

Memoria 116

# Condizioni di risonanza per ruote palettate, definizione del diagramma SAFE

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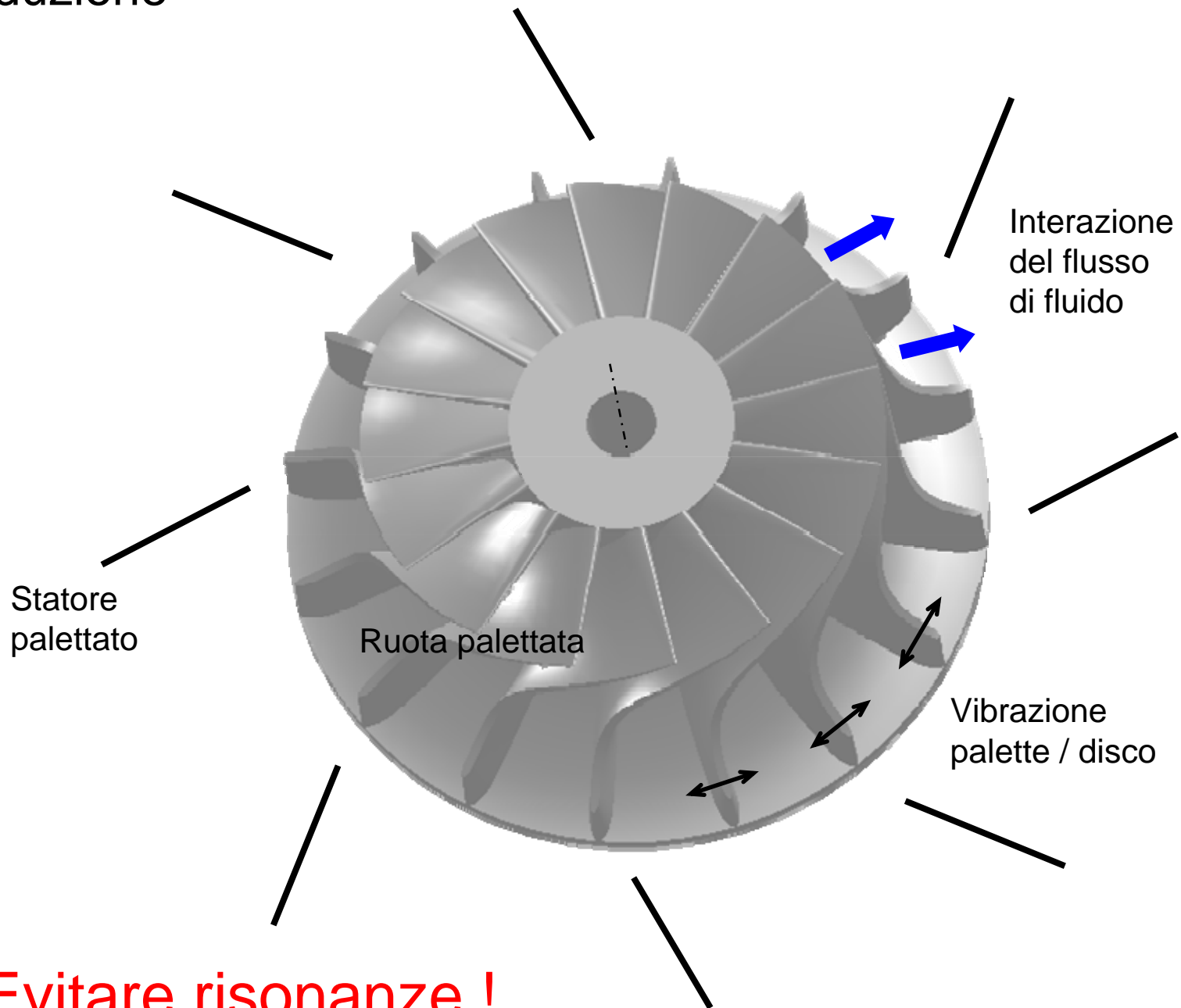
G. Mariotti

GE, Oil & Gas, Nuovo Pignone, Firenze



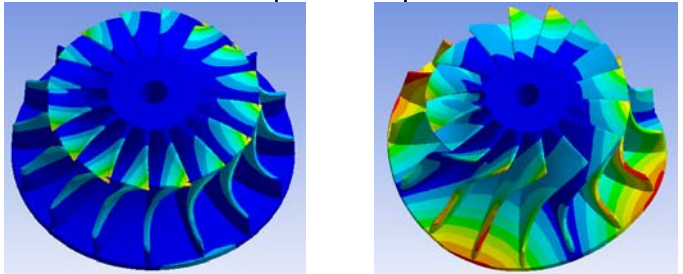
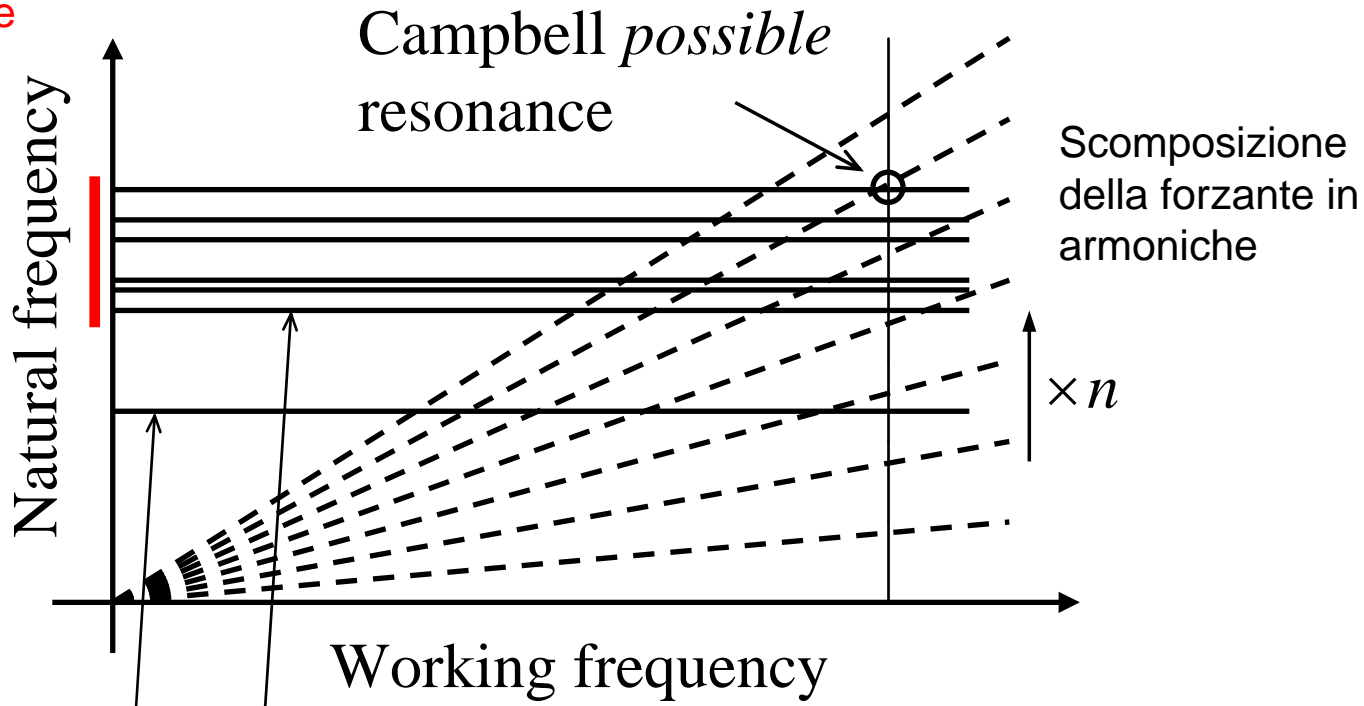
GE imagination at work

# Introduzione



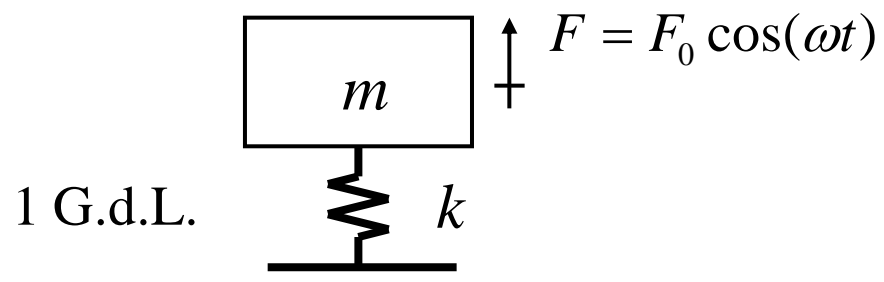
# Approccio diagramma di Campbell

Moltissime frequenze proprie ravvicinate



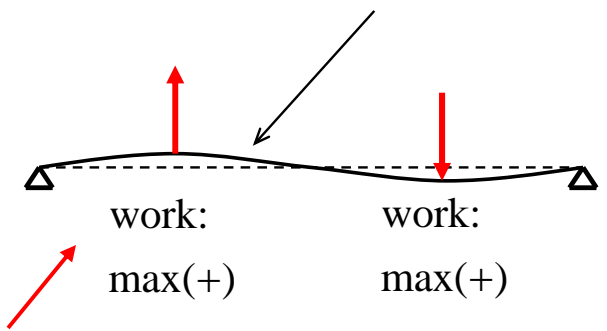
Impossibile evitare coincidenze di frequenza: forzante – modo proprio !

# Forma modo proprio / forzante



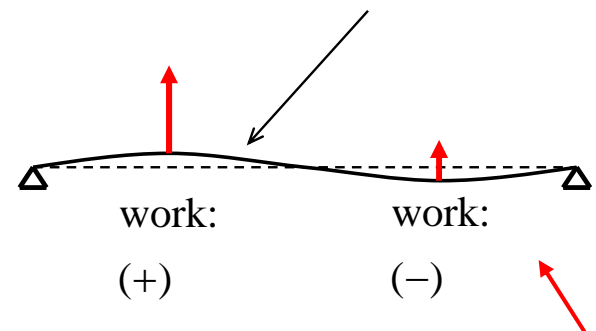
$$\omega_n = \sqrt{\frac{k}{m}}, \quad \omega \rightarrow \omega_n \text{ risonanza}$$

Harmonic forces  $\Omega = \omega_2$       Second natural mode,  $\omega_2$



Risonanza

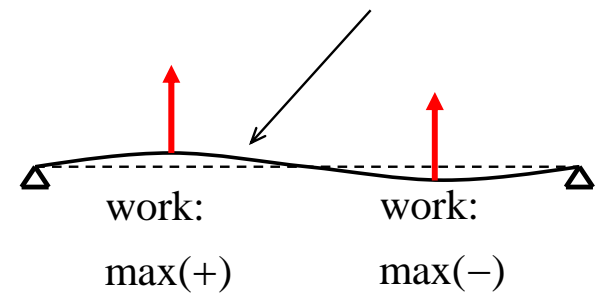
Harmonic forces  $\Omega = \omega_2$       Second natural mode,  $\omega_2$



Parziale risonanza

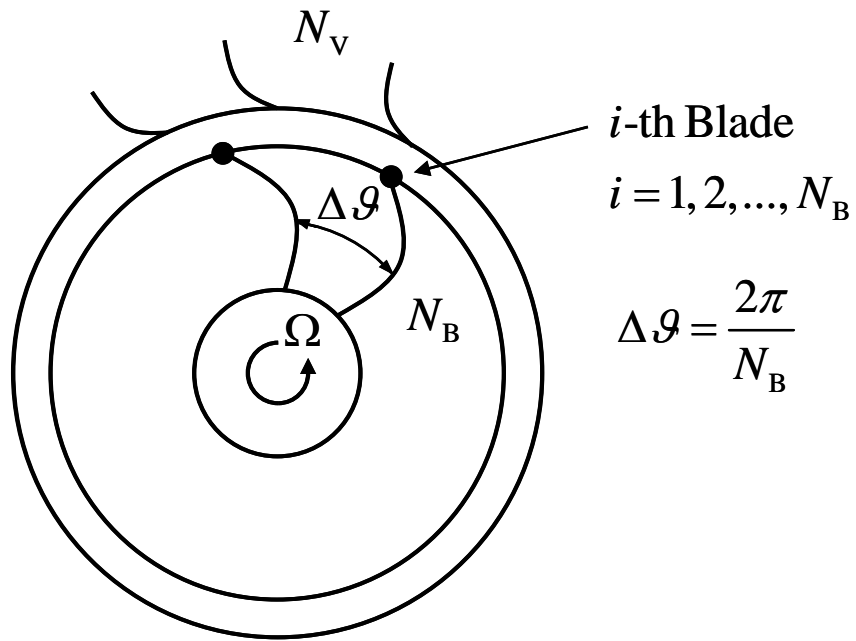
$\infty$  G.d.L.

Harmonic forces  $\Omega = \omega_2$       Second natural mode,  $\omega_2$



No risonanza

# Scomposizione della forzante in armoniche



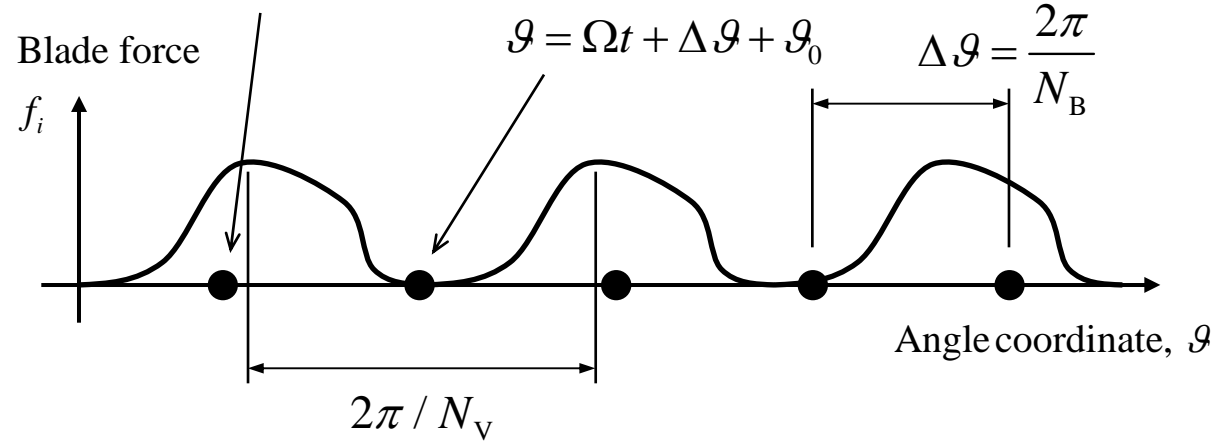
$$\Delta \mathcal{G} = \frac{2\pi}{N_B}$$

$i$ -th traveling blade

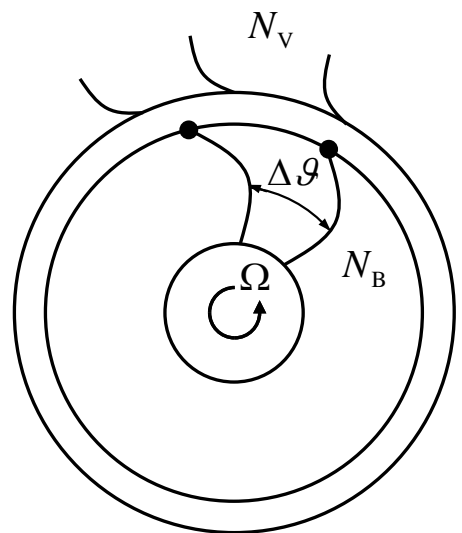
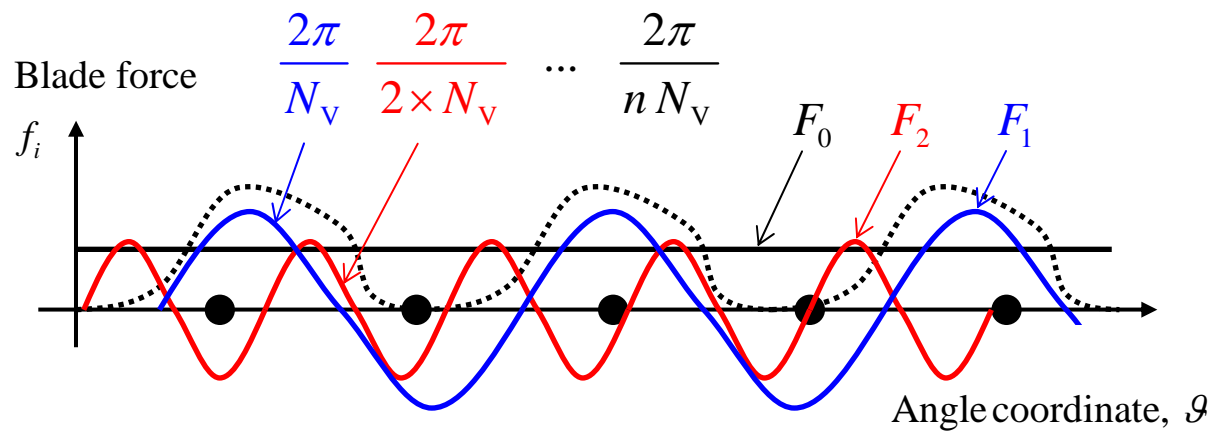
$$\mathcal{G} = \Omega t + \mathcal{G}_0$$

$(i + 1)$ -th traveling blade

$$\mathcal{G} = \Omega t + \Delta \mathcal{G} + \mathcal{G}_0$$



# Scomposizione della forzante in armoniche



$\Omega_1 = N_V \Omega$  ← Prima armonica

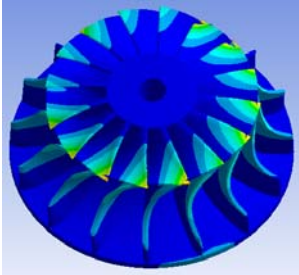
$\Omega_n = n \Omega_1 = n N_V \Omega$  ←  $n$ -esima armonica

Posizione angolare della singola pala  $i$ -esima rispetto allo statore

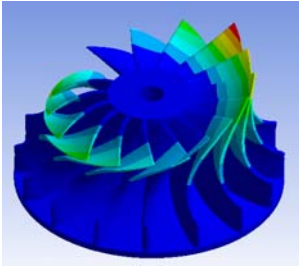
$$f_i = F_0 + \sum_{n=1}^{\infty} F_n \cos(n N_V (\Omega t + i \Delta \vartheta) + \varphi_n)$$

# Scomposizione modale della vibrazione

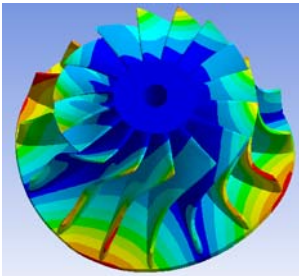
$$x_i = X_0 + \sum_{m=1}^{\infty} X_m \cos(\omega_m t + \varphi_{tm}) \cos(d_m \Delta \vartheta_i + \varphi_{\vartheta m})$$



$d_m = 0$



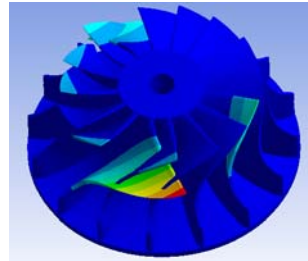
$d_m = 1$



$d_m = 2$

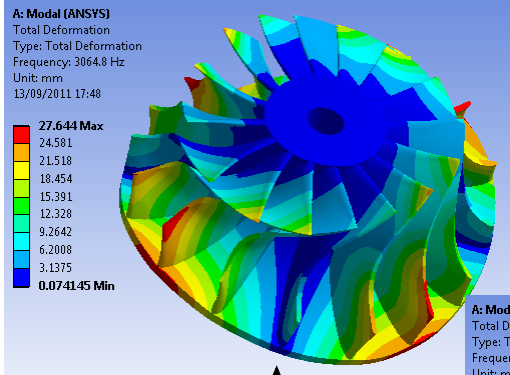
...

...+ altri modi  
NON armonici

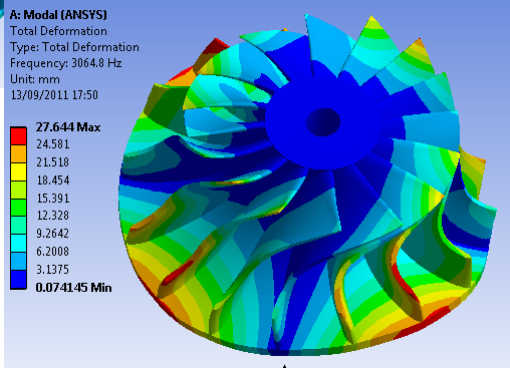
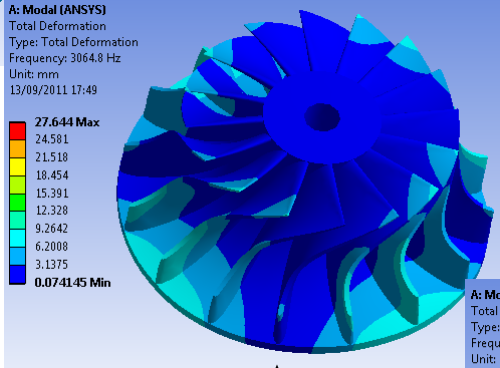
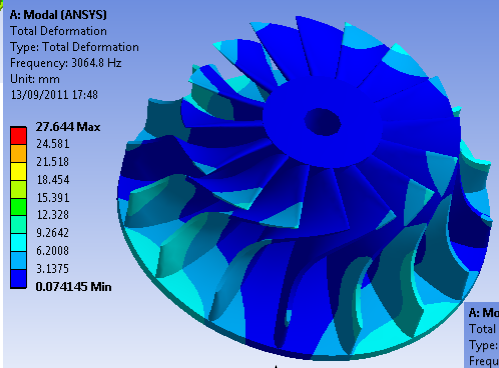


Modi armonici  
(rispetto alla  
coordinata angolare)

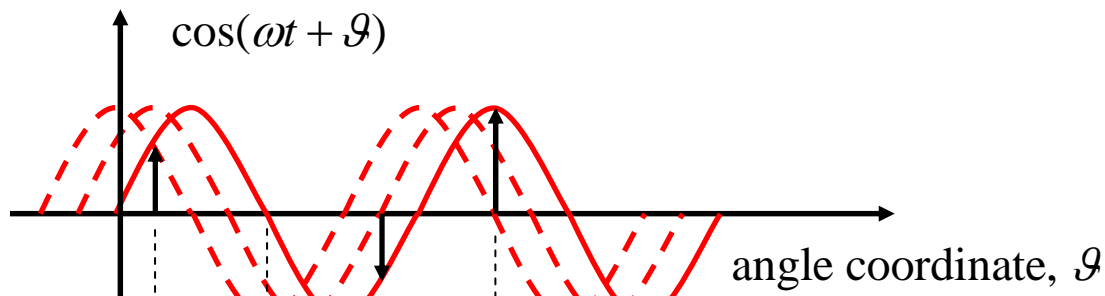
# Esempio di modo a 2 diametri nodali



Diametro nodale  
(spostamento nullo)

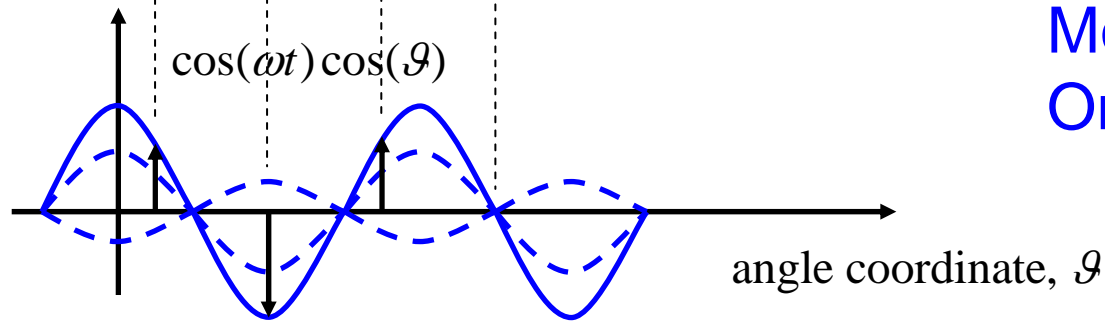


# Modi propri armonici, scomposizione in 2 onde controrotanti



Forzante  
Onda rotante

Blades: 1 2 3 4 ...



Modo proprio armonico  
Onda stazionaria

2 onde controrotanti

$$x_{m,i} = X_m \cos(\omega_m t + \varphi_{tm}) \cos(d_m \Delta \mathcal{G}_i + \varphi_{gm})$$

$$= \frac{X_m}{2} [\cos(-\omega_m t + d_m \Delta \mathcal{G}_i - \varphi_{1m}) + \cos(\omega_m t + d_m \Delta \mathcal{G}_i + \varphi_{2m})]$$

# Lavoro di vibrazione

$$W = \sum_{i=1}^{N_B} W_i \quad \leftarrow \text{Lavoro complessivo su tutte le pale della ruota}$$

$$W_i = \int_t dW_i \quad \leftarrow \text{Lavoro sulla singola pala}$$

$$dW_i = f_i v_i dt \quad \leftarrow \text{Lavoro elementare nel tempo, sulla singola pala}$$

$$v_i = \frac{\partial}{\partial t} x_i = v_i + \sum_{m=1}^{\infty} v_{m,i}$$

$$v_{m,i} = X_m \omega_m \cos(\omega_m t + \varphi_{1m} + \pi / 2) \cos(d_m \Delta \mathcal{G}i + \varphi_{g_m})$$

$$= \frac{X_m}{2} \omega_m [\cos(\omega_m t - d_m \Delta \mathcal{G}i + \varphi'_{1m}) + \cos(\omega_m t + d_m \Delta \mathcal{G}i + \varphi'_{2m})]$$

Componente di velocità  $m$ -esimo,  $i$ -esima pala  
Scomposizione in onde controrotanti

# Lavoro di vibrazione, combinazione di frequenza

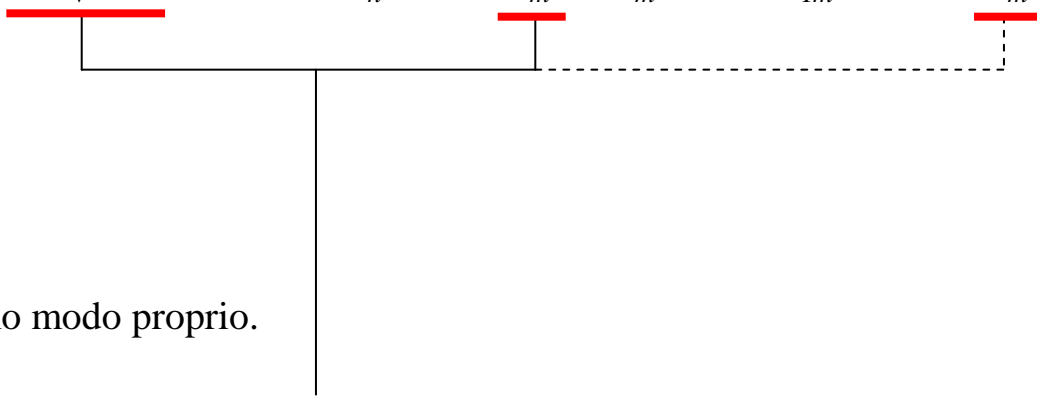
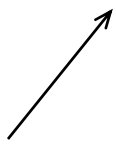
$$f_i v_i = \sum_{m=1}^{\infty} \frac{X_m \omega_m}{2} \cos(nN_V(\Omega t + \Delta \mathcal{G}i) + \varphi_n) [\cos(\omega_m t - d_m \Delta \mathcal{G}i + \varphi'_{1m}) + \cos(\omega_m t + d_m \Delta \mathcal{G}i + \varphi'_{2m})]$$

$$+ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} F_n \frac{X_m \omega_m}{2} \cos(nN_V(\Omega t + \Delta \mathcal{G}i) + \varphi_n) [\cos(\omega_m t - d_m \Delta \mathcal{G}i + \varphi'_{1m}) + \cos(\omega_m t + d_m \Delta \mathcal{G}i + \varphi'_{2m})]$$

Termine a media nulla



Combinazione  
*n*-esima armonica, *m*-esimo modo proprio.



Termine a media non nulla se:

$$\Omega_n = \omega_m$$

(condizione di Campbell)

# Lavoro di vibrazione, combinazione di forma

$$f_i v_i = F_0 \sum_{m=1}^{\infty} \frac{X_m \omega_m}{2} \cos(nN_v (\Omega t + \Delta \mathcal{G}i) + \varphi_n) [\cos(\omega_m t - d_m \Delta \mathcal{G}i + \varphi'_{1m}) + \cos(\omega_m t + d_m \Delta \mathcal{G}i + \varphi'_{2m})]$$

$$+ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} F_n \frac{X_m \omega_m}{2} \cos(nN_v (\Omega t + \Delta \mathcal{G}i) + \varphi_n) [\cos(\omega_m t - d_m \Delta \mathcal{G}i + \varphi'_{1m}) + \cos(\omega_m t + d_m \Delta \mathcal{G}i + \varphi'_{2m})]$$

$$W = \sum_{i=1}^{N_B} W_i$$

Termini di lavoro in fase, fra le varie pale, se:

$$n N_v \Delta \mathcal{G}i = d_m \Delta \mathcal{G}i + k(2\pi), \text{ per ogni } i = 1, 2, \dots, N_B,$$

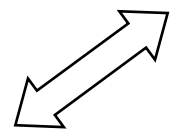
per un qualunque intero  $k = \dots, -2, -1, 0, 1, 2, \dots$

oppure

$$n N_v \Delta \mathcal{G}i = -d_m \Delta \mathcal{G}i + k(2\pi), \text{ per ogni } i = 1, 2, \dots, N_B,$$

per un qualunque intero  $k = \dots, -2, -1, 0, 1, 2, \dots$

$$\frac{n N_v \pm d_m}{N_B} = \text{intero}$$

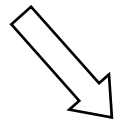


# Condizioni di risonanza

$$n N_V \Omega = \omega_m$$

&

$$\frac{n N_V \pm d_m}{N_B} = \text{intero (positivo negativo o nullo)}$$



$$\frac{n N_V + d_m}{N_B} = \text{intero (positivo negativo o nullo)}$$

or

$$\frac{n N_V - d_m}{N_B} = \text{intero (positivo negativo o nullo)}$$

# Esempio: no combinazione di forma

$N_B = 10$

$N_V = 7$

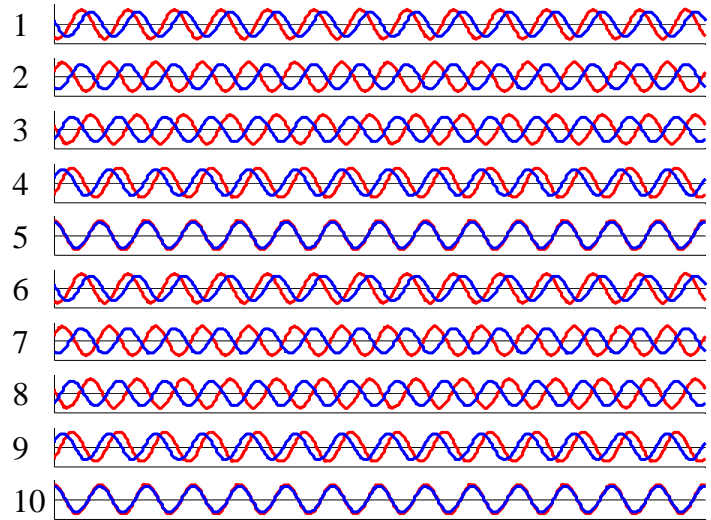
$n = 2$

$d_m = 2$

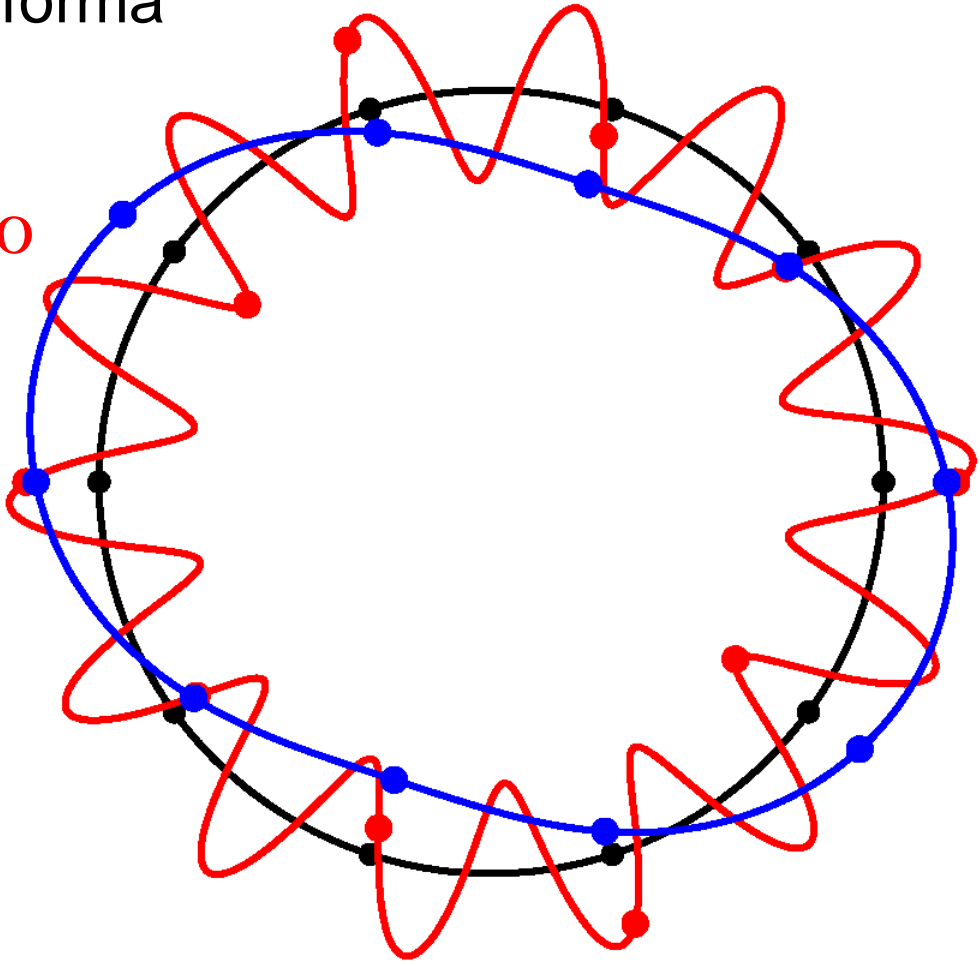
$$\frac{n N_V \pm d_m}{N_B} \neq \text{intero}$$

Condizione sulla frequenza verificata:

$$2N_V \Omega = \omega_2$$



Time



Parziale risonanza

— Modo proprio

— Armonica della forzante

# Esempio: combinazione di forma

$$N_B = 11$$

$$N_V = 5$$

$$n = 4$$

$$d_m = 2$$

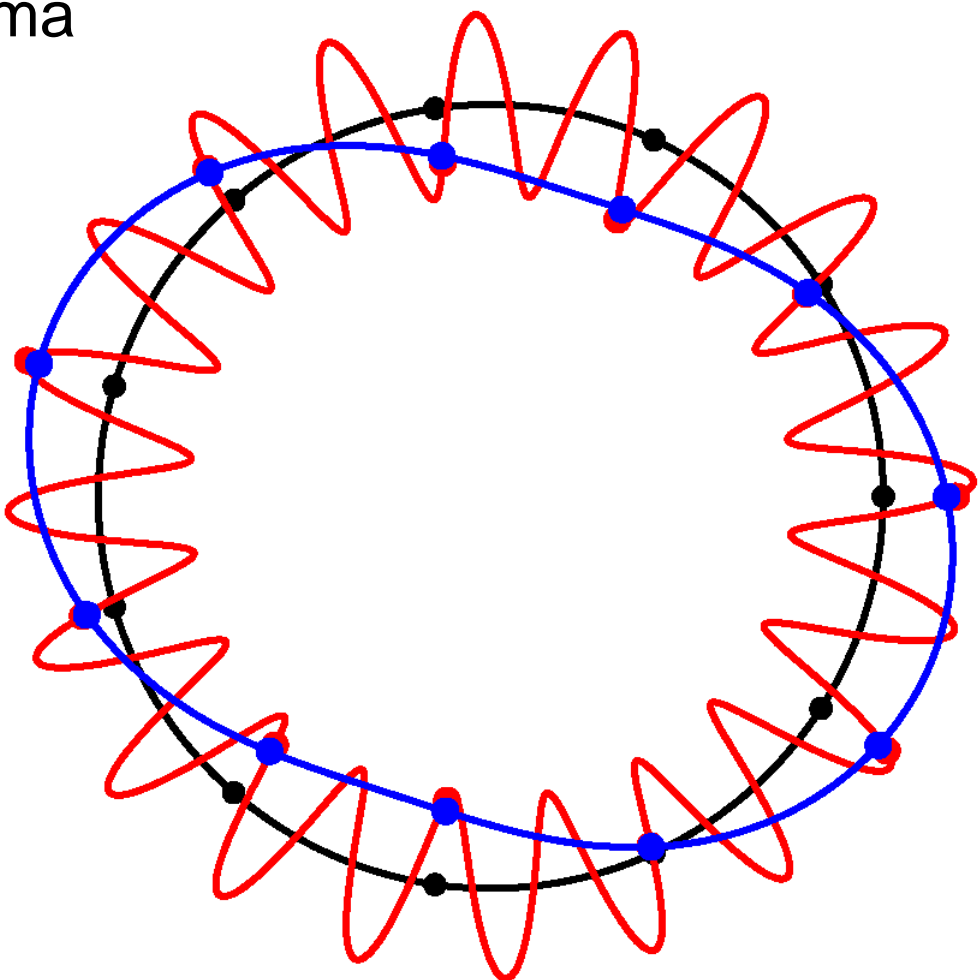
$$\frac{n N_V + d_m}{N_B} = 2$$

Condizione sulla frequenza verificata:

$$4N_V \Omega = \omega_2$$



Time

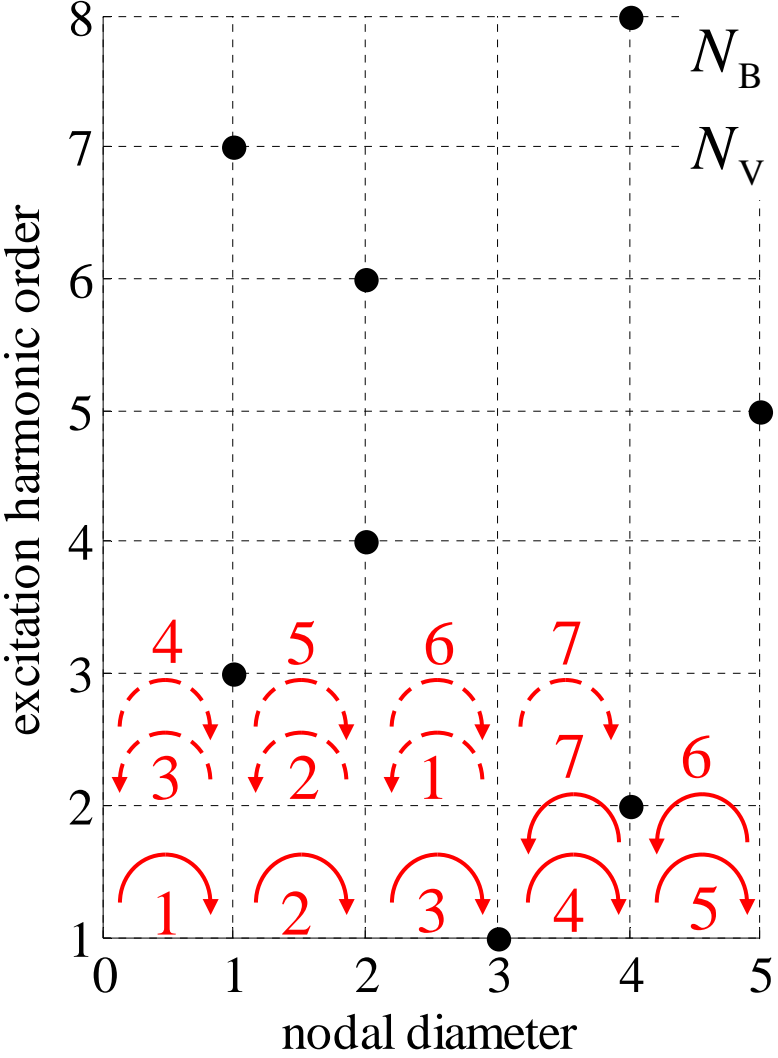


Risonanza

- Modo proprio
- Armonica della forzante

# Mappa di accoppiamento

$$\frac{n N_V \pm d_m}{N_B} = \text{int}$$



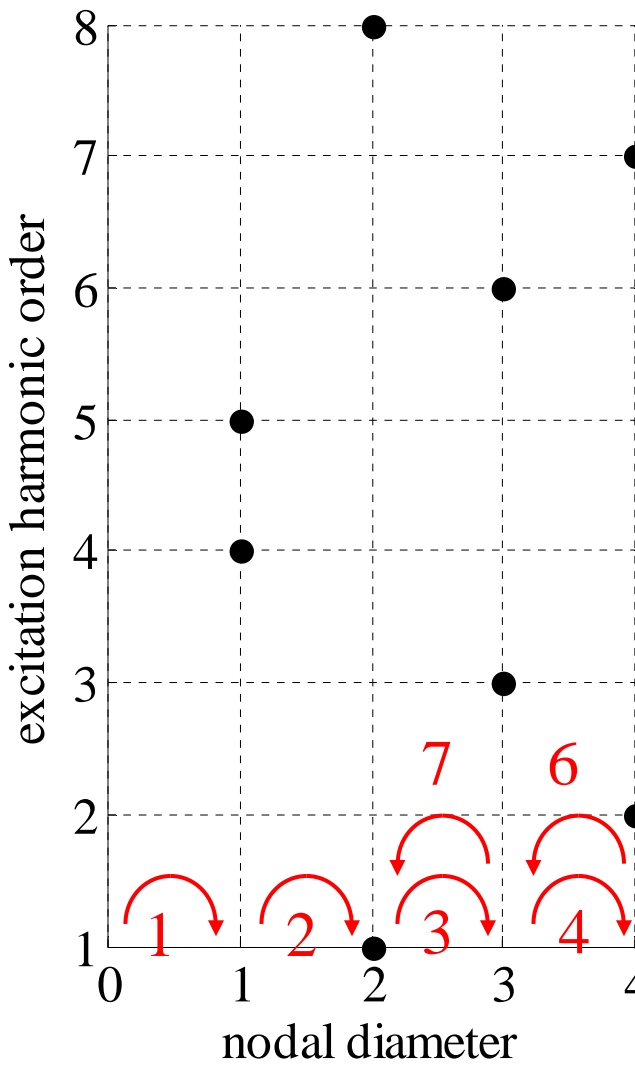
$$N_B = 10$$

$$N_V = 7$$

$$d_{m, \max} = \frac{N_B}{2} (N_B \text{ pari})$$

# Mappa di accoppiamento

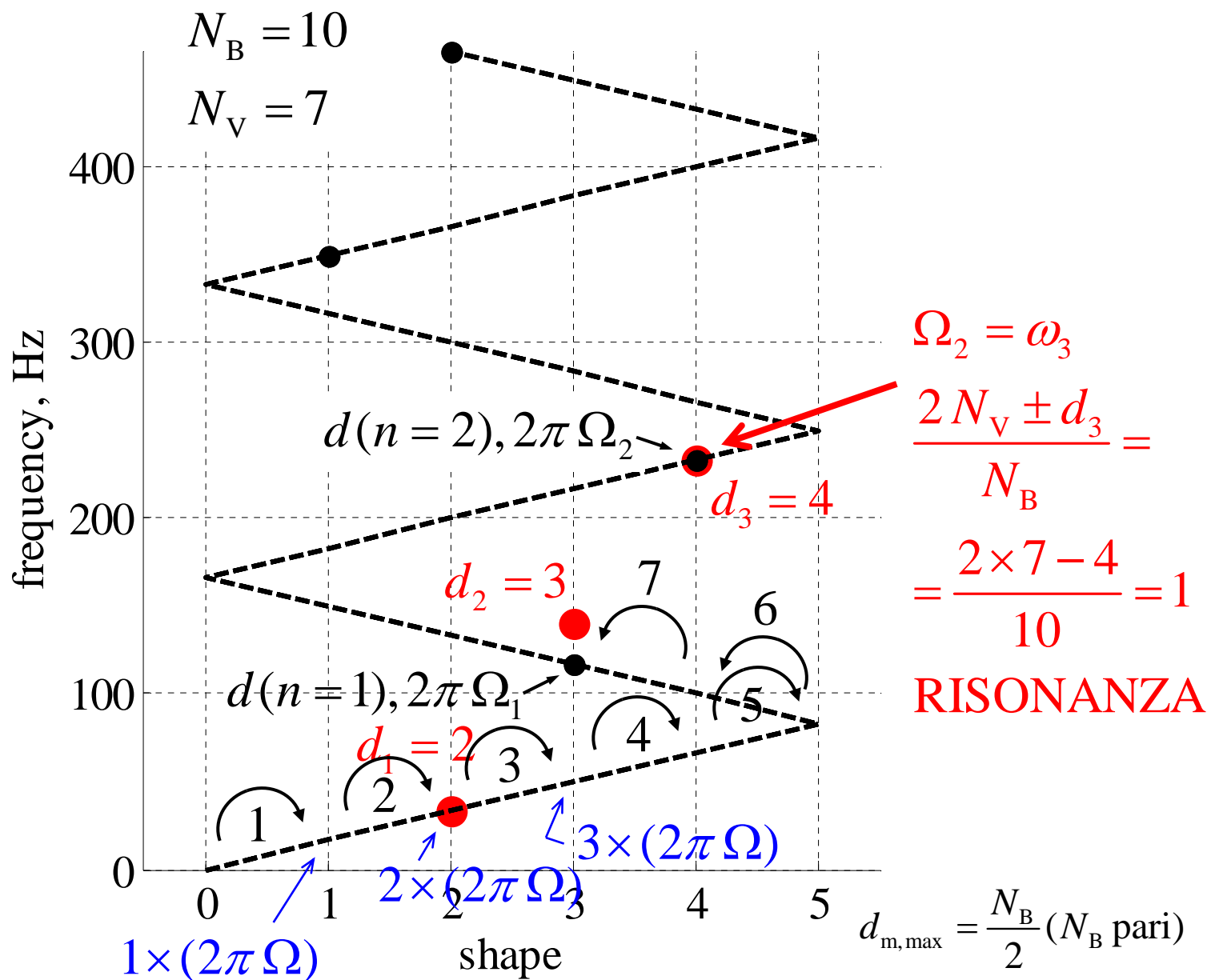
$$\frac{n N_V \pm d_m}{N_B} = \text{int}$$



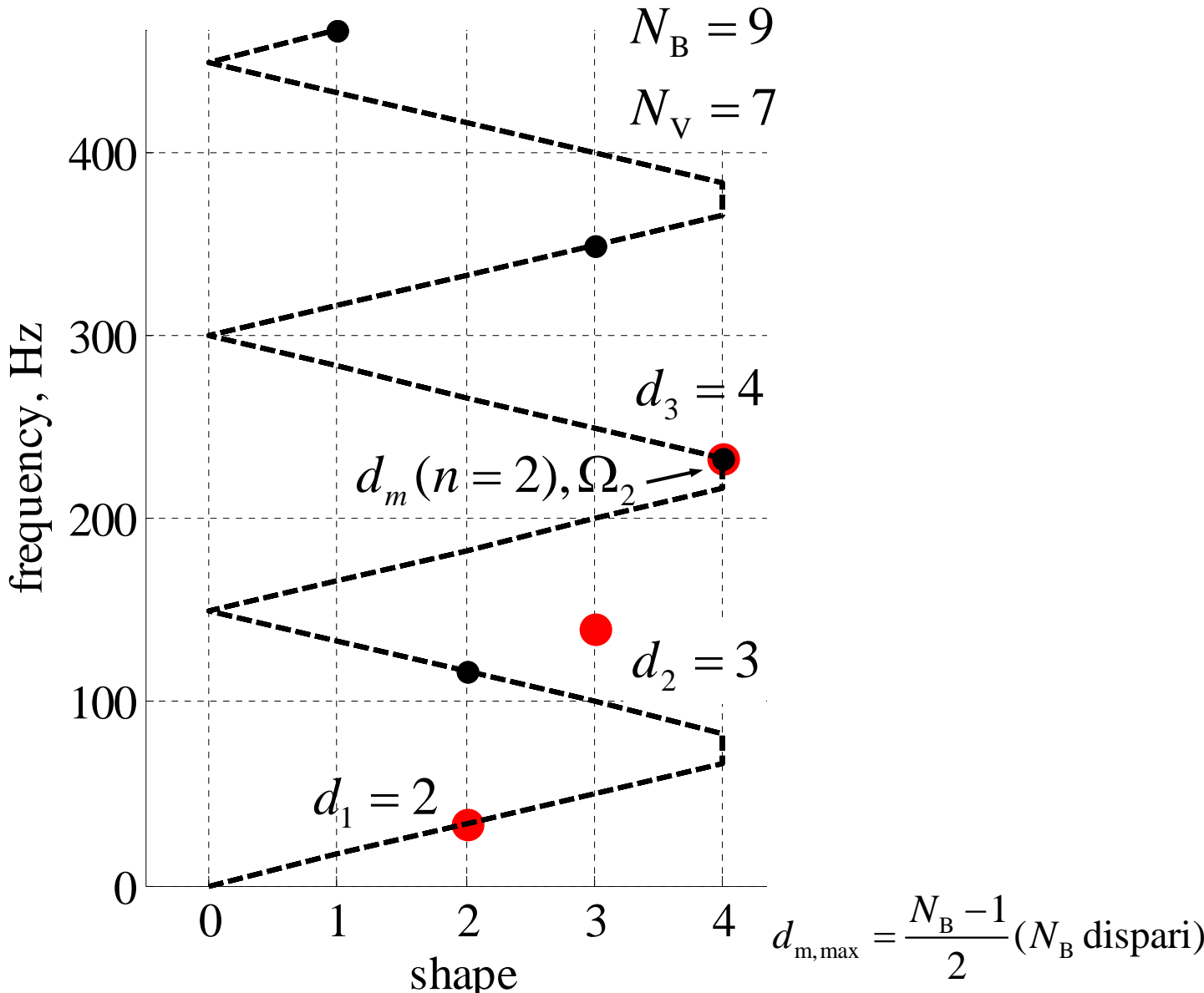
$N_B = 9$   
 $N_V = 7$

$$d_{m, \max} = \frac{N_B - 1}{2} (N_B \text{ dispari})$$

# Diagramma SAFE (Singh's Advanced Frequency Evaluation diagram)



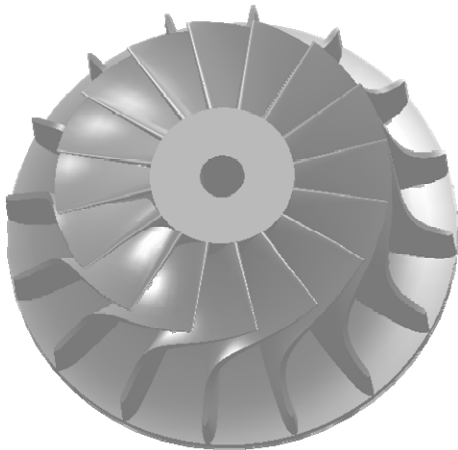
# Diagramma SAFE (Sing's Advanced Frequency Evaluation diagram)



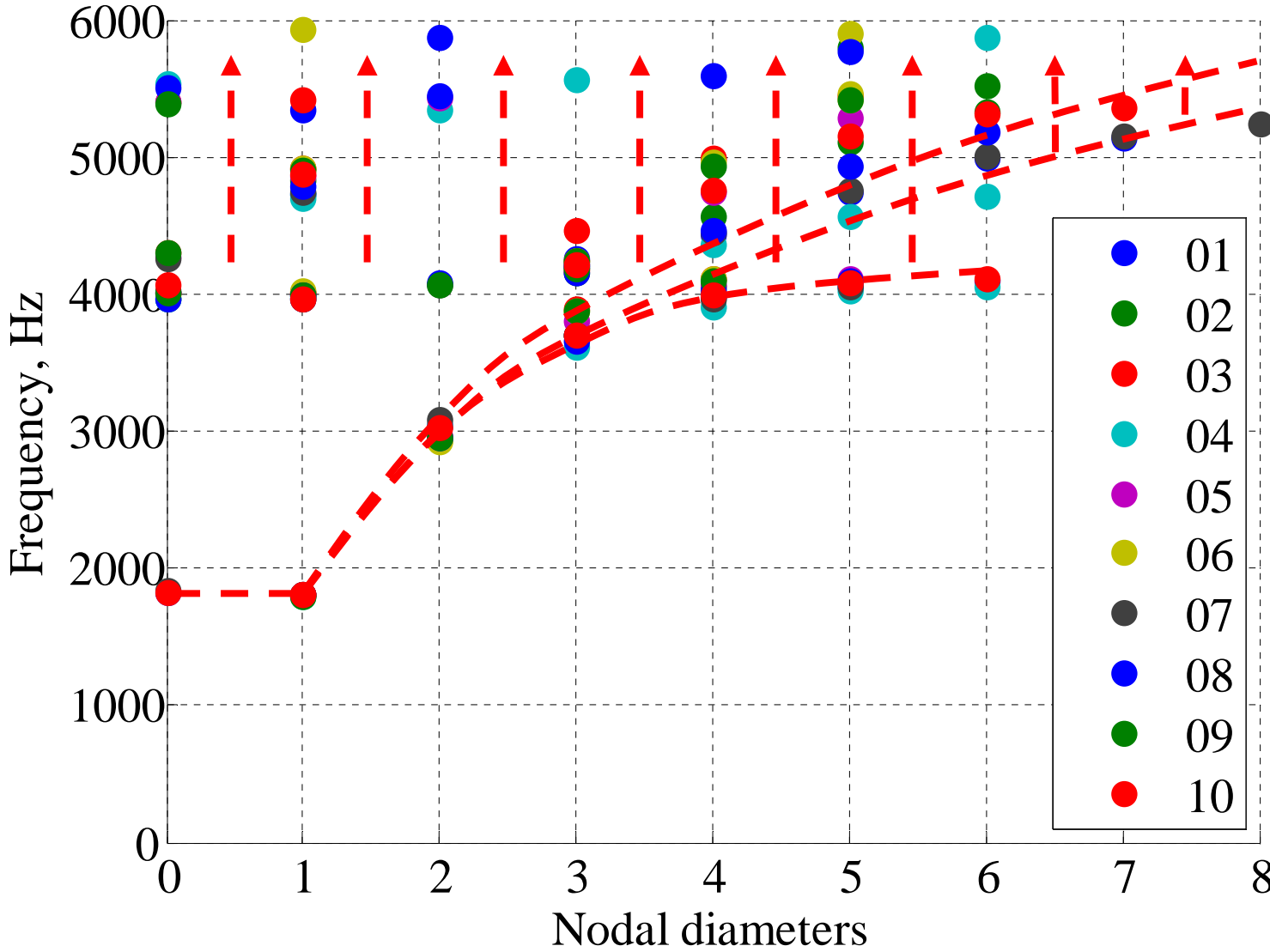
Applicazione:

analisi parametrica, differenti configurazioni ruota / statore

Configurazione Ruota/Statore	Numero di pale $N_B$	Numero di vani $N_V$	Spessore disco [mm]
1	16	4	10
2	13	4	19
3	15	4	28
4	13	7	10
5	17	7	22
6	13	9	28
7	17	11	10
8	13	11	16
9	16	11	27
10	16	9	13



# Distribuzione dei modi propri

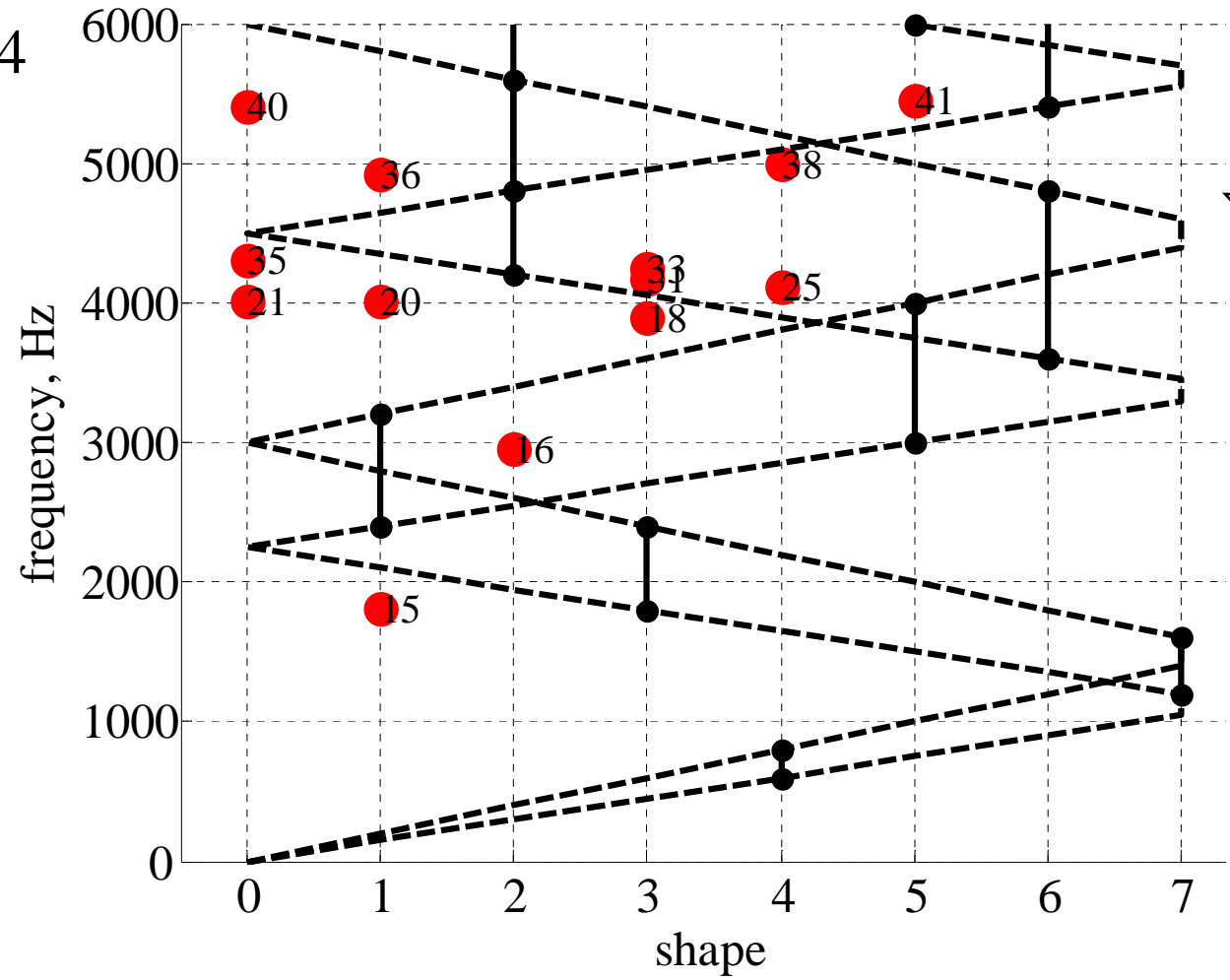


# Configurazione migliore, diagramma SAFE

Config. 3:

$$N_B = 15$$

$$N_V = 4$$



↑  
Armoniche elevate / inevitabili accoppiamenti ma a bassa intensità

↓  
Risonanza evitata per le prime armoniche di maggiore intensità

# Conclusioni

- Diagramma SAFE: coincidenza di frequenza e anche di forma modo proprio / forzante.
- Possibilità di evitare risonanze scegliendo un'opportuna combinazione numero pale / numero vani.
- Basso numero di vani in modo da esaurire le prime armoniche a più alta intensità evitando risonanze, ed entrare nella zona ad elevata densità di modi propri (inevitabile evitare risonanze) con armoniche di ordine superiore e quindi meno intense.