International Conference on Structural Integrity

Optimal notched specimen parameters for accurate fatigue critical distance determination

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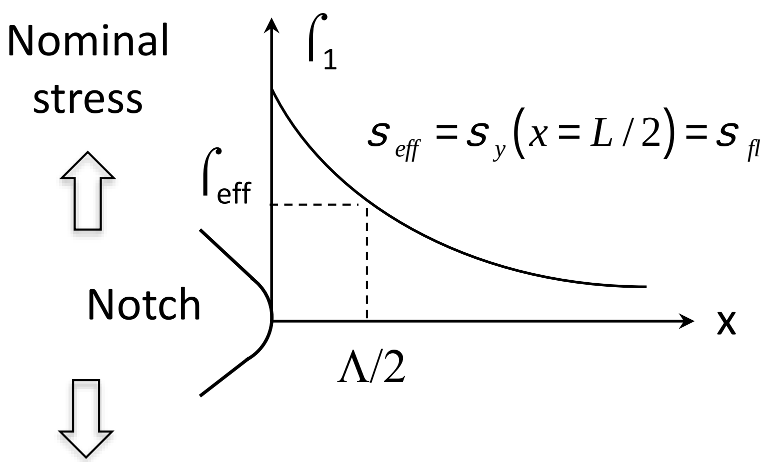
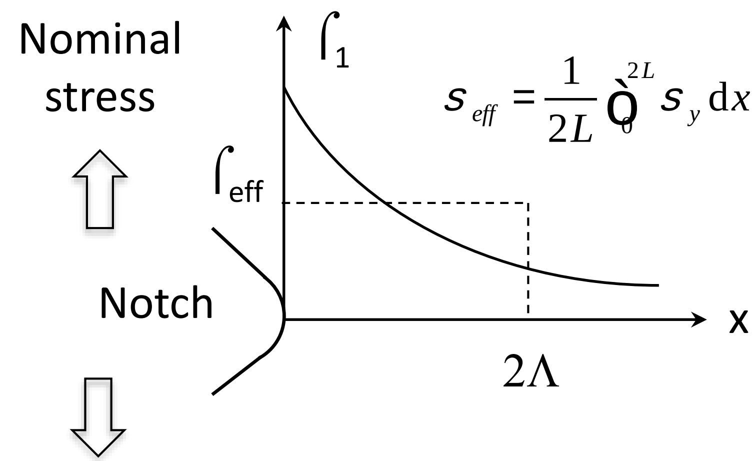
The theory of critical distance (TCD)

The point method (PM)



The line method (LM)



$$L = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{\Delta \sigma_{fl}} \right)^2$$

El-Haddad material length

- L is very sensitive to material characteristics (microstructure, texture, processing routes)
- Crack growth threshold determination through fracture mechanics tests is experimentally challenging

Motivation

- Design of an optimal notched specimen geometry for accurate L determination to circumvent the fracture mechanics test
- Provide a straightforward analytical calculation of L to avoid the FE analysis of each specimen geometry
- Define an effective range where the result is expected to be not largely sensitive to any experimental issue

Outline

- Stress distribution ahead of notches
- Critical distance determination: LM vs. PM
- Sensitivity analysis
- Experiments
- Critical distance evaluation
- Fatigue strength evaluation

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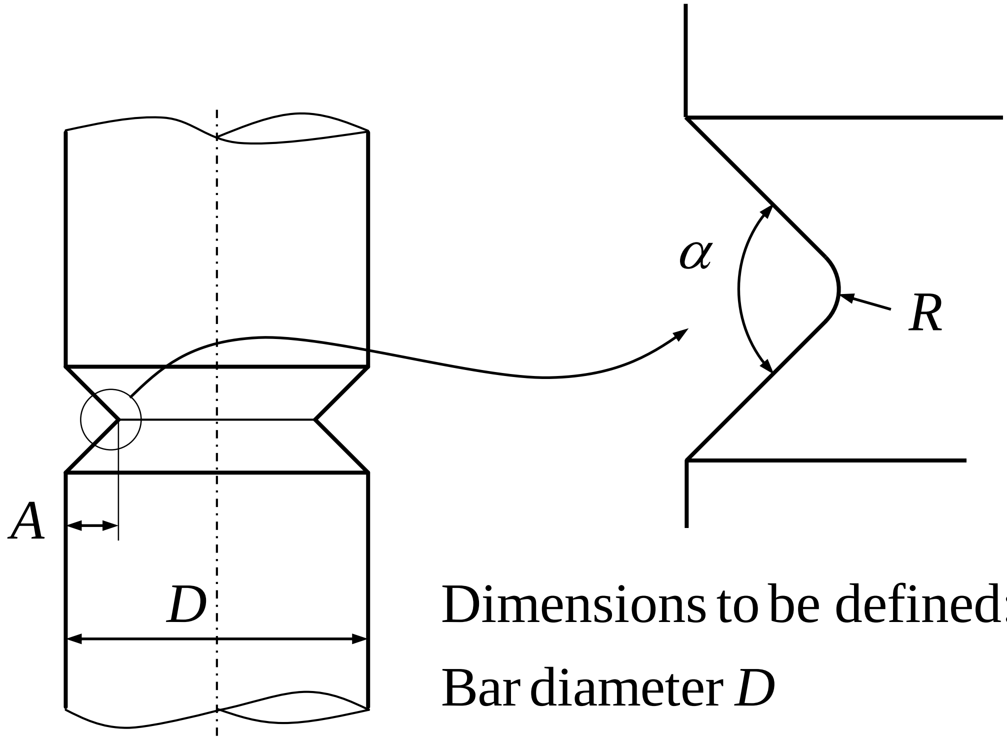
Proposed specimen

- V-notch axisymmetric specimen: easy to manufacture, no boundary effects, no transition from plane stress to plane strain
- Relatively open angle: 90°, 60°
- Sharp root radius



Dimensions to be defined:

Bar diameter D

Notch depth A

Notch angle α

Notch radius R

Stress analysis, (i) sharp notch assumption

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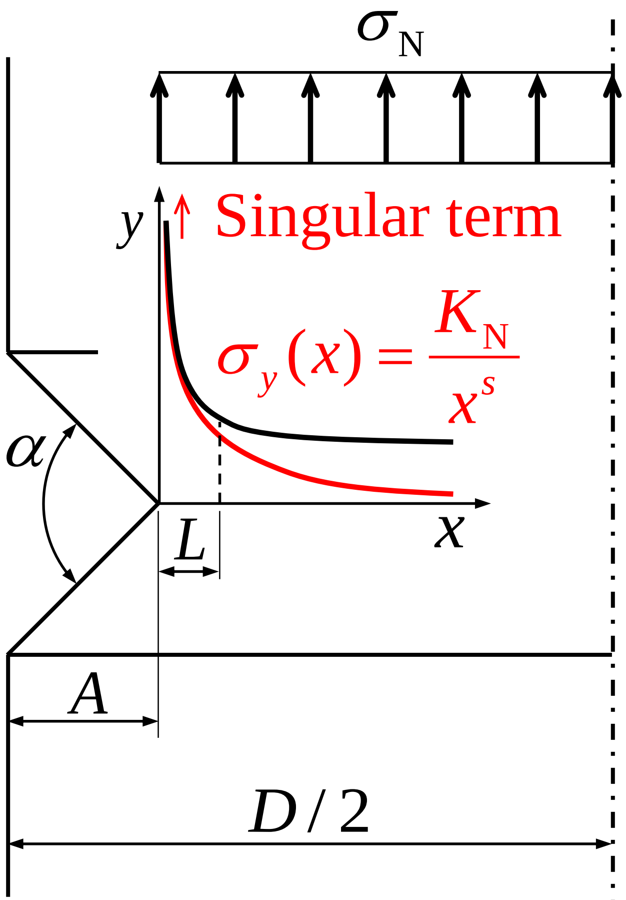
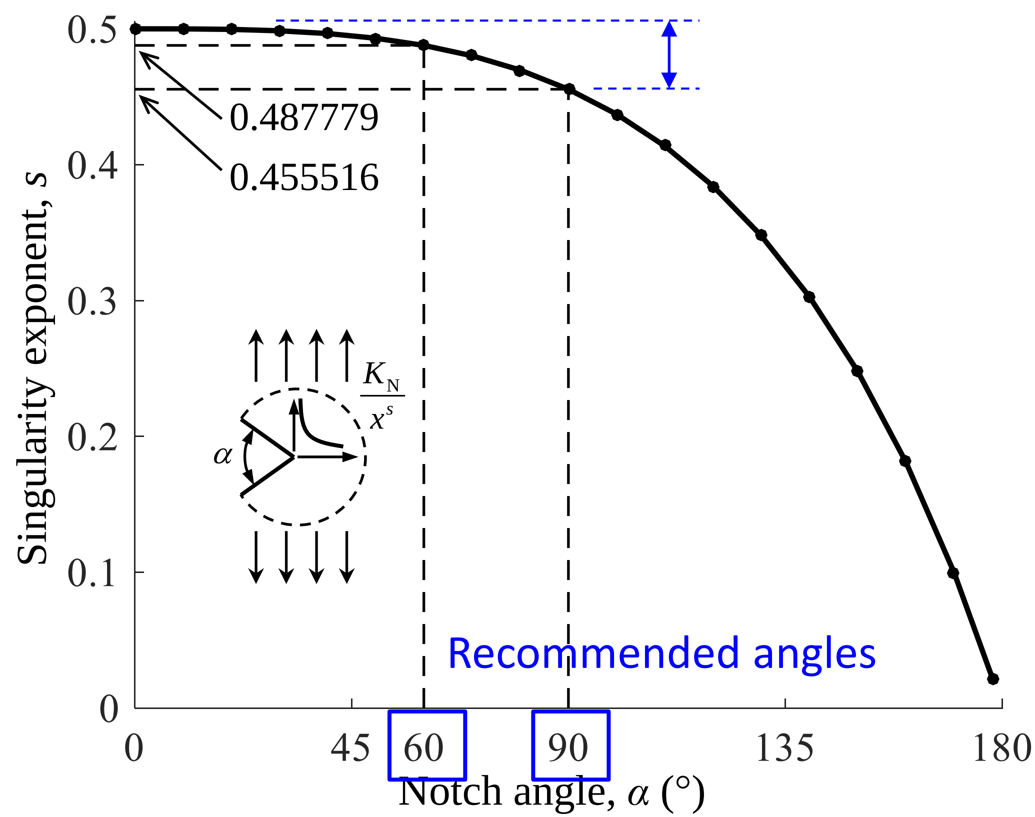
Singular stress field



Williams' exponent close to ½ even for relatively large notch angles



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Singular stress field

Dimensionless form:

$$s_y(x) = \frac{K_N}{x^s} = s_N \frac{K_{N,U}}{x^s} = s_N \frac{K_{N,U} / (D/2)^s}{(x/(D/2))^s}$$

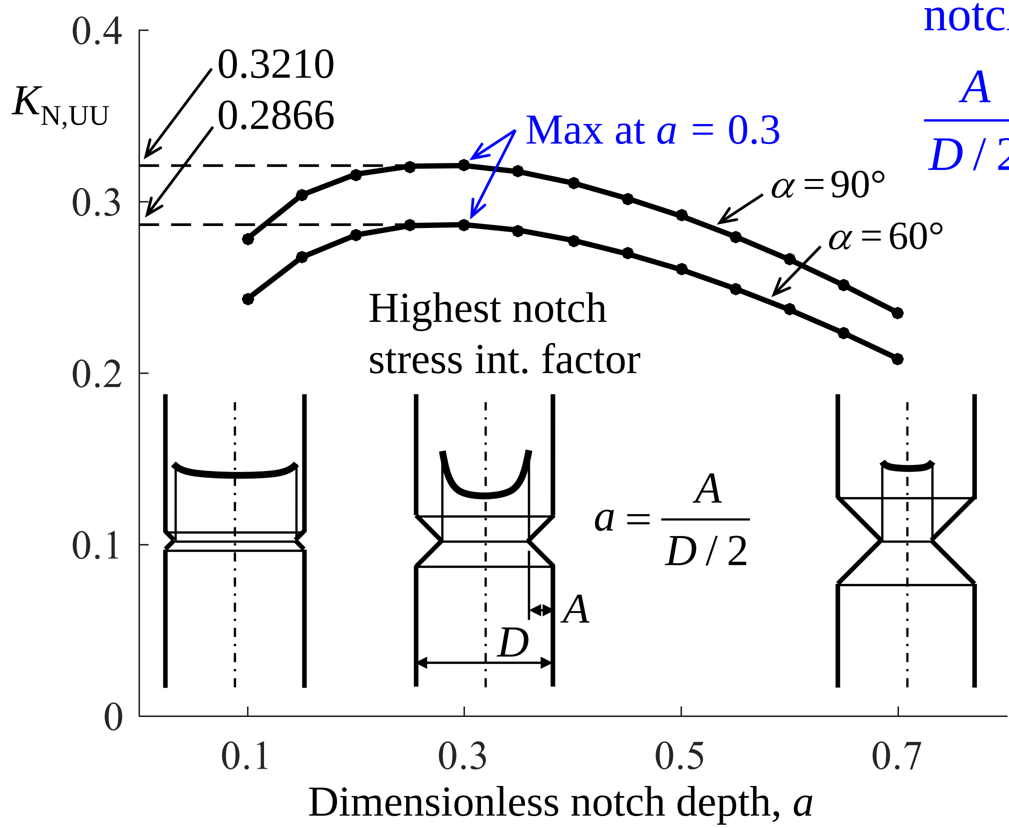
$$s_y(\chi) = s_N \frac{K_{N,UU}}{\chi^s}$$

where:

$$\chi = \frac{x}{D/2}$$

$$K_{N,UU} = \frac{K_{N,U}}{\left(D/2\right)^s} = \frac{K_N}{s_N \left(D/2\right)^s}$$

N-SIF for unitary half diameter and unitary nominal stress



Optimal notch depth:

$$\frac{A}{D/2} = 0.3$$

- At intermediate notch depth a = 0.3 the NSIF is maximum because at a lower depth the notch indentation is just too small, while for the higher depth the peak stress points along the inner ring are too close.

LM critical distance inverse search

Line Method dimensionless form:

$$\frac{1}{2L}\int_0^{2L}\Delta\sigma_y(x)\,\mathrm{d}x = \frac{1}{2l}\int_0^{2l}\Delta\sigma_y(x)\,\mathrm{d}x$$

where: $l = L(D/2)$

Singular term integration:

$$\frac{1}{2l}\int_0^{2l}\Delta\sigma_y(x)\,\mathrm{d}x = \int_0^{2l}\Delta\sigma_\mathrm{N}\frac{K_\mathrm{N,UU}}{x^s} = \frac{\Delta\sigma_\mathrm{N}}{1-s}\frac{K_\mathrm{N,UU}}{(2l)^s}$$

Line Method, average stress equal to fatigue limit:

$$\frac{\Delta\sigma_\mathrm{N}}{1-s}\frac{K_\mathrm{N,UU}}{(2l)^s} = \Delta\sigma_\mathrm{fl}$$

Fatigue stress concentration factor:

$$\frac{1}{1-s}\frac{K_\mathrm{N,UU}}{(2l)^s} = K_\mathrm{f}$$

Critical distance length inverse derivation:

$$l_0 = \frac{1}{2}\left(\frac{K_\mathrm{N,UU}}{(1-s)K_\mathrm{f}}\right)^{1/s}, \quad L_0 = l_0(D/2)$$

LM/PM critical distance inverse search

Line Method length inverse derivation:

$$L_0 = \frac{D}{4}\left(\frac{K_{\mathrm{N,UU}}}{(1-s)K_{\mathrm{f}}}\right)^{1/s}$$

Similar analysis for the Point Method,

length inverse derivation:

$$L_0' = D\left(\frac{K_{\mathrm{N,UU}}}{K_{\mathrm{f}}}\right)^{1/s}$$

"0" stands for singularity derived and ' is for the PM

Notch parameters:

$\alpha = 90°$

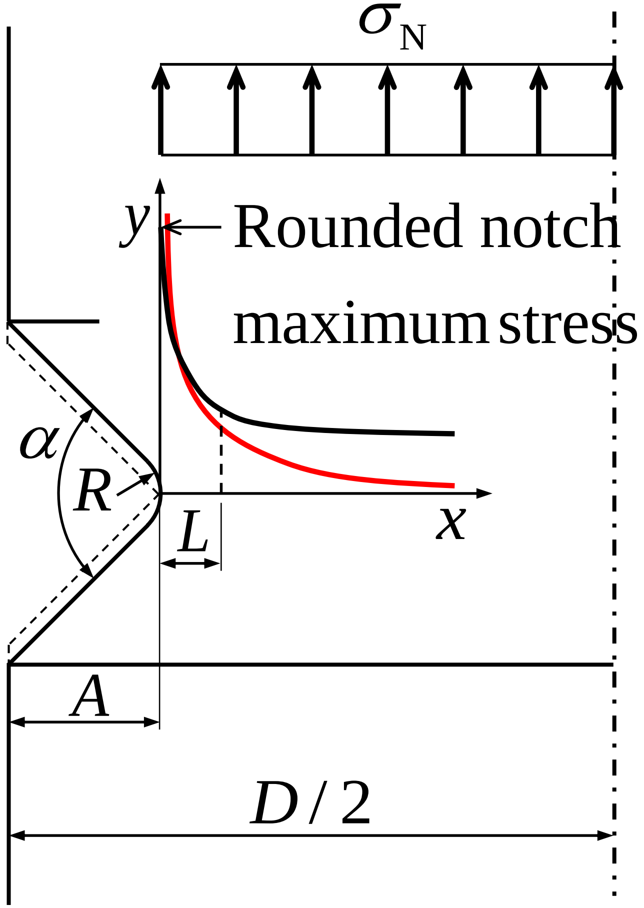
$s = 0.455516 \quad K_{\mathrm{N,UU}} = 0.3210 \; (a = 0.3)$

$\alpha = 60°$

$s = 0.487779 \quad K_{\mathrm{N,UU}} = 0.2866 \; (a = 0.3)$

Stress analysis, (ii) rounded notch tip

Bounded stress field

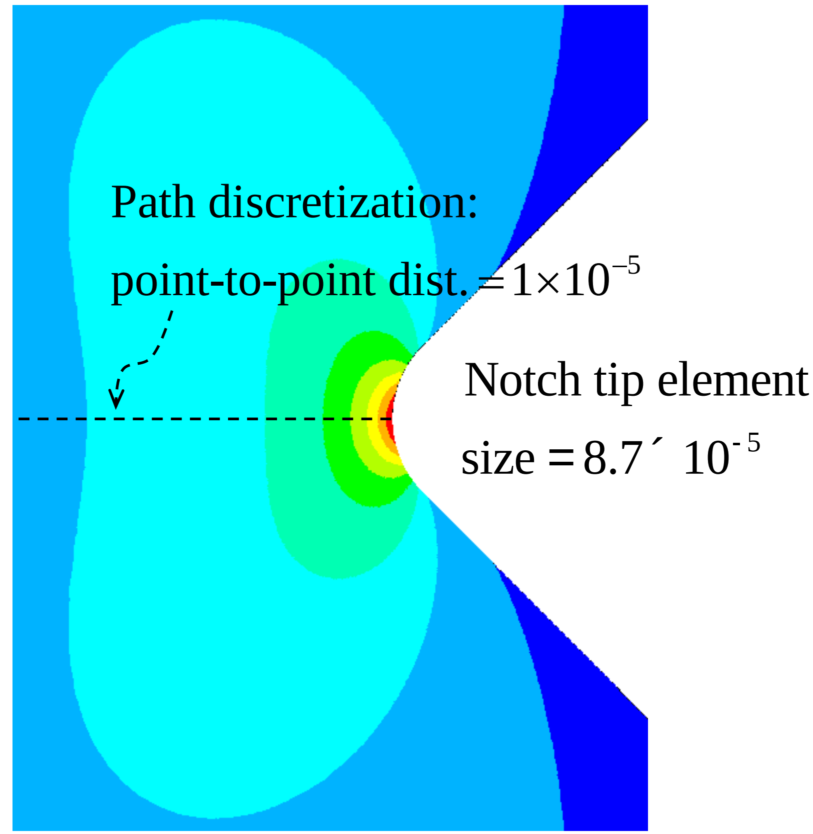


Dimensionless radius:

$$r = R/(D/2)$$

$$\rho = \frac{R}{A} = \frac{r}{a}$$

FE model with unitary half diameter and unitary stress



Performed simulations:

$a = 0.3$,

$r = R/A$

$r = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1.0$

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LM critical distance inverse search

Line Method dimensionless form:

$$\frac{1}{2l}\int_0^{2l}\Delta\sigma_y(\xi)\,\mathrm{d}\xi = \Delta\sigma_{\mathrm{N}}\frac{f(l)}{1-s}\frac{K_{\mathrm{N,UU}}}{(2l)^s}$$

where $f(l)$ is a correction function

Line Method equation:

$$\frac{f(l)}{1-s}\frac{K_{\mathrm{N,UU}}}{(2l)^s} = K_{\mathrm{f}}$$

After introducing l_0:

$$\frac{l}{f(l)^{1/s}} = l_0$$

and the inversion function is defined: $\gamma(\mathrm{l}) = l\,/\,f(l)^{1/s}$

to put the inverse search problem as:

$$\gamma(l) = l_0$$

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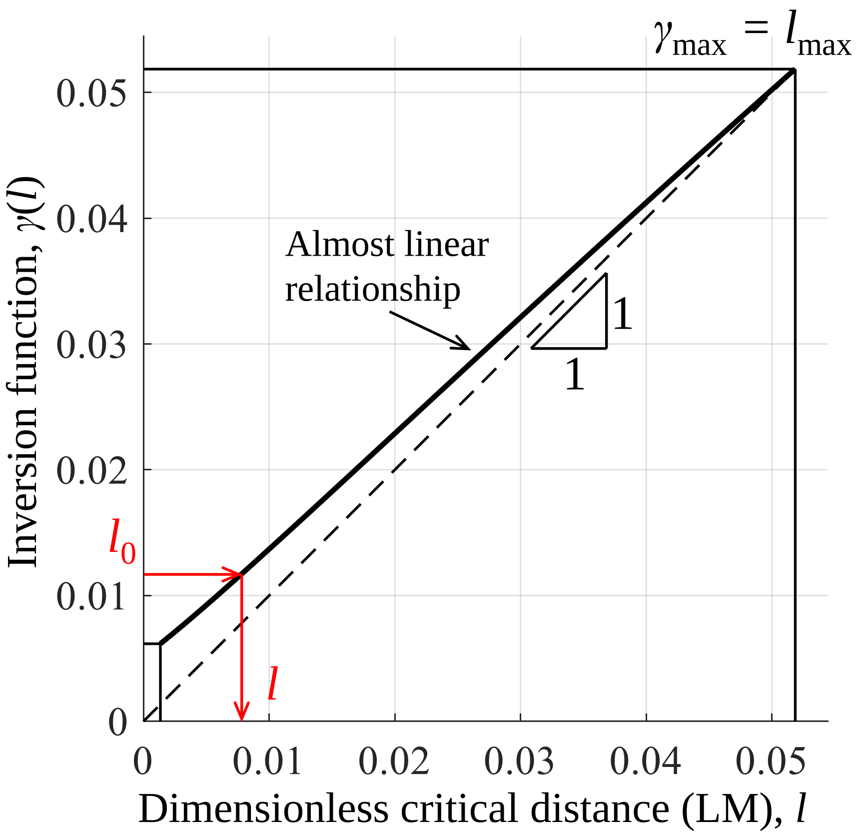
LM critical distance inverse search



Very accurate approx. with a linear model, inverse search:

$$l = l_{min} + \frac{l_0 - \gamma_{min}}{\beta}$$

$$\beta = \frac{\gamma_{max} - \gamma_{min}}{l_{max} - l_{min}}$$

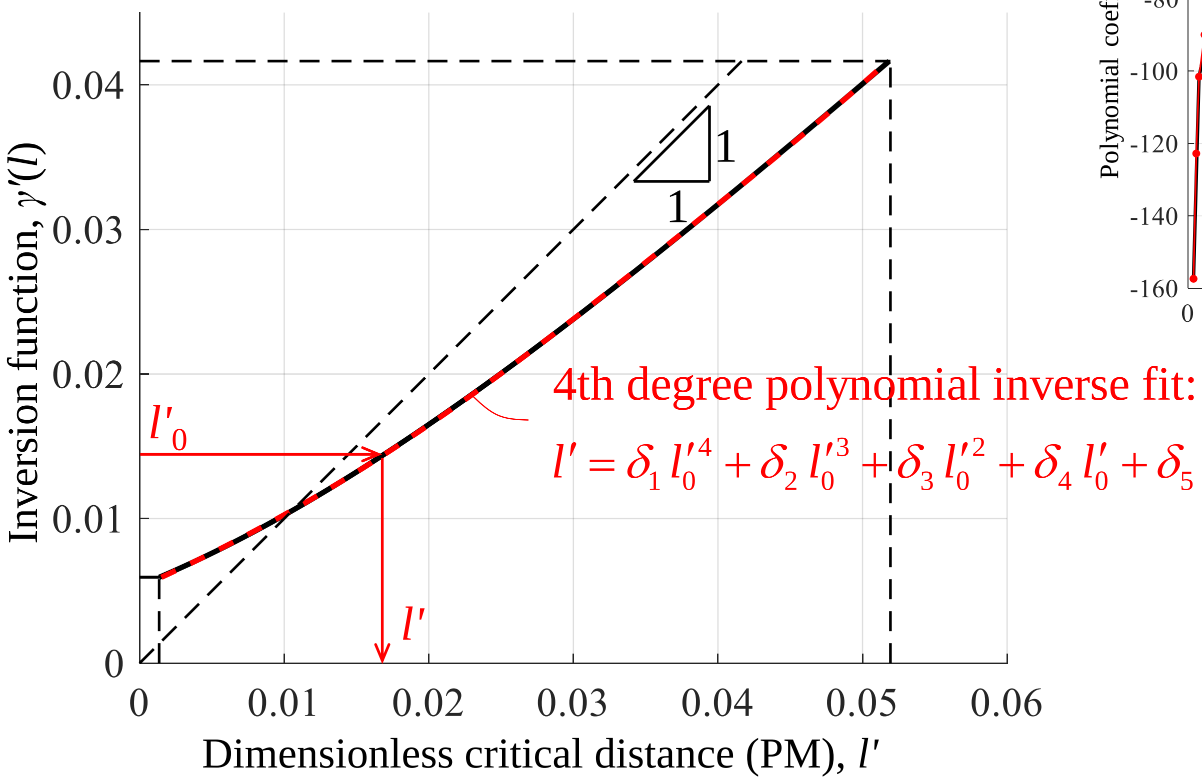
Fit models (ρ functions):

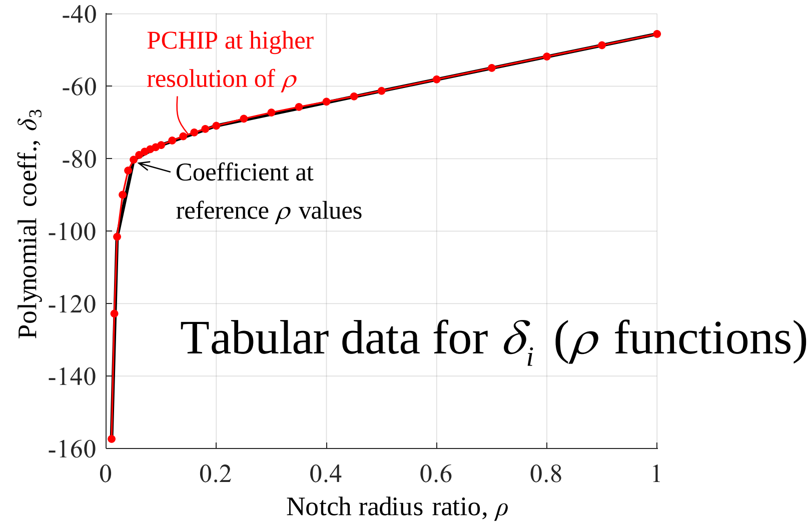
$$l_{min} = p_1\rho^3 + p_2\rho^2 + p_3\rho + p_4$$

$$\gamma_{min} = q_1\rho^3 + q_2\rho^2 + q_3\rho + q_4$$

$$l_{max} = \gamma_{max} = c_1 + c_2\rho^{c_3}$$

PM critical distance inverse search



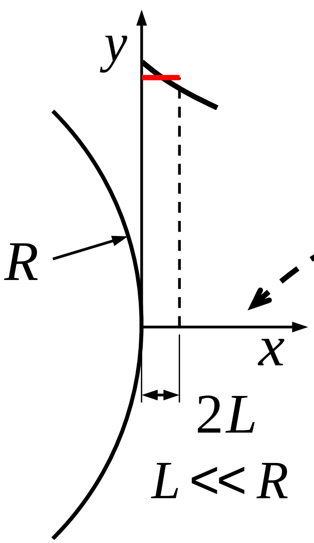


Sensitivity to any experimental variation of K_f

Sensitivity definition:

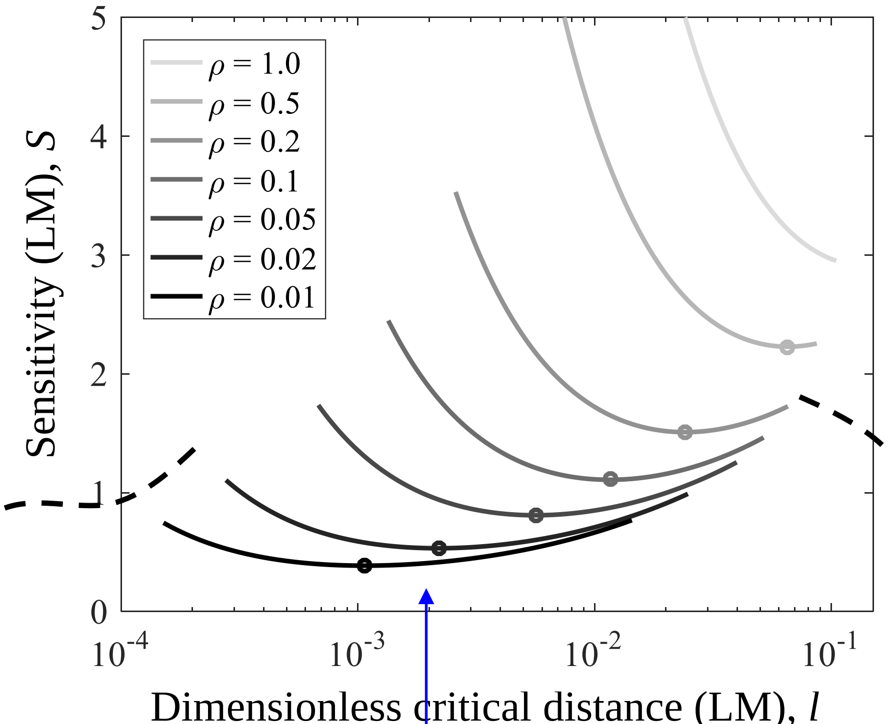
$$S = -\frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}K_\mathrm{f}}$$

Small critical distance wrt notch radius



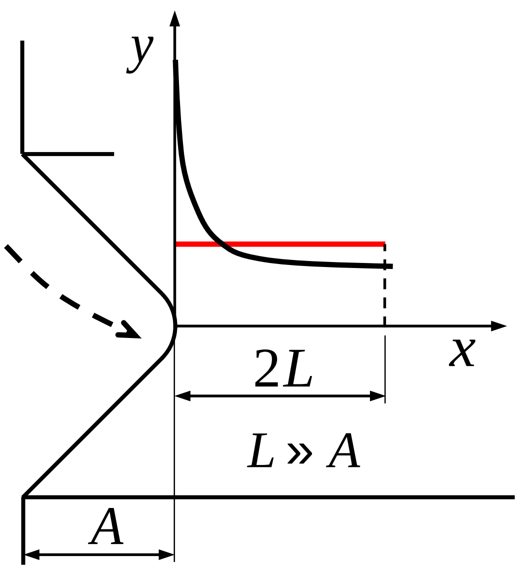
Not sharp enough local radius

LM sensitivity



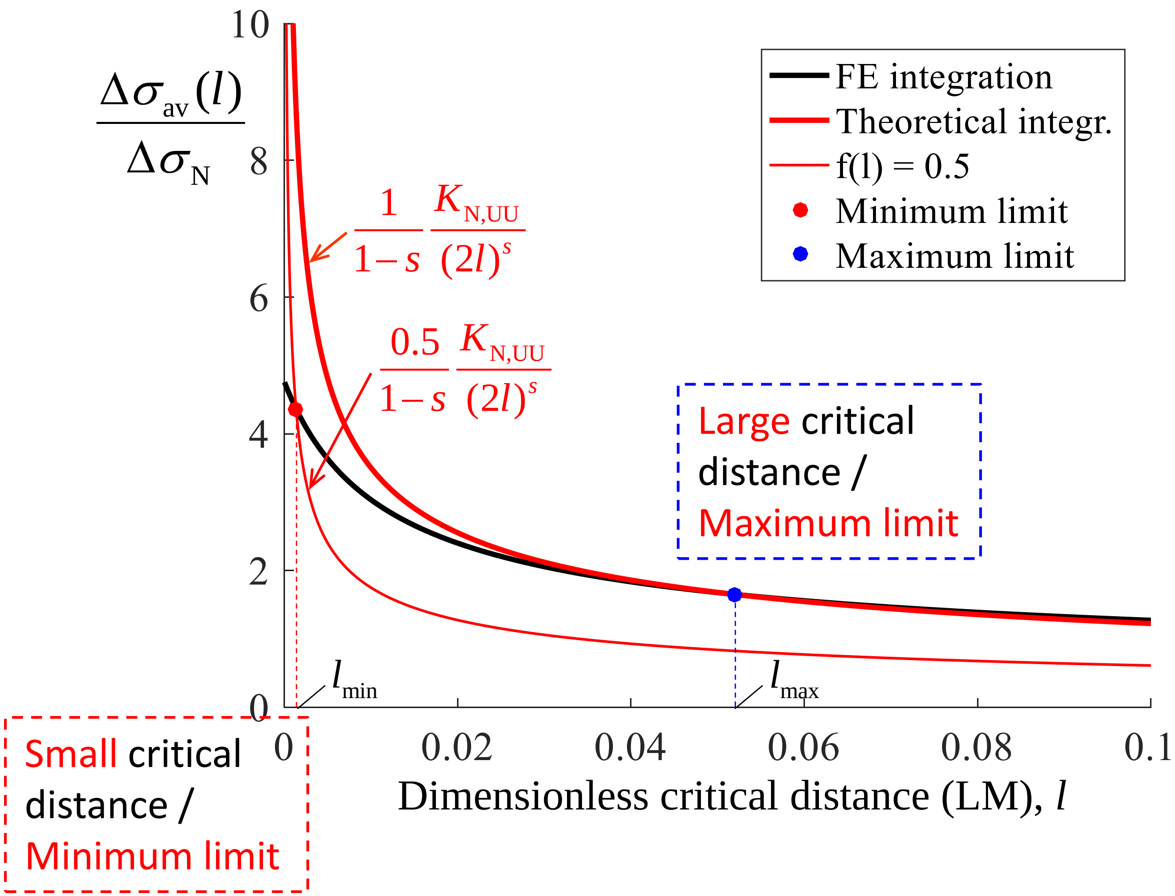
Minimum sensitivity in the range $0.5 < f(l) < 1.0$

Large critical distance wrt diameter size



Small specimen size

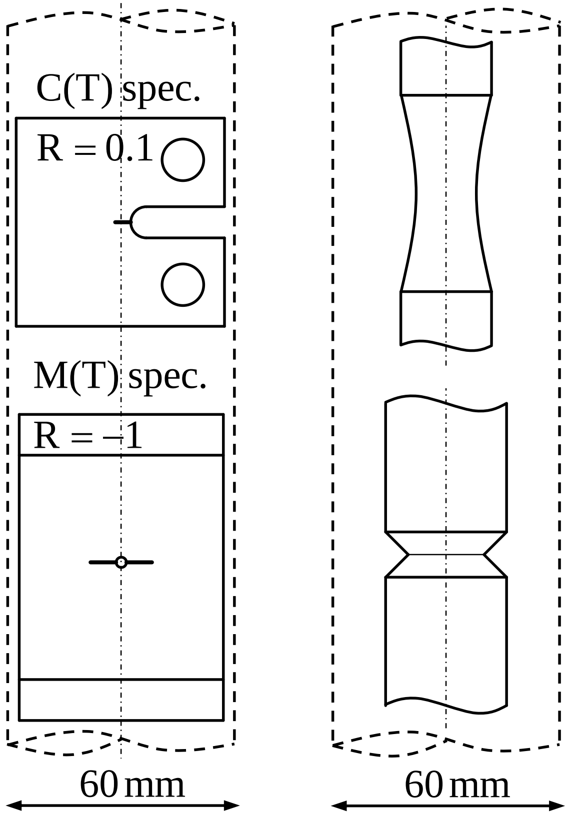
Maximum/minimum limits for the dimensionless critical distance



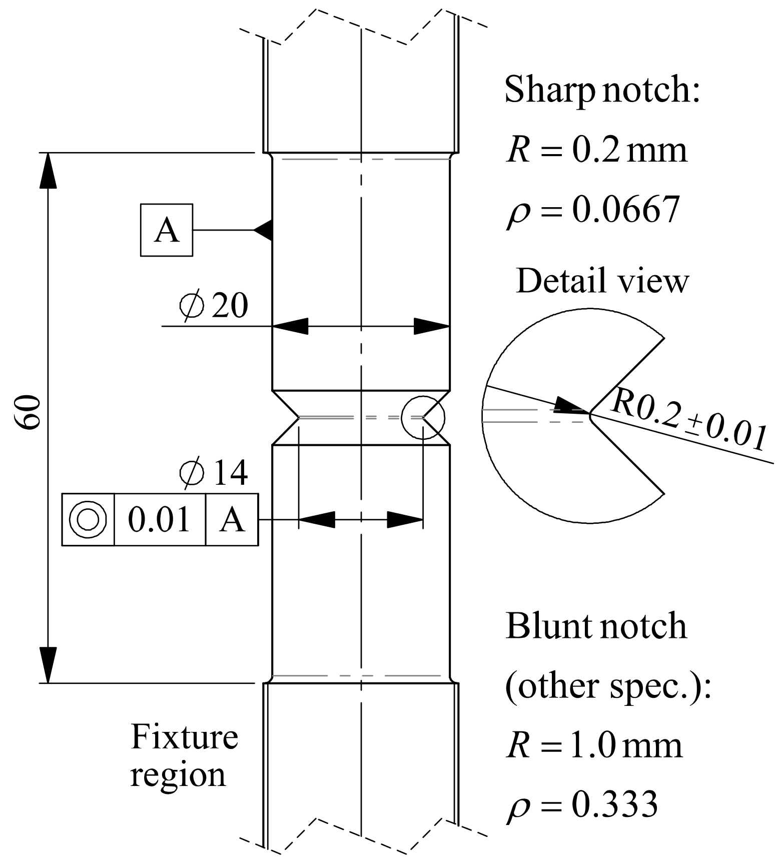
Experimental application, 42CrMo4 - S_U = 875 MPa

Specimen extraction from the same bar supply

Fracture mechanics tests for comparison



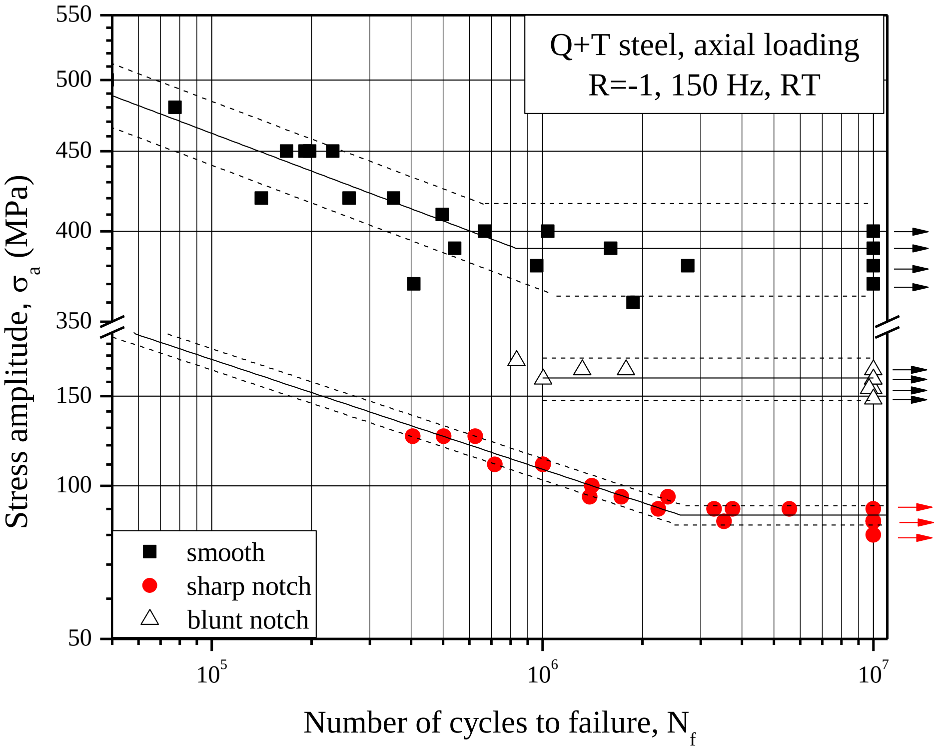
Expected Critical Distance on the order of 0.05 mm, small notch radius 0.2 mm



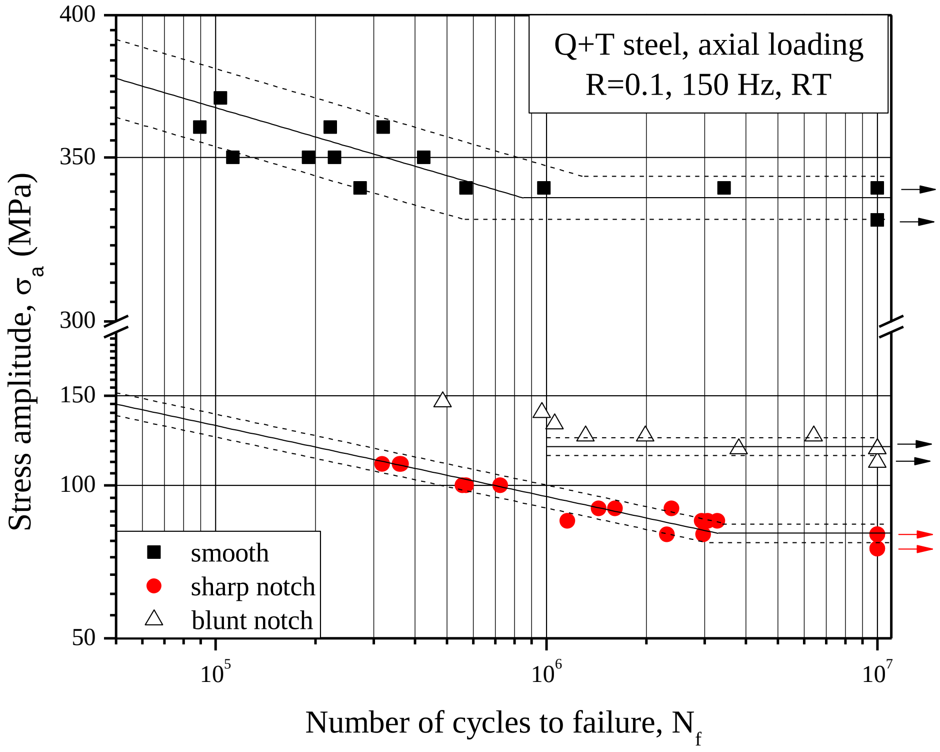
Experimental test results, S-N data

Specimen types: Plain, Blunt (1.0 mm), Sharp (0.2 mm)

Push-pull (R=-1) axial fatigue tests

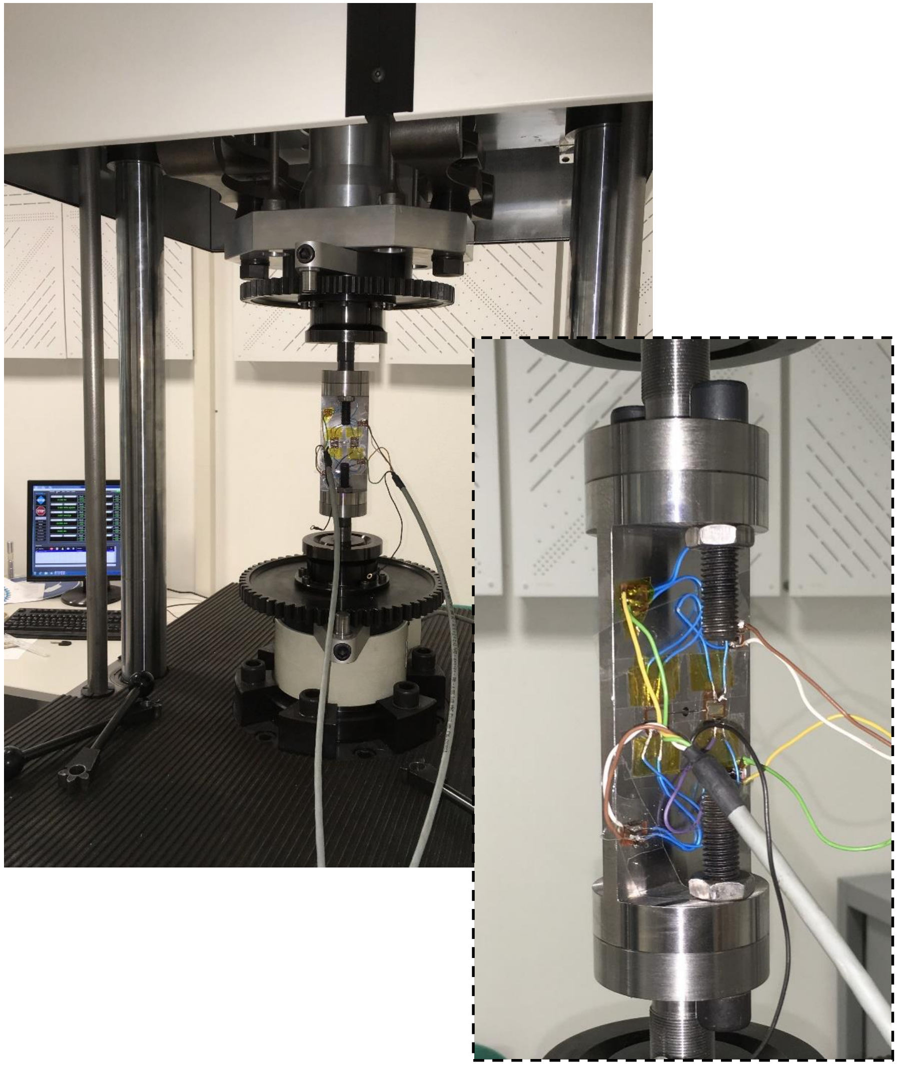


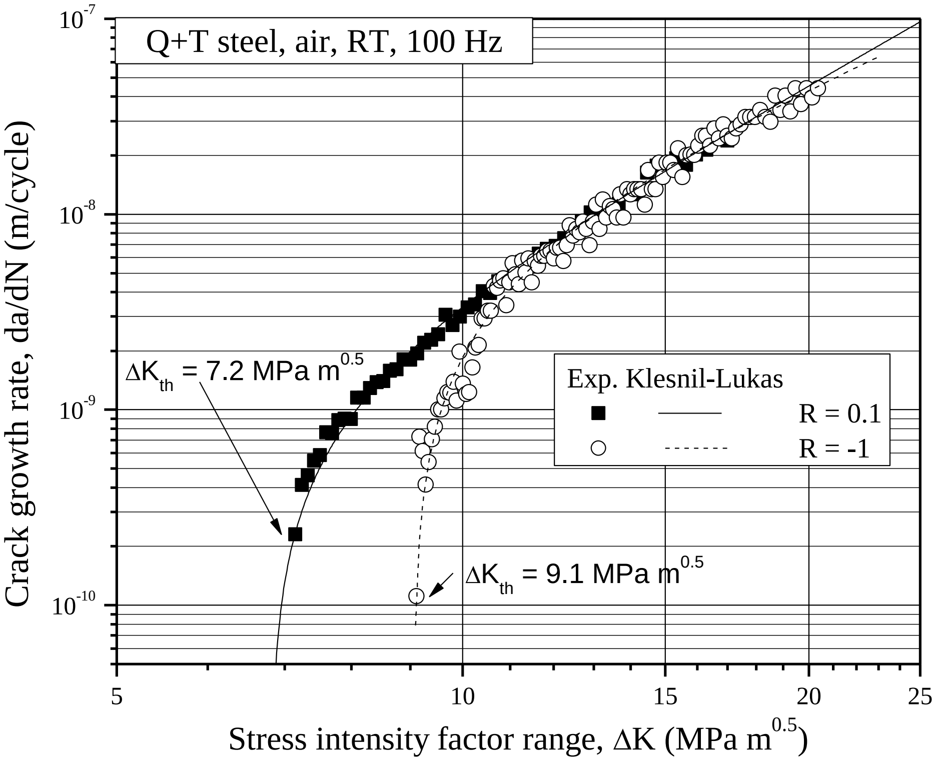
Pulsating (R=0.1) axial fatigue tests



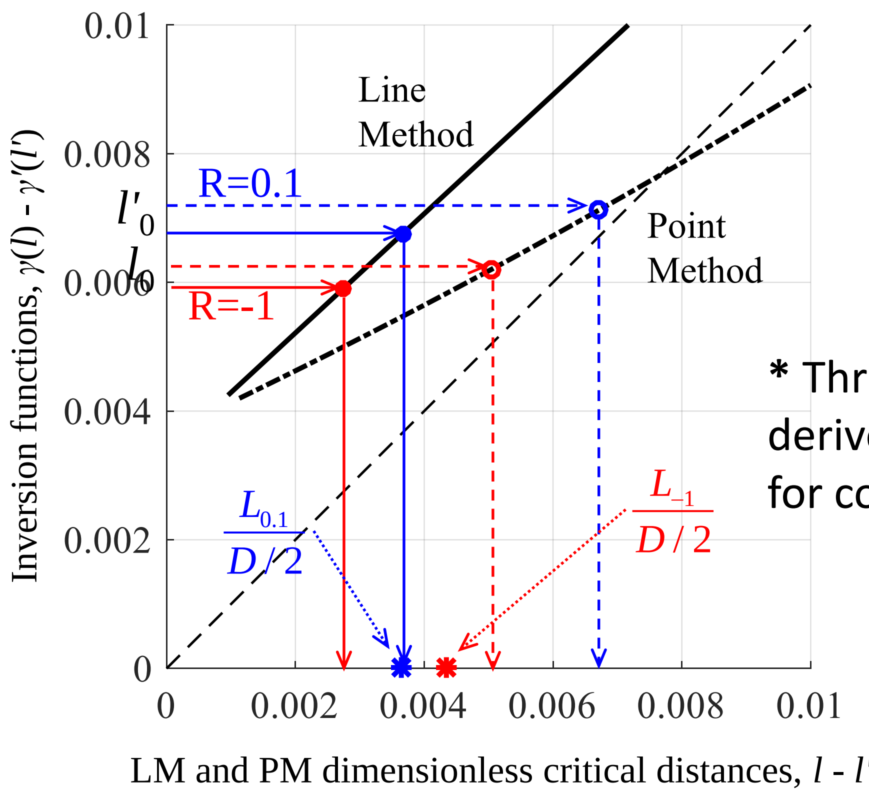
Experimental test results, thresholds

M(T) specimen for negative load ratio





Experimental test results, length comparison



* Threshold derived lengths for comparison

R = –1	Plain - ΔK_{th}, L_{-1} = 0.0433 mm			R = 0.1	Plain - ΔK_{th}, $L_{0.1}$ = 0.0363 mm		
Plain - *Sharp*		Plain - *Blunt*		Plain - *Sharp*		Plain - *Blunt*	
LM	PM	LM	PM	LM	PM	LM	PM
0.0273 mm	0.0505 mm	0.0970 mm	0.1836 mm	0.0367 mm	0.0671 mm	0.0078 mm	0.0063 mm
-36.9%	16.6%	123.8%	323.9%	1.1%	84.7%	-78.5%	-82.5%

Accuracy evaluation based on the strength assessment

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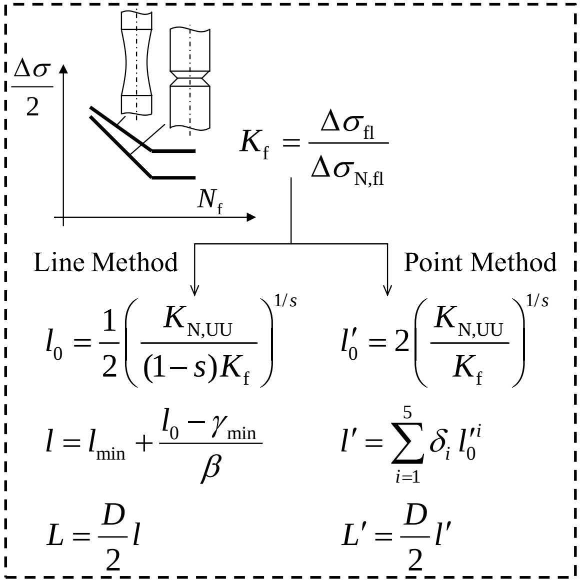
R = –1	ΔK_{th} = 9.1 MPa $m^{0.5}$			R = 0.1	ΔK_{th} = 7.2 MPa $m^{0.5}$		
Plain – *Sharp*		Plain - *Blunt*		Plain - *Sharp*		Plain - *Blunt*	
LM	PM	LM	PM	LM	PM	LM	PM
7.23 MPa $m^{0.5}$	9.82 MPa $m^{0.5}$	13.6 MPa $m^{0.5}$	18.7 MPa $m^{0.5}$	7.24 MPa $m^{0.5}$	9.78 MPa $m^{0.5}$	3.34 MPa $m^{0.5}$	3.01 MPa $m^{0.5}$
-20.6%	8.0%	49.6%	105.9%	0.5%	35.9%	-53.6%	-58.2%

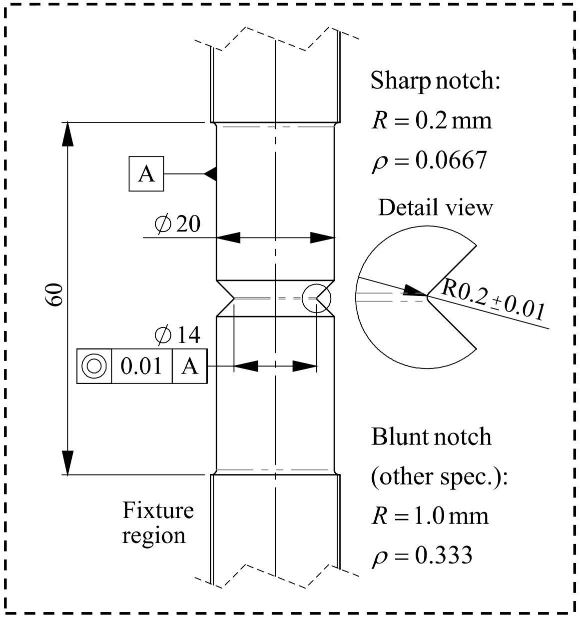
Results obtained with Plain - *Threshold* critical distances							
R = –1, Sharp		R = 0.1, Sharp		R = –1, Blunt		R = 0.1, Blunt	
$\Delta\sigma_{N,fl}/2$ = 87.5 MPa		$\Delta\sigma_{N,fl}/2$ = 80.5 MPa		$\Delta\sigma_{N,fl}/2$ = 163 MPa		$\Delta\sigma_{N,fl}/2$ = 119 MPa	
LM	PM	LM	PM	LM	PM	LM	PM
96.9 MPa	85.0 MPa	80.3 MPa	71.3 MPa	148.4 MPa	143.1 MPa	126.5 MPa	122.8 MPa
10.8%	-2.8%	-0.2%	-11.4%	-9.0%	-12.2%	6.3%	3.2%
Results obtained with Plain - *Blunt* critical distances				Results obtained with Plain - *Sharp* critical distances			
R = –1, Sharp		R = 0.1, Sharp		R = –1, Blunt		R = 0.1, Blunt	
$\Delta\sigma_{N,fl}/2$ = 87.5 MPa		$\Delta\sigma_{N,fl}/2$ = 80.5 MPa		$\Delta\sigma_{N,fl}/2$ = 163 MPa		$\Delta\sigma_{N,fl}/2$ = 119 MPa	
LM	PM	LM	PM	LM	PM	LM	PM
122.5 MPa	130.0 MPa	64.0 MPa	61.6 MPa	143.7 MPa	144.1 MPa	126.6 MPa	126.6 MPa
40.0%	48.6%	-20.5%	-23.4%	-11.8%	-11.6%	6.4%	6.4%

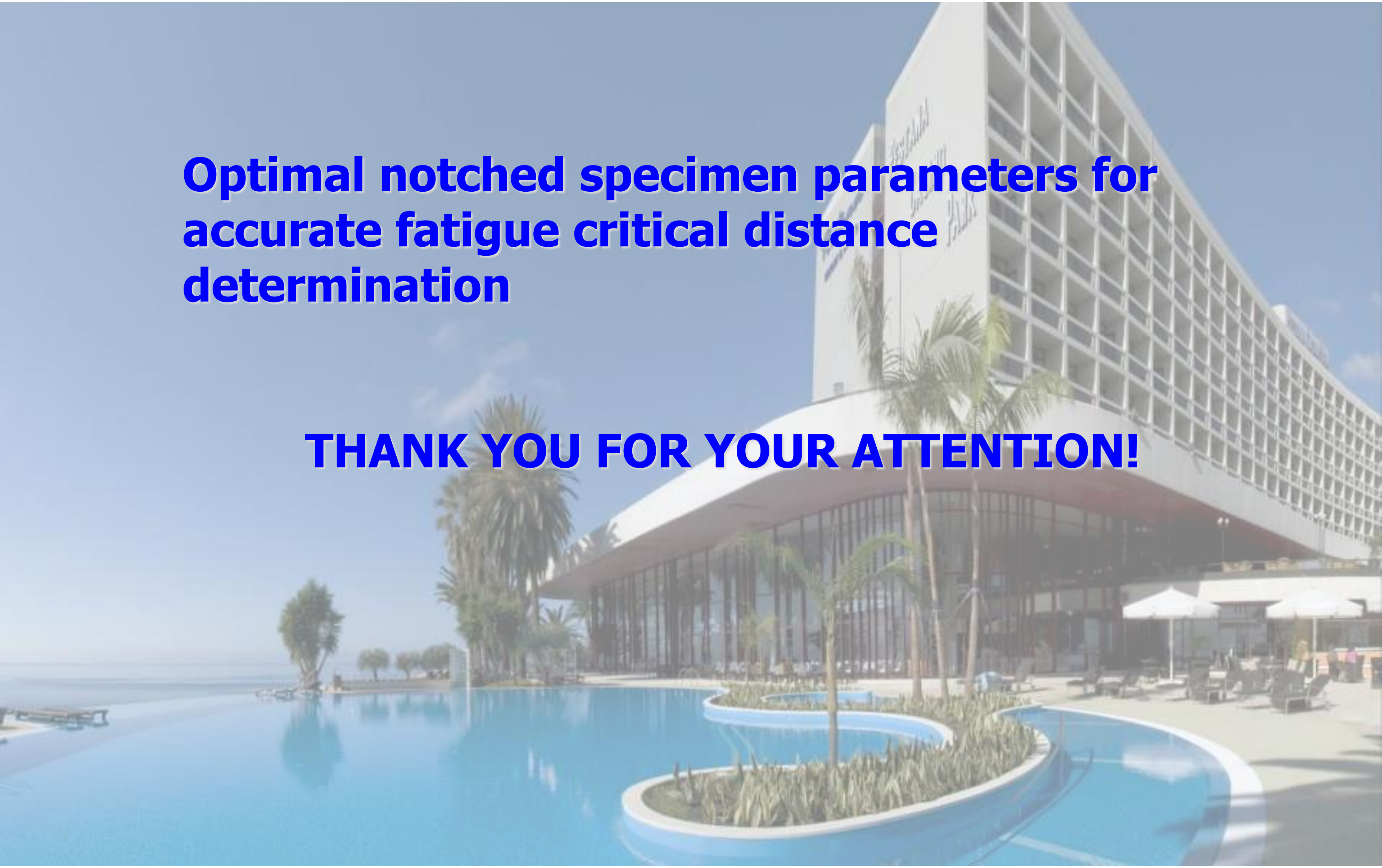
- Not accurate (yellow): blunt for critical distance to evaluate sharper notch strength.
- Accurate (blue): sharp for critical distance to evaluate blunter notch strength.

Conclusions

- V-notched specimen for optimal critical distance inversion search.
- All the dimensions provided and discussed, in particular the notch root radius.
- Analytical procedure to derive the Critical Distance both with Line and Point methods.
- 42CrMo4 Q+T experimental data and comparison provided.
- The obtained critical distances dependent on the method, Point Method much larger than Line Method.
- Small critical distance for this investigated high strength steel: accurate assessments only obtained with the sharp notch or the crack threshold derived critical distances.







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THANK YOU FOR YOUR ATTENTION!