

AIAS 2014

43° Convegno Nazionale

Estensione del metodo integrale, considerando l'effetto di eccentricità, per la valutazione delle tensioni residue con i metodi del foro e della cava anulare

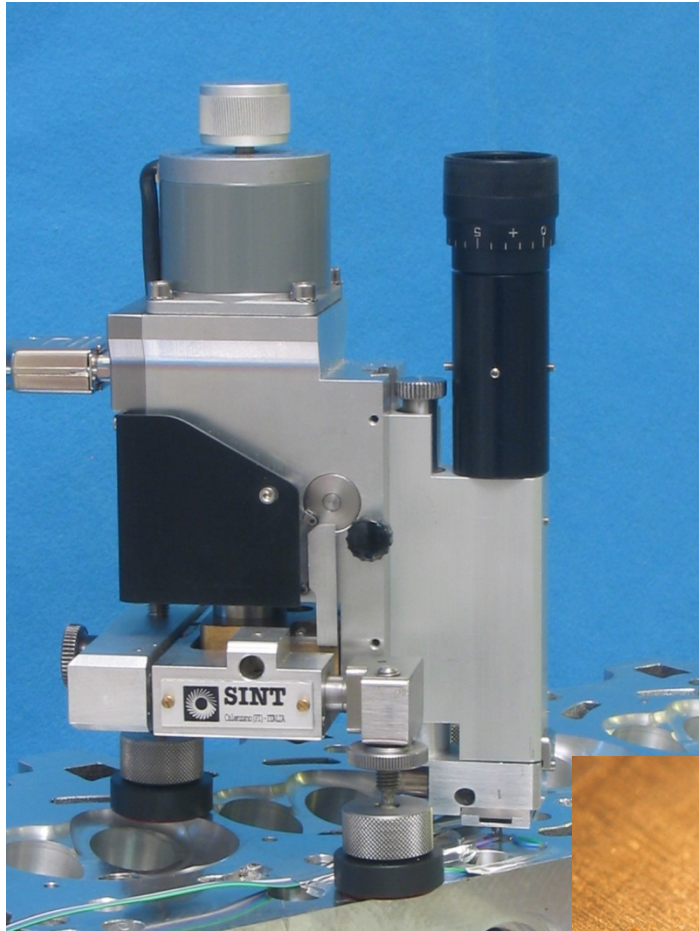
M. Barsanti, M. Beghini, C. Santus



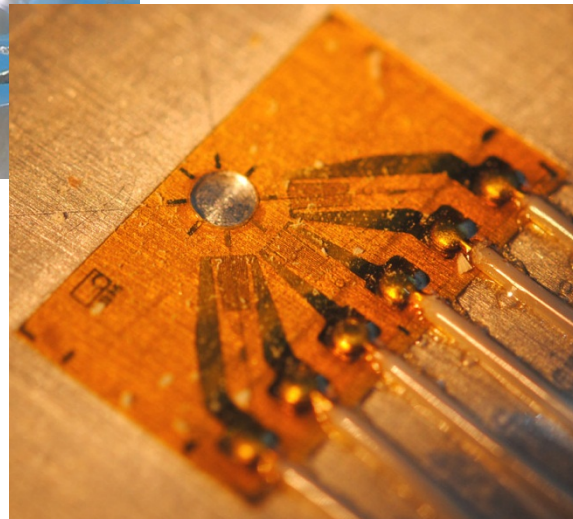
Università di Pisa

DICI – Dipartimento di Ingegneria Civile e Industriale

Introduzione: Metodo del foro / Metodo della cava

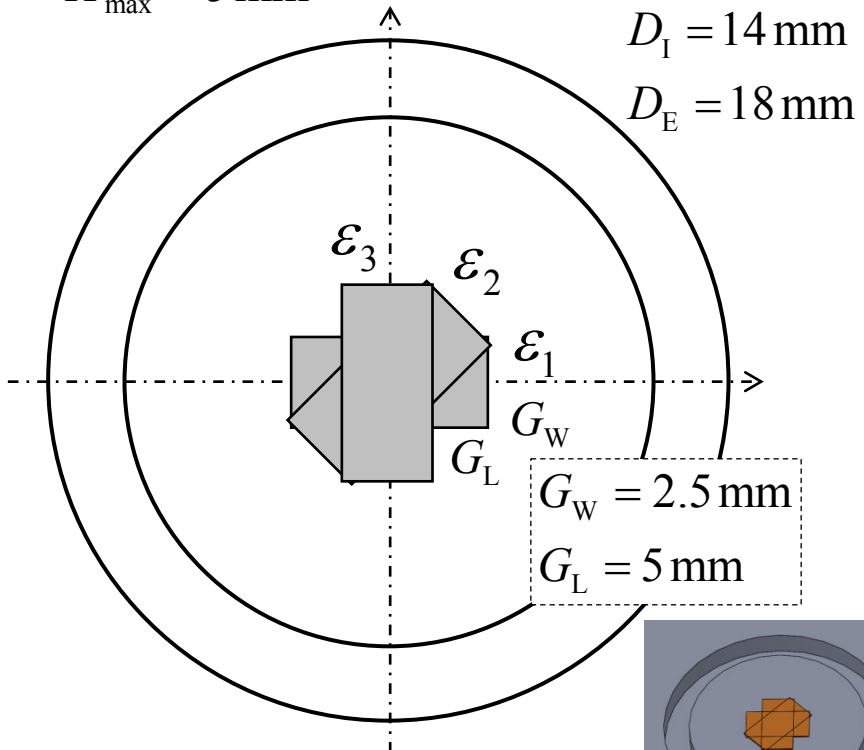
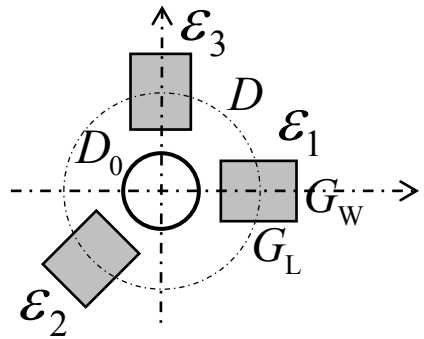
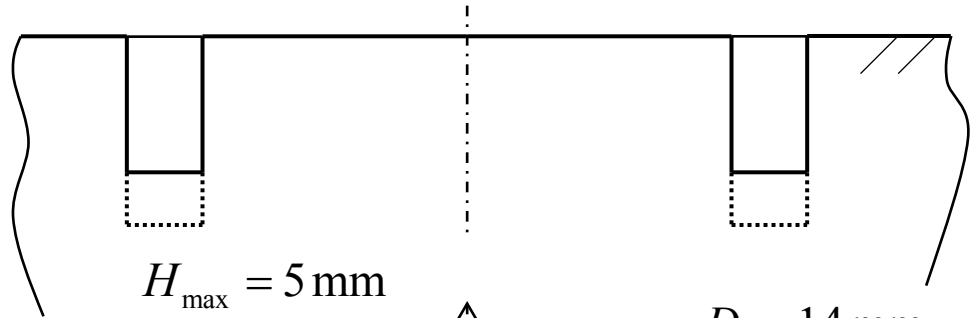
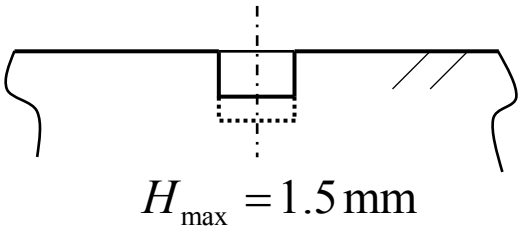


Hole-Drilling

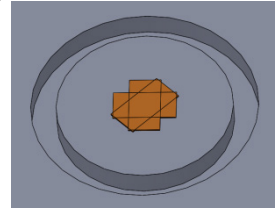
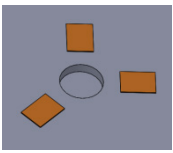


Ring-Core

Introduzione: Metodo del foro / Metodo della cava



$D_0 = 1.7 - 2.0 \text{ mm}$
 $D = 5.13 \text{ mm}$ $D = 5.1 \text{ mm}$
 $G_W = 1.59 \text{ mm}$ $G_W = 0.7 \text{ mm}$
 $G_L = 1.59 \text{ mm}$ $G_L = 1.5 \text{ mm}$
 MM HBM





Designation: E837 – 13a

Standard Test Method for Determining Residual Stresses by the Hole-Drilling Strain- Gage Method¹

This standard is issued under the fixed designation E837; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

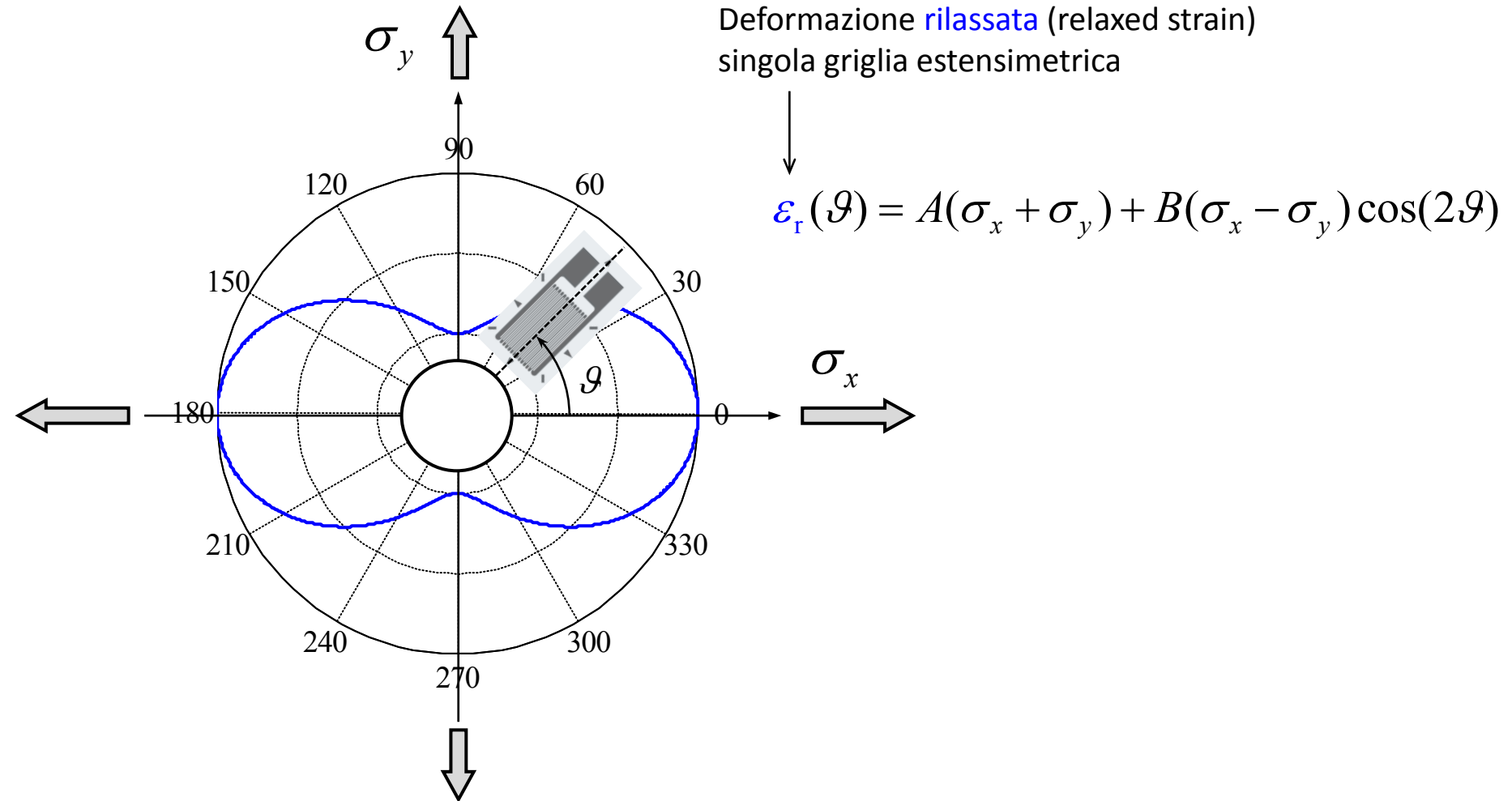
INTRODUCTION

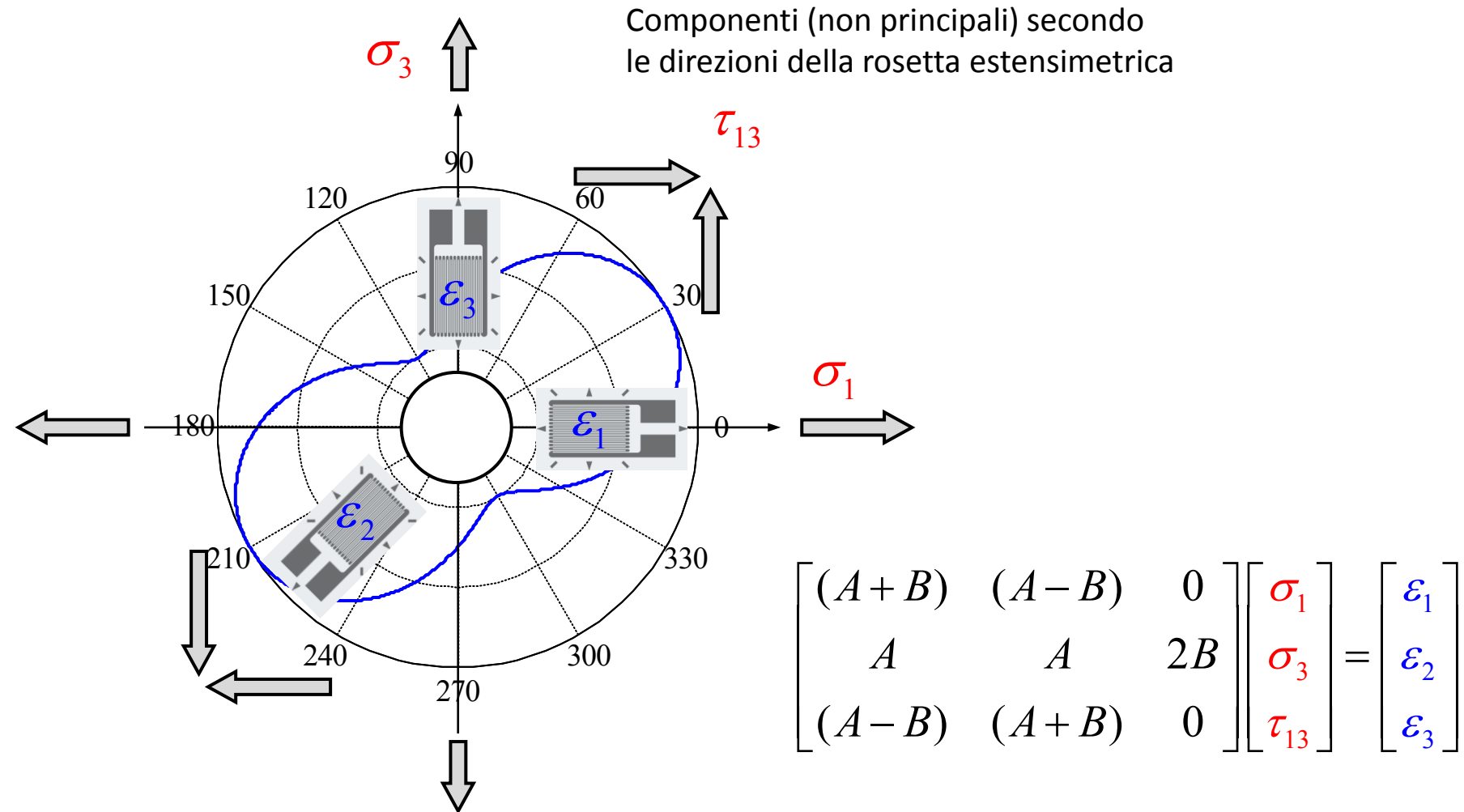
The hole-drilling strain-gage method determines residual stresses near the surface of an isotropic linear-elastic material. It involves attaching a strain rosette to the surface, drilling a hole at the geometric center of the rosette, and measuring the resulting relieved strains. The residual stresses within the removed material are then determined from the measured strains using a series of equations.

“A device that is equipped to drill a hole in the test workpiece in a controlled manner is required. The device must be able to drill a hole aligned concentric with the strain gage circle to within either $\pm 0.004D$ ”

$$D \approx 5 \text{ mm}$$

$$0.004D = 0.02 \text{ mm}$$





$$\begin{bmatrix} (A+B) & (A-B) & 0 \\ A & A & 2B \\ (A-B) & (A+B) & 0 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_3 \\ \tau_{13} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

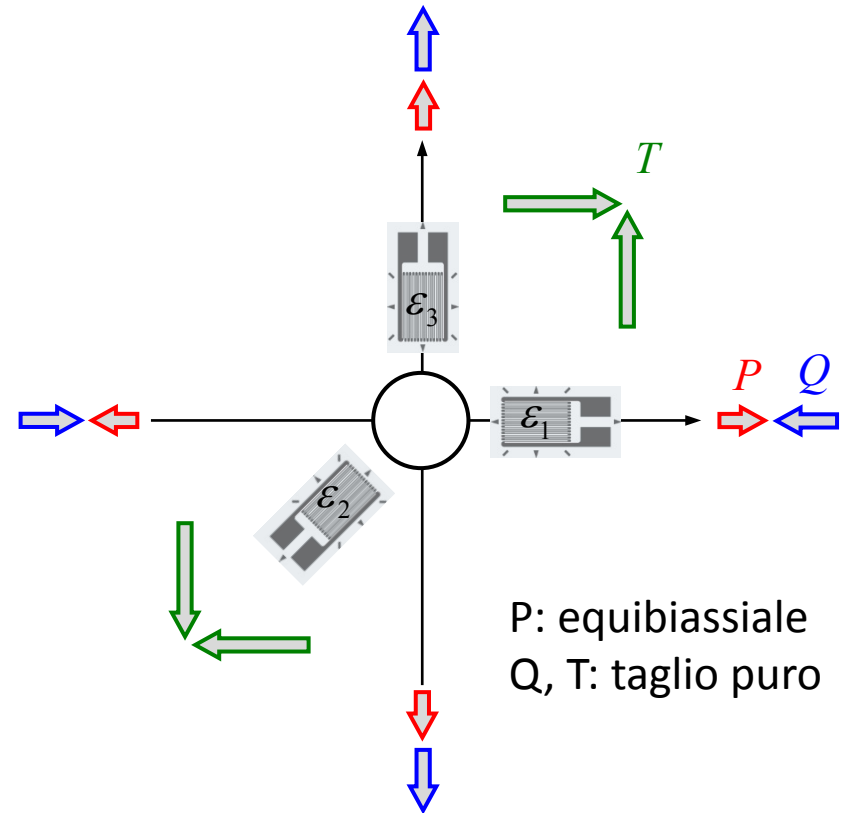
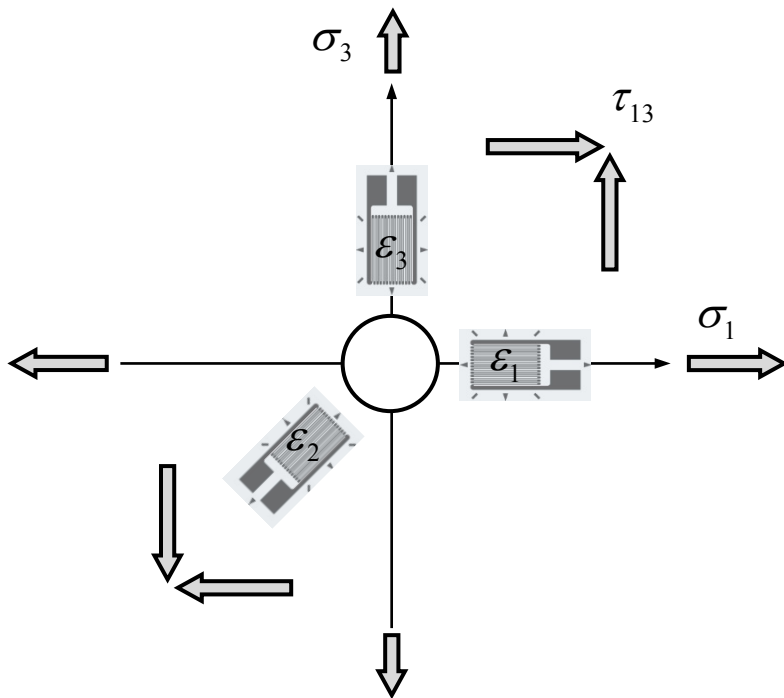
$$p = \frac{\varepsilon_3 + \varepsilon_1}{2} \quad q = \frac{\varepsilon_3 - \varepsilon_1}{2} \quad t = \frac{2\varepsilon_2 - (\varepsilon_3 + \varepsilon_1)}{2} = \varepsilon_2 - p$$

$$P = \frac{\sigma_3 + \sigma_1}{2} \quad Q = \frac{\sigma_3 - \sigma_1}{2} \quad T = \tau_{13}$$

$$\begin{bmatrix} 2A & 0 & 0 \\ 0 & 2B & 0 \\ 0 & 0 & 2B \end{bmatrix} \begin{bmatrix} P \\ Q \\ T \end{bmatrix} = \begin{bmatrix} p \\ q \\ t \end{bmatrix}$$

$$\begin{bmatrix} (A+B) & (A-B) & 0 \\ A & A & 2B \\ (A-B) & (A+B) & 0 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_3 \\ \tau_{13} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

$$\begin{bmatrix} 2A & 0 & 0 \\ 0 & 2B & 0 \\ 0 & 0 & 2B \end{bmatrix} \begin{bmatrix} P \\ Q \\ T \end{bmatrix} = \begin{bmatrix} p \\ q \\ t \end{bmatrix}$$



Disaccoppiamento “p,q,t”

Proprietà dei coefficienti

- Valori negativi (tensioni rilasciate)
- Dipendenza lineare dal modulo di Young
- Dipendenza dal rapporto di Poisson, approx. Plane Stress
- Dipendenza dalla geometria (modellazione FEM)

$$-\frac{1}{E} \begin{bmatrix} a(1+\nu) & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix} \begin{bmatrix} P \\ Q \\ T \end{bmatrix} = \begin{bmatrix} p \\ q \\ t \end{bmatrix}$$

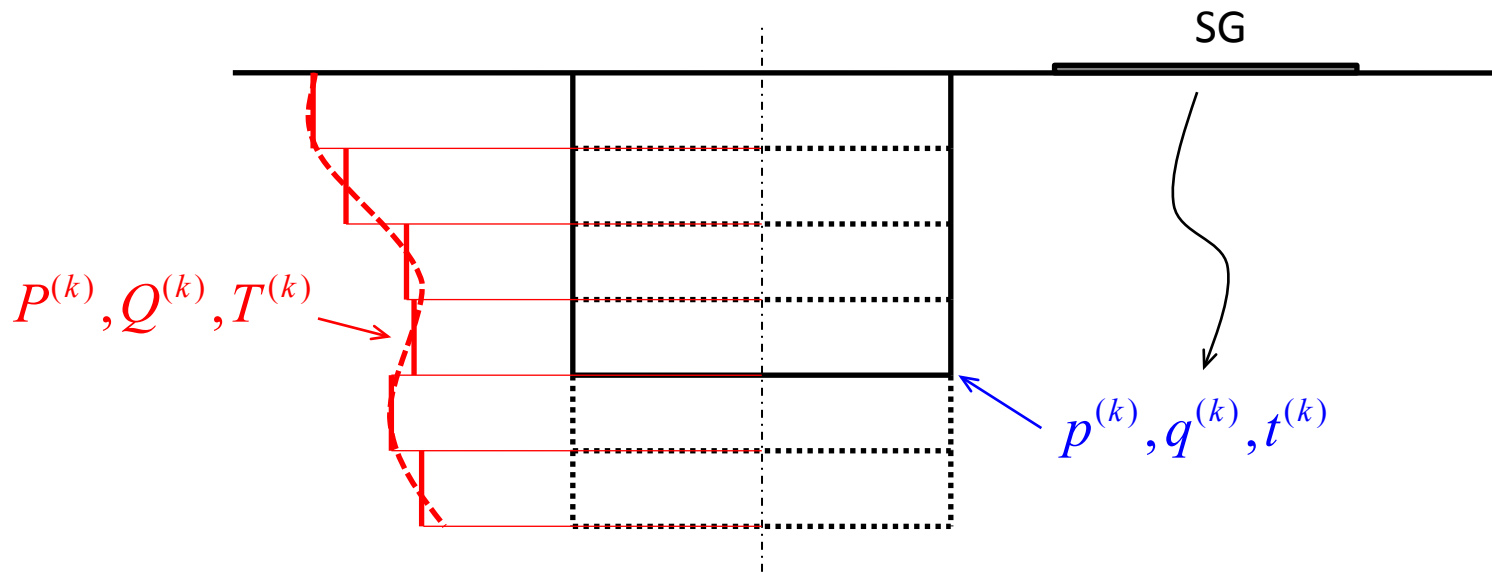


In questa forma a, b dipendono solo dalla geometria:

Diametro del foro, Diametro rosetta, Lunghezza e Larghezza delle griglie

Distribuzione non uniforme di tensione

Metodo integrale, distribuzione delle tensioni costanti a tratti



$$\mathbf{P} = (P^{(1)}, P^{(2)}, \dots, P^{(k)}, \dots, P^{(n)})^T$$

$$\mathbf{Q} = (Q^{(1)}, Q^{(2)}, \dots, Q^{(k)}, \dots, Q^{(n)})^T$$

$$\mathbf{T} = (T^{(1)}, T^{(2)}, \dots, T^{(k)}, \dots, T^{(n)})^T$$

$$\mathbf{p} = (p^{(1)}, p^{(2)}, \dots, p^{(k)}, \dots, p^{(n)})^T$$

$$\mathbf{q} = (q^{(1)}, q^{(2)}, \dots, q^{(k)}, \dots, q^{(n)})^T$$

$$\mathbf{t} = (t^{(1)}, t^{(2)}, \dots, t^{(k)}, \dots, t^{(n)})^T$$

Componente equibiassiale:

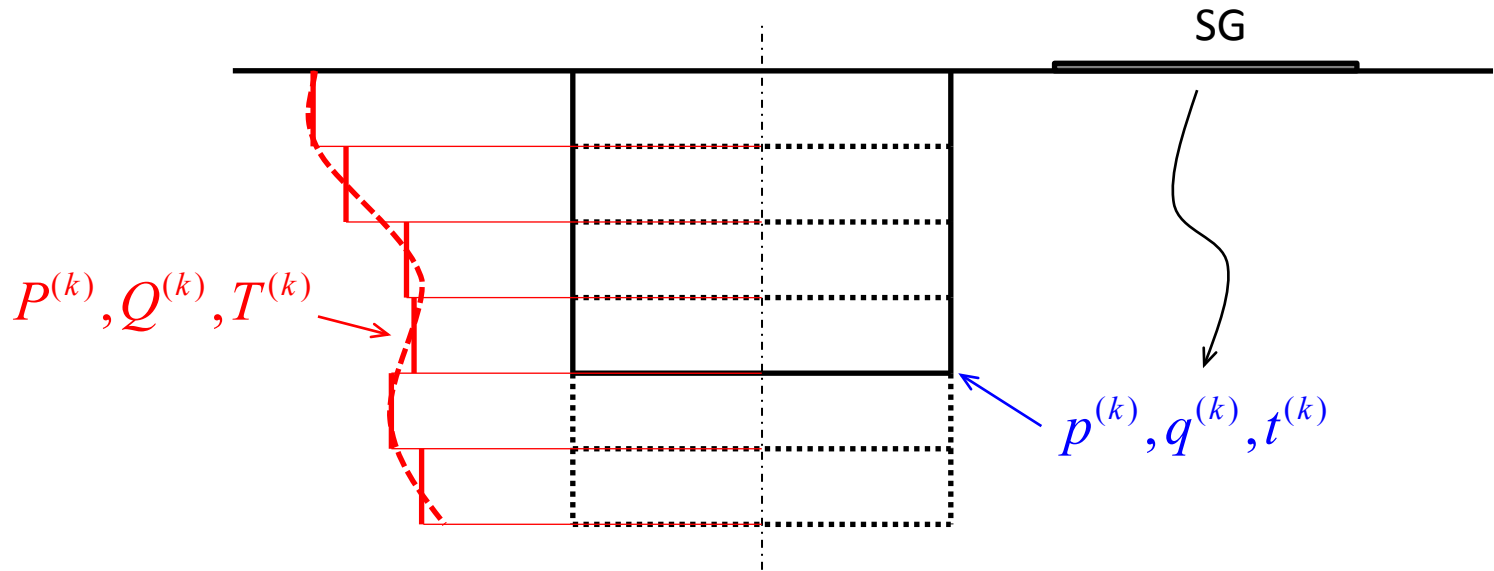
$$-\frac{1+\nu}{E} \mathbf{a} \mathbf{P} = \mathbf{p}$$

Componenti taglio puro:

$$-\frac{1}{E} \mathbf{b} \mathbf{Q} = \mathbf{q} \quad -\frac{1}{E} \mathbf{b} \mathbf{T} = \mathbf{t}$$

Distribuzione non uniforme di tensione

Metodo integrale, distribuzione delle tensioni costanti a tratti



La deformazione rilassata dipende solo dalle tensioni fino alla profondità raggiunta: le matrici \mathbf{a} , \mathbf{b} sono **triangolari inferiori**

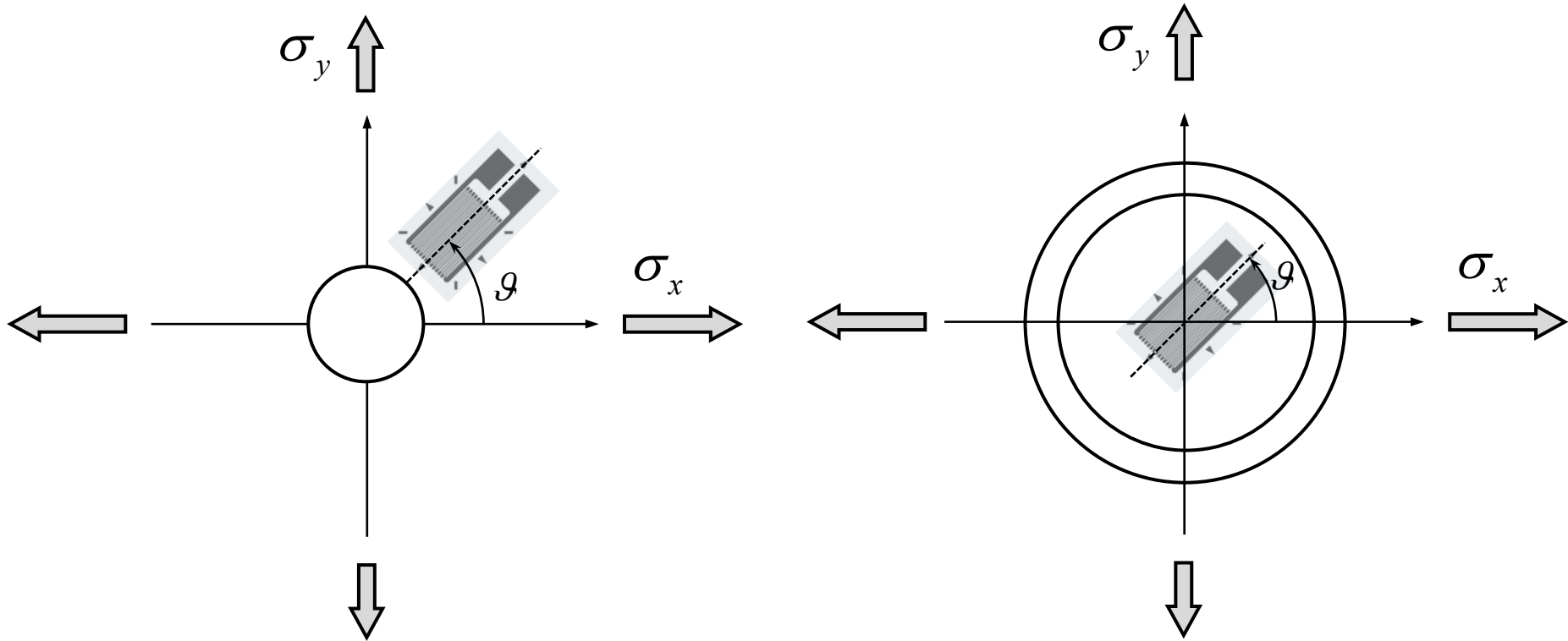
$$\mathbf{a} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Metodo integrale: Foro / Cava anulare

Geometria assial-simmetrica per entrambi i casi (Foro / Cava anulare):

$$\varepsilon_r(\vartheta) = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos(2\vartheta)$$

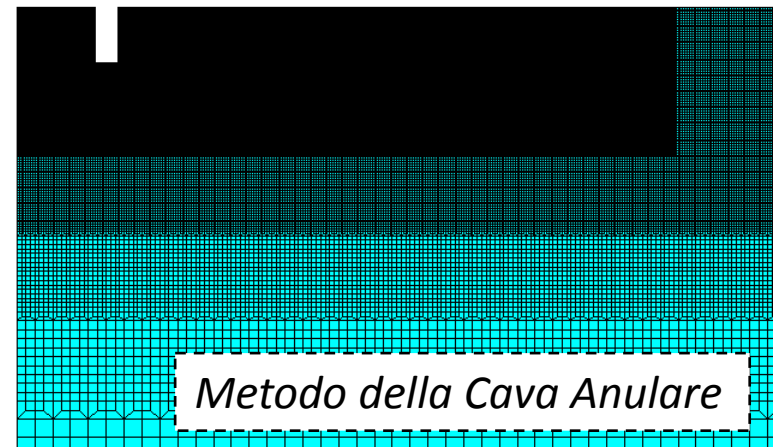
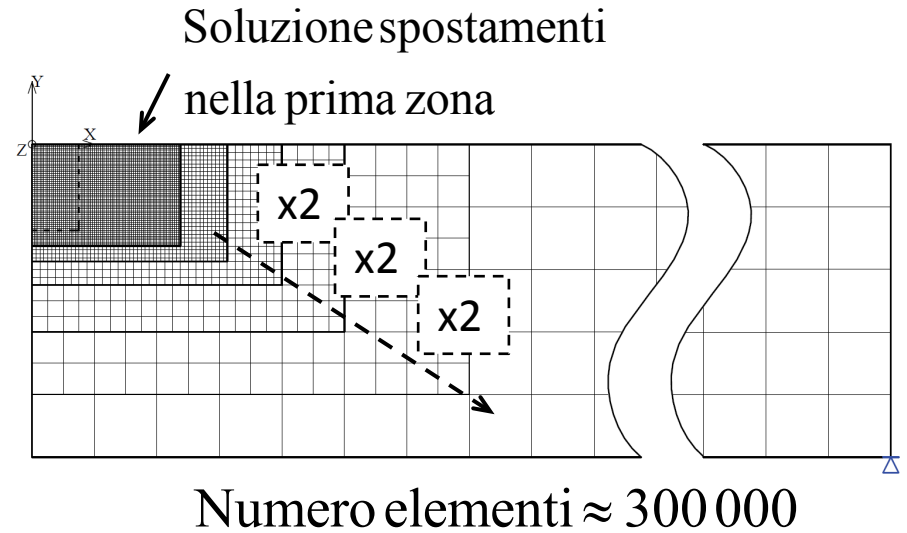
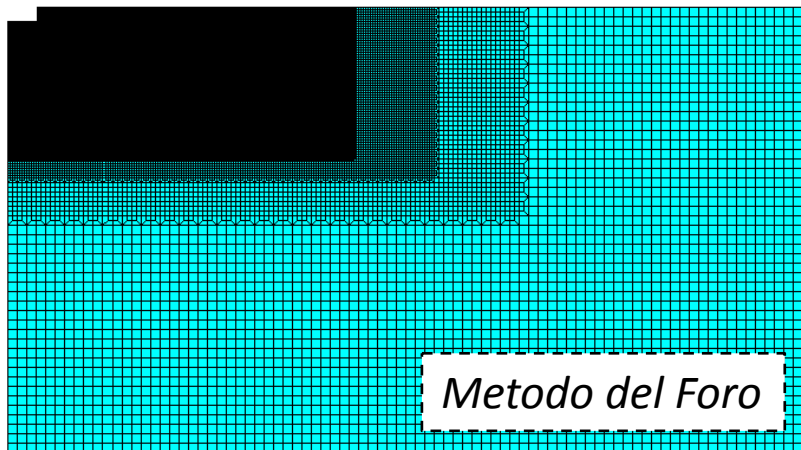
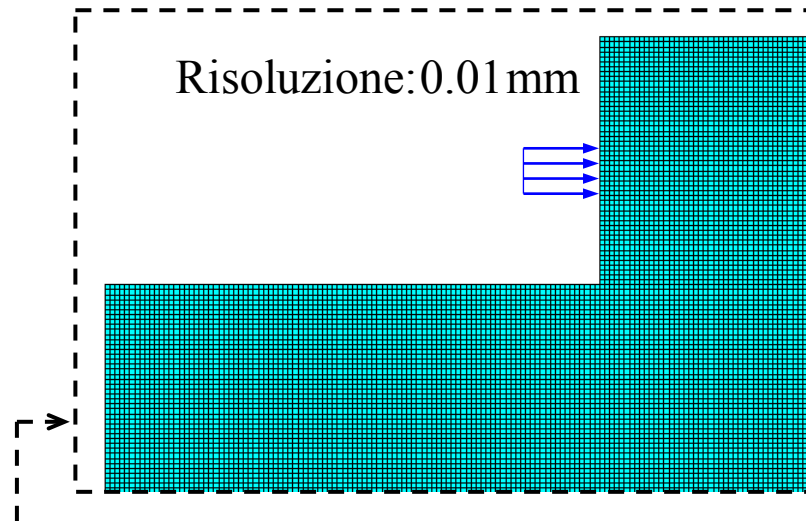


Ok, approccio "p,q,t" anche per la cava.
Opportuni coefficienti delle matrici **a**, **b**

Metodo integrale: Foro / Cava anulare

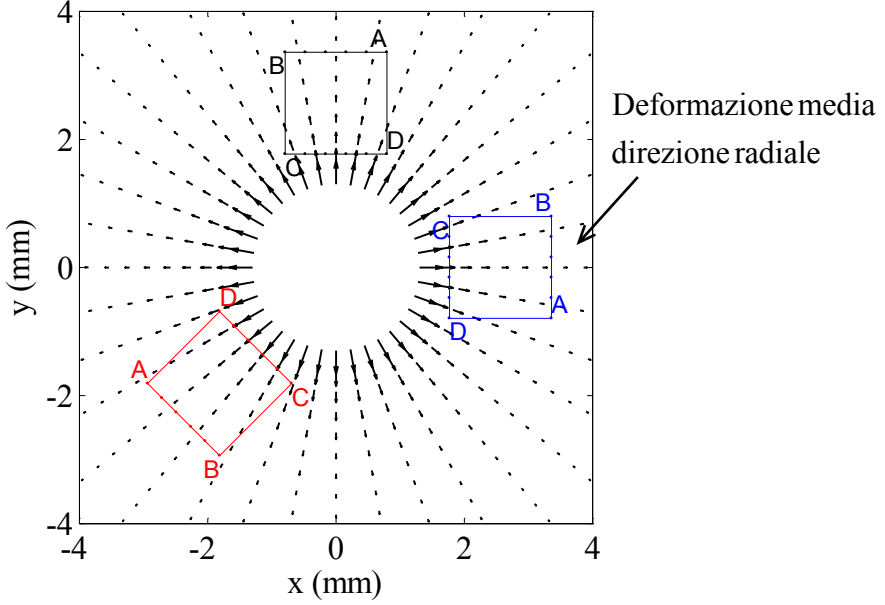
Determinazione dei coefficienti, simulazioni Elementi Finiti

Modello piano, assial-simmetrico *armonico* (ANSYS Plane25)

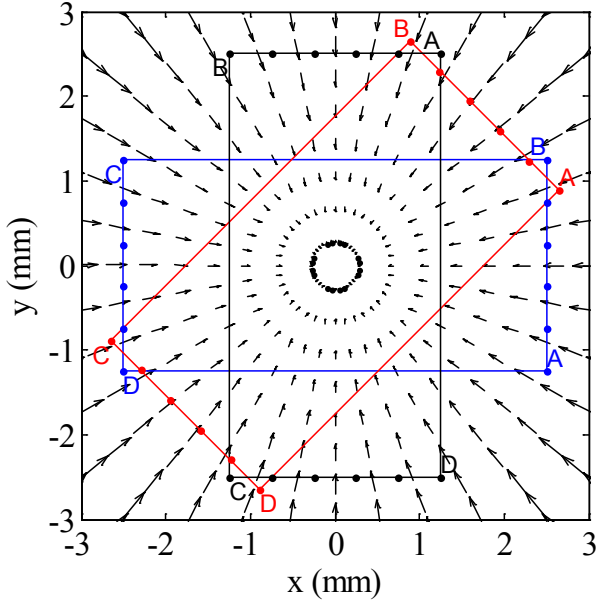


Foro / Cava, campi di spostamento

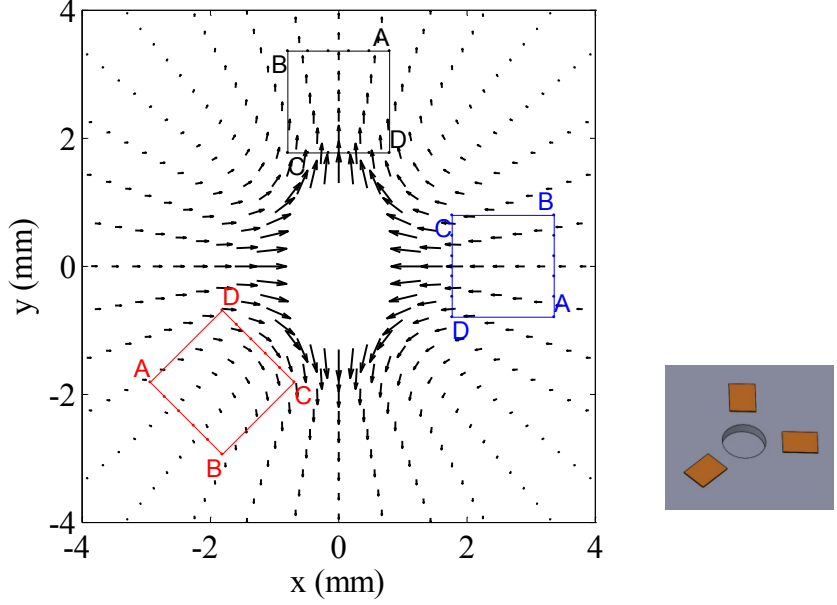
Equibiassiale: P



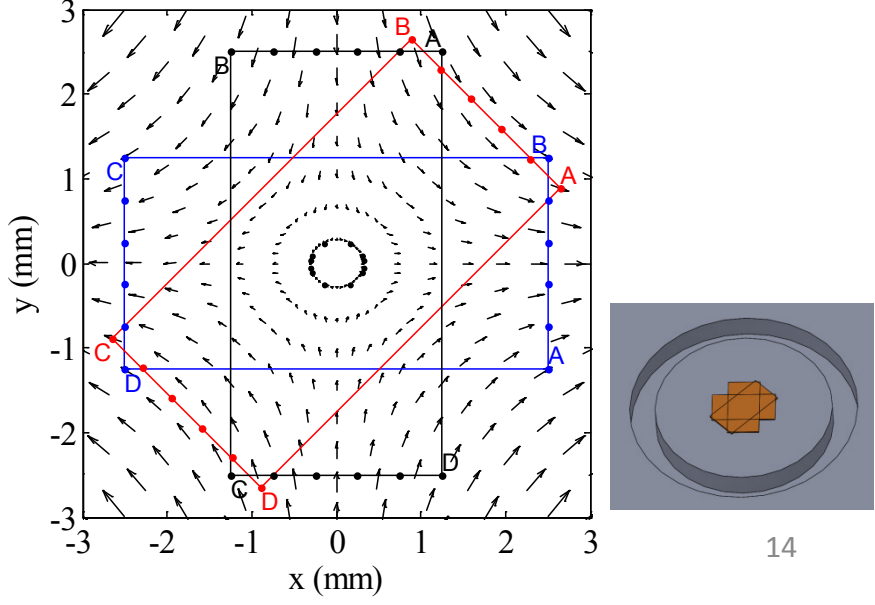
Equibiassiale: P



Taglio puro: Q

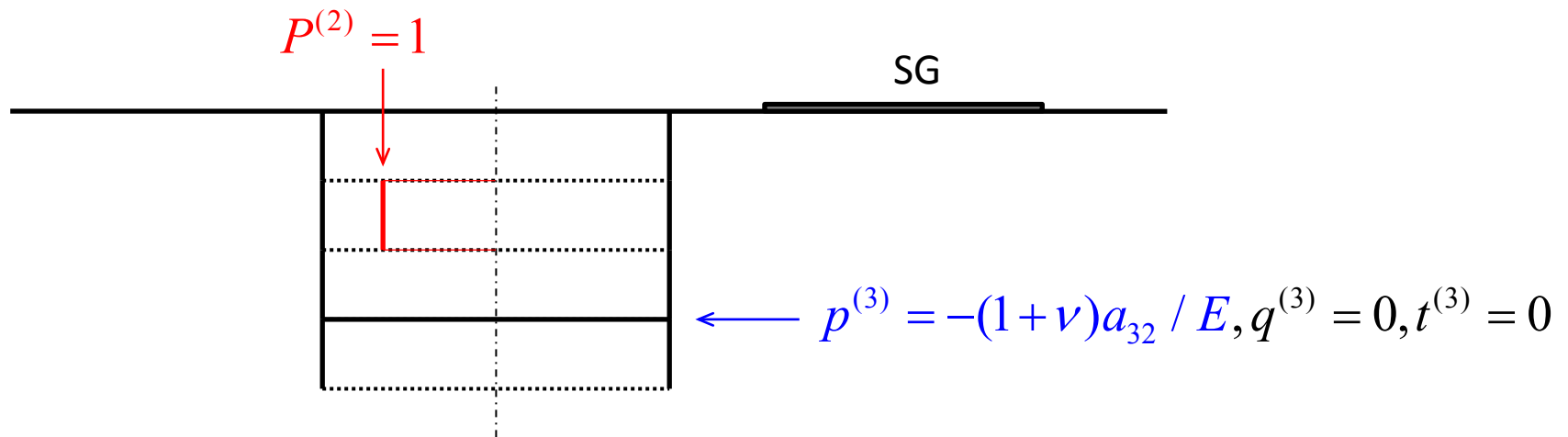


Taglio puro: Q

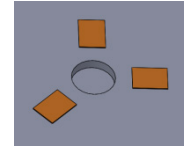


Foro / Cava, derivazione coefficienti matrici **a**, **b**

$$-\frac{1+\nu}{E} \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} P^{(1)} = 0 \\ P^{(2)} = 1 \\ P^{(3)} = 0 \\ P^{(4)} = 0 \end{bmatrix} = \begin{bmatrix} p^{(1)} = 0 \\ p^{(2)} = -(1+\nu)a_{22} / E \\ p^{(3)} = -(1+\nu)a_{32} / E \\ p^{(4)} = -(1+\nu)a_{42} / E \end{bmatrix}$$



Metodo del Foro



ASTME837 – 13a, $D = 5.13$ mm, $D_0 = 2.0$ mm. Step profondità 0.05 mm

Differenze rispetto alla norma : **3 – 7%**

$$a_{ij} (\times 10^3)$$

6.613									
7.932	7.099								
9.040	8.359	7.296							
10.02	9.360	8.540	7.291						
10.89	10.23	9.473	8.521	7.132					
11.65	10.99	10.26	9.400	8.342	6.856				
12.33	11.67	10.95	10.12	9.172	8.036	6.491			
12.93	12.26	11.54	10.74	9.840	8.818	7.633	6.065		
13.45	12.77	12.05	11.27	10.40	9.434	8.367	7.159	5.598	
13.90	13.22	12.50	11.72	10.87	9.940	8.935	7.846	6.640	5.110

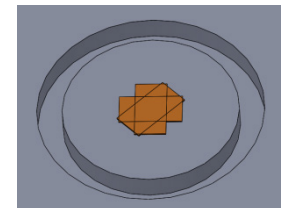
$$b_{ij} (\times 10^3)$$

12.23									
14.10	13.20								
15.70	15.07	13.75							
17.14	16.59	15.67	14.00						
18.44	17.94	17.14	15.95	14.02					
19.61	19.14	18.41	17.38	15.98	13.85				
20.67	20.22	19.53	18.59	17.37	15.79	13.53			
21.61	21.18	20.53	19.64	18.51	17.13	15.43	13.08		
22.44	22.04	21.41	20.56	19.49	18.21	16.71	14.94	12.55	
23.18	22.79	22.18	21.36	20.33	19.12	17.73	16.16	14.34	11.95

$D_I = 14$ mm, $D_E = 18$ mm

Step profondità 1.0 mm

Metodo della Cava Anulare



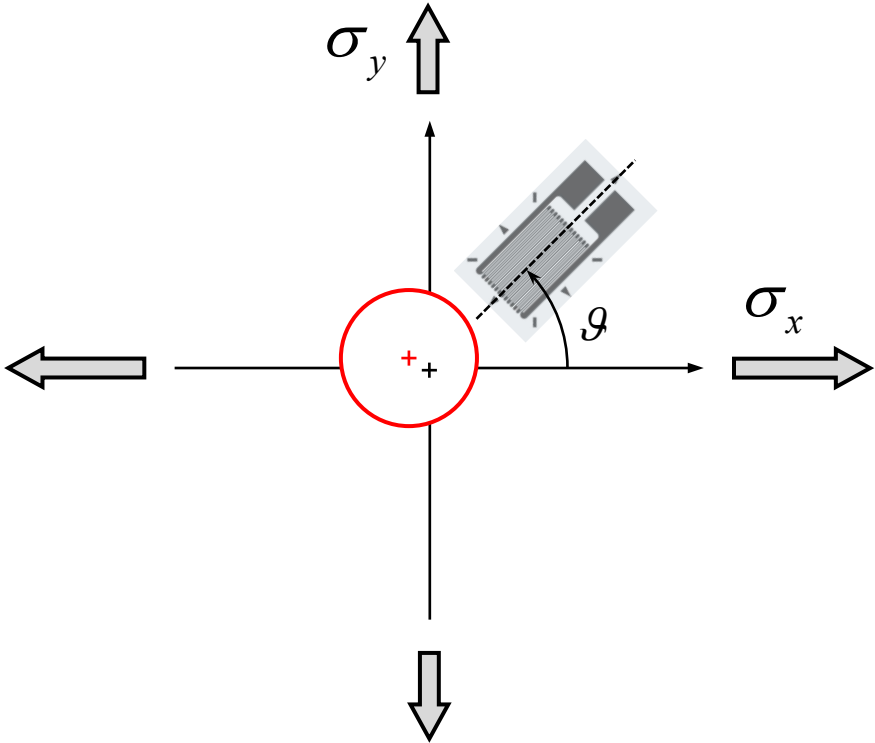
$$a_{ij}$$

0.1145				
0.1966	0.1139			
0.2362	0.1658	0.0815		
0.2523	0.1847	0.1114	0.0451	
0.2575	0.1908	0.1197	0.0598	0.0176

$$b_{ij}$$

0.1382				
0.2668	0.1529			
0.3710	0.2519	0.1279		
0.4462	0.3159	0.1952	0.0872	
0.4945	0.3559	0.2317	0.1286	0.0487

Introduzione dell'eccentricità, matrice correlazione lineare unica



~~$$\varepsilon_r(\vartheta) = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos(2\vartheta)$$~~

Non è più possibile il disaccoppiamento "p,q,t"

$$-\frac{1}{E} \mathbf{A} \mathbf{S} = \mathbf{e}$$

$$\mathbf{S} = (\sigma_1^{(1)}, \sigma_3^{(1)}, \tau_{13}^{(1)}, \dots, \sigma_1^{(n)}, \sigma_3^{(n)}, \tau_{13}^{(n)})^T$$

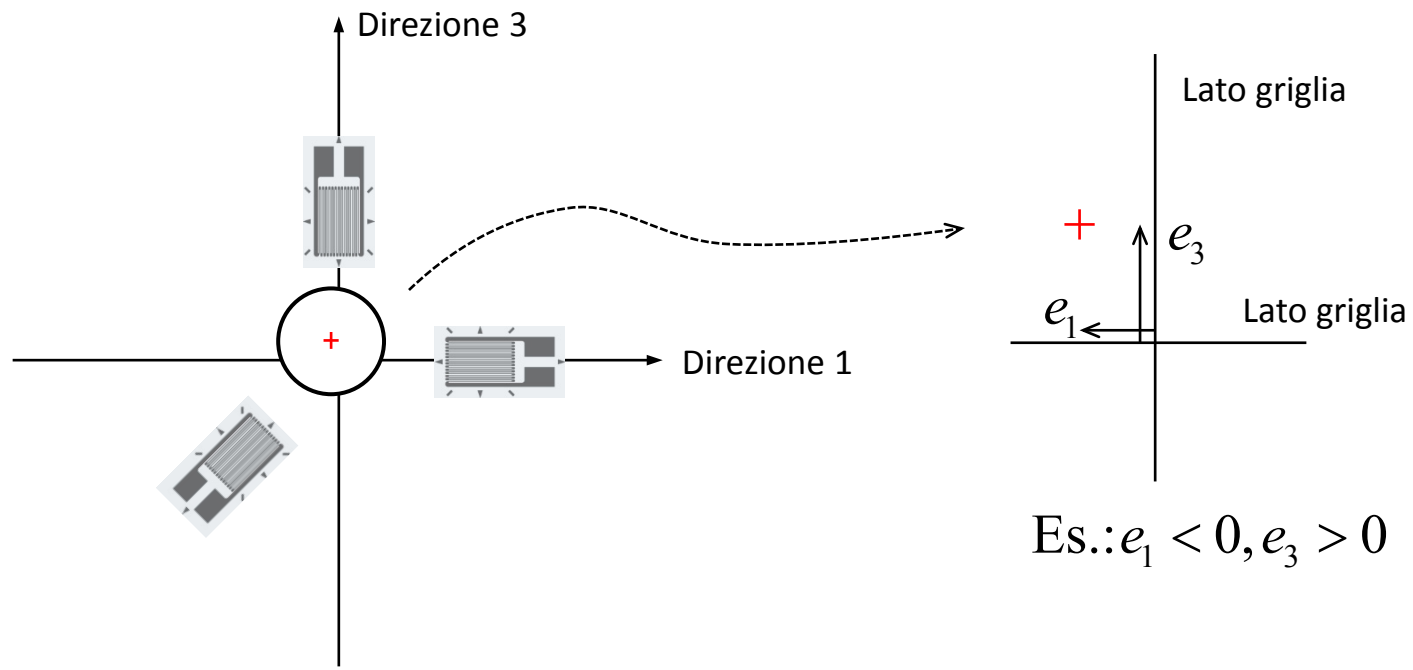
$$\mathbf{e} = (\varepsilon_1^{(1)}, \varepsilon_2^{(1)}, \varepsilon_3^{(1)}, \dots, \varepsilon_1^{(n)}, \varepsilon_2^{(n)}, \varepsilon_3^{(n)})^T$$

$$\mathbf{A} = \begin{bmatrix} A_{11}^{(11)} & A_{12}^{(11)} & A_{13}^{(11)} & 0 & 0 & 0 \\ A_{21}^{(11)} & A_{22}^{(11)} & A_{23}^{(11)} & 0 & 0 & 0 \\ A_{31}^{(11)} & A_{32}^{(11)} & A_{33}^{(11)} & 0 & 0 & 0 \\ \hline A_{11}^{(21)} & A_{12}^{(21)} & A_{13}^{(21)} & A_{11}^{(22)} & A_{12}^{(22)} & A_{13}^{(22)} \\ A_{21}^{(21)} & A_{22}^{(21)} & A_{23}^{(21)} & A_{21}^{(22)} & A_{22}^{(22)} & A_{23}^{(22)} \\ A_{31}^{(21)} & A_{32}^{(21)} & A_{33}^{(21)} & A_{31}^{(22)} & A_{32}^{(22)} & A_{33}^{(22)} \end{bmatrix}$$

Matrice triangolare inferiore a blocchi 3x3

Dipendenza dei coefficienti dalle componenti di eccentricità

$$\begin{aligned}
 A_{ij}^{(h,k)}(e_1, e_3) = & A_{0,ij}^{(h,k)} + \text{Termine ad eccentricità nulla} \\
 & \frac{\partial}{\partial e_1} A_{ij}^{(h,k)} e_1 + \frac{\partial}{\partial e_3} A_{ij}^{(h,k)} e_3 + \text{I}^\circ \text{ ordine} \\
 & \frac{1}{2} \frac{\partial^2}{\partial e_1^2} A_{ij}^{(h,k)} e_1^2 + \frac{1}{2} \frac{\partial^2}{\partial e_3^2} A_{ij}^{(h,k)} e_3^2 + \frac{\partial^2}{\partial e_1 \partial e_3} A_{ij}^{(h,k)} e_1 e_3 + \dots \text{II}^\circ \text{ ordine}
 \end{aligned}$$

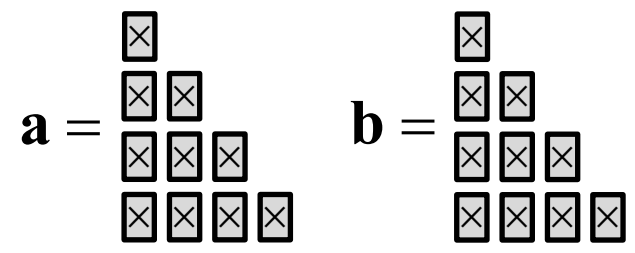
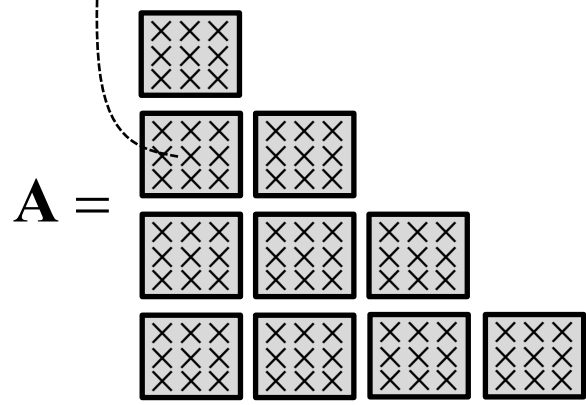


Termine ad eccentricità nulla

Ad eccentricità nulla la matrice \mathbf{A} contiene le stesse informazioni della 2 matrici \mathbf{a} e \mathbf{b} "p,q,t"

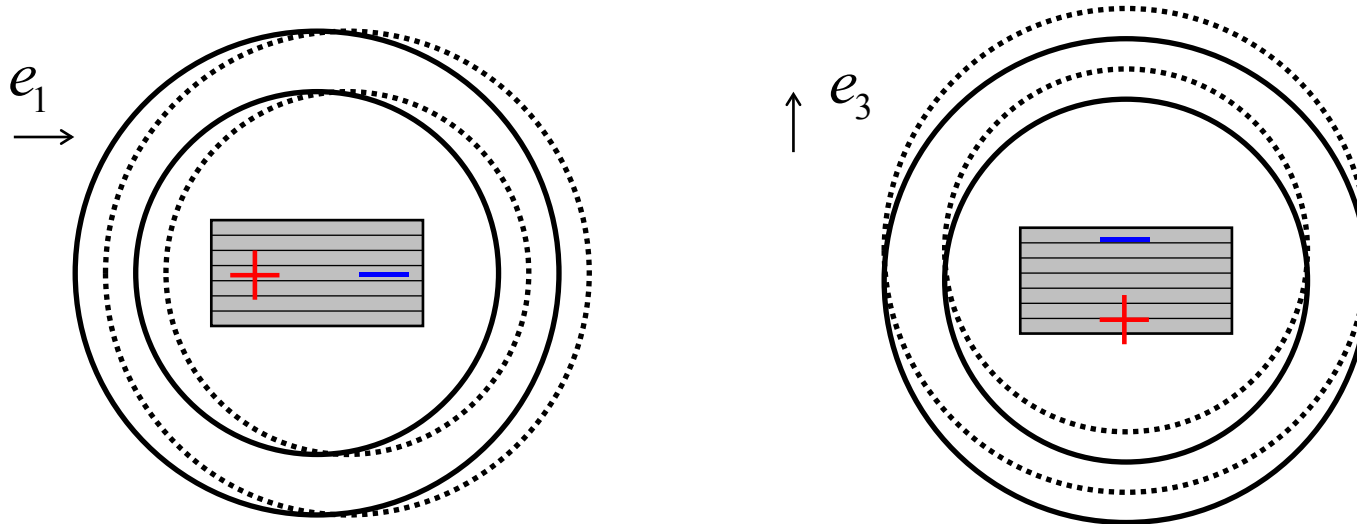
$$\begin{bmatrix} A_{0,11}^{(h,k)} & A_{0,12}^{(h,k)} & A_{0,13}^{(h,k)} \\ A_{0,21}^{(h,k)} & A_{0,22}^{(h,k)} & A_{0,23}^{(h,k)} \\ A_{0,31}^{(h,k)} & A_{0,32}^{(h,k)} & A_{0,33}^{(h,k)} \end{bmatrix} = \begin{bmatrix} \frac{(1+\nu)a_{hk} + b_{hk}}{2} & \frac{(1+\nu)a_{hk} - b_{hk}}{2} & 0 \\ \frac{(1+\nu)a_{hk}}{2} & \frac{(1+\nu)a_{hk}}{2} & b_{hk} \\ \frac{(1+\nu)a_{hk} - b_{hk}}{2} & \frac{(1+\nu)a_{hk} + b_{hk}}{2} & 0 \end{bmatrix}$$

Generico blocco
(anche non diagonale)



Proprietà di simmetria

Cava anulare, assenza dei termini del 1° ordine

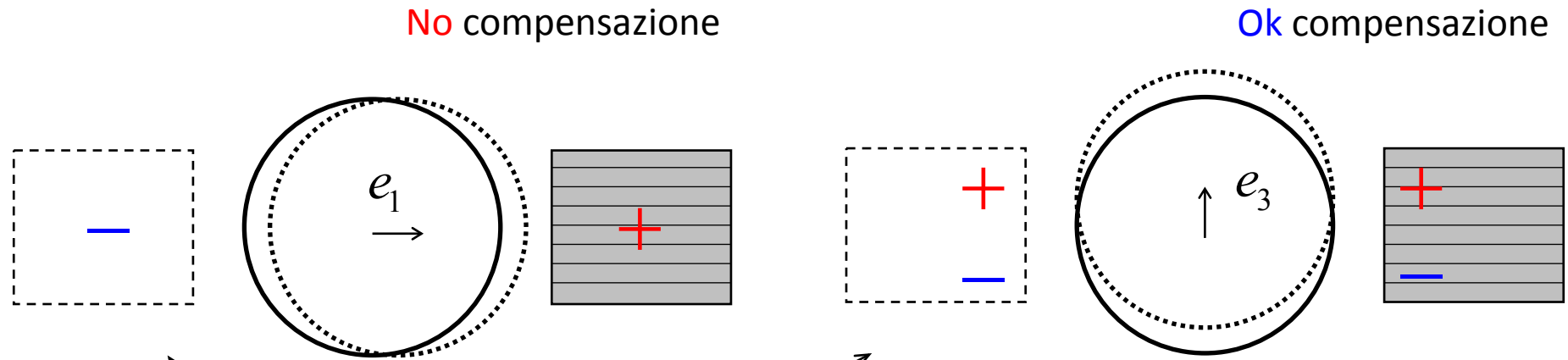


Effetto di compensazione fra due zone opposte della griglia

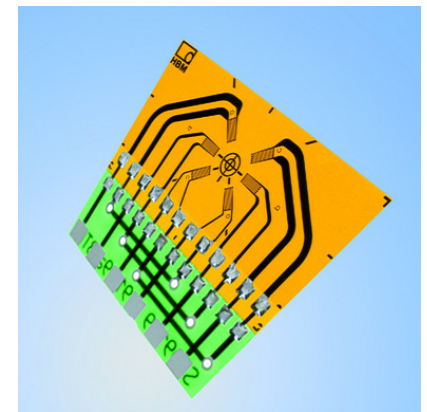
$$A_{ij}^{(h,k)}(e_1, e_3) = A_{0,ij}^{(h,k)} +$$
~~$$\frac{\partial}{\partial e_1} A_{ij}^{(h,k)} e_1 + \frac{\partial}{\partial e_{13}} A_{ij}^{(h,k)} e_3 +$$~~
$$\frac{1}{2} \frac{\partial^2}{\partial e_1^2} A_{ij}^{(h,k)} e_1^2 + \frac{1}{2} \frac{\partial^2}{\partial e_3^2} A_{ij}^{(h,k)} e_3^2 + \frac{\partial^2}{\partial e_1 \partial e_3} A_{ij}^{(h,k)} e_1 e_3 + \dots$$

Proprietà di simmetria

Foro, termini del 1° ordine non compensati



Effetto di compensazione solo se presente griglia diametralmente opposta



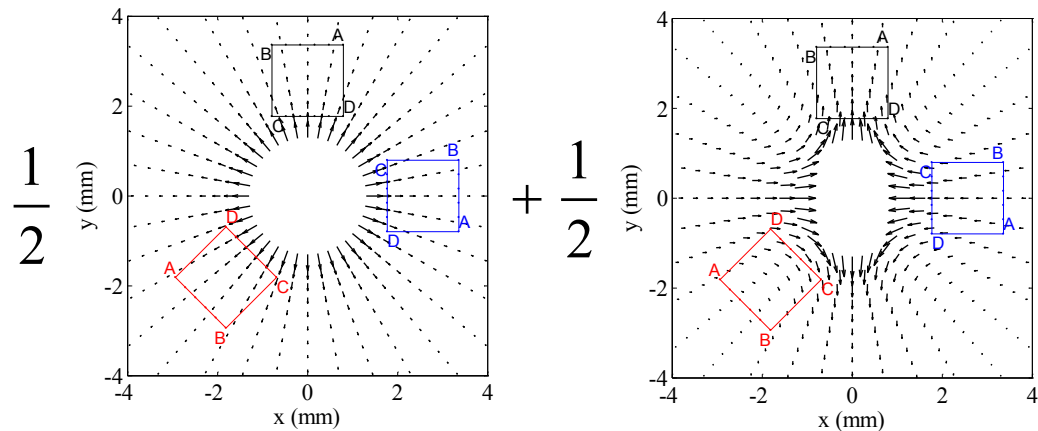
M. Beghini, L. Bertini, C. Santus, A. Benincasa, L. Bertelli, and E. Valentini. Validazione sperimentale di una rosetta a 6 griglie per ridurre l'errore di eccentricità nella misura delle tensioni residue. AIAS XXXIX, 2010. Maratea (PZ)

Foro, derivazione coefficienti della matrice \mathbf{A}

$$-\frac{1}{E} \begin{bmatrix} A_{11}^{(11)} & A_{12}^{(11)} & A_{13}^{(11)} & 0 & 0 & 0 \\ A_{21}^{(11)} & A_{22}^{(11)} & A_{23}^{(11)} & 0 & 0 & 0 \\ A_{31}^{(11)} & A_{32}^{(11)} & A_{33}^{(11)} & 0 & 0 & 0 \\ A_{11}^{(21)} & A_{12}^{(21)} & A_{13}^{(21)} & A_{11}^{(22)} & A_{12}^{(22)} & A_{13}^{(22)} \\ A_{21}^{(21)} & A_{22}^{(21)} & A_{23}^{(21)} & A_{21}^{(22)} & A_{22}^{(22)} & A_{23}^{(22)} \\ A_{31}^{(21)} & A_{32}^{(21)} & A_{33}^{(21)} & A_{31}^{(22)} & A_{32}^{(22)} & A_{33}^{(22)} \end{bmatrix} \begin{bmatrix} \sigma_1^{(1)} = 0 \\ \sigma_3^{(1)} = 0 \\ \tau_{13}^{(1)} = 0 \\ \sigma_1^{(2)} = 1 \\ \sigma_3^{(2)} = 0 \\ \tau_{13}^{(2)} = 0 \end{bmatrix} = \begin{bmatrix} \varepsilon_1^{(1)} = 0 \\ \varepsilon_2^{(1)} = 0 \\ \varepsilon_3^{(1)} = 0 \\ \varepsilon_1^{(2)} = -A_{11}^{(22)} / E \\ \varepsilon_2^{(2)} = -A_{21}^{(22)} / E \\ \varepsilon_3^{(2)} = -A_{31}^{(22)} / E \end{bmatrix}$$

←
Con eccentricità

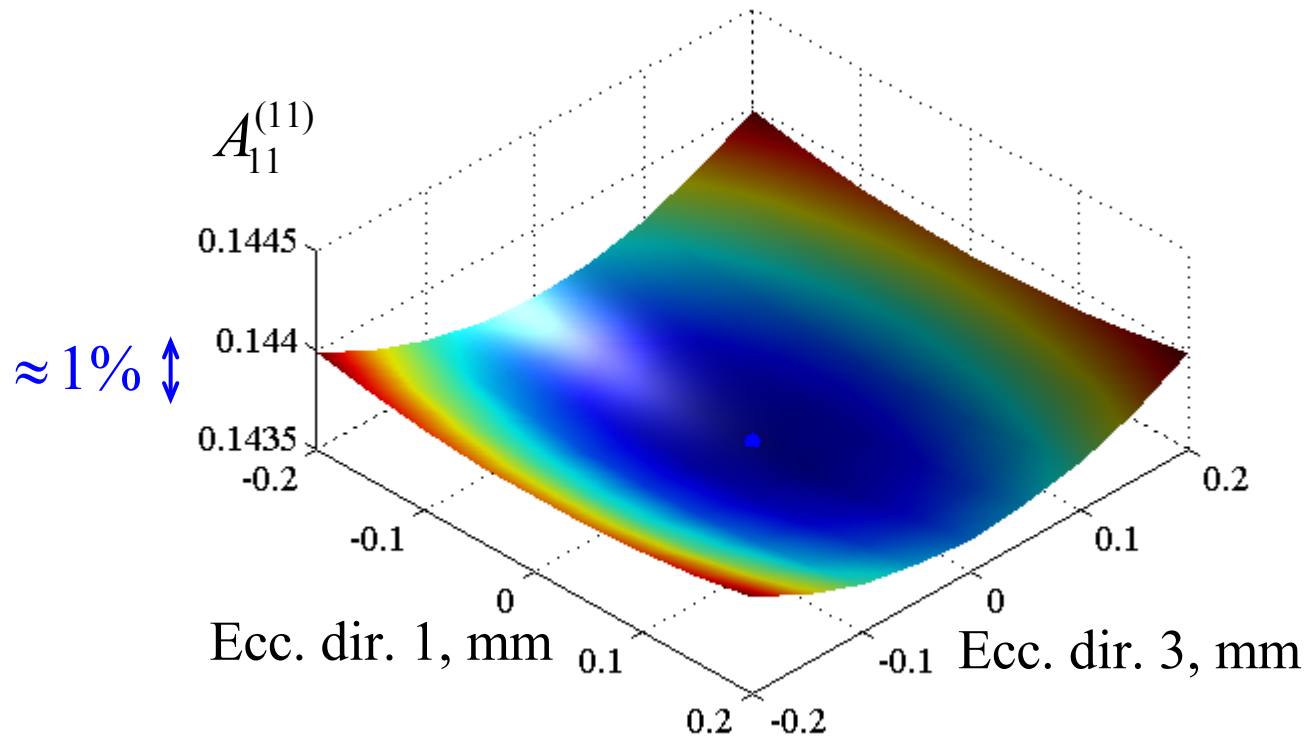
$$\sigma_1 = 1$$



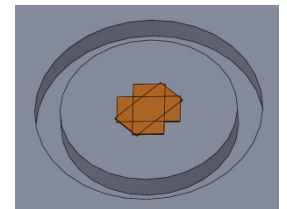
Proprietà di simmetria

Cava anulare, assenza dei termini del I° ordine

$$A_{ij}^{(h,k)}(e_1, e_3) = A_{0,ij}^{(h,k)} + \left[\frac{1}{2} \frac{\partial^2}{\partial e_1^2} A_{ij}^{(h,k)} e_1^2 + \frac{1}{2} \frac{\partial^2}{\partial e_3^2} A_{ij}^{(h,k)} e_3^2 + \frac{\partial^2}{\partial e_1 \partial e_3} A_{ij}^{(h,k)} e_1 e_3 + \dots \right]$$



Termini del II° ordine e superiori trascurabili

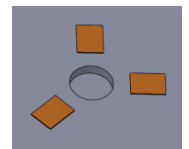
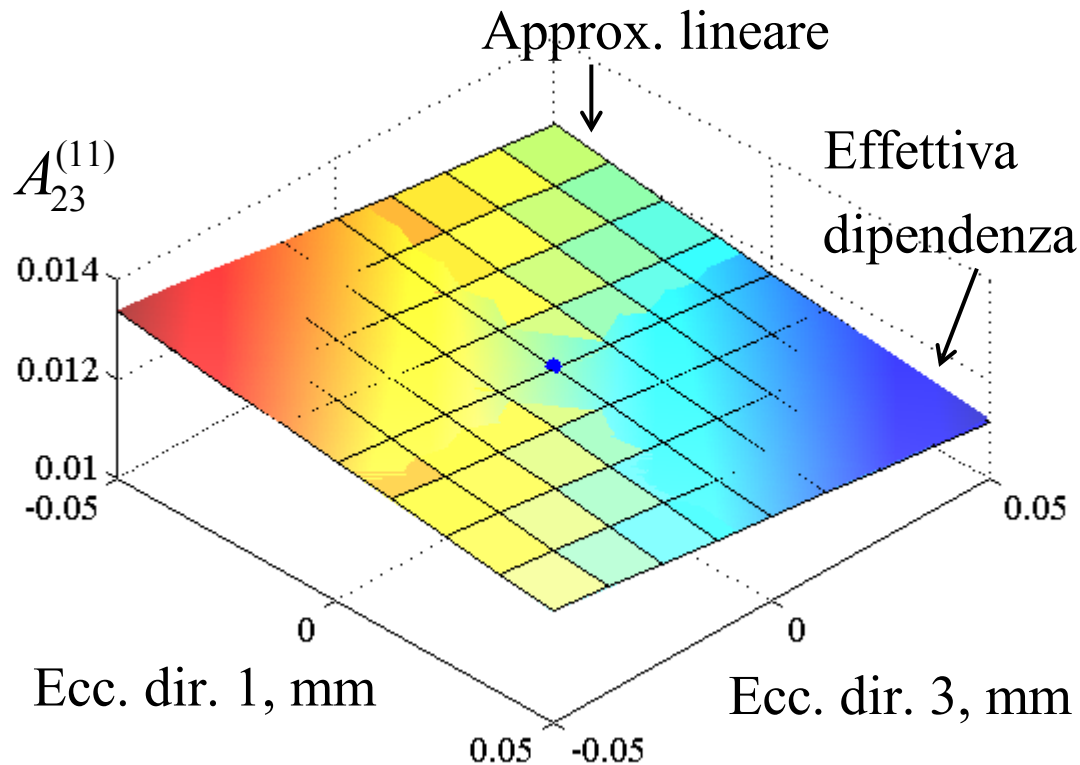


Proprietà di simmetria

Foro, termini del 1° ordine non compensati

$$A_{ij}^{(h,k)}(e_1, e_3) = A_{0,ij}^{(h,k)} + \frac{\partial}{\partial e_1} A_{ij}^{(h,k)} e_1 + \frac{\partial}{\partial e_{13}} A_{ij}^{(h,k)} e_3 + \leftarrow \text{Termini di correzione predominanti}$$
$$\frac{1}{2} \frac{\partial^2}{\partial e_1^2} A_{ij}^{(h,k)} e_1^2 + \frac{1}{2} \frac{\partial^2}{\partial e_3^2} A_{ij}^{(h,k)} e_3^2 + \frac{\partial^2}{\partial e_1 \partial e_3} A_{ij}^{(h,k)} e_1 e_3 + \dots$$

Termini trascurabili



Proprietà di simmetria

Forma matriciale e permutazione dei termini del 1° ordine

$$A_{ij}^{(h,k)}(e_1, e_3) = A_{0,ij}^{(h,k)} + \frac{\partial}{\partial e_1} A_{ij}^{(h,k)} e_1 + \frac{\partial}{\partial e_3} A_{ij}^{(h,k)} e_3 + \dots$$

$$\mathbf{A} = \mathbf{A}_0 + \mathbf{a}_{e_1} \eta_1 + \mathbf{a}_{e_3} \eta_3$$

$$\mathbf{A}_0 = \left[A_{0,ij}^{(h,k)} \right] \quad \mathbf{a}_{e_1} = \left[\alpha_{1,ij}^{(h,k)} \right] \quad \mathbf{a}_{e_3} = \left[\alpha_{3,ij}^{(h,k)} \right]$$

$$\alpha_{1,ij}^{(h,k)} = D \frac{\partial}{\partial e_1} A_{ij}^{(h,k)} \quad \alpha_{3,ij}^{(h,k)} = D \frac{\partial}{\partial e_3} A_{ij}^{(h,k)}$$

$$\eta_1 = \frac{e_1}{D} \quad \eta_3 = \frac{e_3}{D}$$

Proprietà di simmetria

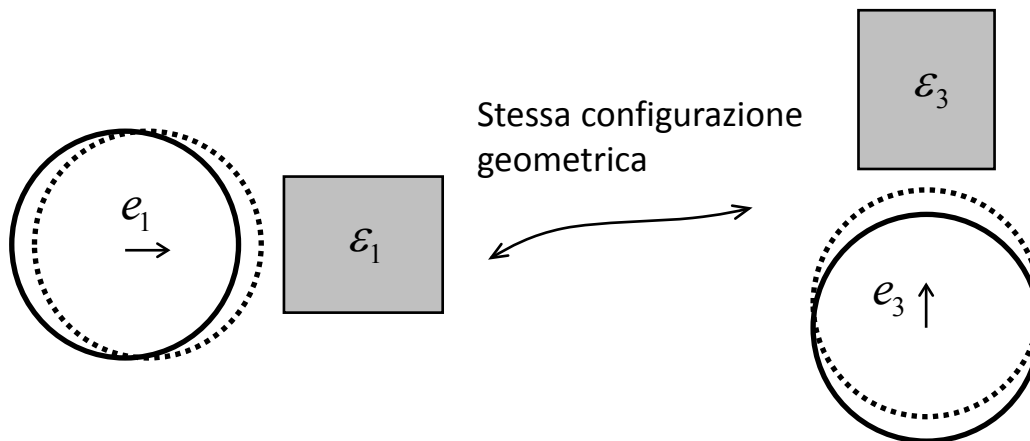
Foro, forma matriciale e permutazione dei termini del 1° ordine

$$\mathbf{A} = \mathbf{A}_0 + \mathbf{a}_{e1} \eta_1 + \mathbf{a}_{e3} \eta_3 = \mathbf{A}_0 + \mathbf{a}_{e1} \eta_1 + \mathbf{P}_s \mathbf{a}_{e1} \mathbf{P}_d \eta_3$$

$$\mathbf{P}_s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

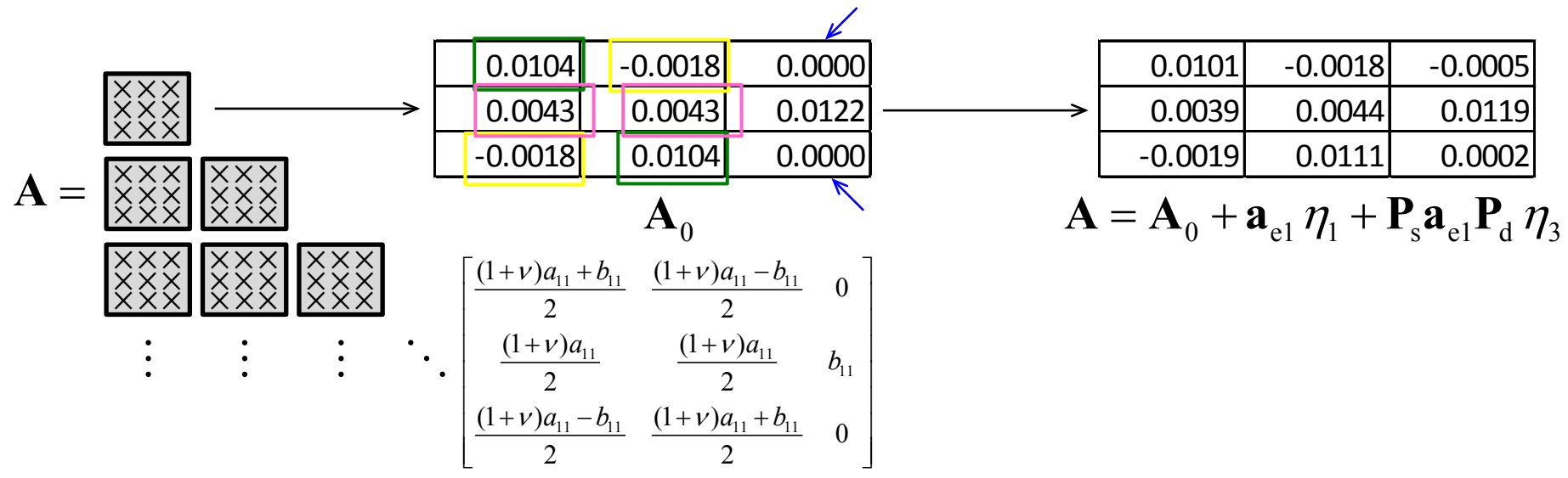
$$\mathbf{P}_d = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

1/0	0	0	0
0	1/0	0	0
0	0	1/0	0
0	0	0	1/0



È sufficiente una sola matrice, es: \mathbf{a}_{e1}

Matrice completa nel caso di eccentricità



$$-\frac{1}{E} \mathbf{A} \mathbf{S} = \mathbf{e}$$

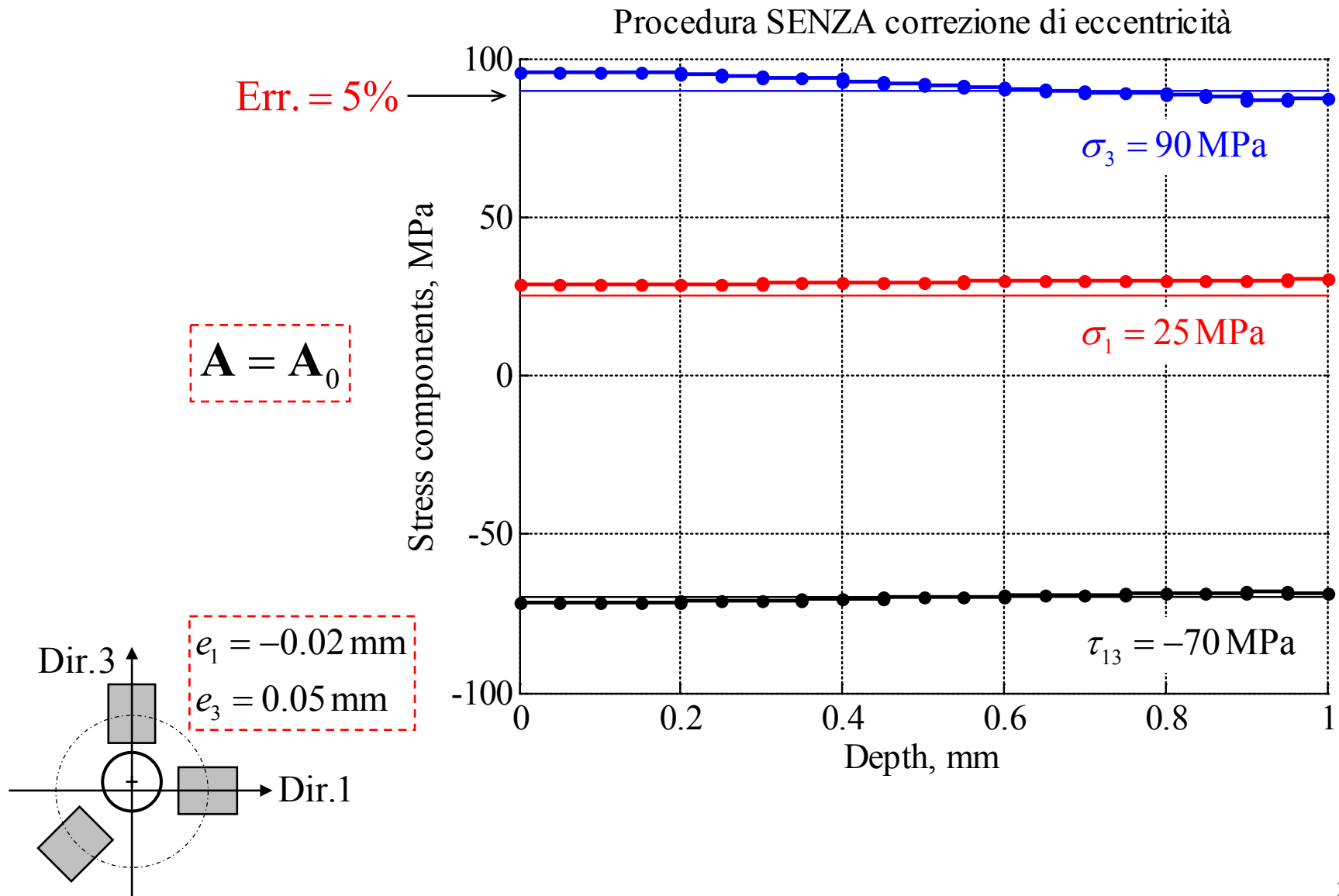
$$\Downarrow$$

$$\mathbf{S} = -E \mathbf{A}^{-1} \mathbf{e}$$

Matrice inversa / pseudo-inversa con es. 4 griglie (no "p,q,t", utili più griglie)

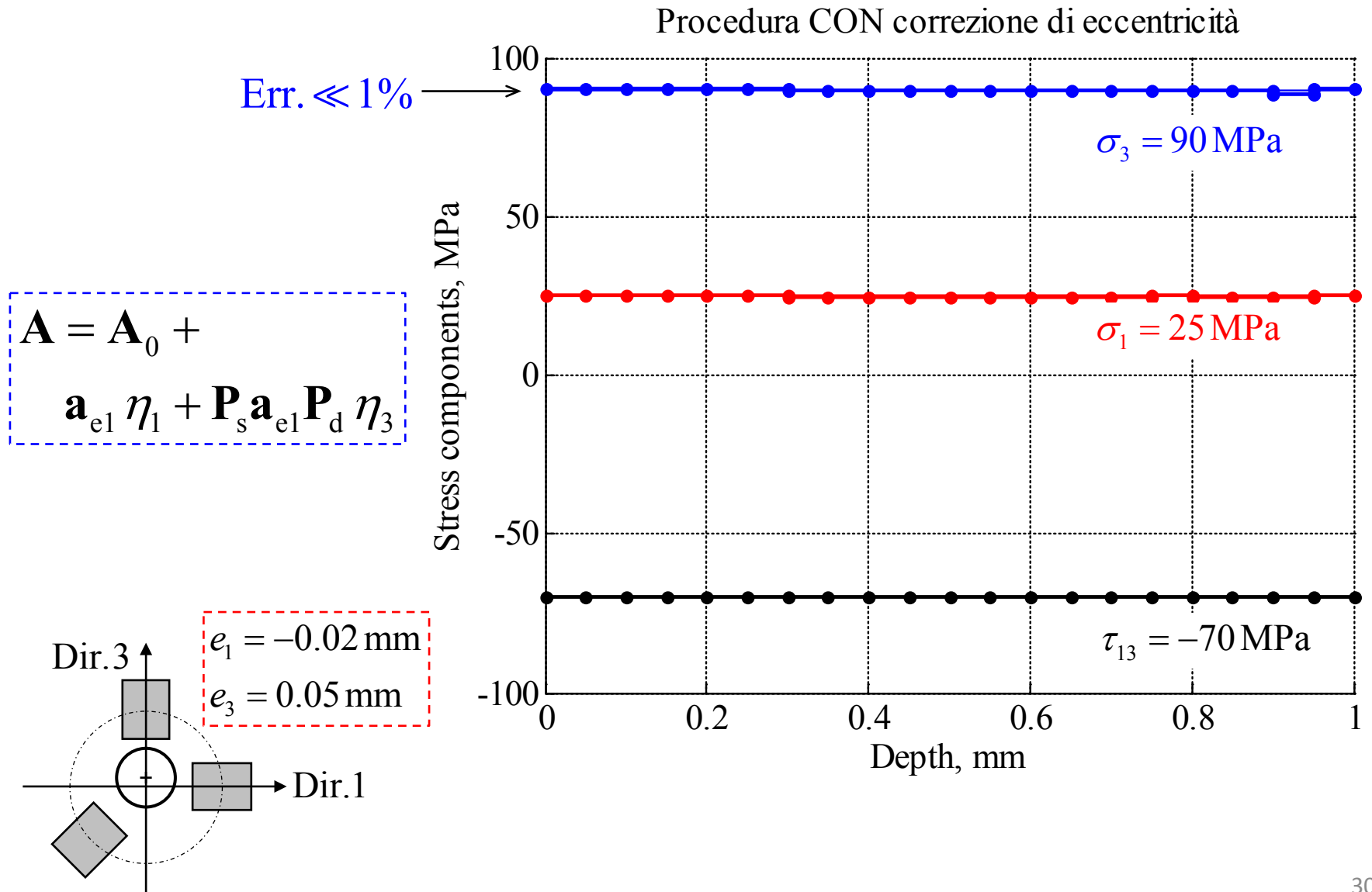
Esempi numerici, effetto della correzione

Metodo del foro, valori di eccentricità tipici della pratica sperimentale

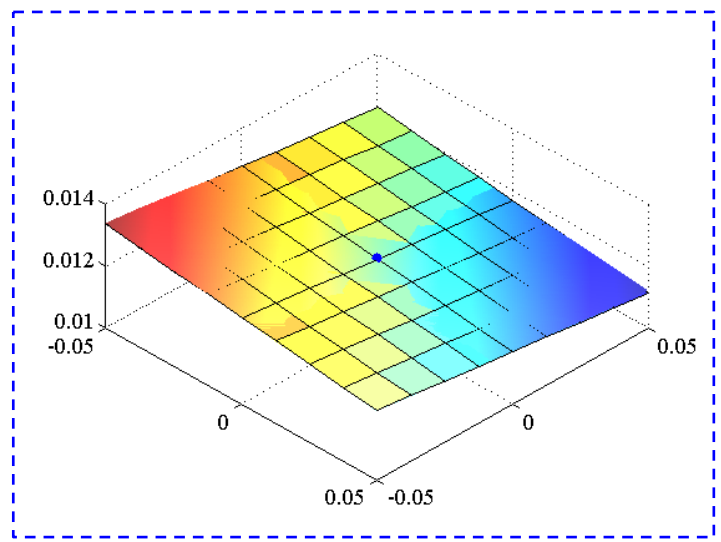
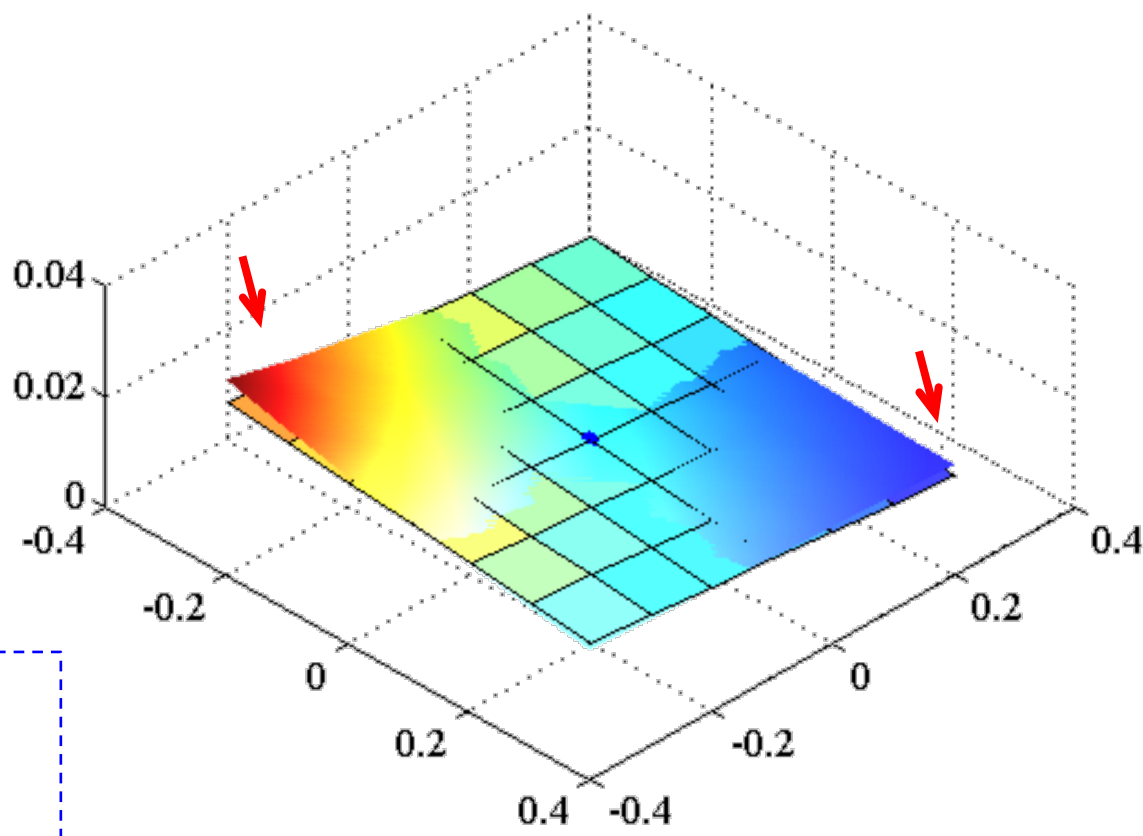
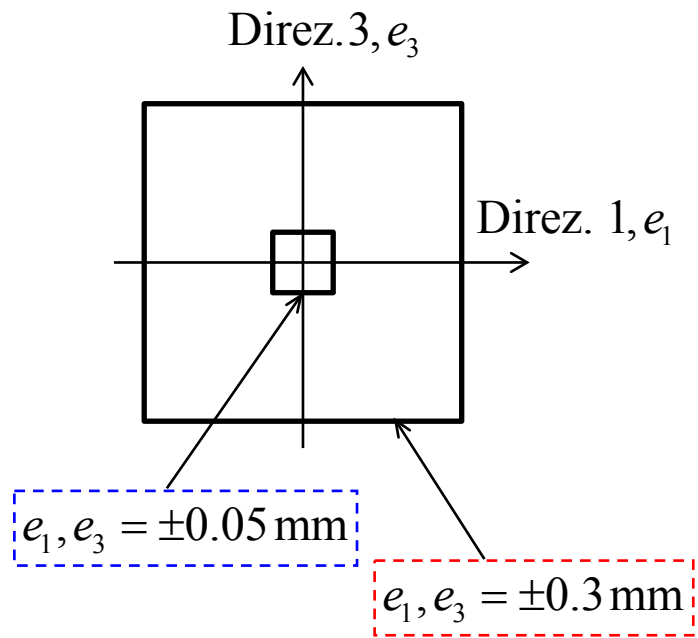


Esempi numerici, effetto della correzione

Metodo del foro, valori di eccentricità tipici della pratica sperimentale

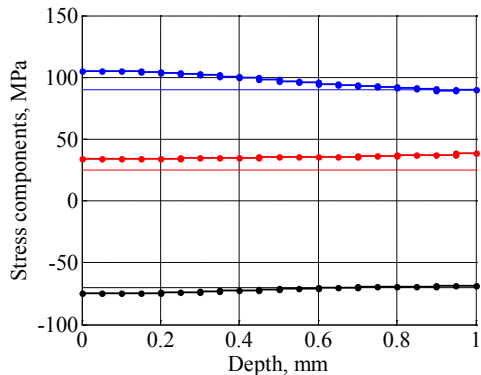


Valori di eccentricità maggiori

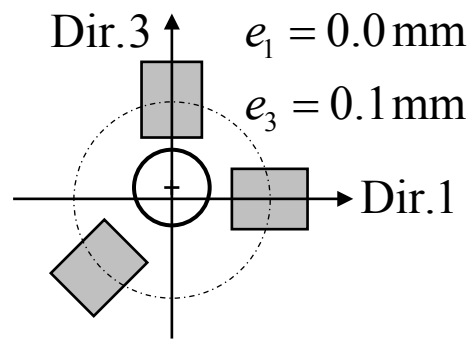
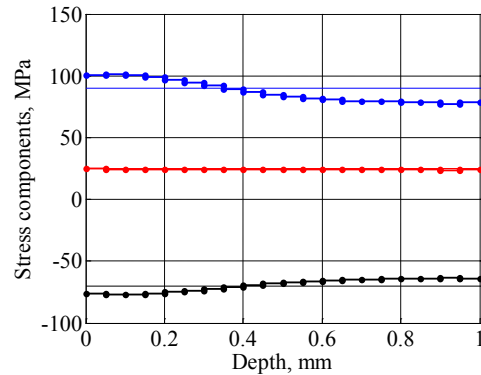
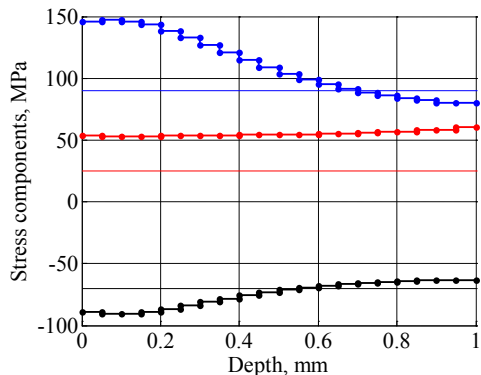
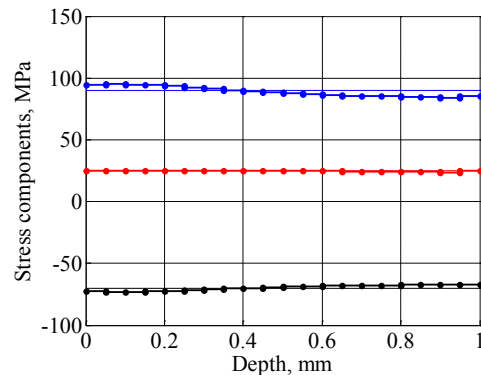
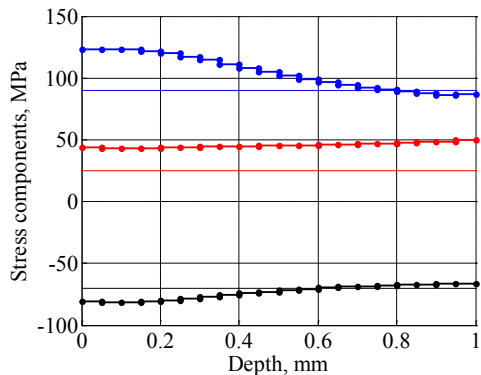
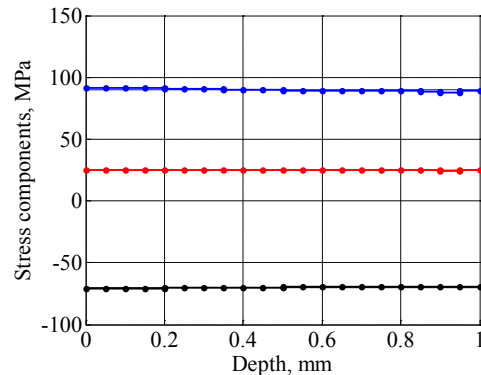


Esempio numerico applicativo

No correzione



Correzione I ordine



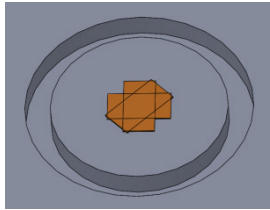
$e_3 = 0.2 \text{ mm}$

Errore sensibile, corr. I ordine insufficiente

$e_3 = 0.3 \text{ mm}$

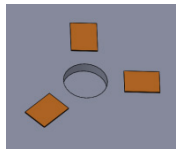
Eccentricità -> approccio “p,q,t” non più possibile $-\frac{1}{E} \mathbf{A} \mathbf{S} = \mathbf{e}$

Metodo della Cava: no termini di 1° ordine $\mathbf{A} = \mathbf{A}_0$



Con buona approssimazione
si può mantenere l'approccio
“p,q,t”

Metodo del Foro: correzione al 1° ordine $\mathbf{A} = \mathbf{A}_0 + \mathbf{a}_{e1} \eta_1 + \mathbf{P}_s \mathbf{a}_{e1} \mathbf{P}_d \eta_3$



Correzione -> Err. <1% per valori comuni/non eccessivi di eccentricità