Optimal notched specimen parameters for accurate fatigue critical distance determination

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The theory of critical distance (TCD)

- \( L \) is very sensitive to material characteristics (microstructure, texture, processing routes)
- Crack growth threshold determination through fracture mechanics tests is experimentally challenging

El-Haddad material length

\[ L = \left( \frac{K_{th}}{f_l} \right)^2 \]
Introduction

Motivation

• Design of an optimal notched specimen geometry for accurate $L$ determination to circumvent the fracture mechanics test

• Provide a straightforward analytical calculation of $L$ to avoid the FE analysis of each specimen geometry

• Define an effective range where the result is expected to be not largely sensitive to any experimental issue

Outline

• Stress distribution ahead of notches

• Critical distance determination: LM vs. PM

• Sensitivity analysis

• Experiments

• Critical distance evaluation

• Fatigue strength evaluation
**Proposed specimen**

- V-notch axisymmetric specimen: easy to manufacture, no boundary effects, no transition from plane stress to plane strain
- Relatively open angle: 90°, 60°
- Sharp root radius

Dimensions to be defined:
- Bar diameter $D$
- Notch depth $A$
- Notch angle $\alpha$
- Notch radius $R$
Stress analysis, (i) sharp notch assumption

**Singular stress field**

\[ \sigma_y(x) = \frac{K_N}{x^s} \]

Singularity exponent, \( s \)

Williams’ exponent close to \( \frac{1}{2} \) even for relatively large notch angles

Recommended angles

<table>
<thead>
<tr>
<th>Notch angle, ( \alpha ) (°)</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williams’ exponent</td>
<td>0.487779</td>
<td>0.455516</td>
</tr>
</tbody>
</table>
**Stress analysis, (i) sharp notch assumption**

**Singular stress field**

Dimensionless form:

$$y(x) = \frac{K_N}{x^s} = \frac{K_{N,U}}{x^s} = \frac{K_{N,U}/(D/2)^s}{(x/(D/2))^s}$$

where:

$$x = \frac{x}{D/2}$$

$$K_{N,UU} = \frac{K_{N,U}}{(D/2)^s} = \frac{K_N}{(D/2)^s}$$

N-SIF for unitary half diameter and unitary nominal stress

- At intermediate notch depth $a = 0.3$ the NSIF is maximum because at a lower depth the notch indentation is just too small, while for the higher depth the peak stress points along the inner ring are too close.
**LM critical distance inverse search**

Line Method dimensionless form:

\[
\frac{1}{2L} \int_0^{2L} y(x) \, dx = \frac{1}{2l} \int_0^{2l} y(\ ) \, dx
\]

where: \( l = L(D / 2) \)

Singular term integration:

\[
\frac{1}{2l} \int_0^{2l} y(\ ) \, dx = \int_0^{2l} \frac{K_{N,UU}}{s} \, dx = \frac{N}{1} \frac{K_{N,UU}}{s} (2l)^s
\]

Line Method, average stress equal to fatigue limit:

\[
\frac{N}{1} \frac{K_{N,UU}}{s} (2l)^s = \text{fat}
\]

Fatigue stress concentration factor:

\[
\frac{1}{s} \frac{K_{N,UU}}{(2l)^s} = K_f
\]

Critical distance length inverse derivation:

\[
l_0 = \frac{1}{2} \left( \frac{K_{N,UU}}{s} (1/s) K_f \right)^{1/s}, \quad L_0 = l_0 (D / 2)
\]
Stress analysis, (i) sharp notch assumption

**LM/PM critical distance inverse search**

Line Method length inverse derivation:

\[
L_0 = \frac{D}{4} \left( \frac{K_{N,UU}}{(1 - s)K_f} \right)^{1/s}
\]

Similar analysis for the Point Method, length inverse derivation:

\[
L'_0 = D \left( \frac{K_{N,UU}}{K_f} \right)^{1/s}
\]

"0" stands for singularity derived and ' is for the PM

Notch parameters:

\[
\begin{align*}
\alpha &= 90^\circ \\
&\quad s = 0.455516 \quad K_{N,UU} = 0.3210 \quad (a = 0.3) \\
\alpha &= 60^\circ \\
&\quad s = 0.487779 \quad K_{N,UU} = 0.2866 \quad (a = 0.3)
\end{align*}
\]
Stress analysis, (ii) rounded notch tip

### Bounded stress field

- **Rounded notch maximum stress**
- **Dimensionless radius:**
  \[ r = \frac{R}{D/2} \]
  \[ \rho = \frac{R}{A} = \frac{r}{a} \]

- **Performed simulations:**
  \[ a = 0.3, \]
  \[ = \frac{R}{A} \]
  \[ = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1.0 \]

- FE model with unitary half diameter and unitary stress
  - Path discretization: point-to-point dist. = \( 1 \times 10^{-5} \)
  - Notch tip element size = \( 8.7 \times 10^{-5} \)
Stress analysis, (ii) rounded notch tip

**LM critical distance inverse search**

Line Method dimensionless form:

\[
\frac{1}{2l} \int_0^{2l} \Delta \sigma_y(\xi) \, d\xi = \Delta \sigma_N \frac{f(l) \, K_{N,UU}}{1 - s \, (2l)^s}
\]

where \(f(l)\) is a correction function

Line Method equation:

\[
\frac{f(l) \, K_{N,UU}}{1 - s \, (2l)^s} = K_f
\]

After introducing \(l_0\):

\[
\frac{l}{f(l)^{1/s}} = l_0
\]

and the inversion function is defined: \(\gamma(l) = l / f(l)^{1/s}\)

to put the inverse search problem as:

\[
\gamma(l) = l_0
\]
Stress analysis, (ii) rounded notch tip

**LM critical distance inverse search**

Very accurate approx. with a linear model, inverse search:

\[ l = l_{\text{min}} + \frac{l_0 - \gamma_{\text{min}}}{\beta} \]

\[ \beta = \frac{\gamma_{\text{max}} - \gamma_{\text{min}}}{l_{\text{max}} - l_{\text{min}}} \]

Fit models (\( \rho \) functions):

\[ l_{\text{min}} = p_1 \rho^3 + p_2 \rho^2 + p_3 \rho + p_4 \]

\[ \gamma_{\text{min}} = q_1 \rho^3 + q_2 \rho^2 + q_3 \rho + q_4 \]

\[ l_{\text{max}} = \gamma_{\text{max}} = c_1 + c_2 \rho^{c_3} \]
Stress analysis, (ii) rounded notch tip

**PM critical distance inverse search**

- **Inversion function:**
  \[ \gamma'(l) = \delta_1 l_0^4 + \delta_2 l_0^3 + \delta_3 l_0^2 + \delta_4 l_0 + \delta_5 \]

- **Dimensionless critical distance (PM),** \( l' \)

- **Notch radius ratio,** \( \rho \)

- **PCHIP at higher resolution of \( \rho \)**

- **Coefficient at reference values**

- **Tabular data for \( \delta_i (\rho \text{ functions}) \)**
Inaccurate inverse search configurations

Sensitivity to any experimental variation of $K_f$

Sensitivity definition:

$$S = -\frac{1}{L} \frac{dL}{dK_f}$$

Small critical distance wrt notch radius

Not sharp enough local radius

Minimum sensitivity in the range $0.5 < f(l) < 1.0$

Large critical distance wrt diameter size

Small specimen size
Accurate inverse search range

Maximum/minimum limits for the dimensionless critical distance

\[ \frac{\Delta \sigma_{av}(l)}{\Delta \sigma_N} = \frac{1}{1 - s} \frac{K_{N,UU}}{(2l)^s}  \]

\[ 0.5 \frac{K_{N,UU}}{1 - s} \]

Small critical distance / Minimum limit

Large critical distance / Maximum limit

FE integration

Theoretical integr.

\( f(l) = 0.5 \)

Minimum limit

Maximum limit
Experimental application, 42CrMo4 - $S_U = 875$ MPa

Specimen extraction from the same bar supply

Fracture mechanics tests for comparison

Expected Critical Distance on the order of 0.05 mm, small notch radius 0.2 mm

Sharp notch:
$R = 0.2$ mm
$\rho = 0.0667$

Detail view

Blunt notch (other spec.):
$R = 1.0$ mm
$\rho = 0.333$
Experimental test results, S-N data

Specimen types: Plain, Blunt (1.0 mm), Sharp (0.2 mm)

Push-pull (R=-1) axial fatigue tests

Pulsating (R=0.1) axial fatigue tests

Stress amplitude, $\sigma_a$ (MPa)

Number of cycles to failure, $N_f$
Experimental test results, thresholds

M(T) specimen for negative load ratio

- Stress intensity factor range, $\Delta K$ (MPa m$^{0.5}$)
  - $\Delta K_{th} = 7.2$ MPa m$^{0.5}$
  - $\Delta K_{th} = 9.1$ MPa m$^{0.5}$

- Crack growth rate, $da/dN$ (m/cycle)

- Q+T steel, air, RT, 100 Hz

- Exp. Klesnil-Lukas
- $R = 0.1$
- $R = -1$

Graph showing crack growth rate vs. stress intensity factor range.
Experimental test results, length comparison

LM and PM dimensionless critical distances, $l - l'$

<table>
<thead>
<tr>
<th>R = -1</th>
<th>Plain - $\Delta K_{th}$, $L_{-1} = 0.0433$ mm</th>
<th>R = 0.1</th>
<th>Plain - $\Delta K_{th}$, $L_{0.1} = 0.0363$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain - $\text{Sharp}$</td>
<td>Plain - $\text{Blunt}$</td>
<td>Plain - $\text{Sharp}$</td>
<td>Plain - $\text{Blunt}$</td>
</tr>
<tr>
<td>LM</td>
<td>PM</td>
<td>LM</td>
<td>PM</td>
</tr>
<tr>
<td>0.0273 mm</td>
<td>0.0505 mm</td>
<td>0.0970 mm</td>
<td>0.1836 mm</td>
</tr>
<tr>
<td>-36.9%</td>
<td>16.6%</td>
<td>123.8%</td>
<td>323.9%</td>
</tr>
</tbody>
</table>

- Threshold derived lengths for comparison

- Line Method
- Point Method

$R = -1$ Plain - $\Delta K_{th}$, $L_{-1} = 0.0433$ mm

$R = 0.1$ Plain - $\Delta K_{th}$, $L_{0.1} = 0.0363$ mm

Plain - Sharp
Plain - Blunt
LM
PM
Accuracy evaluation based on the strength assessment

<table>
<thead>
<tr>
<th>R = –1</th>
<th>ΔK_{th} = 9.1 MPa m^{0.5}</th>
<th>R = 0.1</th>
<th>ΔK_{th} = 7.2 MPa m^{0.5}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain – Sharp</td>
<td>Plain - Blunt</td>
<td>Plain - Sharp</td>
<td>Plain - Blunt</td>
</tr>
<tr>
<td>LM 7.23 MPa m^{0.5}</td>
<td>PM 9.82 MPa m^{0.5}</td>
<td>LM 7.24 MPa m^{0.5}</td>
<td>PM 9.78 MPa m^{0.5}</td>
</tr>
<tr>
<td>-20.6%</td>
<td>8.0%</td>
<td>0.5%</td>
<td>-53.6%</td>
</tr>
<tr>
<td>49.6%</td>
<td>105.9%</td>
<td>35.9%</td>
<td>-58.2%</td>
</tr>
</tbody>
</table>

Results obtained with Plain - Threshold critical distances

<table>
<thead>
<tr>
<th>R = –1, Sharp</th>
<th>R = 0.1, Sharp</th>
<th>R = –1, Blunt</th>
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<tbody>
<tr>
<td>Δσ_{N,fl/2} = 87.5 MPa</td>
<td>Δσ_{N,fl/2} = 80.5 MPa</td>
<td>Δσ_{N,fl/2} = 163 MPa</td>
<td>Δσ_{N,fl/2} = 119 MPa</td>
</tr>
<tr>
<td>LM 96.9 MPa</td>
<td>PM 85.0 MPa</td>
<td>LM 148.4 MPa</td>
<td>PM 143.1 MPa</td>
</tr>
<tr>
<td>-20.6%</td>
<td>-2.8%</td>
<td>-9.0%</td>
<td>-12.2%</td>
</tr>
<tr>
<td>49.6%</td>
<td>-0.2%</td>
<td>-11.4%</td>
<td>6.3%</td>
</tr>
<tr>
<td>61.6%</td>
<td>-11.6%</td>
<td>6.4%</td>
<td>6.4%</td>
</tr>
</tbody>
</table>

Results obtained with Plain - Blunt critical distances

<table>
<thead>
<tr>
<th>R = –1, Sharp</th>
<th>R = 0.1, Sharp</th>
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</tr>
<tr>
<td>LM 122.5 MPa</td>
<td>PM 130.0 MPa</td>
<td>LM 143.7 MPa</td>
<td>PM 144.1 MPa</td>
</tr>
<tr>
<td>40.0%</td>
<td>48.6%</td>
<td>-11.8%</td>
<td>-11.6%</td>
</tr>
<tr>
<td>-20.5%</td>
<td>-23.4%</td>
<td>6.4%</td>
<td>6.4%</td>
</tr>
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</table>

Not accurate (yellow): blunt for critical distance to evaluate sharper notch strength.
Accurate (blue): sharp for critical distance to evaluate blunter notch strength.
Conclusions

- V-notched specimen for optimal critical distance inversion search.
- All the dimensions provided and discussed, in particular the notch root radius.
- Analytical procedure to derive the Critical Distance both with Line and Point methods.
- 42CrMo4 Q+T experimental data and comparison provided.
- The obtained critical distances dependent on the method, Point Method much larger than Line Method.
- Small critical distance for this investigated high strength steel: accurate assessments only obtained with the sharp notch or the crack threshold derived critical distances.
Optimal notched specimen parameters for accurate fatigue critical distance determination

THANK YOU FOR YOUR ATTENTION!