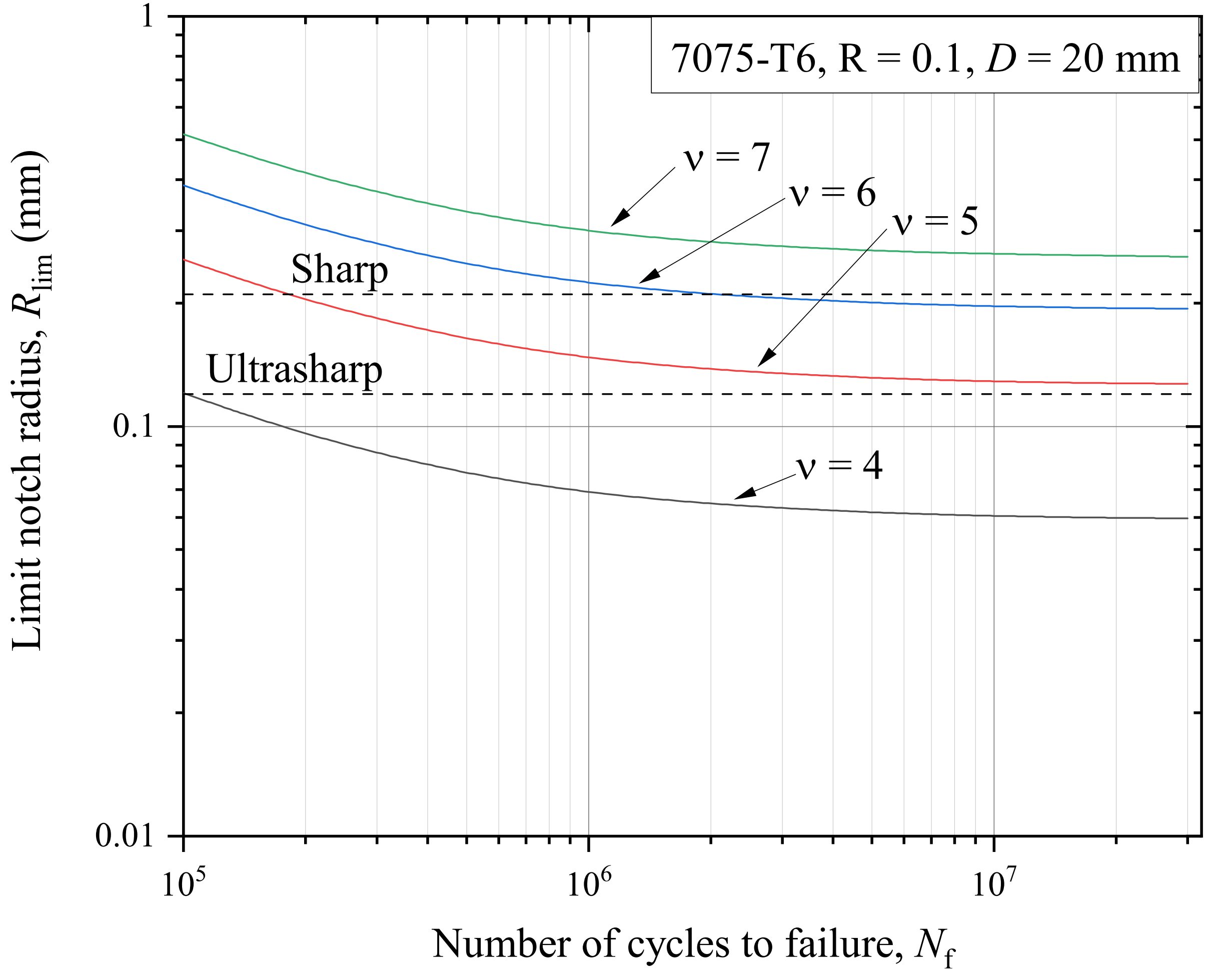
If the notched specimen radius is higher than $R_{\lim}$ the assessment of L has an excessive standard deviation

Limit notch radius for an accurate assessment of the critical distance

Load ratio R = -1



Load ratio R = 0.1



NCV = 5, or higher, can be obtained with the Ultrasharp specimen, and among the two load ratios, R = 0.1 requires a sharper radius at equal NCV

Fatigue strength prediction of blunter specimens

Specimens used for L assessment and prob. distr.

- Plain specimen
- Notched specimen blunt
- Notched specimen sharp
- Notched specimen ultra-sharp
- Crack threshold C(T) / M(T) specimen

Fatigue strength of these specimens assessed, along with their statistical distribution

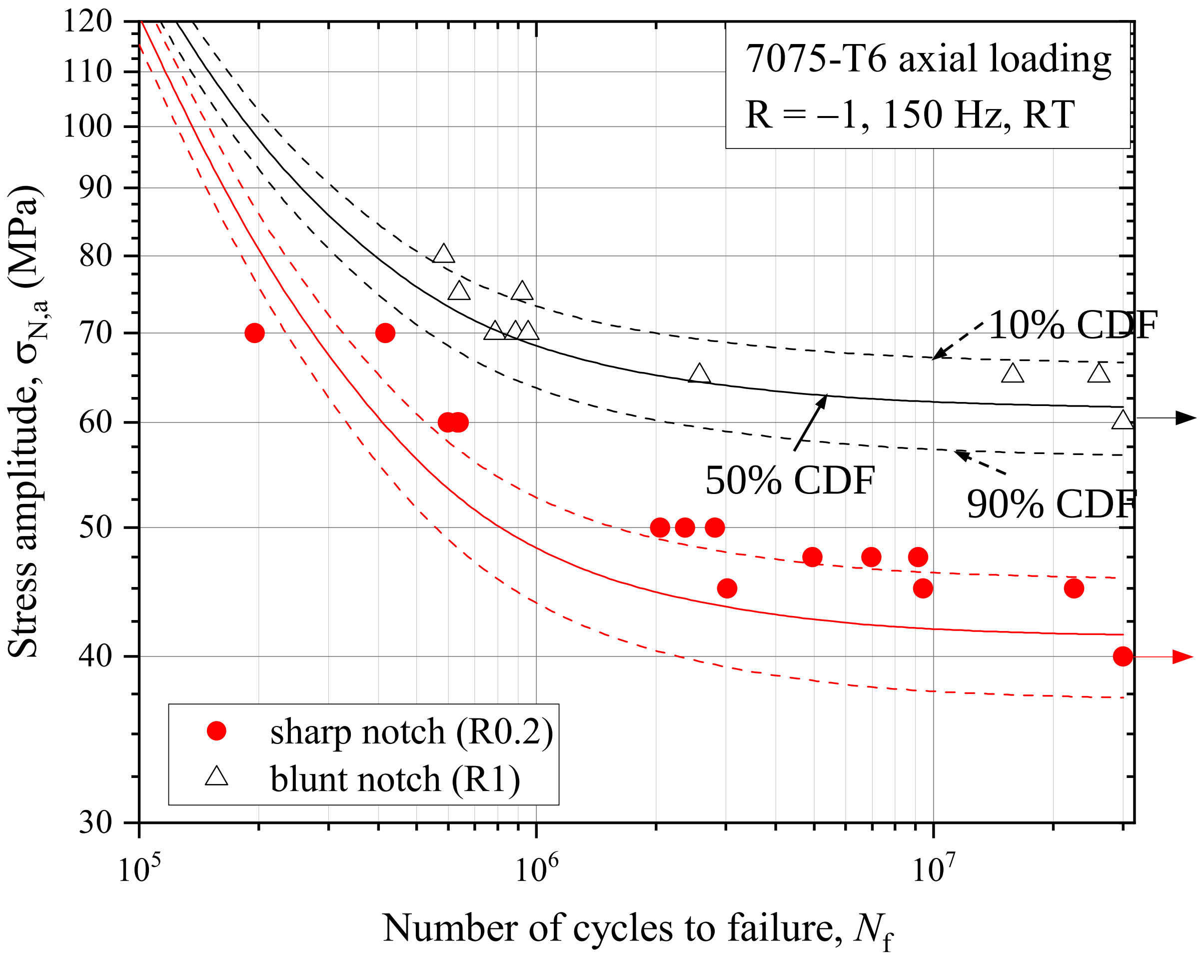
Plain specimen $\Delta\sigma_{\mathrm{fl}}$

Critical distance L

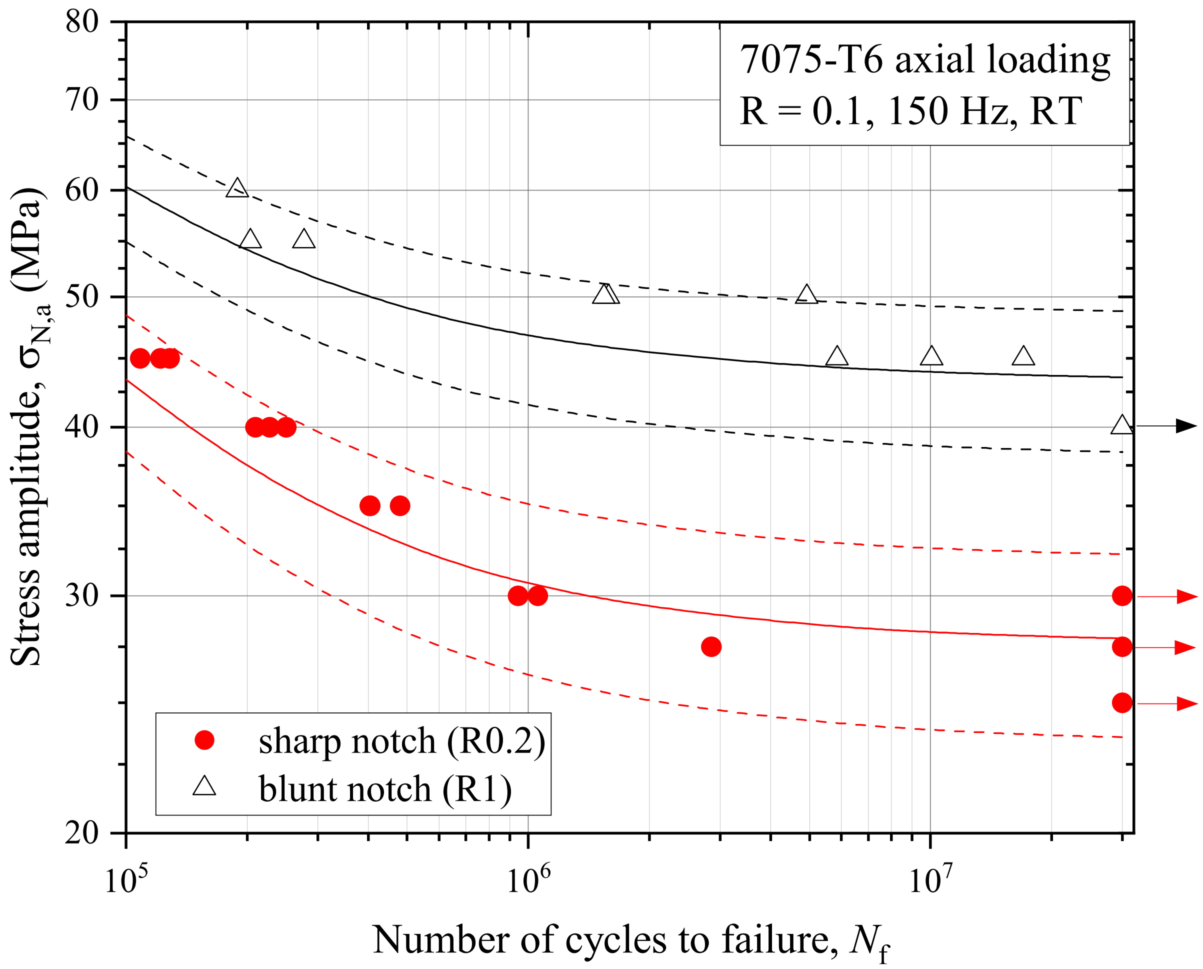
Notched specimen $\Delta\sigma_{\mathrm{N,fl}}$

Fatigue strength prediction of blunter specimens

Load ratio R = -1



Load ratio R = 0.1



Accurate prediction on the entire finite life regime, both in terms of mean values and trends, and also statistical distributions

Conclusions

- The critical distance shows a Skew normal (not symmetric) distribution.
- This distribution is Right-skewed, i.e. the skewness is positive.
- This procedure is extended to the fatigue finite life regime, the distribution trend depends on the load ratio.
- A limit (maximum) radius of the notched specimen is found and its value was larger for the finite life than at the fatigue limit.
- Blunter specimens are assessed, and the probability distribution is in agreement with the experimental data scatter on the entire high cycle fatigue regime.