Sensibility analysis of the fatigue critical distance values assessed by combining plain and notched cylindrical specimens

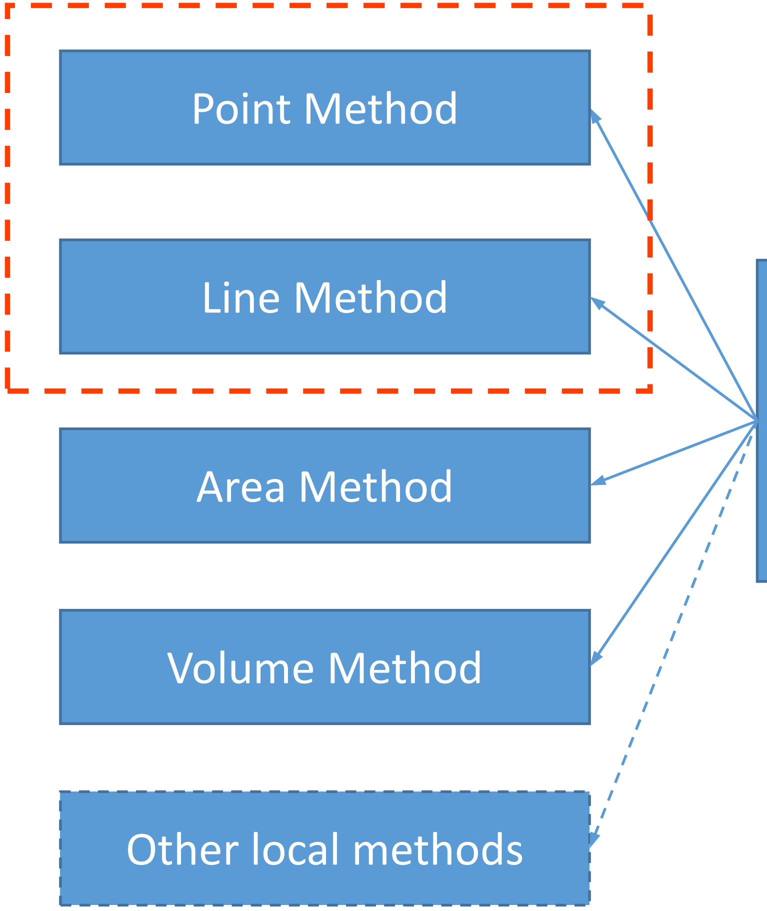
Santus C.[(a)], Taylor D.[(b)], Benedetti M.[(c)]

(a) University of Pisa

(b) Trinity College Dublin

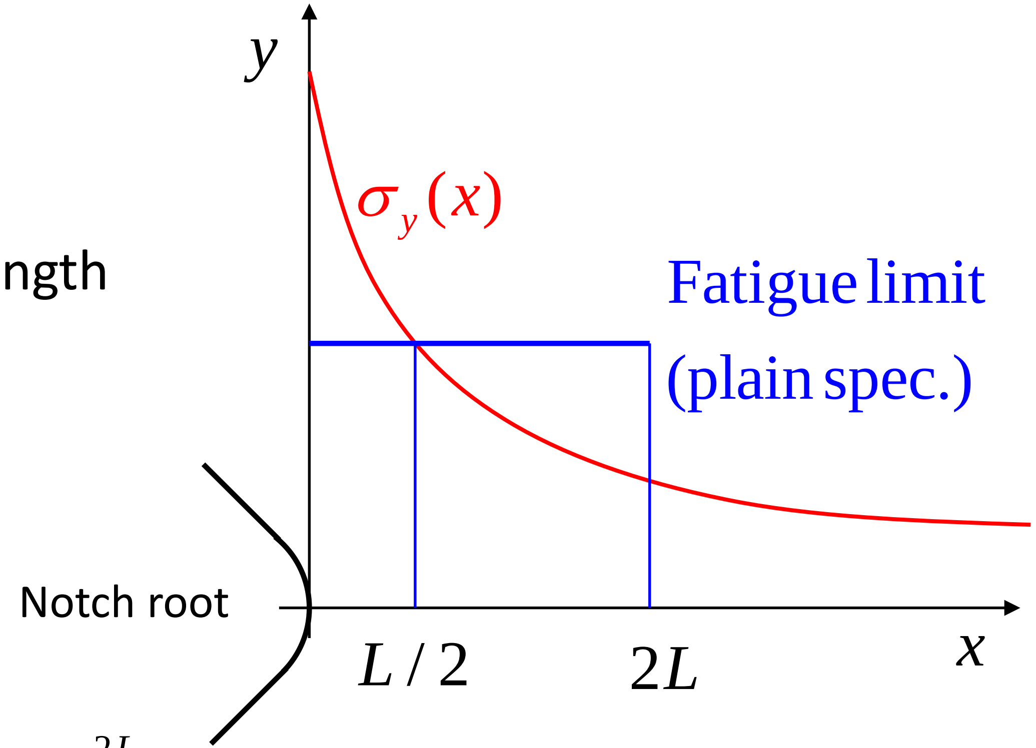
(c) University of Trentio

The Theory of Critical Distances (fatigue)



Critical Distance or
Material characteristic length

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\Delta \sigma_{\mathrm{fl}}}\right)^2$$

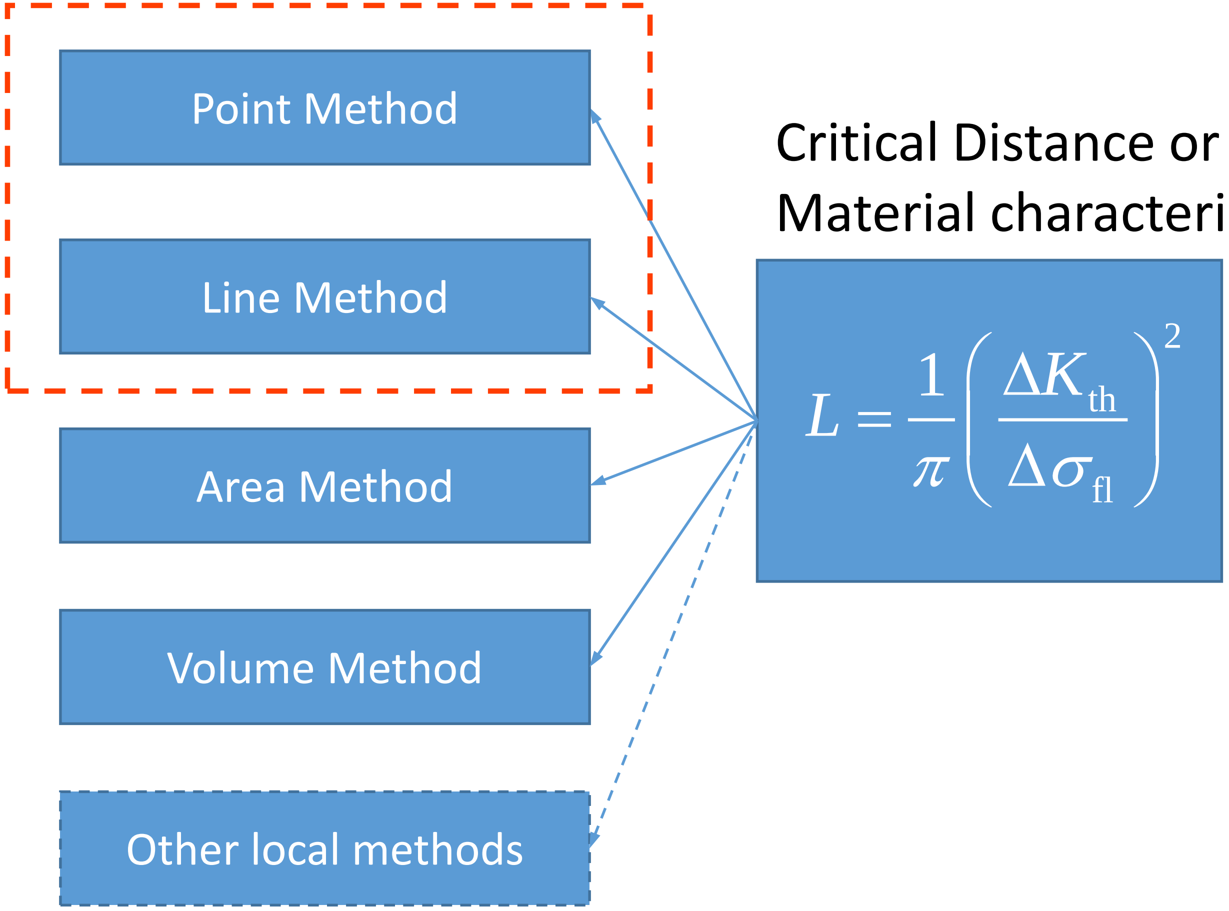


$$\int_0^{2L} \Delta\sigma_y(x)\,\mathrm{d}x = \Delta\sigma_{\mathrm{fl}} \quad \text{(Line Method, LM)}$$

$$\Delta\sigma_y(L/2) = \Delta\sigma_{\mathrm{fl}} \quad \text{(Point Method, PM)}$$

The Theory of Critical Distances (fatigue)

Critical Distance or
Material characteristic length



$$L = \frac{1}{\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\Delta \sigma_{\mathrm{fl}}}\right)^2$$

How to obtain L?

- Literature (similar materials)
- Experiment ($\Delta\sigma_{\mathrm{fl}}$, ΔK_{th})

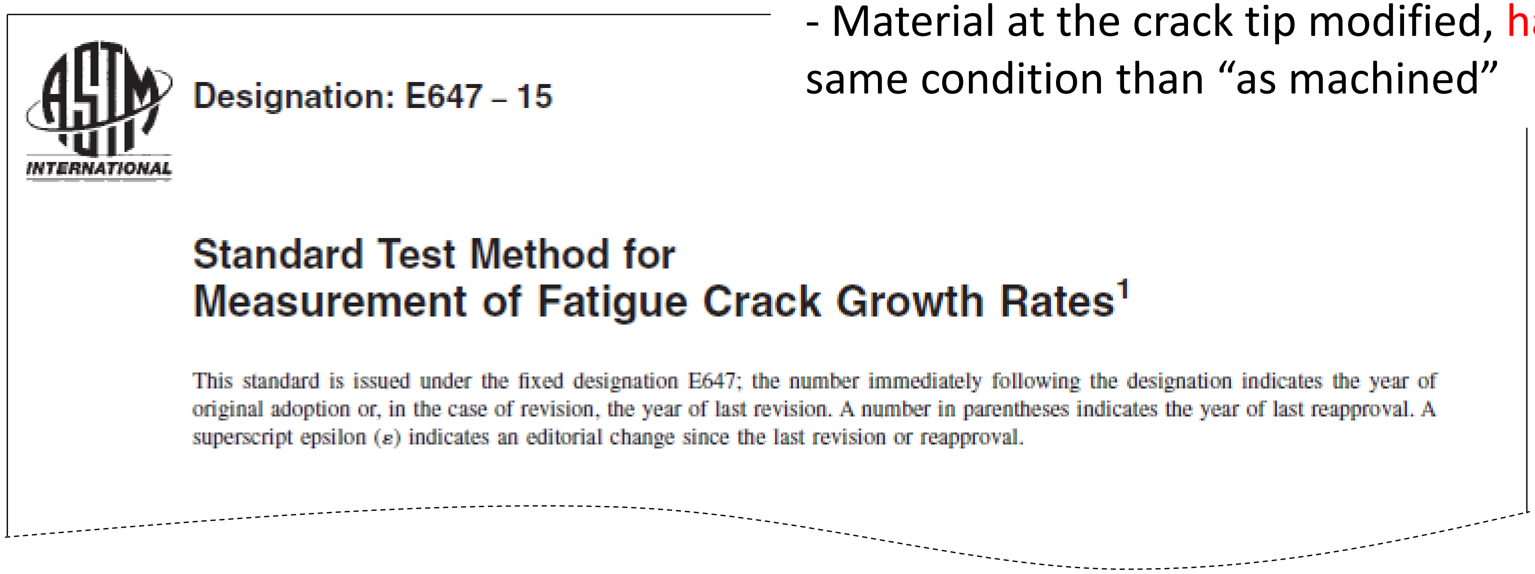
Experimental determination of the SIF Threshold full range

ΔK_{th}

- Challenging experimental procedure though ASTM standard, crack length measure required

- Specimen C(T), or M(T), may not fit into the samples, especially for material supply in bars

- Material at the crack tip modified, hardened and/or damaged, thus not same condition than “as machined”



INTERNATIONAL

Designation: E647 – 15

Standard Test Method for Measurement of Fatigue Crack Growth Rates[1]

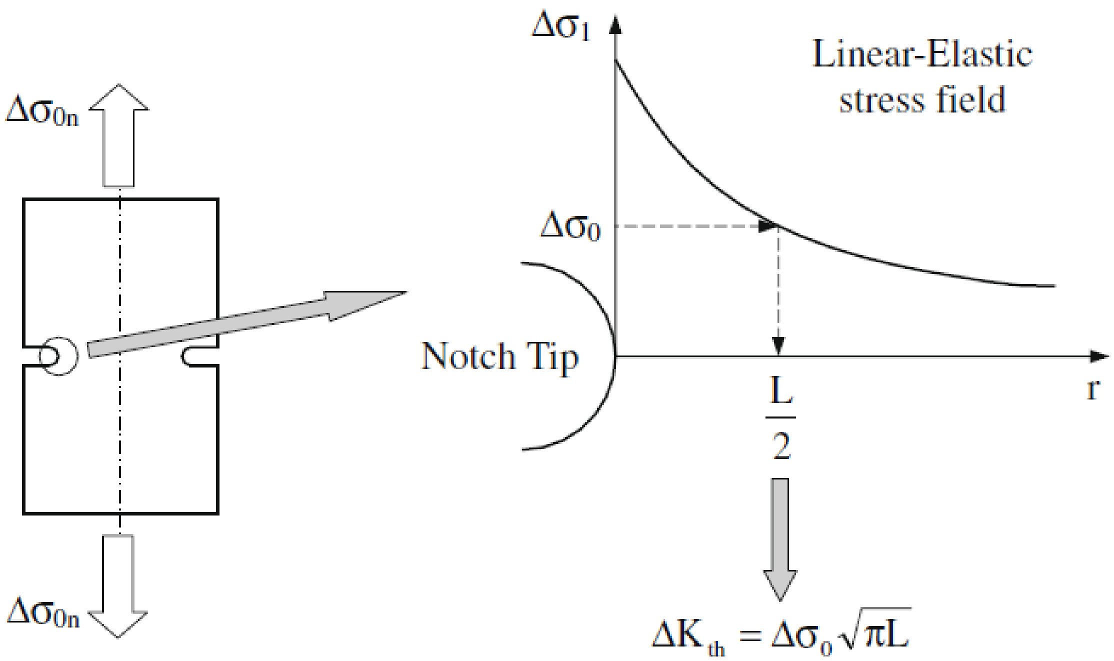
This standard is issued under the fixed designation E647; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Inverse search without SIF Threshold, literature examples

[1] L. Susmel, D. Taylor, The Theory of Critical Distances as an alternative experimental strategy for the determination of KIc and DKth, Engineering Fracture Mechanics 77 (9) (2010) 1492–1501. doi:10.1016/j.engfracmech.2010.04.016

[2] W. Li, L. Susmel, H. Askes, F. Liao, T. Zhou, Assessing the integrity of steel structural components with stress raisers using the Theory of Critical Distances, Engineering Failure Analysis 70 (2016) 73–89. doi:10.1016/j.engfailanal.2016.07.007

[…]



Inverse search, proposed procedure

Motivations

- Definition of an optimized specimen to circumvent the Fracture Mechanics test
- Provide an analytical procedure to avoid the FE analysis of each specimen geometry
- Obtain an effective range where the resulting critical distance is expected to be not largely sensitive to any experimental issue

Application, Steel 42MoCr4 Q+T

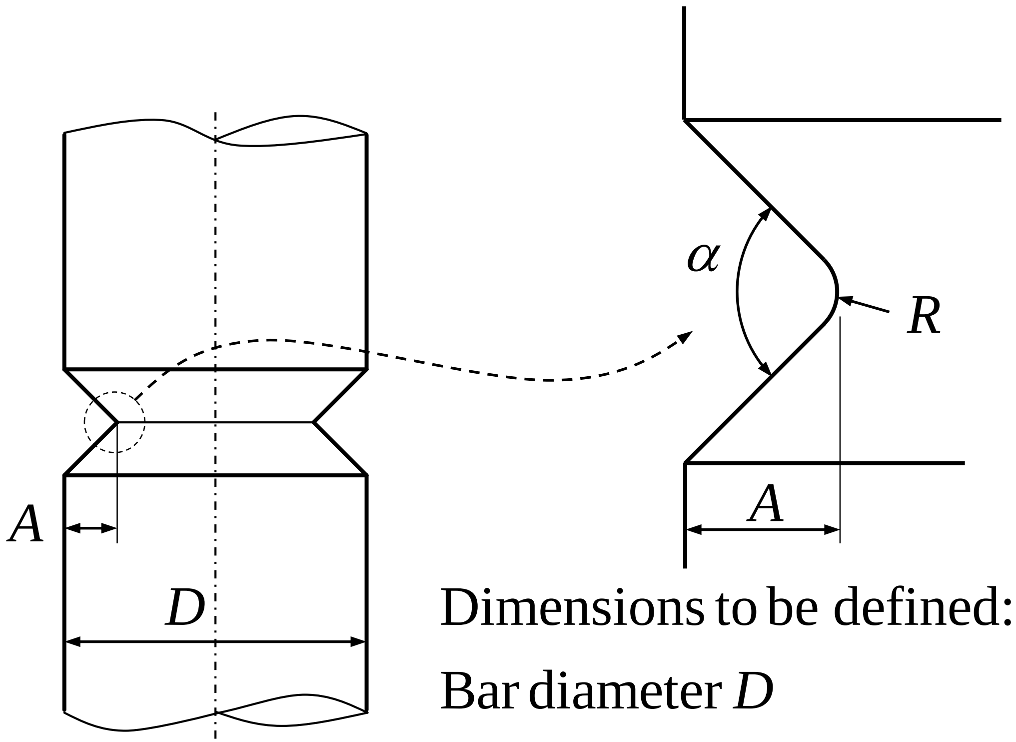
- Experimental data and comparison with the Fracture Mechanics derived lengths
- Fatigue strength assessment, accuracy and sensitivity analyses

Proposed specimen

- V-notch axisymmetric specimen:
easy to manufacture, no boundary effects

- Relatively open angle: 90°, 60°

- Sharp root radius



Dimensions to be defined:

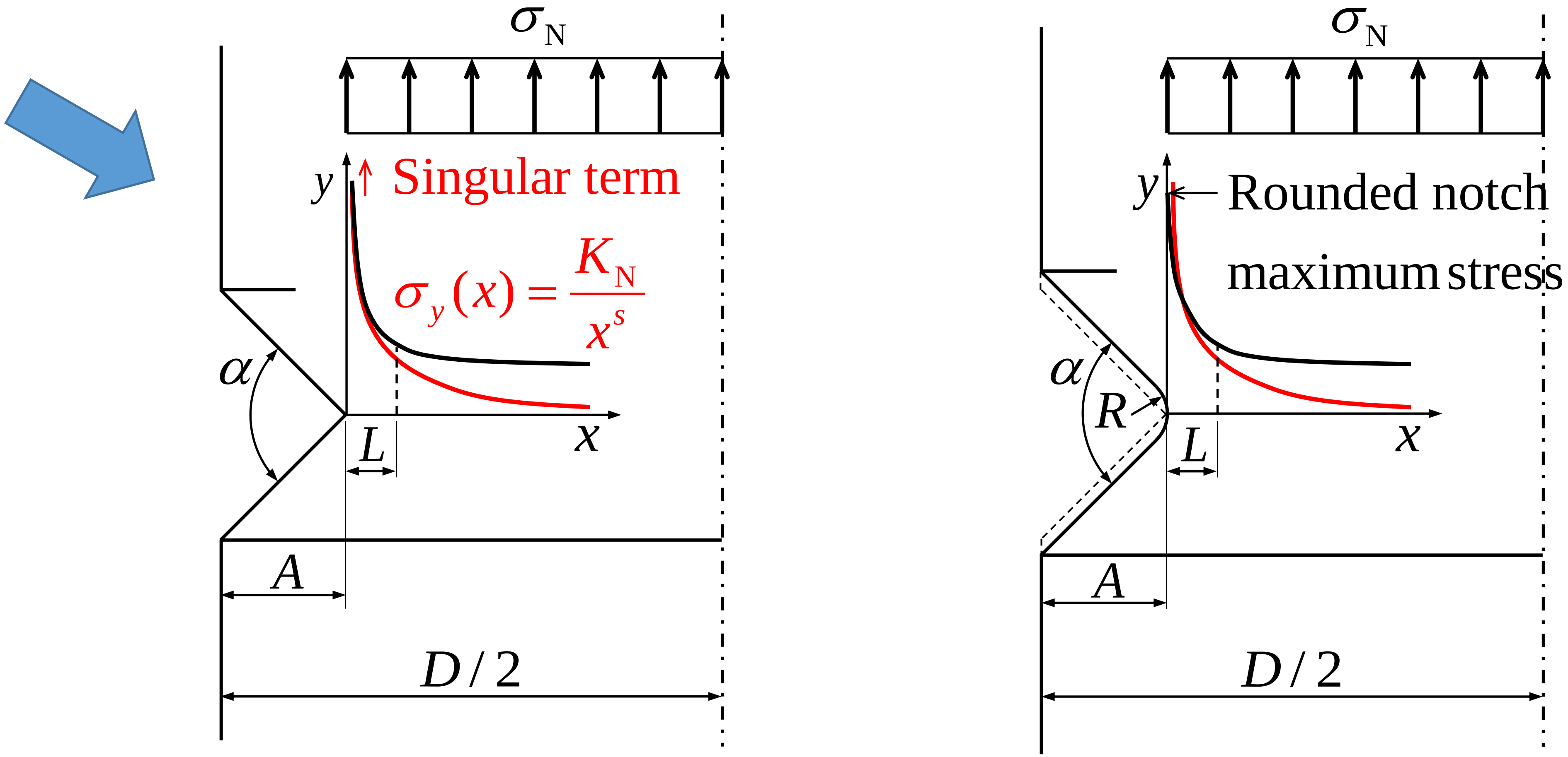
Bar diameter D

Notch depth A

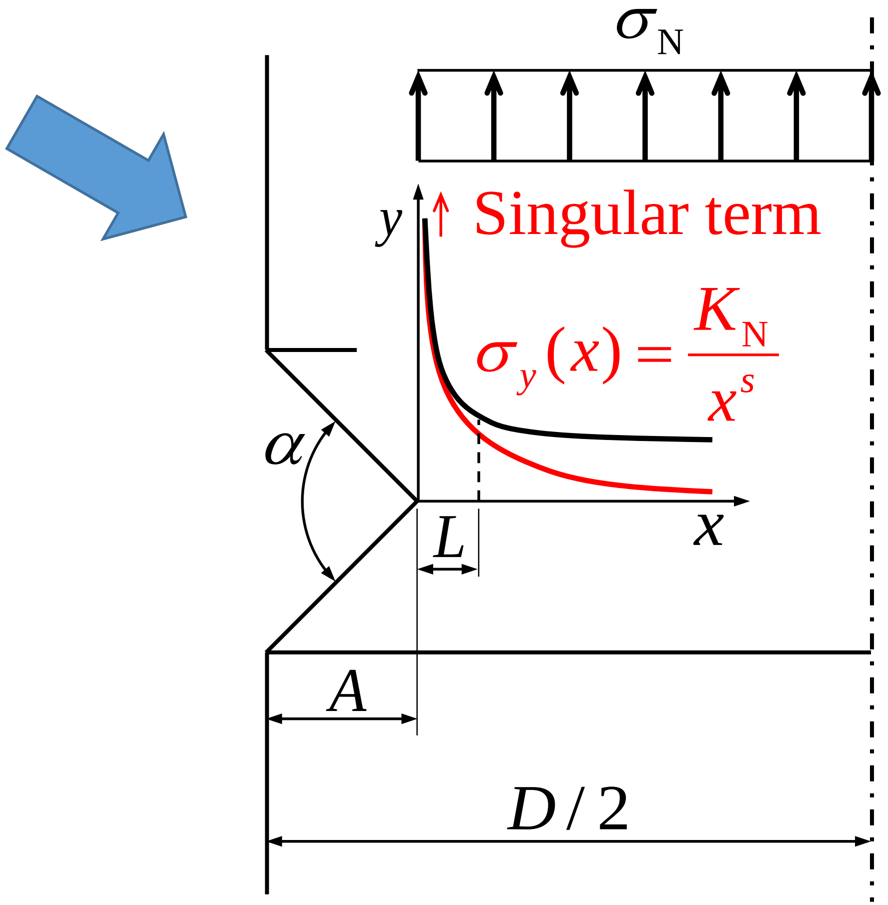
Notch angle α

Notch radius R

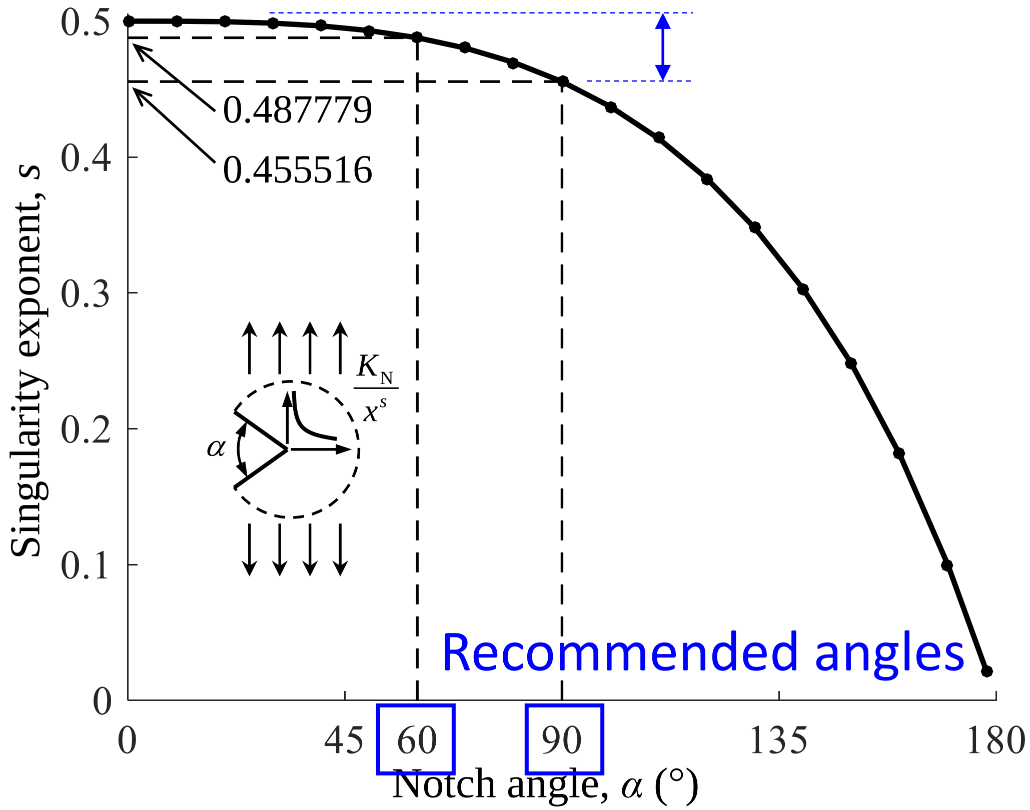
Stress analysis, (i) sharp notch assumption



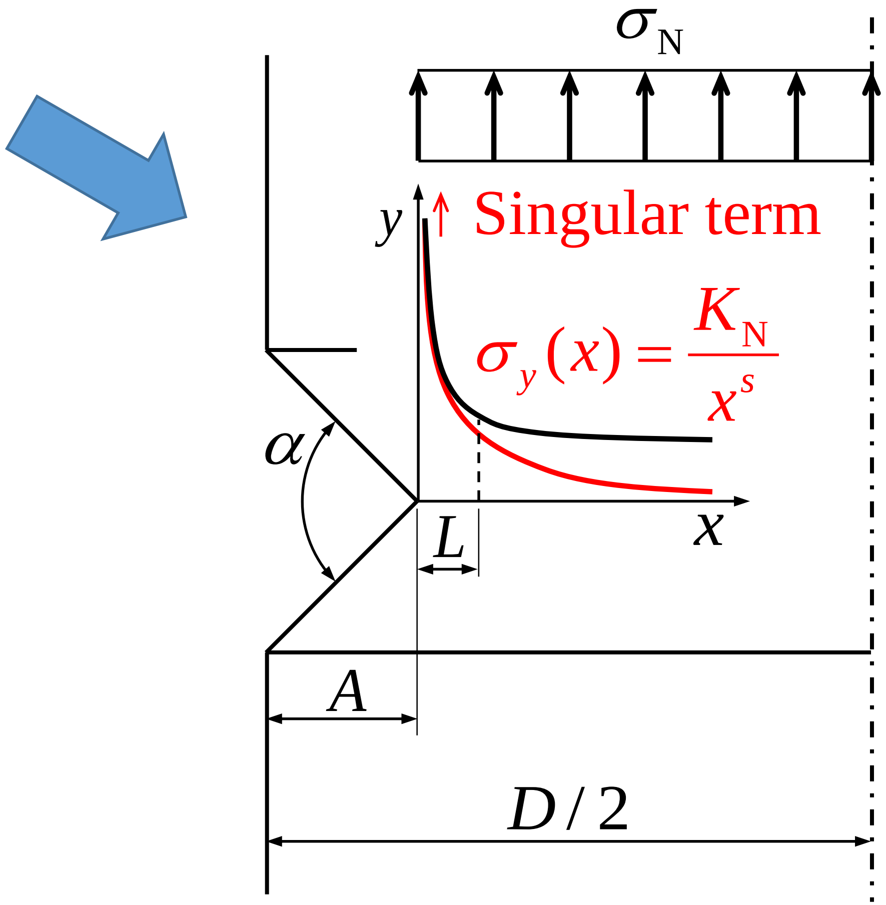
Stress analysis, (i) sharp notch assumption



Williams' exponent close to ½ even for relatively large notch angles



Stress analysis, (i) sharp notch assumption



Dimensionless form:

$$\sigma_y(x) = \frac{K_N}{x^s} = \sigma_N \frac{K_{N,U}}{x^s} = \sigma_N \frac{K_{N,U} / (D/2)^s}{(x/(D/2))^s}$$

$$\sigma_y(\xi) = \sigma_N \frac{K_{N,UU}}{\xi^s}$$

where:

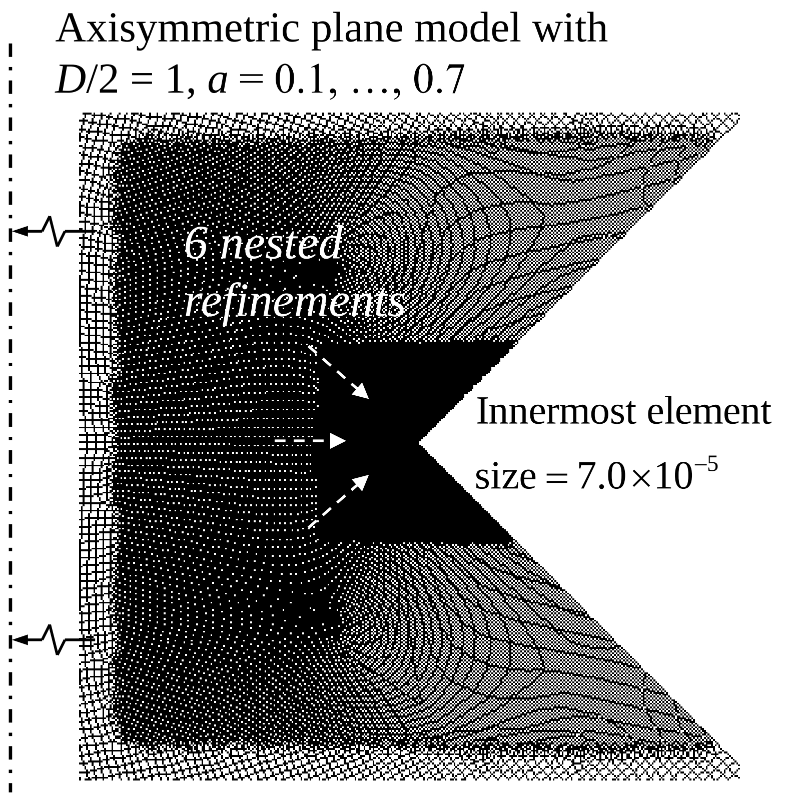
$$\xi = \frac{x}{D/2}$$

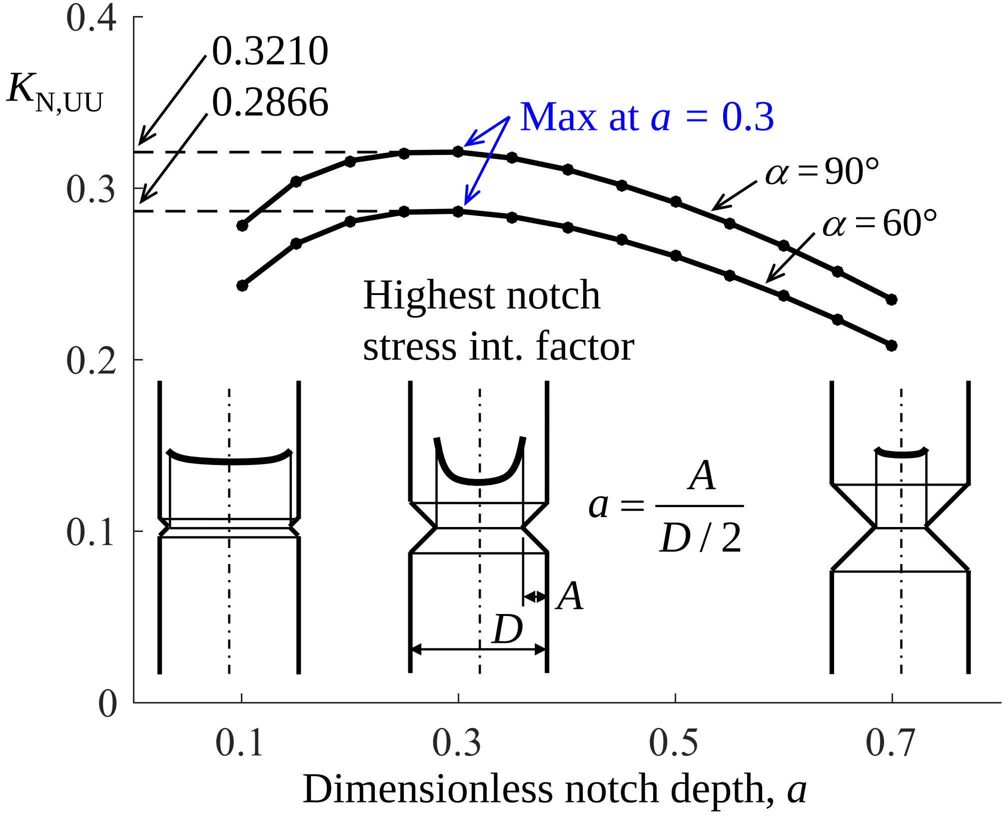
N-SIF for unitary half diameter and unitary nominal stress

$$K_{N,UU} = \frac{K_{N,U}}{(D/2)^s} = \frac{K_N}{\sigma_N (D/2)^s}$$

Stress analysis, (i) sharp notch assumption

FE model with unitary half diameter and unitary stress





Optimal notch depth:

$$\frac{A}{D/2} = 0.3$$

Stress analysis, (i) sharp notch assumption

LM critical distance inverse search

Line Method dimensionless form:

$$\frac{1}{2L}\int_0^{2L}\Delta\sigma_y(x)\,\mathrm{d}x=\frac{1}{2l}\int_0^{2l}\Delta\sigma_y(\xi)\,\mathrm{d}\xi$$

where: $l = L/(D/2)$

Singular term integration:

$$\frac{1}{2l_0}\int_0^{2l_0}\Delta\sigma_y(\xi)\,\mathrm{d}\xi=\int_0^{2l_0}\Delta\sigma_\mathrm{N}\frac{K_\mathrm{N,UU}}{\xi^s}=\frac{\Delta\sigma_\mathrm{N}}{1-s}\frac{K_\mathrm{N,UU}}{(2l_0)^s}$$

"0" stands for singularity

Line Method, average stress equal to fatigue limit:

$$\frac{\Delta\sigma_\mathrm{N}}{1-s}\frac{K_\mathrm{N,UU}}{(2l_0)^s}=\Delta\sigma_\mathrm{fl}$$

Fatigue stress concentration factor:

$$\frac{1}{1-s}\frac{K_\mathrm{N,UU}}{(2l_0)^s}=K_\mathrm{f}$$

Critical distance length inverse derivation:

$$l_0=\frac{1}{2}\left(\frac{K_\mathrm{N,UU}}{(1-s)K_\mathrm{f}}\right)^{1/s},\quad L_0=l_0\,(D/2)$$

Stress analysis, (i) sharp notch assumption

LM/PM critical distance inverse search

Line Method length inverse derivation:

$$L_0 = \frac{D}{4}\left(\frac{K_{\mathrm{N,UU}}}{(1-s)K_{\mathrm{f}}}\right)^{1/s}$$

Similar analysis for the Point Method, length inverse derivation:

$$L_0' = D\left(\frac{K_{\mathrm{N,UU}}}{K_{\mathrm{f}}}\right)^{1/s}$$

Notch parameters:

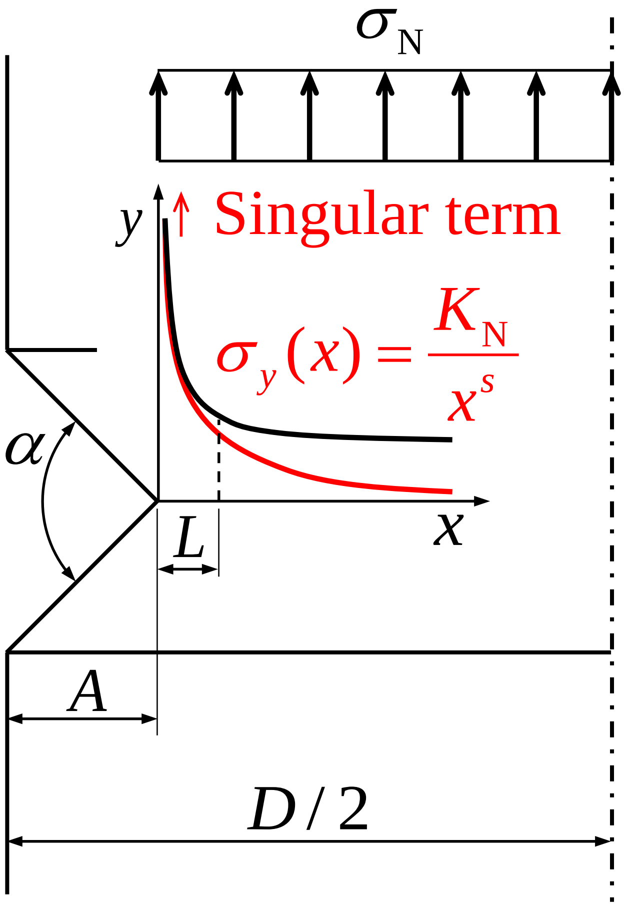
$\alpha = 90°$

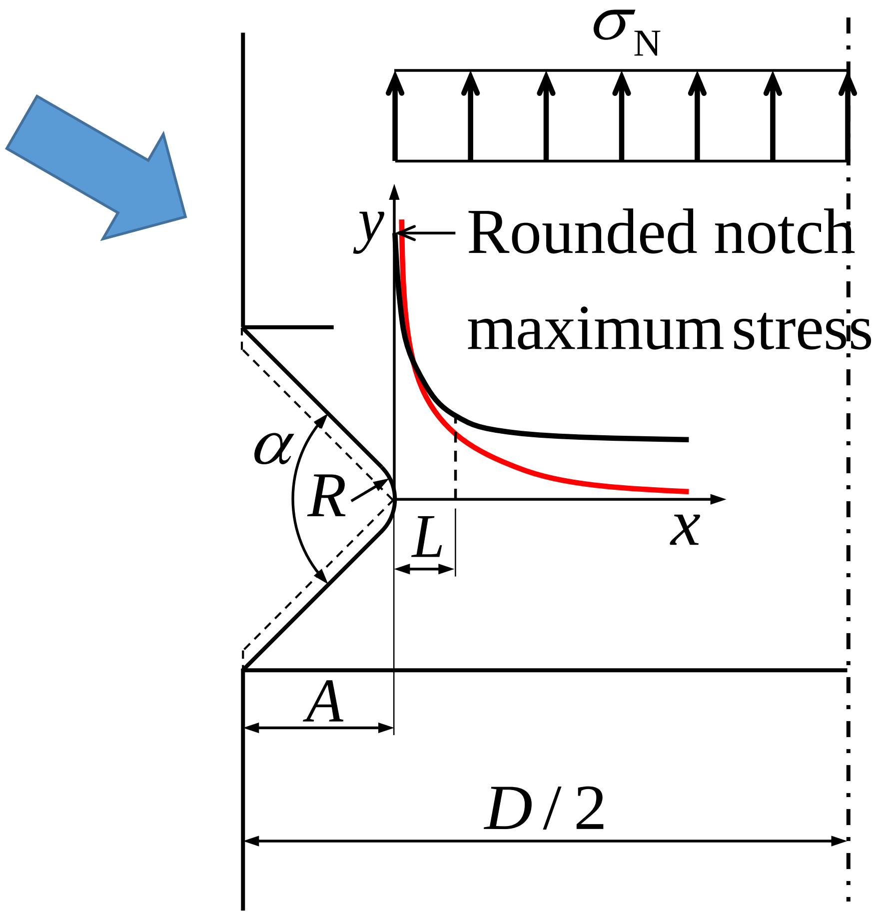
$s = 0.455516 \quad K_{\mathrm{N,UU}} = 0.3210\ (a = 0.3)$

$\alpha = 60°$

$s = 0.487779 \quad K_{\mathrm{N,UU}} = 0.2866\ (a = 0.3)$

Stress analysis, (ii) rounded notch





Dimensionless radius:

$$r = R/(D/2)$$

$$\rho = \frac{R}{A} = \frac{r}{a}$$

Stress analysis, (ii) rounded notch

LM critical distance inverse search

Line Method dimensionless form:

$$\frac{1}{2l}\int_0^{2l}\Delta\sigma_y(\xi)\,\mathrm{d}\xi = \Delta\sigma_\mathrm{N}\,\frac{f(l)}{1-s}\,\frac{K_\mathrm{N,UU}}{(2l)^s}$$

where $f(l)$ is a correction function

Line Method equation:

$$\frac{f(l)}{1-s}\,\frac{K_\mathrm{N,UU}}{(2l)^s} = K_\mathrm{f}$$

After introducing l_0:

$$\frac{l}{f(l)^{1/s}} = l_0$$

Inversion function is defined: $\gamma(l) = l\,/\,f(l)^{1/s}$

Inverse search problem:

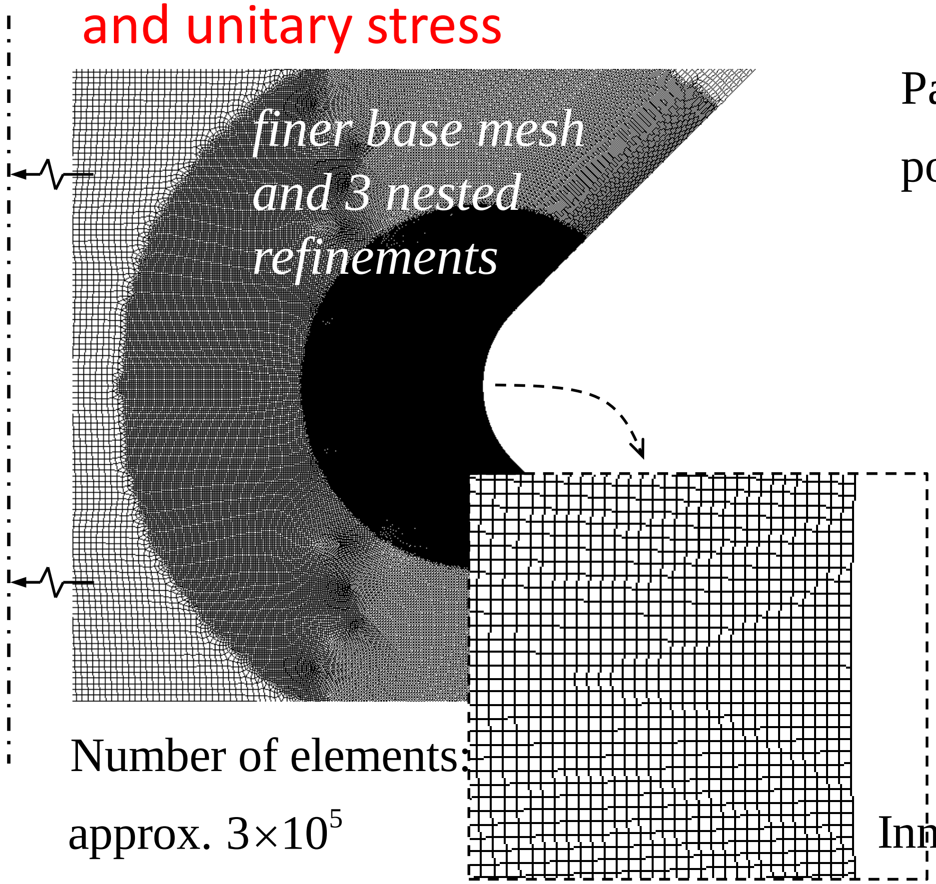
$$\gamma(l) = l_0$$

where:

l_0 known from K_f, l is the unknown

Stress analysis, (ii) rounded notch

FE model with unitary half diameter and unitary stress

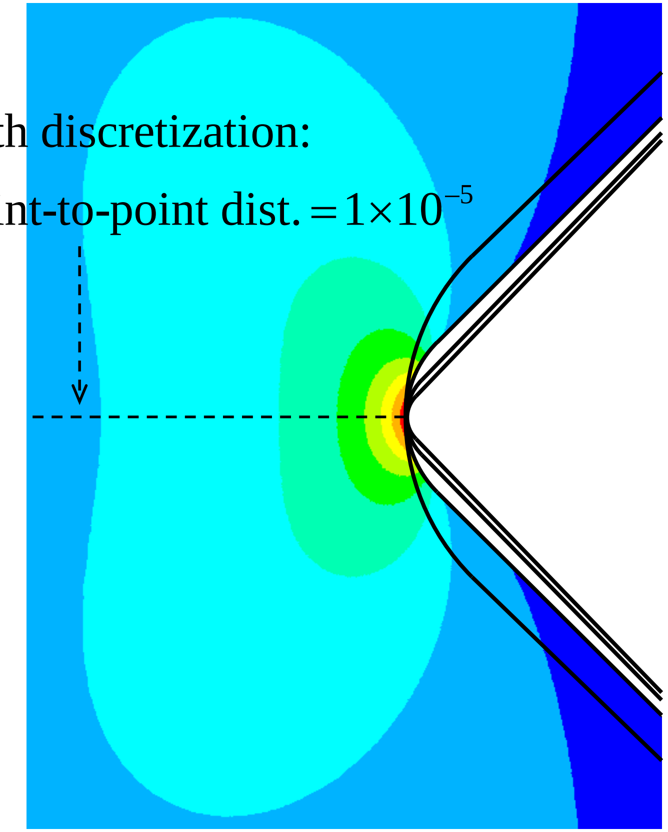


Number of elements: approx. 3×10^{5}

Innermost element size $= 8.7\times10^{-5}$

Path discretization:

point-to-point dist. $= 1\times10^{-5}$



Performed simulations:

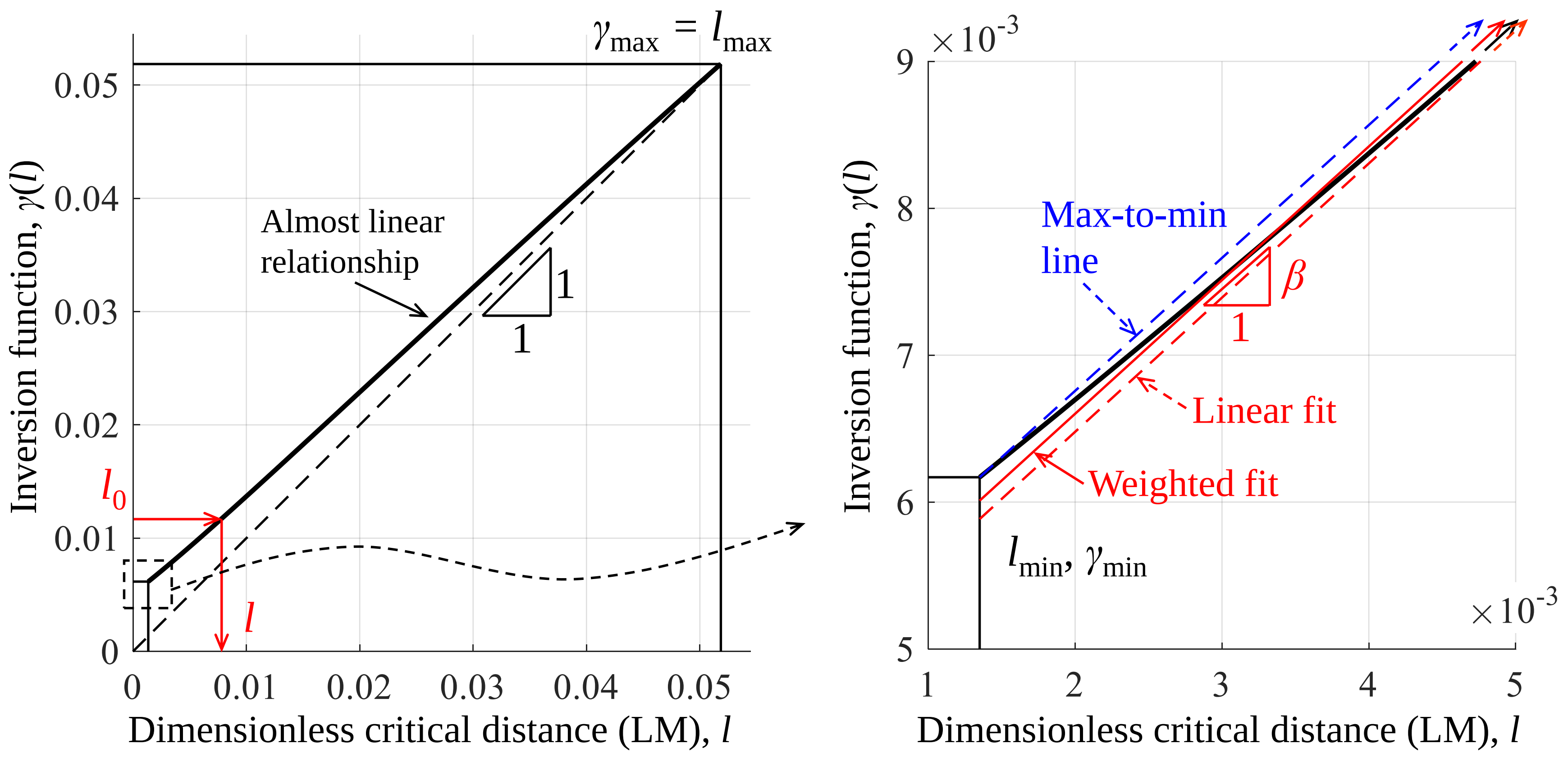
$a = 0.3,$

$\rho = R / A$

$\rho = 0.01,\ 0.02,\ 0.05,\ 0.1,\ 0.2,\ 0.5,\ 1.0$

as the "Euro series"

LM inverse search



Very accurate approx. with a linear model, inverse search:

$$\beta = \frac{\gamma_{max} - \gamma_{min}}{l_{max} - l_{min}}$$

$$l = l_{min} + \frac{l_0 - \gamma_{min}}{\beta}$$

Fit models (ρ functions):

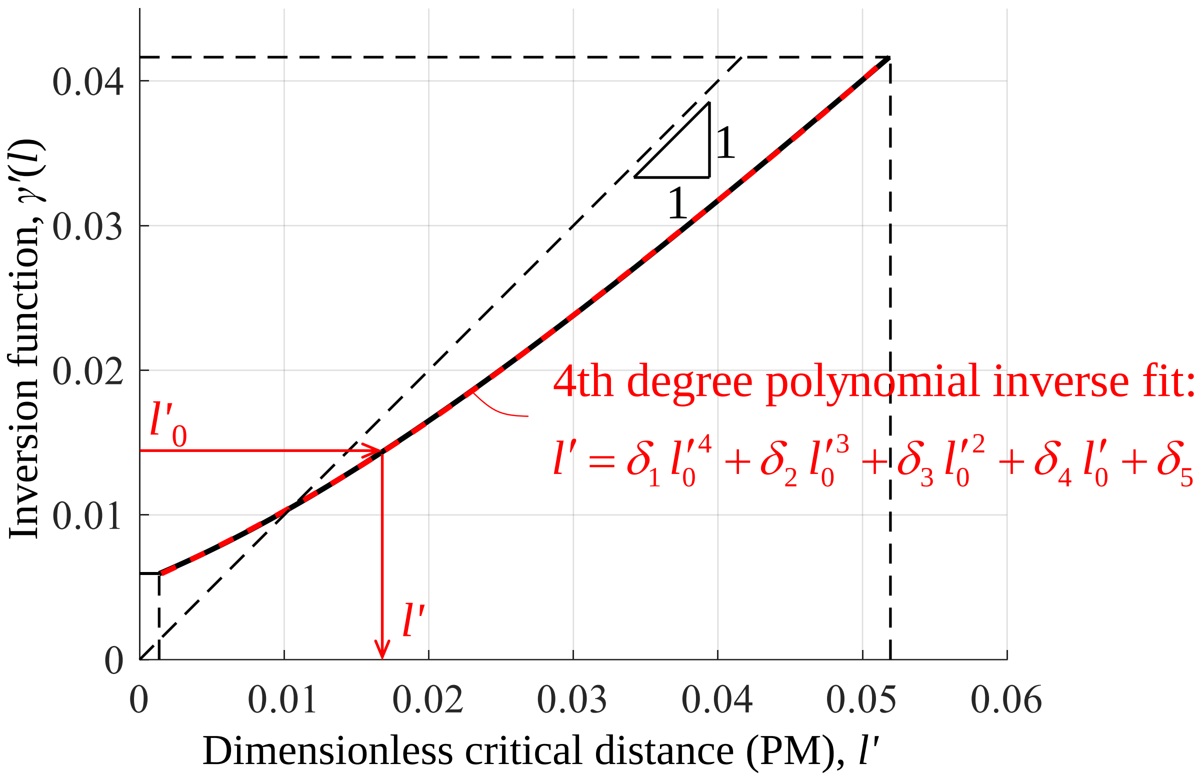
$$l_{min} = p_1\rho^3 + p_2\rho^2 + p_3\rho + p_4$$

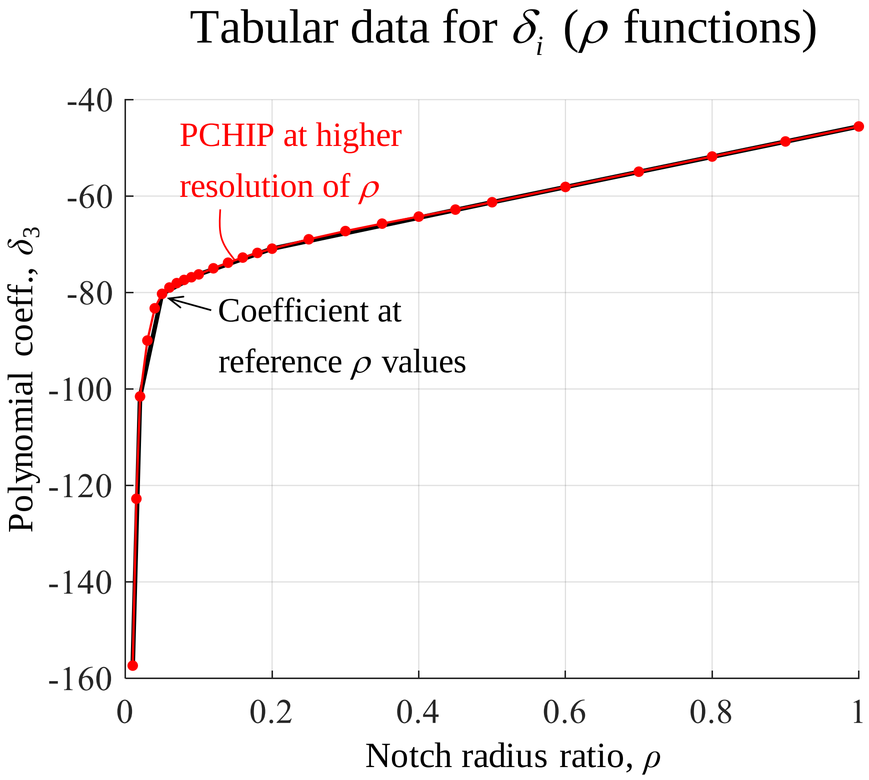
$$\gamma_{min} = q_1\rho^3 + q_2\rho^2 + q_3\rho + q_4$$

$$l_{max} = \gamma_{max} = c_1 + c_2\rho^{c_3}$$

p_i, q_i, c_i provided

PM inverse search





LM/PM inverse search/ direct problem

Inverse search

Line Method

$$l_0 = \frac{1}{2}\left(\frac{K_{\mathrm{N,UU}}}{(1-s)K_{\mathrm{f}}}\right)^{1/s}$$

$$l = l_{\min} + \frac{l_0 - \gamma_{\min}}{\beta}$$

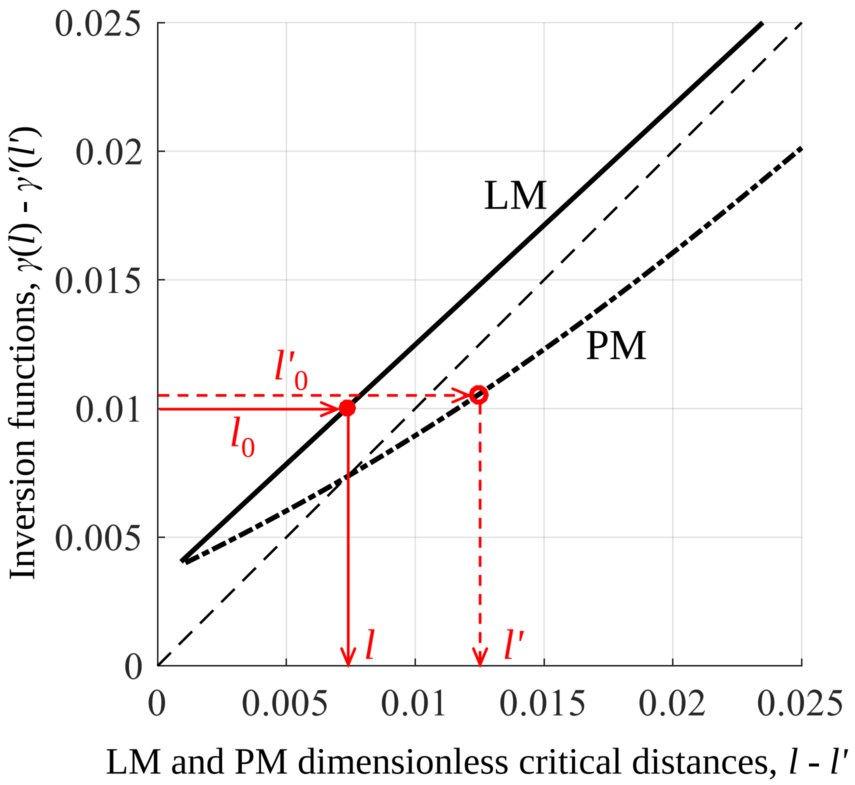
$$L = \frac{D}{2} l$$

Point Method

$$l'_0 = 2\left(\frac{K_{\mathrm{N,UU}}}{K_{\mathrm{f}}}\right)^{1/s}$$

$$l' = \sum_{i=1}^{5} \delta_i \, l_0'^{\,i}$$

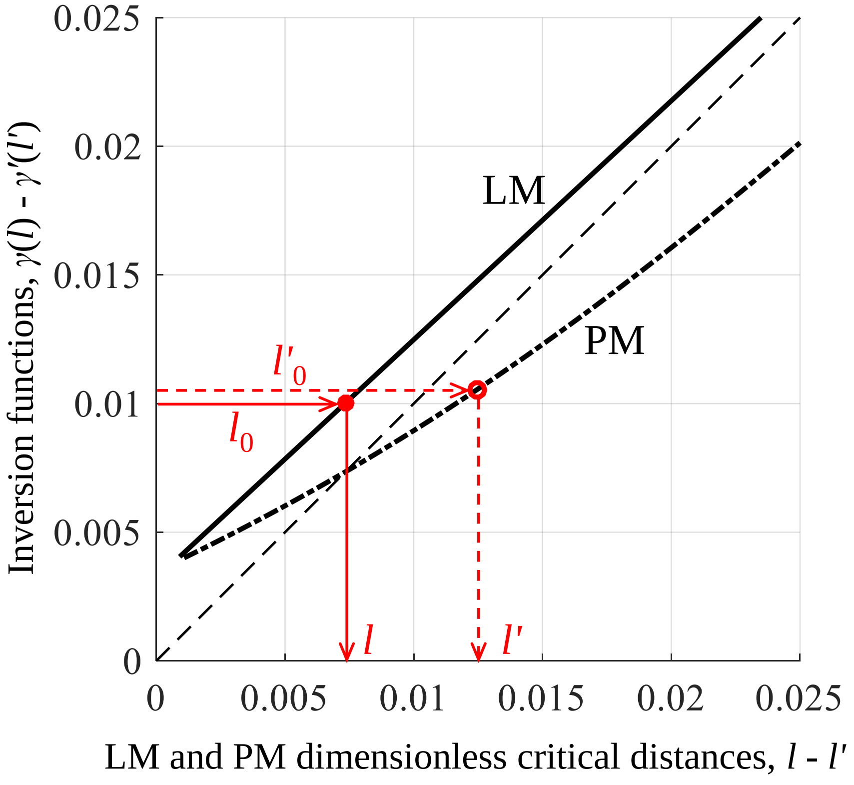
$$L' = \frac{D}{2} l'$$



$$K_{\mathrm{f}} \rightarrow l_0 \rightarrow l \rightarrow L$$

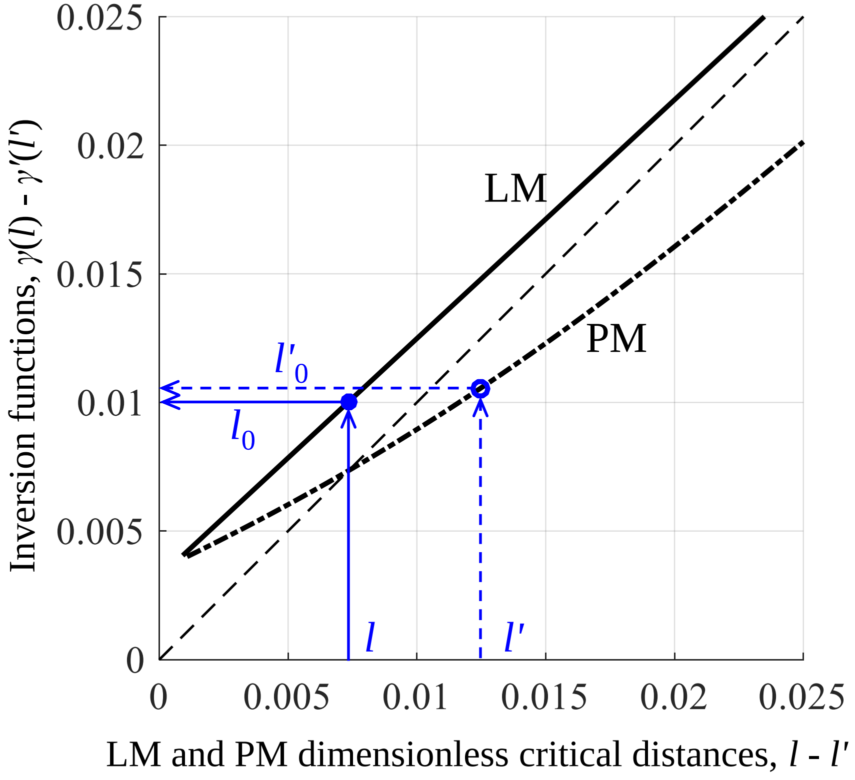
LM/PM inverse search/ direct problem

Inverse search



$$K_f \rightarrow l_0 \rightarrow l \rightarrow L$$

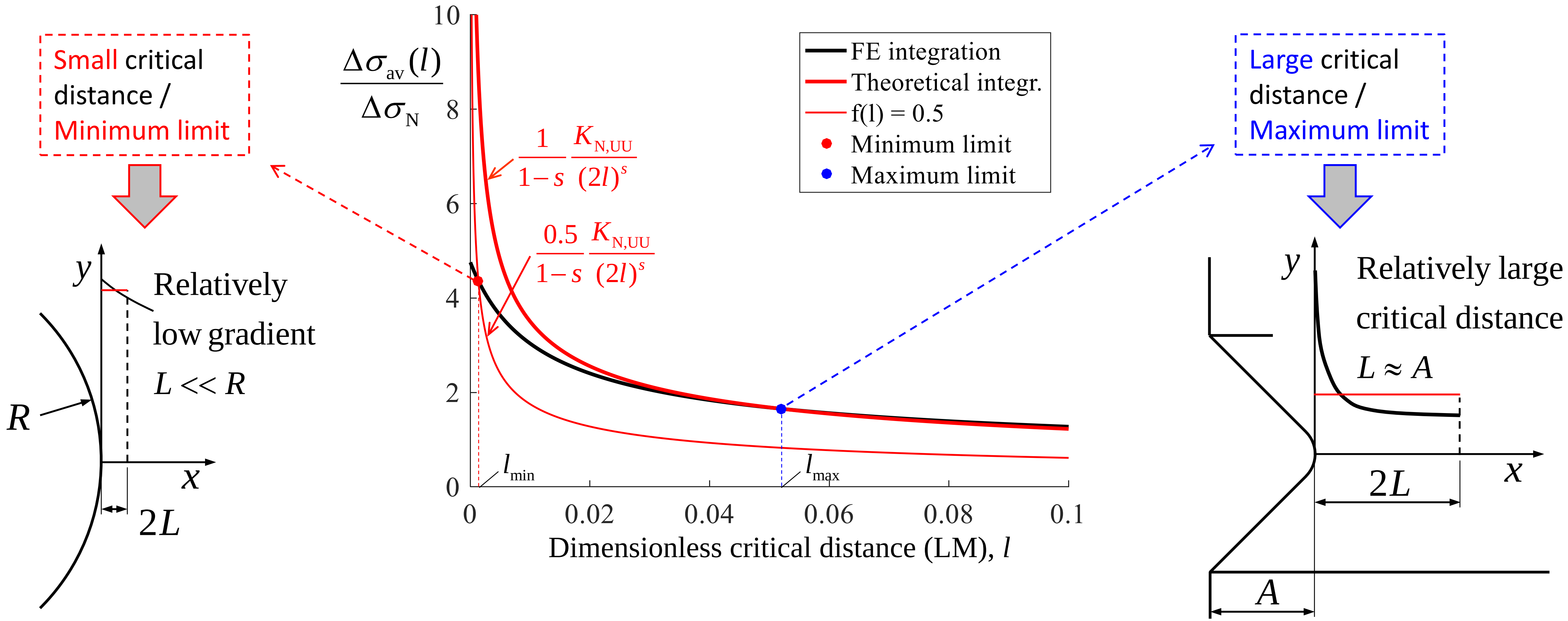
"Direct" problem: *L* is known and then the strength of the specimen is assessed



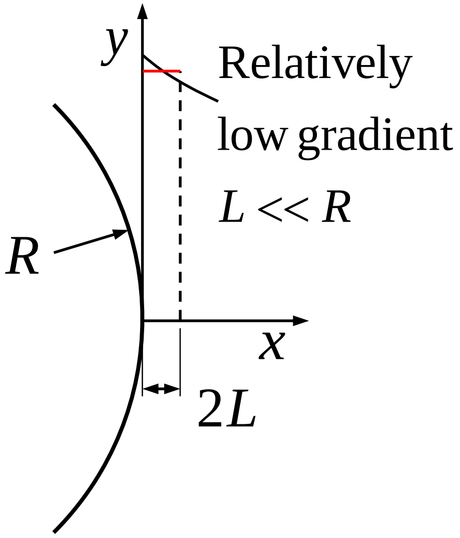
$$L \rightarrow l \rightarrow l_0 \rightarrow K_f$$

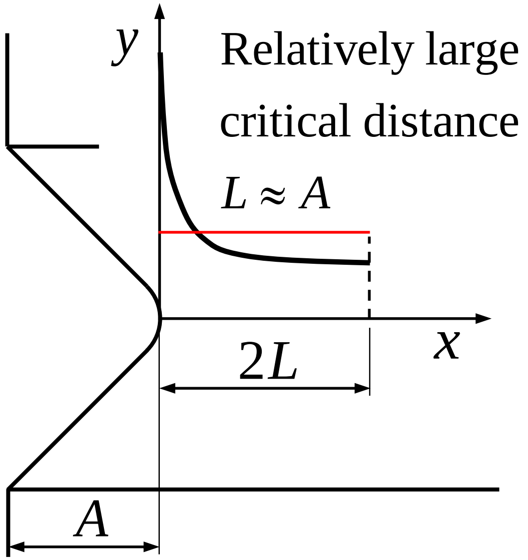
Accurate inverse search range, defined with the Corr. Function

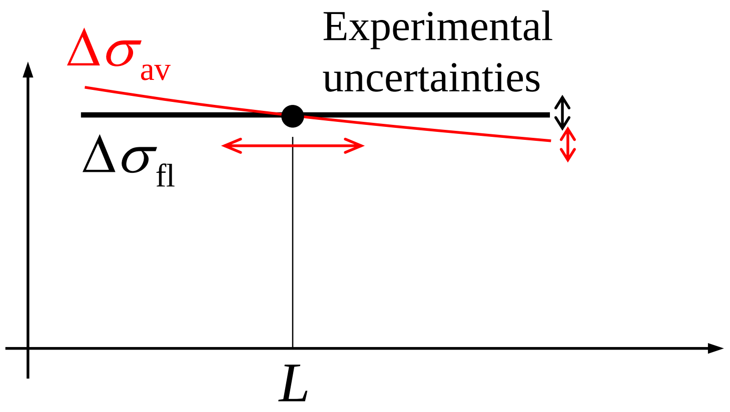
Maximum/Minimum limits for the dimensionless critical distance



Inaccurate inverse search configurations







Small critical distance /
Not sharp enough local radius

Large critical distance /
Small specimen size

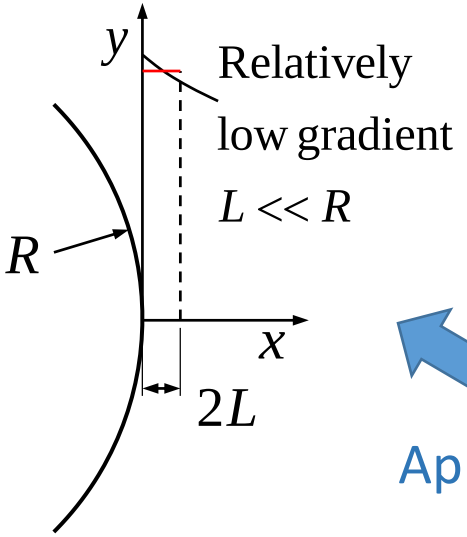
Ill-conditioned inverse search:
Low slope average stress, large variation of the deduced critical distance induced by experimental uncertainties

Sensitivity to any experimental variation of K_f

Sensitivity definition:

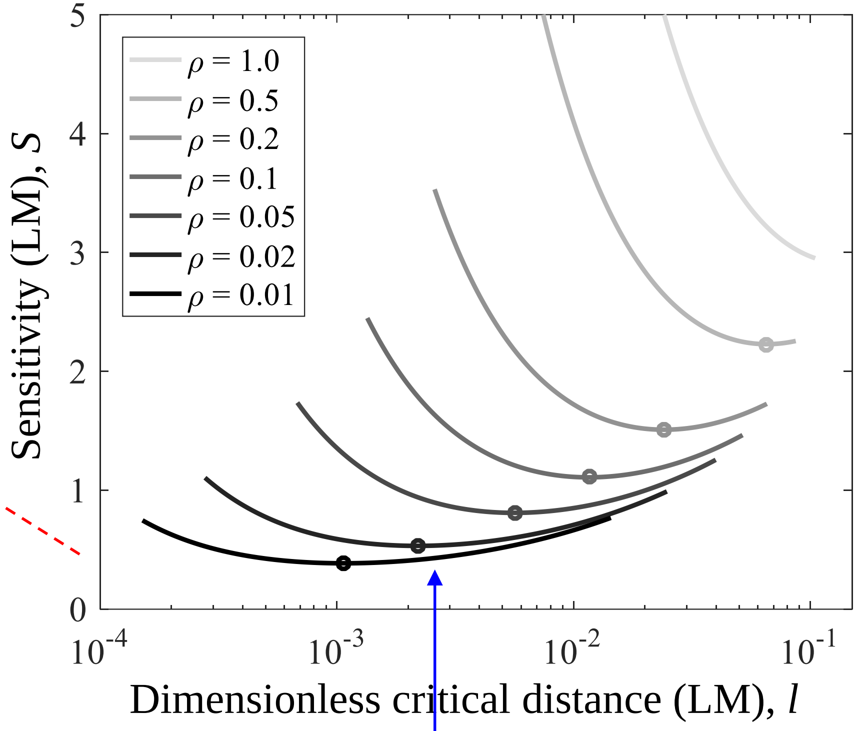
$$S = -\frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}K_f}$$

Small critical distance / Minimum limit



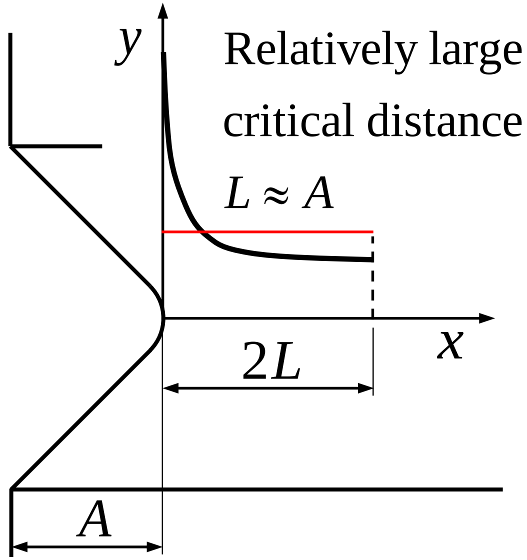
Application

LM sensitivity



Minimum sensitivity in the range $0.5 < f(l) < 1.0$: l_{min} - l_{max}

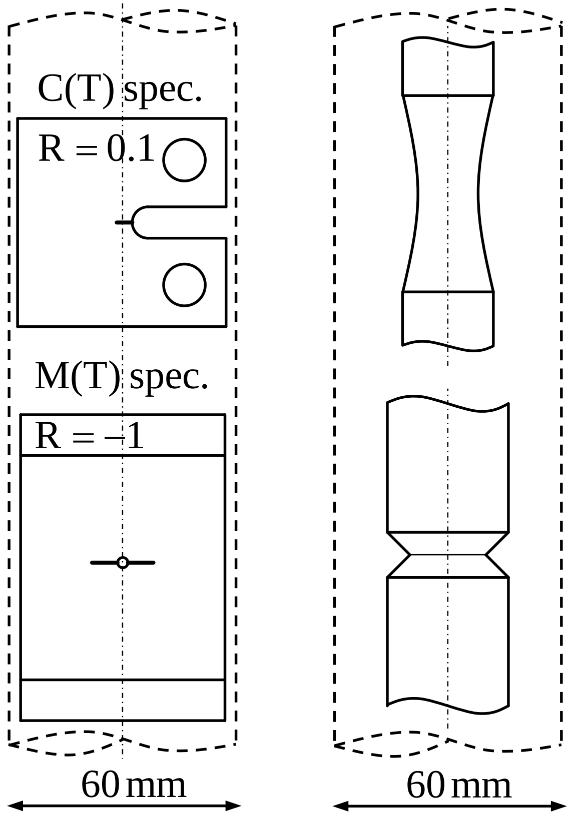
Large critical distance / Maximum limit



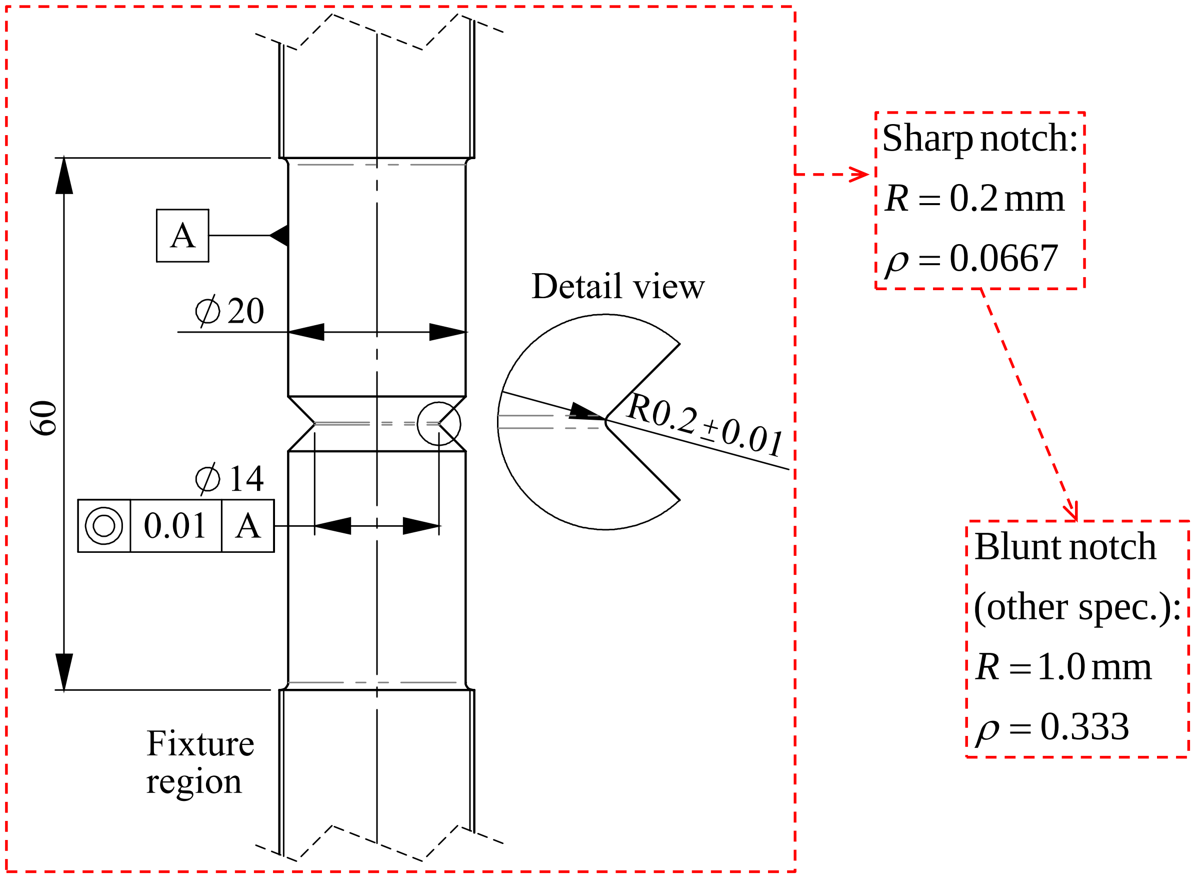
Experimental application, 42CrMo4 Q+T - S_U = 875 MPa

Specimen extraction
from the same bar supply

Fracture mechanics tests for comparison

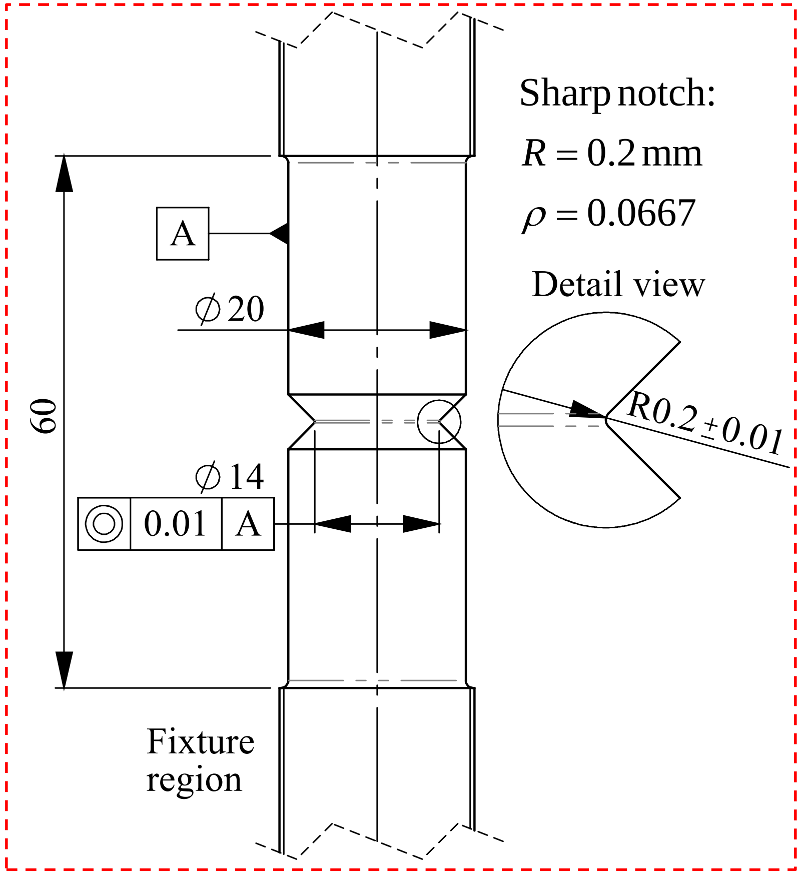


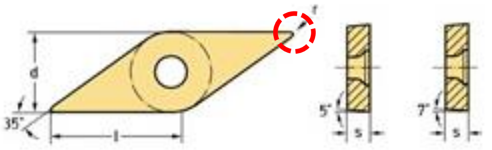
Expected Critical Distance in the order of 0.05 mm, small notch radius 0.2 mm



Experimental application, 42CrMo4 - S_U = 875 MPa

Expected Critical Distance in the order of 0.05 mm, small notch radius 0.2 mm

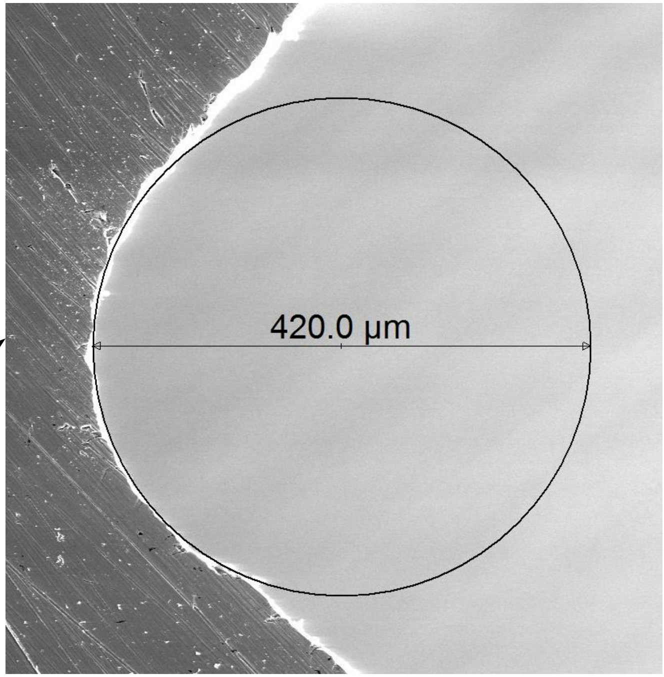




Section and SEM verification

Nominal: $R = 0.2\,\text{mm}$

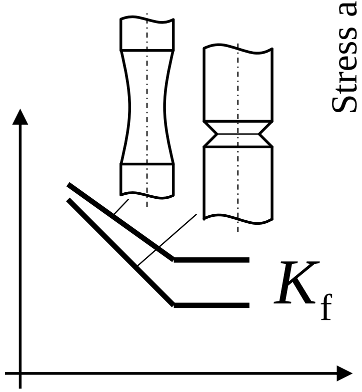
Actual: $R = 0.21\,\text{mm}$

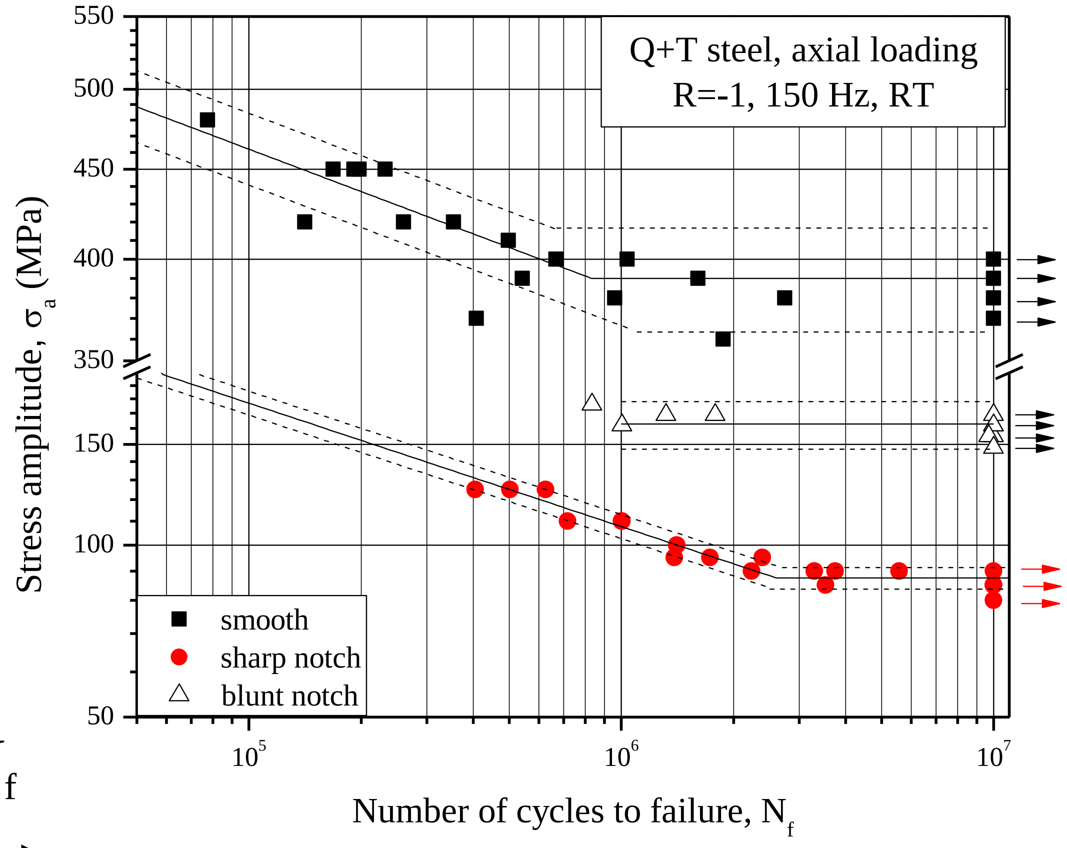


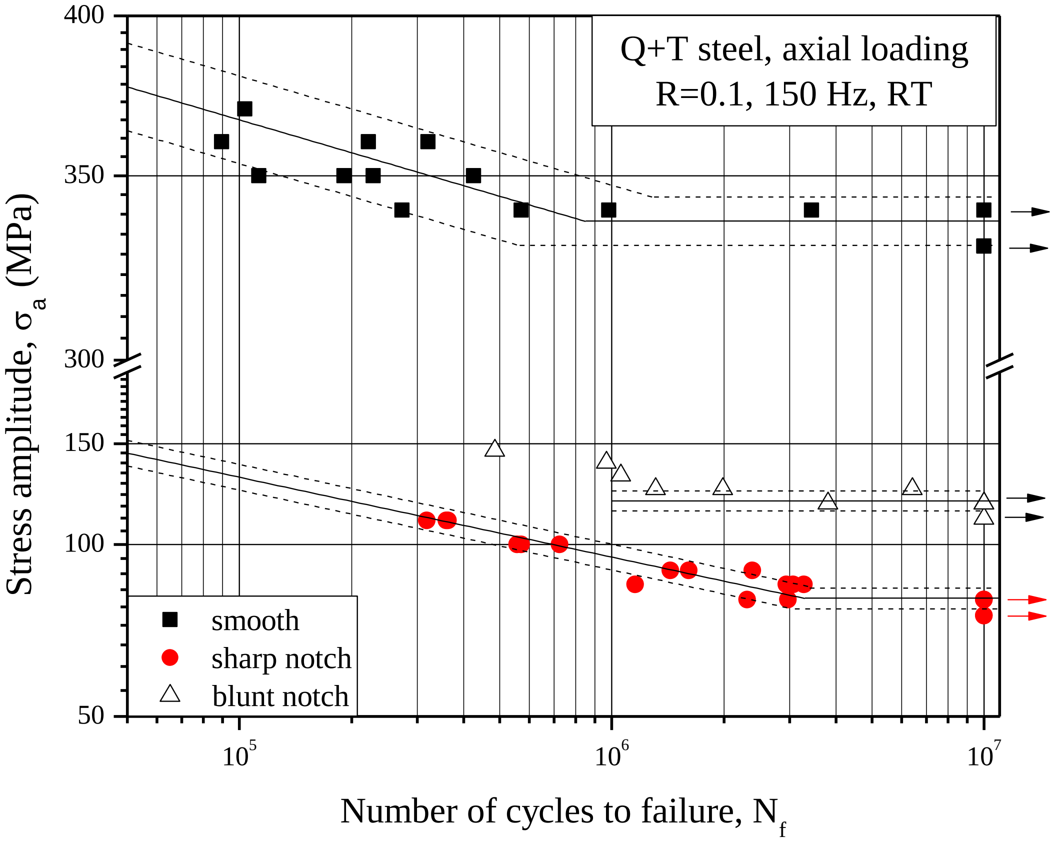
Experimental test results, S-N data

Load ratios: R = 0.1, R = -1

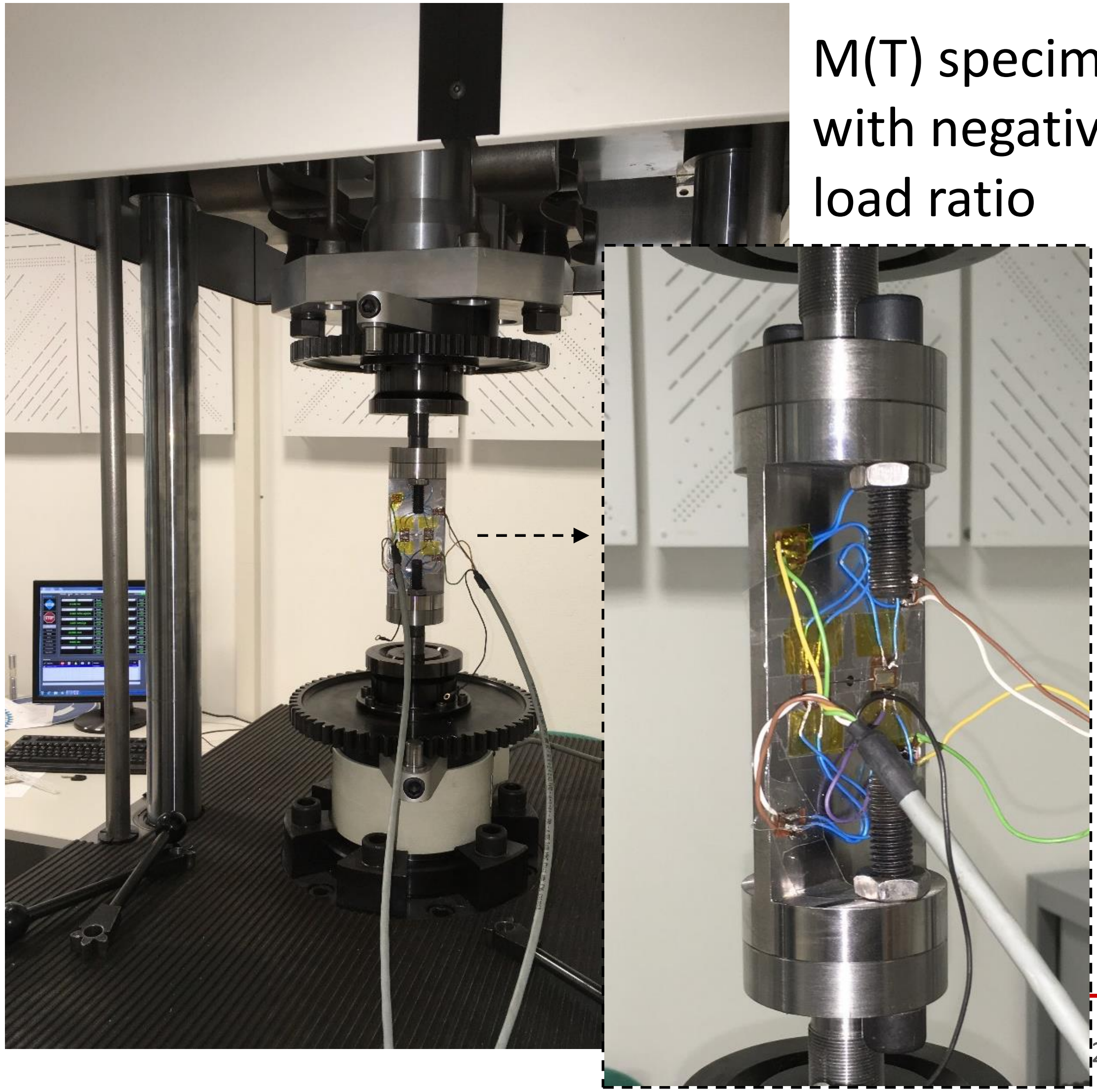
Specimen types: Plain, Sharp (0.2 mm), Blunt (1.0 mm)



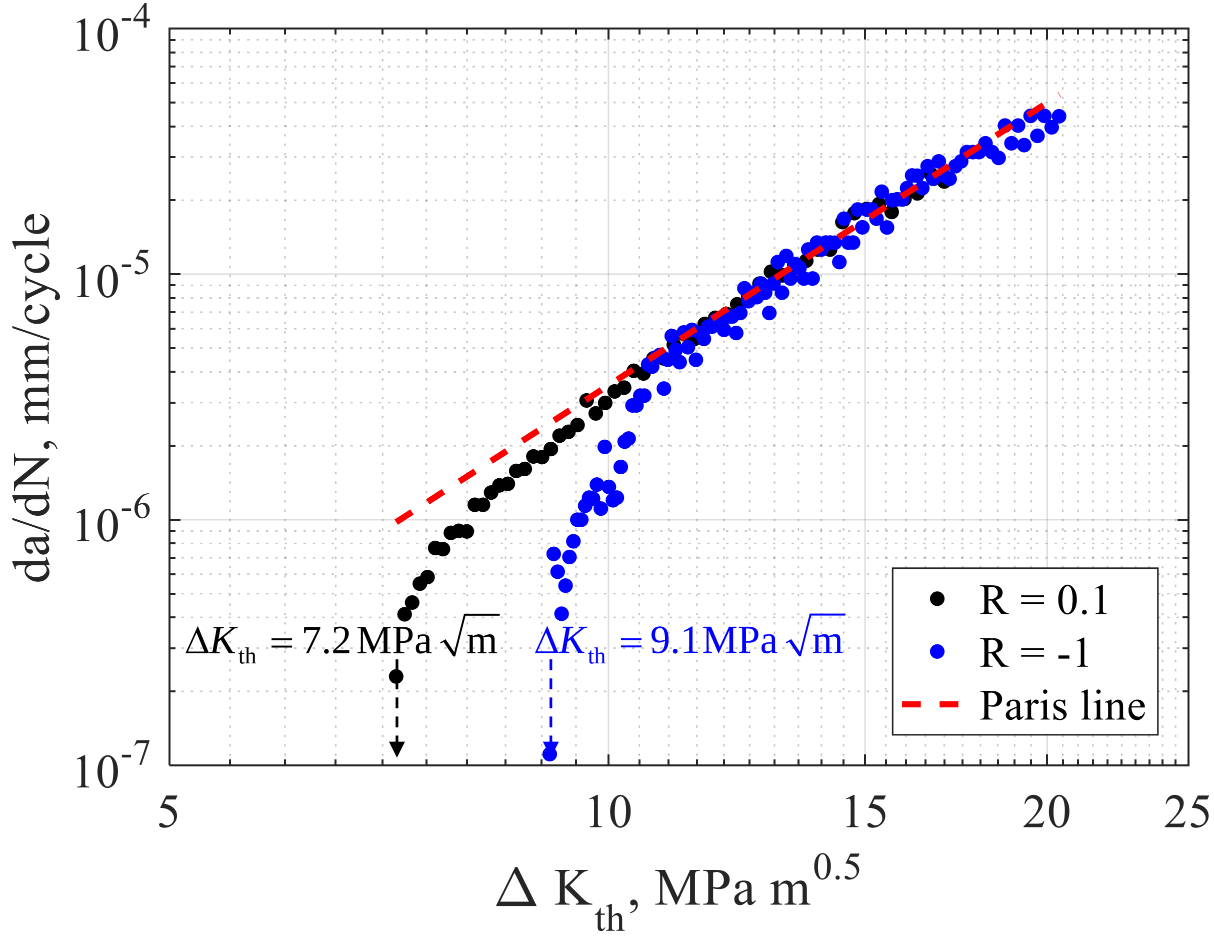




Experimental test results, Thresholds

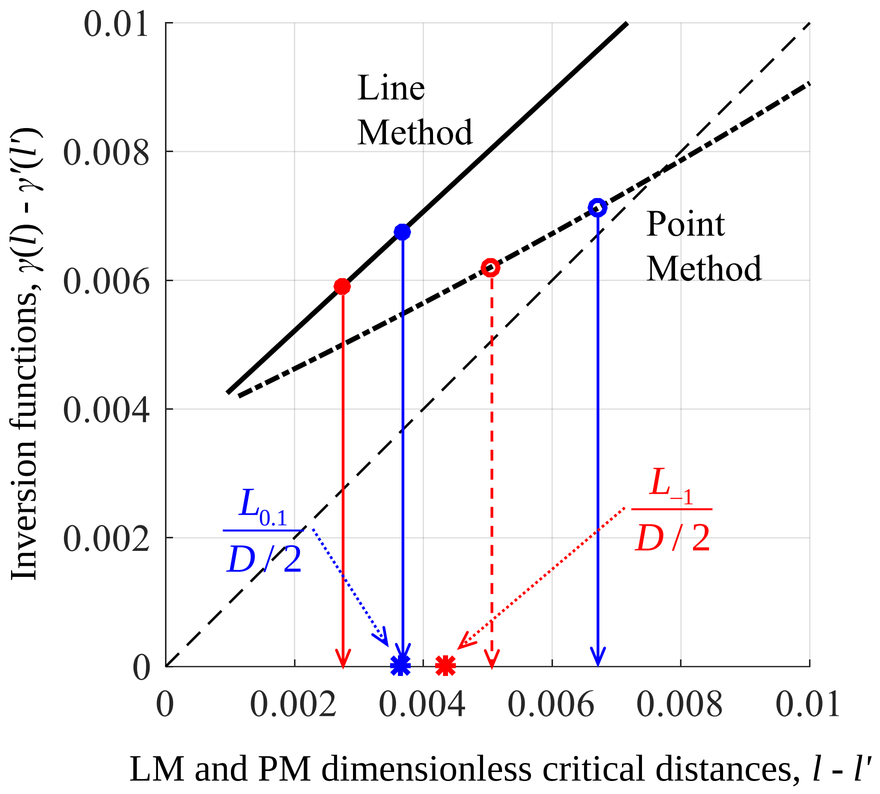


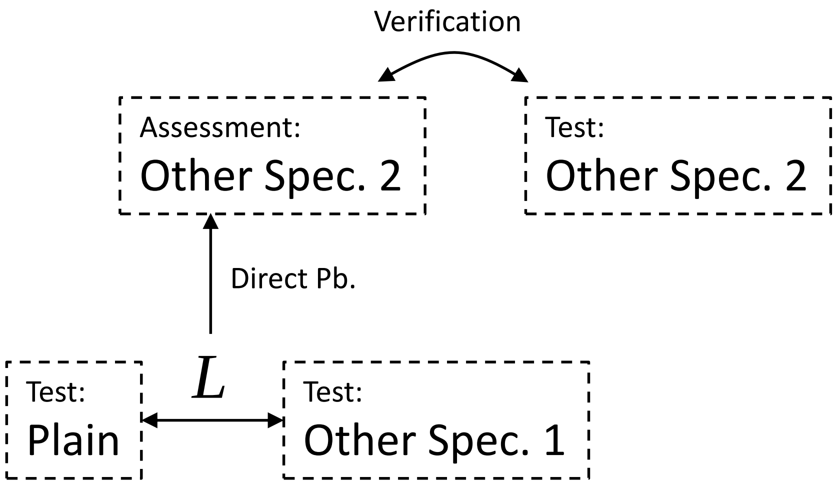
M(T) specimen with negative load ratio



Experimental test results, Critical Distance comparisons

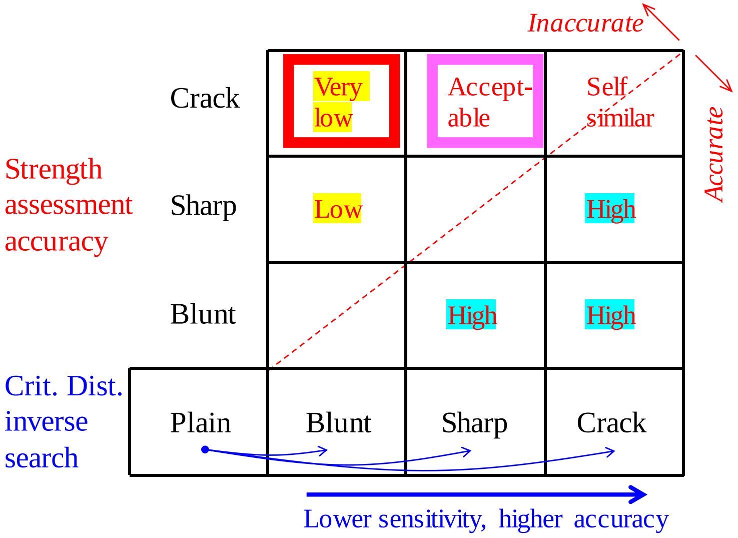
* Threshold derived lengths for comparison





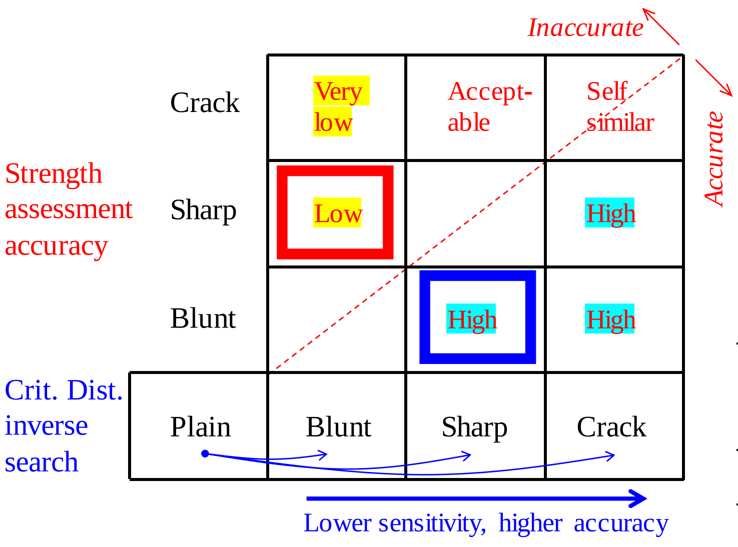
R = –1	Plain - ΔK_{th}, L_{-1} = 0.0433 mm			R = 0.1	Plain - ΔK_{th}, $L_{0.1}$ = 0.0363 mm		
Plain - *Sharp*		Plain - *Blunt*		Plain - *Sharp*		Plain - *Blunt*	
LM	PM	LM	PM	LM	PM	LM	PM
0.0273 mm	0.0505 mm	0.0970 mm	0.1836 mm	0.0367 mm	0.0671 mm	0.0078 mm	0.0063 mm
-36.9%	16.6%	123.8%	323.9%	1.1%	84.7%	-78.5%	-82.5%

Accuracy evaluation based on the strength assessment



R = –1	ΔK_{th} = 9.1 MPa $m^{0.5}$			R = 0.1	ΔK_{th} = 7.2 MPa $m^{0.5}$		
Plain – *Sharp*		Plain - *Blunt*		Plain - *Sharp*		Plain - *Blunt*	
LM	PM	LM	PM	LM	PM	LM	PM
7.23 MPa $m^{0.5}$	9.82 MPa $m^{0.5}$	13.6 MPa $m^{0.5}$	18.7 MPa $m^{0.5}$	7.24 MPa $m^{0.5}$	9.78 MPa $m^{0.5}$	3.34 MPa $m^{0.5}$	3.01 MPa $m^{0.5}$
-20.6%	8.0%	49.6%	105.9%	0.5%	35.9%	-53.6%	-58.2%

Accuracy evaluation based on the strength assessment



Results obtained with Plain - *Threshold* critical distances							
R = –1, Sharp		R = 0.1, Sharp		R = –1, Blunt		R = 0.1, Blunt	
$\Delta\sigma_{N,fl}/2$ = 87.5 MPa		$\Delta\sigma_{N,fl}/2$ = 80.5 MPa		$\Delta\sigma_{N,fl}/2$ = 163 MPa		$\Delta\sigma_{N,fl}/2$ = 119 MPa	
LM	PM	LM	PM	LM	PM	LM	PM
96.9 MPa	85.0 MPa	80.3 MPa	71.3 MPa	148.4 MPa	143.1 MPa	126.5 MPa	122.8 MPa
10.8%	-2.8%	-0.2%	-11.4%	-9.0%	-12.2%	6.3%	3.2%

Results obtained with Plain - *Blunt* critical distances				Results obtained with Plain - *Sharp* critical distances			
R = –1, Sharp		R = 0.1, Sharp		R = –1, Blunt		R = 0.1, Blunt	
$\Delta\sigma_{N,fl}/2$ = 87.5 MPa		$\Delta\sigma_{N,fl}/2$ = 80.5 MPa		$\Delta\sigma_{N,fl}/2$ = 163 MPa		$\Delta\sigma_{N,fl}/2$ = 119 MPa	
LM	PM	LM	PM	LM	PM	LM	PM
122.5 MPa	130.0 MPa	64.0 MPa	61.6 MPa	143.7 MPa	144.1 MPa	126.6 MPa	126.6 MPa
40.0%	48.6%	-20.5%	-23.4%	-11.8%	-11.6%	6.4%	6.4%

Accuracy evaluation based on the strength assessment

Accuracy map

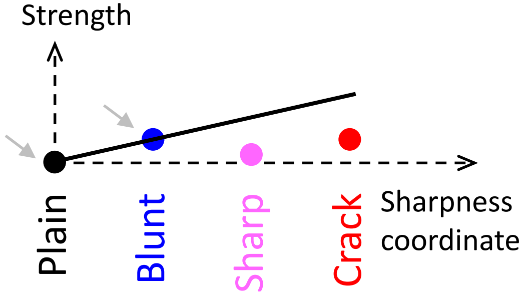
Inaccurate ↗ / *Accurate* ↘

Strength assessment accuracy				
	Crack	Very low	Accept-able	Self similar
	Sharp	Low		High
	Blunt		High	High
Crit. Dist. inverse search	Plain	Blunt	Sharp	Crack

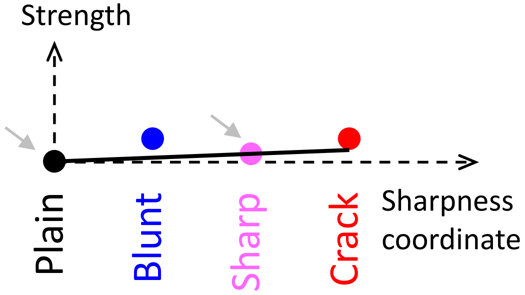
Lower sensitivity, higher accuracy →

Not accurate:
Blunt for critical distance to evaluate Sharper notch strength

Recommended:
Sharp for critical distance to evaluate Blunter notch strength

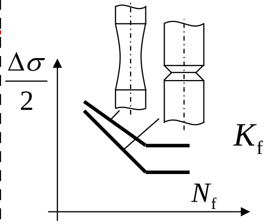


"Lever effect"



Conclusions

- Standard proposal for the fatigue Critical Distance inverse search:
  - Optimal V-notched specimen for critical distance inversion search
  - Specimen dimensions and ratios provided and discussed
  - Analytical procedure with coefficients available, no FE analysis required
- 42CrMo4 Q+T experimental example (small Critical Distance)
- Accurate assessments only obtained by evaluating the Critical Distance with a sharp notch



$$K_f = \frac{\Delta\sigma_{fl}}{\Delta\sigma_{N,fl}}$$

Line Method

$$l_0 = \frac{1}{2}\left(\frac{K_{N,UU}}{(1-s)K_f}\right)^{1/s}$$

$$l = l_{min} + \frac{l_0 - \gamma_{min}}{\beta}$$

$$L = \frac{D}{2} l$$

Point Method

$$l'_0 = 2\left(\frac{K_{N,UU}}{K_f}\right)^{1/s}$$

$$l' = \sum_{i=1}^{5} \delta_i l_0'^{i}$$

$$L' = \frac{D}{2} l'$$